

Coupling Brane Fields to Bulk Supergravity

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In this note we present a simple, general prescription for coupling brane localized fields to bulk supergravity. We illustrate the procedure by considering 6D $N=2$ bulk supergravity on a 2D orbifold, with brane fields localized at the fixed points. The resulting action enjoys the full 6D $N=2$ symmetries in the bulk, and those of 4D $N=1$ supergravity at the brane positions.

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I. INTRODUCTION

Consider a bulk supergravity theory in higher dimensions ($D > 4$), in which the extra dimensions are compactified on an orbifold. The orbifold action has a number of fixed points, and certain fields may be localized at these points. In this way, we can construct a 4D brane. In the following we shall use the terms brane and fixed point interchangeably.

At the fixed points, part of the higher dimensional gravitational- and super-symmetries are broken explicitly. For instance, half of the supersymmetry generators are projected out. But the subset of symmetries corresponding to 4D $N=1$ supergravity survive. Therefore, we can recast the bulk theory at the brane in such a way that keeps 4D $N=1$ symmetries manifest. 4D $N=1$ supergravity is very well understood, and its general couplings to matter was worked out in [1]. So, we can use the machinery of 4D $N=1$ supergravity to couple the bulk theory at the brane to the localized fields. The result will be a bulk plus brane action, which enjoys the higher dimensional symmetries away from the branes; the fields on the brane transform instead only under 4D $N=1$.

We take our inspiration from [2–5] who reformulated higher dimensional globally supersymmetric theories in terms of 4D $N=1$ superfields or their components. Previous work on brane-bulk couplings in local 5D models includes [6, 7]. One might have believed that an off-shell description of the bulk supergravity be necessary to construct general couplings to brane fields [29]. This is because on-shell, the supersymmetry algebra closes only up to the equations of motion, and so care must be taken if we introduce new terms to the action. But we only add new interactions at the brane positions where the $N=2$ supersymmetry is explicitly broken, and there we can invoke by now text-book results of [1] on 4D $N=1$ supergravity.

In detail, having recast the bulk theory at the brane into the form of 4D $N=1$ supergravity, we use the tem-

plate of on-shell 4D $N=1$ supergravity with general matter couplings to write down the couplings at the brane. These general matter couplings were indeed first derived via an off-shell formulation, but having established the general on-shell Lagrangian there is no need to refer back to the off-shell one. Therefore, contrary to standard lore, we are able to simply use the component on-shell descriptions of both the higher dimensional theory in the bulk and the 4D theory at the branes to construct a general bulk-brane theory with the required symmetries. On one hand, our method allows one to avoid the tedious (and possibly never-ending) on-shell Noether procedure, and on the other hand we can apply the method to cases where no off-shell description is available, *e.g.* in 10D supergravity.

We shall illustrate these ideas by coupling 4D $N=1$ brane fields to 6D $N=2$ supergravity (with 8 supercharges). That is, we also consider higher codimension branes. One such model was constructed by [9–11], who used the Noether method to iteratively find appropriate brane-bulk couplings in the action and supersymmetry transformation laws. Our prescription has the advantage that we can immediately introduce arbitrary 4D $N=1$ brane fields and their interactions, and moreover, it is also easy to write down the action up to four fermion terms.

Our motivation is to provide a field theory setup in which we can study scenarios like Supersymmetric Large Extra Dimensions [12], or orbifold-GUTs [13], allowing for non-trivial dynamics for gravity, bulk and brane matter. These constructions have provided interesting ways to approach long-standing problems in cosmology and particle physics, and find more fundamental descriptions within string theory. For example, the brane fields may represent a field theory description of the twisted sectors that arise in string orbifold compactifications. An intermediate 6D compactification [14] is particularly interesting in that context, since anisotropic orbifolds allow an understanding of the mild hierarchy between the GUT and Planck scales.

Bearing in mind this purpose we shall make a few assumptions in order to simplify our analysis and presentation. Most of the work in our construction goes in rearranging the bulk theory at the brane in terms of 4D N=1 supergravity. The constraint that odd fields, and internal derivatives of even and odd fields, are vanishing at the brane simplifies this task considerably. In this way, we do not obtain all the possible couplings between the bulk fields and our brane fields. We do however obtain the simplest ones that are necessary for consistency with the symmetries, by which (charged) brane fields must couple to the 4D metric (and gauge fields) and their N=1 supersymmetry partners. Moreover, we are able to immediately couple any possible brane fields to the bulk.

II. 6D N=2 BULK SUPERGRAVITY

We take a minimal on-shell field content in the bulk; a supergravity-tensor multiplet $(e_M^A, B_{MN}, \varphi, \Psi_M^i, \chi^i)$, a $U(1)$ vector multiplet (A_M, λ^i) and a charged bulk hypermultiplet (Φ^α, ζ^a) . We take as the target quaternionic manifold of the hyperscalars the canonical example $\frac{Sp(1,1)}{Sp(1) \times Sp(1)_R}$. The action is (see Appendix A for our conventions) [15]:

$$\begin{aligned} S_B = \int d^6 X e \left[-\frac{1}{2\kappa^2} R + \frac{1}{2\kappa^2} \partial_M \varphi \partial^M \varphi \right. \\ \left. - \frac{1}{4} e^\varphi F_{MN} F^{MN} + \frac{1}{12} e^{2\varphi} G_{MNP} G^{MNP} \right. \\ \left. + \frac{1}{2\kappa^2} g_{\alpha\beta}(\Phi) D_M \Phi^\alpha D^M \Phi^\beta - \frac{1}{2\kappa^4} e^{-\varphi} v(\Phi) \right. \\ \left. + \text{fermions} \right] \end{aligned} \quad (1)$$

