Full quantum treatment of spin-dependent beam-beam processes at linear colliders

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Abstract. Depolarisation processes at future linear colliders need to be understood as precisely as possible. To that end a theoretical consideration of the spin flip process and its radiative corrections is presented here. The spin flip process contains a divergence and it is useful to repeat the calculation of its transition rate using a coordinate system which makes the physical nature of the divergence apparent. It is argued that the radiative corrections to the spin flip process should be considered within the Furry Picture. The Electron Self Energy in the external field is being explicitly re-examined in order to establish the presence of UV divergences and the procedure required to remove them. A calculation of the Vertex Correction in an external field is being performed and results obtained so far for special kinematics are consistent with known results.

1. Introduction

In order to do precision spin physics at future linear colliders currently undergoing R&D (namely the ILC and CLIC), precise spin tracking is required. The main sources of uncertainty in the state of the luminosity weighted polarisation at the Interaction Point (IP) occur in the sensitivity of the Compton polarimeters complimented by annihilation data [1], in the beam delivery system due to ground motion induced misalignment [2], and at the IP itself due to the beam-beam processes [3].

The work here is concerned with the theoretical development of the beam-beam processes as regards depolarisation. The two main processes are the quantum spin flip (Sokolov-Ternov) process and the anomalous magnetic moment contribution to the classical spin precession in the T-BMT equation. Both are derived from the photon emission from a polarised electron (Beamstrahlung) in the presence of the strong electromagnetic fields associated with charge bunches at collision.

The Beamstrahlung is calculated semi-classically in the Furry picture using exact solutions of the Dirac Equation in the classical potential of the strong bunch fields. The Beamstrahlung transition rate contain divergences which have not yet been properly treated [4]. The divergences require consideration of extra processes within the Furry picture which remove divergent terms.

2. The spin dependent Beamstrahlung

The transition probability W^e of the radiation of a polarized electron of momentum p with spin vector s_v in a strong constant crossed field is [5]

$$W^e(p_i \to p_f, \gamma) \propto \int_0^\infty \frac{du}{(1+u)^2} \left[\text{Ai}_1(z) + \frac{2+2u+u^2}{z(1+u)} \text{Ai}'(z) - \frac{e}{m^3} \frac{F^{*\mu\nu} p_\mu s_\nu}{1+u} z \text{Ai}(z) \right] \qquad (1)$$

$$\text{where} \quad z \propto u$$

The difficulty with obtaining numerical values for the this transition probability (and thus in calculating the extent of the depolarisation) is in the second term which goes to infinity at the lower bound of the integration over u. There is an associated theoretical difficulty in dealing with this divergence - its nature is not obvious and therefore neither is the Regularization/Renormalization procedure that is required.

A clue is provided by consideration of the Optical Theorem which relates the Electron Self Energy $\Sigma^e(p)$ to the probability of radiation (equation 2)

$$\Im(\Sigma^e(p)) = 2W^e(p \to p, \gamma) \qquad (2)$$

Since the Optical Theorem proceeds purely from the unitarity of the S-matrix, and the structure of the S-matrix is unaffected by working within the Furry Picture, then it is reasonable to expect that the Electron Self Energy in a constant crossed external field will provide a term that removes the divergence in the Beamstrahlung transition probability.

3. The Electron Self Energy in a constant crossed field

A calculation of the Electron Self Energy in a constant crossed external field (figure 1) yielded an expression consistent with what was expected from the Optical theorem [5]. However two conceptual difficulties in the calculation remain. The first is concerned with the nature of the divergence that is corrected by the Self Energy. A careful consideration of the Beamstrahlung transition probability calculation and in particular the integration over final state momenta (say $\vec{k}$) is required. A Cartesian coordinate system ($dk_x\, dk_y\, dk_z$) was chosen to reduce the the Phase Integral to an integration over a ratio of scalar products denoted by the symbol u. The Beamstrahlung transition probability diverges at lower bound of the u integration [4]. In order to understand the nature of the divergence (IR and/or collinear) it is desireable to redo the Phase Integral calculation in a shperical polar coordinate system.

The second difficulty is the appearance or otherwise of the expected UV divergence. The literature is divided on the question of divergences in bound state problems. Whereas some find that there are UV divergences with respect to the electron interaction with the vacuum [5, 6], others find none of the usual divergences [7]. An independent calculation is underway to satisfy these uncertainties.
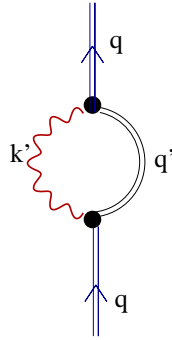
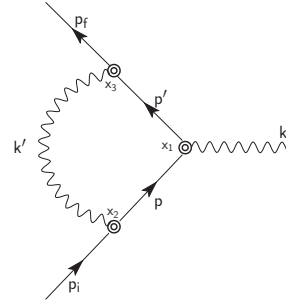Figure 1: The Electron Self Energy in an external field.

Figure 2: The Furry Picture Vertex Correction with dressed verticies.

4. The Vertex Correction in a constant crossed field

Once a cancellation procedure for divergences within the Furry Picture at lowest order has been found, one would like to establish the cancellation to all orders. Assuming a Bloch-Nordiseck type procedure is required then an explicit calculation of the Vertex Correction in an external field Γ^e_μ (see figure 3) is required. Such a calculation involves explicit solutions for the electron embedded in the external field resulting in dressed vertices, $\gamma^e_\mu(p, p')$,

$$-ie\Gamma^e_\mu = 2ie^2 \int_{-\infty}^{\infty} \frac{d^4p}{(2\pi)^4} \, dl \, dr \, ds \, \delta^4(p_i + lk - p_f - k_f)$$

$$\bullet \; \gamma^e(p_f, p')_\nu \; \frac{\not{p}' + m}{p'^2 - m^2} \; \gamma^e(p', p)_\mu \; \frac{\not{p} + m}{p^2 - m^2} \; \gamma^e(p, p_i)_\nu \; \frac{1}{k'^2} \tag{3}$$

$$\text{where} \quad p' \to p - k_f + rk \quad , \quad k' \to p_i - p + sk$$

This expression has been calculated for special kinematics in which the radiated photon is parallel to the external field wave vector. For such a case no UV divergence exists and the resulting expression contains a part consistent with the known expression for the anomalous magnetic moment in a constant crossed field [8]. Work is continuing to extend the calculation to the general case.

5. Conclusion

Precision spin physics at a future linear collider requires a precise understanding of the state of polarisation at collision and any depolarisation losses due to misalignment of machine elements or beam-beam processes. For the strong fields present in the beam-beam processes at the Interaction Point, theoretical calculation which take into account the effect of the field exactly, are necessary. Such calculations are performed in the Furry Picture and a consideration of the lowest order process within this picture poses several theoretical challenges.

The Beamstrahlung process (and the Electron Self Energy in an external field) both contain a divergence which appear to cancel. However the physical nature of the divergence needs clarification as does the appearance of any UV divergences. Furthermore it is highly desirable to show explicitly that divergences within the Furry Picture cancel to all orders. To that end the Vertex Correction in an external field is in the process of being calculated and a detailed reexamination of the Electron Self Energy calculation is underway. Eventually this work will be reapplied to simulation of linear collider machine designs to determine any reassessment of the expected depolarisation.

References

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