where the covariant derivative of the hyperscalars is:

$$D_M \Phi^\alpha = \partial_M \Phi^\alpha - g A_M \xi^\alpha \quad (2)$$

with $\xi^\alpha = (T\Phi)^\alpha$ the Killing vectors, and T the Hermitian generator of the gauge group. The dependence of the potential on the hyperscalars is given by:

$$v = P^x P^x \quad (3)$$

with P^x the so-called Killing prepotentials, and x running over the adjoint of the composite $Sp(1)_R$. The prepotentials depend on the spin-connection on the target manifold, \mathcal{W}_α^x , and the Killing vectors, as:

$$P^x = g \mathcal{W}_\alpha^x \xi^\alpha \quad (4)$$

and we give details on the target geometry in Appendix B. The Kalb-Ramond field strength is given by $G_{MNP} = \partial_M B_{NP} + \frac{\kappa}{\sqrt{2}} F_{MN} A_P + 2 \text{ perms}$.

The fermionic supersymmetry transformations are (we shall always present up to fermion bilinears only):

$$\delta \Psi_M^i = \frac{\sqrt{2}}{\kappa} D_M \epsilon^i - \frac{1}{24} e^\varphi G_{NLR} \Gamma^{NLR} \Gamma_M \epsilon^i$$

$$\begin{aligned} \delta \chi^i &= -\frac{i}{\kappa \sqrt{2}} \partial_M \varphi \Gamma^M \epsilon^i - \frac{i}{12} e^\varphi G_{MNL} \Gamma^{MNL} \epsilon^i \\ \delta \lambda^i &= -\frac{1}{2\sqrt{2}} e^{\varphi/2} F_{MN} \Gamma^{MN} \epsilon^i - \frac{\sqrt{2}}{\kappa^2} e^{-\varphi/2} P^x (T^x \epsilon)^i \\ \delta \zeta^a &= \frac{i\sqrt{2}}{\kappa} (D_M \Phi^\alpha) V_\alpha^{ai} \Gamma^M \epsilon_i. \end{aligned} \quad (5)$$

Here, V_α^{ai} is the vielbein on the target space manifold, carrying the tangent space indices $a = 1, 2$ and $i = 1, 2$, which run over the fundamental of the composite $Sp(1)$'s. All spinors are symplectic-Majorana Weyl, with the gravitino and gaugino being left-handed and the dilatino and hyperinos being right-handed. The gravitini, Killing spinor, gaugini, and dilatini are all in the fundamental of $Sp(1)_R$, whereas the gaugini and hyperini are charged under the physical $U(1)$. The covariant derivative acting on the Killing spinor is given by:

$$D_M \epsilon^i = \partial_M \epsilon^i + \frac{1}{4} \omega_M^{AB} \Gamma_{AB} \epsilon^i + (D_M \Phi^\alpha) \mathcal{W}_\alpha^x T_j^{xi} \epsilon^j. \quad (6)$$

We can always go to complex-Weyl spinors by defining $\epsilon = \epsilon^1 + i\epsilon^2$ and so on. We record the fermionic part of the action and bosonic supersymmetry transformations in Appendix C. Below we set $\kappa = 1$.

Finally, let us note that as well as the gravitational- and super-symmetries, our model has two kinds of gauge symmetries. Under the $U(1)$ gauge symmetry, not only do the gauge fields and hypermultiplets transform, but also the Kalb-Ramond field due to Cherns-Simons term in its Kalb-Ramond field strength (the latter must clearly be gauge invariant):

$$\begin{aligned} A_M &\rightarrow A_M + \partial_M \Lambda \\ B_{MN} &\rightarrow B_{MN} - \frac{\kappa}{\sqrt{2}} F_{MN} \Lambda. \end{aligned} \quad (7)$$

Furthermore, there is an independent Kalb-Ramond gauge symmetry, whose transformation is:

$$B_{MN} \rightarrow B_{MN} + \partial_{[M} \Lambda_{N]}. \quad (8)$$

III. THE ORBIFOLD

Let us now consider the bulk theory on a orbifold, M/Z_2 . M is a smooth, 2D manifold, for instance it could have topology \mathbb{R}^2 , or T^2 being a torus or a deformed torus. We can assign the following parities with respect to the point group Z_2 . For the bosonic fields we choose:

$$\begin{aligned} \text{even} : & \quad g_{\mu\nu}, g_{mn}, \varphi, B_{\mu\nu}, B_{mn}, A_\mu, \Phi^1, \Phi^3 \\ \text{odd} : & \quad g_{\mu m}, B_{\mu m}, A_m, \Phi^2, \Phi^4, \end{aligned} \quad (9)$$

and we can re-write the internal metric as:

$$g_{mn} = \frac{r^2}{\tau_2} \begin{pmatrix} -1 & -\tau_1 \\ -\tau_1 & -\tau_1^2 - \tau_2^2 \end{pmatrix}. \quad (10)$$

For the fermions, it is useful to first decompose the 6D complex Weyl spinors into 4D ones as (see Appendix A

for gamma matrix conventions):

$$\Psi_M = \begin{pmatrix} \Psi_{L\mu} \\ \Psi_{R\mu} \end{pmatrix}, \begin{pmatrix} \Psi_{Lm} \\ \Psi_{Rm} \end{pmatrix},$$

$$\lambda = \begin{pmatrix} \lambda_L \\ \lambda_R \end{pmatrix}, \chi = \begin{pmatrix} \chi_R \\ \chi_L \end{pmatrix}, \zeta = \begin{pmatrix} \zeta_R \\ \zeta_L \end{pmatrix}, \quad (11)$$

and similarly for the 6D supersymmetry parameter, ϵ :

$$\epsilon = \begin{pmatrix} \epsilon_L \\ \epsilon_R \end{pmatrix}. \quad (12)$$

Then the corresponding fermionic parity assignments are:

$$\begin{aligned} \text{even:} & \quad \Psi_{L\mu}, \Psi_{Rm}, \chi_R, \lambda_L, \zeta_R, \epsilon_L \\ \text{odd:} & \quad \Psi_{R\mu}, \Psi_{Lm}, \chi_L, \lambda_R, \zeta_L, \epsilon_R. \end{aligned} \quad (13)$$

Notice that, although we have arranged the fields according to how they transform under 4D general coordinate invariance, since we allow them to depend on both external coordinates, x^μ , and internal coordinates, y^m , we are *not* dimensionally reducing. Indeed, the fields r, τ_1 and τ_2 carry all the degrees of freedom of the extra dimensional components in the 6D metric.

At the orbifold fixed points, a subset of the 6D N=2 symmetries are explicitly broken. In particular, the supersymmetry transformations generated by the Killing spinor ϵ_R are projected out, leaving only N=1 supersymmetry. This is because the supersymmetry parameter ϵ_R is a continuous odd function, and thus must be vanishing at the fixed points. Part of the 6D gravitational symmetries are similarly broken, for example the general coordinate transformations given by $x^m \rightarrow x^m + \xi^m$. However, the symmetries corresponding to N=1 4D supergravity survive, and some additional ones. These include part of the $U(1)$ gauge symmetries (7) and Kalb-Ramond gauge symmetries (8), since $\partial_\mu \Lambda$ and $\partial_{[5} \Lambda_{6]}$ are non-vanishing on the brane.

In the interests of simplicity we shall further assume that the odd bulk fields, internal derivatives of even fields and internal derivatives of odd fields are vanishing at the brane positions (unless the symmetries require otherwise) [30]. In the absence of brane sources, the first two of our conditions would be consequences of the orbifold parity symmetry, but couplings to brane sources may induce discontinuities, in the presence of which odd fields can be non-trivial at the fixed points [16]. With our assumptions, therefore, we will not obtain the most general couplings between bulk and brane fields. We will, however, be able to couple general brane fields.

One other comment on our constraints is in order, which is that they also limit the possible background solutions that can be studied. The constraint that $\partial_m g_{\mu\nu} = 0$ at the branes excludes some warp factors, but those typically encountered in 6D brane world models [17–19] are allowed.

IV. BULK THEORY AT THE BRANES

At the fixed points, the symmetries of 4D N=1 supergravity survive. Therefore, the fields there assemble into on-shell N=1 supermultiplets. For instance, the 4D scalars organize into complex scalar components of N=1 chiral supermultiplets as $S = \frac{1}{2}(s + ia)$, $T = \frac{1}{2}(t + ib)$, $U = \frac{1}{2}(\tau_2 + i\tau_1)$, $Z = \Phi^1 + i\Phi^3$ [9, 20], where we defined the scalars $s = r^2 e^\varphi$ and $t = r^2 e^{-\varphi}$, and the pseudo-scalars a, b via:

$$G_{\mu\nu\rho} = \frac{r^{-4} e^{-2\varphi}}{\sqrt{2}} \epsilon_{\mu\nu\rho\lambda} \partial^\lambda a,$$

$$\partial_\mu b = \frac{1}{\sqrt{2}} \partial_{[\mu} B_{\dot{5}\dot{6}]} \quad (14)$$

Here, $\epsilon_{\mu\nu\rho\sigma}$ is the 4D Levi-Civita tensor. Notice that we kept internal derivatives of the odd Kalb-Ramond field components, since we must ensure invariance under the surviving parts of the Kalb-Ramond gauge symmetry (8). They do not however carry independent physical degrees of freedom. The fermionic components of the chiral supermultiplets will be given by three linear combinations of the fermions Ψ_{Rm} and χ_R as:

$$\begin{aligned} \psi^S &= \frac{r^{3/2} e^\varphi}{2} (\chi_R + \Psi_{R\dot{5}} - i\Psi_{R\dot{6}}) \\ \psi^T &= \frac{r^{3/2} e^{-\varphi}}{2} (-\chi_R + \Psi_{R\dot{5}} - i\Psi_{R\dot{6}}) \\ \psi^U &= \frac{r^{-1/2} \tau_2}{2} (\Psi_{R\dot{5}} + i\Psi_{R\dot{6}}) \end{aligned} \quad (15)$$

and:

$$\psi^Z = -\frac{(1 - |Z|^2)}{2r^{1/2}} \zeta_R. \quad (16)$$

Meanwhile, A_μ and λ_L make up a N=1 vector multiplet. We will now observe all this from the susy transformations.

Consider how the fermions at the branes transform under the N=1 supersymmetry that survives. We write the corresponding supersymmetry parameter as:

$$\epsilon = \begin{pmatrix} \epsilon_L(x) \\ 0 \end{pmatrix}. \quad (17)$$

It is a straightforward if laborious exercise to then rewrite the transformations (5) in terms of 4D fields defined above. Remember that at the branes the odd fields and internal derivatives of odd and even fields vanish.

At the same time, we Weyl rescale to the 4D Einstein frame, taking $g_{\mu\nu} \rightarrow r^{-2} g_{\mu\nu}$. Diagonalizing the gravitino kinetic term, we find that the effective 4D gravitino on the brane is the linear combination $\psi_{L\mu} = \Psi_{L\mu} + \frac{1}{2} \Gamma_\mu \Gamma^m \Psi_{Rm}$. Moreover, we perform the following chiral rotations on the fermions, in order to obtain canonical kinetic terms; $\psi_{L\mu} \rightarrow \psi_{L\mu}/r^{1/2}$ and $\lambda_L \rightarrow r^{3/2} e^{\varphi/2} \lambda_L$, together with $\epsilon_L \rightarrow \sqrt{2} \epsilon_L / r^{1/2}$. We also find it convenient to scale out the volume factor in the internal metric, $g_{mn} \rightarrow r^2 g_{mn}$.

After some beautiful cancellations, we find the following:

$$\begin{aligned}
\delta\psi_{L\mu} &= 2D_\mu\epsilon_L + \frac{i}{2}\left(\frac{\partial_\mu a}{s} + \frac{D_\mu b}{t} + \frac{\partial_\mu \tau_1}{\tau_2}\right)\epsilon_L \\
&\quad + \frac{1}{1-|Z|^2}(ZD_\mu\bar{Z} - \bar{Z}D_\mu Z)\epsilon_L \\
\delta\lambda_L &= -\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon_L + i\frac{2g}{s}\frac{|Z|^2}{1-|Z|^2}\epsilon_L \\
\delta\psi^S &= -i\partial_\mu S\gamma^\mu\epsilon_L \\
\delta\psi^T &= -i\partial_\mu T\gamma^\mu\epsilon_L \\
\delta\psi^U &= -i\partial_\mu U\gamma^\mu\epsilon_L \\
\delta\psi^Z &= -iD_\mu Z\gamma^\mu\epsilon_L
\end{aligned} \tag{18}$$

where the complex scalar, Z , has charge +1 with respect to the $U(1)$ gauge symmetry and so:

$$D_\mu Z = \partial_\mu Z - igA_\mu Z. \tag{19}$$

These 4D N=1 local supersymmetry transformations, along with the bulk Lagrangian evaluated at the brane positions, fall naturally within the general structure of 4D N=1 supergravity developed in [1]. Indeed, at the branes, the bulk Lagrangian can be moulded into the form:

$$\begin{aligned}
\mathcal{L}_{Bb} = & e_4 \left[-\frac{1}{2}R_{(4)} + K_{i\bar{j}}D_\mu\phi^i D^\mu\bar{\phi}^{\bar{j}} - \frac{1}{4}\text{Re}H F_{\mu\nu}F^{\mu\nu} \right. \\
& \left. - \frac{1}{8}(\text{Re}H)^{-1}(K_i T_j^i \phi^j + h.c.)^2 \right. \\
& \left. + \text{fermions} \right].
\end{aligned} \tag{20}$$

where e_4 and $R_{(4)}$ are the volume tensor density and Ricci scalar associated with the 4D metric $g_{\mu\nu}$. We have written the scalar components of the N=1 chiral supermultiplets as $\phi^i = S, T, U, Z$, and the Kähler potential is:

$$\begin{aligned}
K = & -\log(T + \bar{T}) - \log(S + \bar{S}) - \log(U + \bar{U}) \\
& -2\log(1 - Z\bar{Z}).
\end{aligned} \tag{21}$$

Playing its role in the component Lagrangian, K is a function of the scalar fields, and as usual, subscripts on K indicate derivatives with respect to the corresponding complex scalar. The gauge kinetic function, again a function of the scalar fields, can be identified as $H = 2S$. Finally, the last term in the bosonic Lagrangian takes the (on-shell) form of a D-term potential due to the charged scalar. The supersymmetry transformations similarly fall into the template of [1]:

$$\begin{aligned}
\delta\psi_{L\mu} &= 2D_\mu\epsilon_L - \frac{1}{2}(K_i D_\mu\phi^i - K_{\bar{i}} D_\mu\bar{\phi}^{\bar{i}})\epsilon_L \\
\delta\lambda_L &= -\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon_L + \frac{i}{2}(\text{Re}H)^{-1}(K_i T_j^i \phi^j + h.c.)\epsilon_L \\
\delta\psi^i &= -iD_\mu\phi^i\gamma^\mu\epsilon_L.
\end{aligned} \tag{22}$$

V. BULK-BRANE COUPLINGS

Having written the bulk theory at the brane in the standard form of on-shell 4D N=1 supergravity, we can

immediately couple any collection of on-shell 4D N=1 brane fields localized at the fixed points, $y^m = y_b^m$. This is because the general couplings in 4D N=1 supergravity have long been well understood, and these are the symmetries to be obeyed at the fixed points. Indeed, the total Lagrangian at the fixed points, with contributions from the bulk and the brane fields, must take the form:

$$\begin{aligned}
\mathcal{L}_B|_b + \mathcal{L}_b\delta^{(2)}(0) = & e_4 \left[-\frac{1}{2}R_{(4)} + K_{i\bar{j}}D_\mu\phi^i D^\mu\bar{\phi}^{\bar{j}} \right. \\
& \left. - \frac{1}{4}\text{Re}H_{(a)}F_{\mu\nu}^{(a)}F^{(a)\mu\nu} \right. \\
& \left. - V_D - V_F + \text{fermions} \right]
\end{aligned} \tag{23}$$

where now ϕ^i and $A_\mu^{(a)}$ include any brane fields as well as the bulk fields above, and we formally keep track of the localization with delta-function distributions $\delta^{(2)}(y^m - y_b^m) \equiv \delta(y^5 - y_b^5)\delta(y^6 - y_b^6)$, where the superscript $^{(2)}$ indicates that we have 2D delta-functions. In the Lagrangian, there is a sum over gauge indices (a) , and the D-term potential is written as:

$$V_D = \frac{1}{8}(\text{Re}H_{(a)})^{-1} \left(K_i T_j^{(a)i} \phi^j + h.c. \right)^2. \tag{24}$$

We saw above that it may already have non-trivial contributions at the classical level from the bulk. There may be further contributions to the scalar potential from the brane fields, to the D-term potential and to an F-term potential, which we write in terms of the superpotential, W (a function of the scalar fields), as:

$$V_F = e^K \left(K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right). \tag{25}$$

The supersymmetry transformations at the brane positions are similarly the standard ones of N=1 4D supergravity. For the fermions, we have:

$$\begin{aligned}
\delta\psi_{L\mu} &= 2D_\mu\epsilon_L - \frac{1}{2}(K_i D_\mu\phi^i - K_{\bar{i}} D_\mu\bar{\phi}^{\bar{i}})\epsilon_L \\
&\quad - e^{K/2}W\gamma_\mu\bar{\epsilon}_L \\
\delta\lambda_L^{(a)} &= -\frac{1}{2}F_{\mu\nu}^{(a)}\gamma^{\mu\nu}\epsilon_L \\
&\quad + \frac{i}{2}(\text{Re}H)^{-1}(K_i T_j^{(a)i} \phi^j + h.c.)\epsilon_L \\
\delta\psi^i &= -iD_\mu\phi^i\gamma^\mu\epsilon_L - e^{K/2}K^{i\bar{j}}D_{\bar{j}}\bar{W}\bar{\epsilon}_L,
\end{aligned} \tag{26}$$

and this clearly includes new terms, with respect to the original bulk transformations, depending on the brane fields.

Let us discuss two simple explicit examples, to illustrate the generality of the scheme. First, consider a brane localized chiral supermultiplet, (Q, ψ_Q) , with charge +1 under the bulk $U(1)$ gauge symmetry and a canonical kinetic term. The total Lagrangian at the brane positions takes the form (23), with:

$$\begin{aligned}
K = & -\log(T + \bar{T}) - \log(S + \bar{S}) - \log(U + \bar{U}) \\
& -2\log(1 - Z\bar{Z}) + Q\bar{Q}\delta^{(2)}(0)
\end{aligned}$$

$$H = 2S \quad \text{and} \quad W = 0. \quad (27)$$

Thus, the Lagrangian for the brane fields is given explicitly by:

$$\mathcal{L}_b = e_4 \left[g^{\mu\nu} D_\mu Q D_\nu \bar{Q} - \frac{1}{2s} g^2 |Q|^4 \delta^{(2)}(y^m - y_b^m) - \frac{2g^2}{s} |Q|^2 \frac{|Z|^2}{1 - |Z|^2} + \text{fermions} \right], \quad (28)$$

and we see that gauge invariance and local N=1 supersymmetry requires the charged brane fields to couple not only to $g_{\mu\nu}$ and A_μ but also to s and Z . It is also easy to observe from (26) that there are new brane localized field contributions to the supersymmetry transformations of bulk fields, $\psi_{L\mu}$ and λ_L (there are also new brane localized contributions to $\delta\psi_{L\mu}$, $\delta\lambda_L$ and $\delta\psi^i$ at bilinear order in the fermions, as can be read from the text-books).

Take care that we have written the above couplings in terms of the Weyl rescaled metric, corresponding to the 4D Einstein frame. It is in this frame that the bulk Lagrangian at the brane position takes the standard 4D N=1 form. If we want the couplings in terms of the original 6D Einstein frame we must perform the inverse rescaling, which leads to a further coupling between the brane fields and bulk field r .

The total Lagrangian is clearly invariant (up to total derivatives and the field equations) under the 6D N=2 local supersymmetries, with the brane localized fields transforming only under the 4D N=1 subset. In detail, the Lagrangian and supersymmetry transformations are each composed of two parts; the original bulk supergravity interactions and the brane localized ones. The original supersymmetry variations of the bulk Lagrangian clearly cancel, since they are those of 6D N=2 supergravity. The new brane localized contributions to the variation of the Lagrangian, which arise both due to new terms in the Lagrangian and the supersymmetry transformations, are by construction within the form of 4D N=1 supergravity with general matter couplings. So they cancel too. We have checked this explicitly for our simple example. Therefore, the total action:

$$\mathcal{S}_B + \mathcal{S}_b \quad (29)$$

with \mathcal{S}_B given in (1) and

$$\mathcal{S}_b = \int d^6x \mathcal{L}_b \delta^{(2)}(y^m - y_b^m), \quad (30)$$

is invariant under the full 6D N=2 symmetries, with the brane fields Q, ψ^Q transforming only under a 4D N=1 subset of them.

Moreover, it also follows easily that the supersymmetry algebra closes up to the field equations. Observe again that the new brane localized terms that we have added in the supersymmetry transformations and field equations correspond to standard interactions in 4D N=1 supergravity. When computing the commutators of the supersymmetry transformations on the various fields, we

obtain purely bulk contributions and new brane localized contributions. Applying the equations of motion introduces further bulk and brane localized contributions. Finally, putting together all the terms, the bulk contributions close exactly as in 6D N=2 supergravity, and the brane localized contributions close exactly as in 4D N=1 supergravity.

Notice the presence of a singular, delta-function squared term in the action, which is required for the invariance of the action under the supersymmetry transformations, and which is typical in supersymmetric scenarios with localized fields (see *e.g.* [3, 4, 6, 7, 9–11, 21, 25]). We shall discuss this a little more in our closing remarks.

As a second example, let us consider pure 6D N=2 supergravity in the bulk, and introduce a brane localized $U(1)$ gauge multiplet, (A_μ, λ) , and a charged brane chiral supermultiplet, (Q, ψ_Q) . We allow the complex scalar Q to have a kinetic coupling to the bulk complex field, T . This model will allow us to compare our construction with the example worked out in the literature via the Noether method [9]. It will prove convenient to redefine the scalar component of the chiral supermultiplet, T , at the brane as:

$$T = t + |Q|^2 \delta^{(2)}(0) + ib. \quad (31)$$

Notice that this implies a brane localized field contribution to the fermion ψ^T :

$$\psi^T = \frac{r^{3/2} e^{-\varphi}}{2} (-\chi_R + \Psi_{R\dot{5}} - i\Psi_{R\dot{6}}) + \left(\bar{Q}\psi^Q + Q\psi^{\bar{Q}} \right) \delta^{(2)}(0). \quad (32)$$

The total Lagrangian at the brane can be chosen such that:

$$\begin{aligned} K &= -\log(T + \bar{T} - 2|Q|^2 \delta^{(2)}(0)) \\ &\quad -\log(S + \bar{S}) - \log(U + \bar{U}) \\ H &= 1 \delta^{(2)}(0) \quad \text{and} \quad W = 0. \end{aligned} \quad (33)$$

Then, the subsequent component Lagrangian for the brane fields is given explicitly by:

$$\begin{aligned} \mathcal{L}_b &= e_4 \left[e^\varphi D_\mu Q D^\mu \bar{Q} - \frac{i}{2} e^{2\varphi} D_\mu b (Q D^\mu \bar{Q} - \bar{Q} D^\mu Q) \right. \\ &\quad - \frac{1}{4} e^{2\varphi} r^{-2} (Q D_\mu \bar{Q} - \bar{Q} D_\mu Q) \\ &\quad \times (Q D^\mu \bar{Q} - \bar{Q} D^\mu Q) \delta^{(2)}(y^m - y_b^m) \\ &\quad \left. - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} e^{-2\varphi} g^2 |Q|^4 + \text{fermions} \right], \end{aligned} \quad (34)$$

where $D_\mu Q = \partial_\mu Q + ig A_\mu Q$, and here we have re-Weyl rescaled back to the 6D Einstein metric.

The final result for the total action, $\mathcal{S}_B + \mathcal{S}_b$, agrees with the one found in [9], [31]. Moreover, it is now easy to complete the theory up to four fermion terms, and to show that the bulk plus brane action does indeed have all the required symmetries: 6D N=2 in the bulk, 4D N=1 on the brane.

VI. CONCLUSIONS

The idea that fields may be localized on a brane has played an important role in many aspects of fundamental high energy physics and cosmology for more than a decade. However, building explicit, detailed models which realize this idea remains technically challenging, especially within the well-motivated framework of supergravity. In the present letter, we have shown how to construct bulk plus brane actions which incorporate the symmetries of 6D N=2 supergravity away from the branes and 4D N=1 supergravity at the brane positions. The power of our approach is that we do not need to enter into extremely lengthy and messy computations to check the supersymmetry invariance and closure of the algebra each time we add new brane localized fields and interactions. Instead, one can simply write down N=1 preserving interactions between the bulk supergravity and brane localized fields because the general matter couplings for 4D N=1 supergravity are known, and the bulk theory at the brane positions can be recast into that form.

It is this latter step that represents the technical challenge in our proposal, and we reduce it by making some simplifying assumptions for the behaviour of bulk fields at the branes, including that the internal derivatives are vanishing there. We are still able to couple general brane fields to the bulk theory, *e.g.* it becomes very easy to extend the model of Ref [9], first constructed by the Noether method. However, it would certainly be interesting to relax those assumptions, to allow the most general bulk-brane couplings possible. Of course, in principle, it should indeed be possible and interesting to rewrite the whole 6D N=2 bulk theory keeping only the N=1 supersymmetries manifest (see [8] for some first steps).

Our focus has been on the bosonic part of the action and its fermionic supersymmetry transformations, since this is most interesting part. The completion to the fermionic action and bosonic transformations is of course guaranteed by supersymmetry, and can be read from the text-books. At the same time, our analysis has been entirely classical. At the quantum level, there are generically gauge anomalies in the bulk and on the brane, as well as bulk gravitational anomalies, and these provide restrictions on the bulk and brane matter contents.

The results presented allow us to build field theory models describing, for example, the low energy dynamics of orbifold string compactifications. For instance, our construction is rich enough to build supergravity realizations of the orbifold-GUT models in [13]. Orbifold string compactifications have proved remarkably successful in the quest for a fundamental origin of the MSSM [22], but their dynamical aspects remain to be understood. Ref [23, 24] suggest that a non-trivial dynamics for the brane localized fields may help in stabilizing the bulk moduli, using a toy globally supersymmetric 6D model. We may now study such issues taking into proper account the consequences of dynamical gravity.

As was to be expected from previous work (starting

with [21]), our supergravity actions describing the brane localized fields suffer from the presence of delta-function squared terms, which are indeed required by supersymmetry. It is generally believed that such singularities would be resolved in a full quantum gravity treatment. At the same time, we can pragmatically try to live with them within the effective field theory [21] (see [3] for a useful representation of them). For example, we expect them to play an important role in the cancellation of divergences ensured by supersymmetry [3]. This was shown explicitly for a global 5D model in [3, 25], and also for a 5D supergravity model in [7]. It would furthermore be important to develop techniques to construct non-trivial background compactifications and subsequent low energy 4D effective field theories, despite these singularities. For recent work on the backreaction of codimension-two branes in the absence of brane matter see [26]. Finally, for some insights regarding the subtleties of supersymmetry in singular spaces, we refer to [27].

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Appendix A: Conventions

Our signature is mostly minus and we take MTW conventions for the curvature tensors. 6D spacetime coordinates are X^M , 4D ones are x^μ and 2D ones y^m . Tangent space indices are, respectively, A, B, \dots ; α, β, \dots and $a, b, \dots = \dot{5}, \dot{6}$.

We build the 4D gamma matrices from the Pauli matrices,

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (\text{A1})$$

as follows:

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -\mathbb{1}_2 & 0 \\ 0 & \mathbb{1}_2 \end{pmatrix}. \quad (\text{A2})$$

In turn, the 6D gamma matrices are:

$$\Gamma^\alpha = \begin{pmatrix} \gamma^\alpha & 0 \\ 0 & \gamma^\alpha \end{pmatrix}, \quad \Gamma^{\dot{5}} = \begin{pmatrix} 0 & i\gamma_5 \\ i\gamma_5 & 0 \end{pmatrix}, \quad \Gamma^{\dot{6}} = \begin{pmatrix} 0 & \gamma_5 \\ -\gamma_5 & 0 \end{pmatrix}, \quad (\text{A3})$$

with the chirality matrix

$$\Gamma_7 = \begin{pmatrix} \gamma_5 & 0 \\ 0 & -\gamma_5 \end{pmatrix}. \quad (\text{A4})$$

The 2D Levi-Civita tensor density is ϵ_{ab} , with $\epsilon_{\dot{5}\dot{6}} = 1$.

Appendix B: Hyper Target Geometry

The hyperscalars in N=2 6D supergravity coordinatize a quaternionic manifold, and we shall take as a canonical example the manifold $\frac{Sp(1,1)}{Sp(1) \times Sp(1)_R}$. The geometry of this class of manifolds is described in detail in [28]. With our four hyperscalars, we can compose a quaternion, $t = \Phi^1 \mathbf{1} + \Phi^2 \mathbf{i} + \Phi^3 \mathbf{j} + \Phi^4 \mathbf{k}$, where we have introduced the following 2×2 basis for the quaternions:

$$\begin{aligned} \mathbf{i} &= \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, & \mathbf{j} &= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \\ \mathbf{k} &= \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}, & \mathbf{1} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \end{aligned} \quad (B1)$$

The $Sp(1)_R$ spin-connection and vielbein are given in terms of the quaternion by:

$$\begin{aligned} \mathcal{W}_\alpha^{ij} &= \frac{1}{2} \gamma^{-2} (\partial_\alpha t^\dagger t - t^\dagger \partial_\alpha t) \\ V_\alpha^{ai} &= \gamma^{-1} (I - t t^\dagger)^{-1/2} \partial_\alpha t. \end{aligned} \quad (B2)$$

The target manifold metric is then given explicitly by:

$$g_{\alpha\beta} = \frac{2}{1 - |\Phi|^2} \delta_{\alpha\beta}, \quad (B3)$$

with the shorthand $|\Phi|^2 = (\Phi^1)^2 + (\Phi^2)^2 + (\Phi^3)^2 + (\Phi^4)^2$. Meanwhile, we choose the hypermultiplet to be charged under the bulk $U(1)$ gauge symmetry, such that the Killing vector $\xi^\alpha = (T\Phi)^\alpha$ is:

$$\xi^\alpha = \begin{pmatrix} -\Phi^3 \\ \Phi^4 \\ \Phi^1 \\ -\Phi^2 \end{pmatrix}. \quad (B4)$$

The Killing prepotentials are then given by:

$$P^x T^x ij \equiv P^{ij} = g \frac{|\Phi|^2}{1 - |\Phi|^2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (B5)$$

Using $Tr T^x T^y = -\frac{1}{2} \delta^{xy}$, we find that the 6D scalar potential function is:

$$v(\Phi) = 4g^2 \frac{|\Phi|^4}{(1 - |\Phi|^2)^2} \quad (B6)$$

Appendix C: Fermionic Action and Bosonic SUSY Transformations

The fermionic part of the action for minimal 6D supergravity, to bilinear order, is:

$$S_F = \int d^6 X e_6 \left[-\frac{1}{2} i \bar{\Psi}_M \Gamma^{MNP} D_N \Psi_P + \frac{1}{2} i \bar{\chi} \Gamma^M D_M \chi \right.$$

$$\begin{aligned} &+ \frac{1}{2} i \bar{\lambda} \Gamma^M D_M \lambda + \frac{1}{2} i \bar{\zeta}^a \Gamma^M D_M \zeta_a \\ &+ \frac{1}{4} \bar{\chi} \Gamma^N \Gamma^M \Psi_N \partial_M \varphi + \frac{1}{2} \bar{\Psi}_M \Gamma^N \Gamma^M \zeta^a V_{\alpha ai} D_N \Phi^a \\ &- \frac{\kappa}{48\sqrt{2}} i e^\varphi G_{MNP} (-\bar{\chi} \Gamma^{MNP} \chi + \lambda \Gamma^{MNP} \lambda \\ &\quad + \bar{\zeta}^a \Gamma^{MNP} \zeta_a \\ &\quad - \bar{\Psi}_L \Gamma_{[L} \Gamma^{MNP} \Gamma_{S]} \Psi^S + 2 i \bar{\Psi}_L \Gamma^{MNP} \Gamma^L \chi) \\ &+ \frac{\kappa}{8} i e^{\varphi/2} F_{PQ} (\bar{\Psi}_M \Gamma^{PQ} \Gamma^M \lambda - i \chi \Gamma^{PQ} \lambda) \\ &+ e^{-\varphi/2} i \kappa^{-1} \sqrt{\frac{1}{2}} (\bar{\Psi}_M \Gamma^M T^x \lambda C^x \\ &\quad + i \bar{\chi} T^x \lambda C^x + 2 i \zeta^a \lambda^i V_{\alpha ai} \xi^a) \Big]. \quad (C1) \end{aligned}$$

Meanwhile, the supersymmetry transformations for the bosonic fields, up to fermion bilinears, are:

$$\begin{aligned} \delta e_M^A &= -i \frac{\kappa}{\sqrt{2}} \bar{\epsilon} \Gamma^A \Psi_M \\ \delta \varphi &= \frac{\kappa}{\sqrt{2}} \bar{\epsilon} \chi \\ \delta B_{MN} &= -\frac{1}{2} i e^{-\varphi} (\bar{\epsilon} \Gamma_M \Psi_N - \bar{\epsilon} \Gamma_N \Psi_M - i \bar{\epsilon} \Gamma_{MN} \chi) \\ &\quad + \sqrt{2} \kappa A_{[M} \delta A_{N]} \\ \delta A_M &= -\sqrt{\frac{1}{2}} i e^{-\varphi/2} \bar{\epsilon} \Gamma_M \lambda \\ \delta \Phi^a &= -\frac{\kappa}{\sqrt{2}} V_{ai}^\alpha \bar{\epsilon}^i \zeta^a. \end{aligned} \quad (C2)$$

Appendix D: Metric Decomposition

Here, we write down the decomposition of the bulk curvature tensors and connections, which appear in the intermediate steps of our calculations. Recalling the Weyl rescalings, we decompose the 6D metric as:

$$g_{MN} = \begin{pmatrix} r^{-2} g_{\mu\nu} & 0 \\ 0 & r^2 g_{mn} \end{pmatrix}, \quad (D1)$$

with the corresponding vierbiens, e_μ^a and

$$e_m^a = \begin{pmatrix} -\frac{1}{\sqrt{\tau_2}} & 0 \\ -\frac{\tau_1}{\sqrt{\tau_2}} & \sqrt{\tau_2} \end{pmatrix}. \quad (D2)$$

The relevant components of the 6D spin-connection, ω_M^{AB} , are then:

$$\begin{aligned} \omega_\mu^{\alpha\beta} &= \omega_\mu^{\alpha\beta} [e_\mu^\alpha] + e^{\nu\beta} e_\mu^\alpha \partial_\nu r^{-1} - e^{\nu\alpha} e_\mu^\beta \partial_\nu r^{-1} \\ \omega_\mu^{\alpha b} &= \text{internal derivs of } r \\ \omega_\mu^{\dot{5}\dot{6}} &= -\omega_\mu^{\dot{6}\dot{5}} = \frac{\partial_\mu \tau_1}{2\tau_2} \\ \omega_m^{\alpha\beta} &= 0 \\ \omega_m^{\alpha a} &= r e^{\mu\alpha} \begin{pmatrix} -\frac{\partial_\mu \tau_2}{2\tau_2^{3/2}} & -\frac{\partial_\mu \tau_1}{2\tau_2^{3/2}} \\ \frac{\tau_2 \partial_\mu \tau_1 - \tau_1 \partial_\mu \tau_2}{2\tau_2^{3/2}} & -\frac{\tau_1 \partial_\mu \tau_1 + \tau_2 \partial_\mu \tau_2}{2\tau_2^{3/2}} \end{pmatrix} \\ &\quad + e^{\mu\alpha} e_m^a r \partial_\mu r \\ \omega_5^{\dot{5}\dot{6}} &= -\omega_5^{\dot{6}\dot{5}} = \text{internal derivs of } r, \tau_1, \tau_2 \\ \omega_6^{\dot{5}\dot{6}} &= -\omega_6^{\dot{6}\dot{5}} = \text{internal derivs of } r, \tau_1, \tau_2. \end{aligned} \quad (D3)$$

Subsequently, the 6D Ricci scalar decomposes as:

$$R = r^2 R_{(4)}[g_{\mu\nu}] - \frac{1}{2\tau_2^2} g^{\mu\nu} \partial_\mu \tau_2 \partial_\nu \tau_2 - \frac{1}{2\tau_2^2} g^{\mu\nu} \partial_\mu \tau_1 \partial_\nu \tau_1 - \frac{1}{r^4} g^{\mu\nu} \partial_\mu r^2 \partial_\nu r^2$$

$$+\text{internal derivs of } r, \tau_1, \tau_2. \quad (\text{D4})$$

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- [31] Indeed we can correct a typo there in Equation (8), where the brane current should read $j_\mu = \frac{i}{\sqrt{2}} (\bar{Q} D_\mu Q - D_\mu \bar{Q} Q) + \text{fermions}$.