Electroproduction of the $N^*(1535)$ Resonance at Large Momentum Transfer

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We report on the first lattice calculation of light-cone distribution amplitudes of the $N^*(1535)$ resonance, which are used to calculate the transition form factors at large momentum transfers using light-cone sum rules. In the region $Q^2 > 2$ GeV^2, where the light-cone expansion is expected to converge, the results appear to be in good agreement with the experimental data.

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Introduction.—Electroproduction of nucleon resonances has long been recognized as an important tool in the exploration of the nucleon structure at different scales. Quantum chromodynamics (QCD) predicts [1–4] that at large momentum transfer the transition form factors become increasingly dominated by the contribution of the valence Fock state with small transverse separation between the partons. There is a growing consensus that perturbative QCD (pQCD) factorization based on hard gluon exchange is not reached at present energies; however, the emergence of quarks and gluons as the adequate degrees of freedom is expected to happen earlier, at $Q^2 \sim$ a few GeV^2. In this transition region the form factors can be measured to high accuracy; see, e.g., [5,6].

The major problem is that any attempt at a quantitative description of form factors in the transition region must include soft nonperturbative contributions which correspond to the overlap integrals of the soft wave functions; see, e.g., [7,8]. In particular, models of generalized parton distributions (GPDs) usually are chosen such that the experimental data on form factors are described by the soft contributions alone; cf. [9–11]. A subtle point for these semiphenomenological approaches is to avoid double counting of hard rescattering contributions "hidden" in the model-dependent hadron wave functions or GPD parametrizations. An approach that is more directly connected to QCD is based on the light-cone sum rules (LCSRs) [12,13]. This technique is attractive because in LCSRs "soft" contributions to the form factors are calculated as an expansion in terms of the momentum fraction distributions of partons at small transverse separations, dubbed distribution amplitudes (DAs), which are the same quantities that enter the calculation in pQCD, and there is no double counting. Thus the LCSRs provide one with the most direct relation of the hadron form factors and DAs that is available at present, with no other nonperturbative parameters. Unfortunately, with the exception of the $\Delta(1232)$ resonance, up to now there exists almost no information on the DAs of nucleon resonances. Thus pQCD predictions [14,15] cannot be quantified, and the LCSRs cannot be used as well.

Moments of the DAs can, however, be calculated on the lattice [16]. In this work we suggest a synthetic approach combining the constraints on DAs from a lattice calculation with LCSRs to calculate the form factors. As the first demonstration of this strategy, we consider the electroproduction of $N^*(1535)$, the parity partner of the nucleon. This is a special case because lattice calculations of baryon correlation functions always yield results for baryons of both parities $J^P = 1/2^+$ and $J^P = 1/2^-$ (see, e.g., [17,18]), so in fact the results for $N^*(1535)$ appear to be a by-product of our calculation of the nucleon DAs [19,20], to which we refer for further technical details. We find that the shapes of the nucleon and N^* DAs are rather different. A preliminary account of this study was presented in Ref. [21]. In this Letter, we further use our results on the DAs to calculate the helicity amplitudes $A_{1/2}(Q^2)$ and $S_{1/2}(Q^2)$ for the electroproduction of $N^*(1535)$ in the LCSR approach.

Distribution amplitudes.—In full analogy to the nucleon, the leading-twist ($=3$) DA of the $N^*(1535)$ resonance can be defined from a matrix element of a nonlocal light-ray operator that involves quark fields of given helicity $q^{\uparrow(\downarrow)} = (1/2)(1 \pm \gamma_5)q$ (cf. [22]):

$$\langle 0|\epsilon^{ijk}[u_i^\uparrow(a_1 n)C\not{n}u_j^\downarrow(a_2 n)]\not{n}d_k^\uparrow(a_3 n)|N^*(P)\rangle = \frac{1}{2}f_{N^*}P\cdot n\not{n}u^\uparrow(P)\int[dx]e^{-iP\cdot n\sum x_i a_i}\varphi_{N^*}(x_i).$$

Here P_μ ($P^2 = m_{N^*}^2$) is the N^* momentum, $u(P)$ the Dirac spinor in relativistic normalization, n_μ an arbitrary lightlike

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vector ($n^2 = 0$), and C the charge-conjugation matrix. The variables x_1, x_2, and x_3 have the meaning of the momentum fractions carried by the three valence quarks, and the integration measure is defined as $\int[dx] = \int_0^1 dx_1 dx_2 dx_3 \delta(\sum x_i - 1)$. The Wilson lines that ensure gauge invariance are inserted between the quarks; they are not shown for brevity.

On the lattice, one can calculate moments of the DA $\varphi^{lmn} = \int[dx] x_1^l x_2^m x_3^n \varphi(x_i)$, which are related to matrix elements of local three-quark operators with covariant derivatives; see [20] for details. The normalization is $\varphi^{000} = 1$, which corresponds to the definition of the coupling f_{N^*} as

$$\langle 0|\epsilon^{ijk}(u_i C \not{n} u_j)\gamma_5 \not{n} d_k|N^*(P)\rangle = f_{N^*} P\cdot n \gamma_5 \not{n} u(P). \quad (1)$$

There exist three independent subleading twist-4 distribution amplitudes $\Phi_4^{N^*}$, $\Psi_4^{N^*}$, and $\Xi_4^{N^*}$ (as for the nucleon). They can be defined as (cf. [22,23])

$$\langle 0|\epsilon^{ijk}[u_i^\uparrow(a_1 n) C \not{n} u_j^\downarrow(a_2 n)]\not{P} d_k^\uparrow(a_3 n)|N^*(P)\rangle = \frac{1}{4} P\cdot n \not{P} u^\uparrow(P)\int[dx]e^{-iP\cdot n\sum x_i a_i}[f_{N^*}\Phi_4^{N^*,WW}(x_i) + \lambda_1^*\Phi_4^{N^*}(x_i)],$$

$$\langle 0|\epsilon^{ijk}[u_i^\uparrow(a_1 n) C \not{n}\gamma_\perp \not{P} u_j^\downarrow(a_2 n)]\gamma^\perp \not{n} d_k^\uparrow(a_3 n)|N^*(P)\rangle = -\frac{1}{2} P\cdot n \not{n} m_{N^*} u^\uparrow(P)\int[dx]e^{-iP\cdot n\sum x_i a_i}[f_{N^*}\Psi_4^{N^*,WW}(x_i) - \lambda_1^*\Psi_4^{N^*}(x_i)],$$

$$\langle 0|\epsilon^{ijk}[u_i^\uparrow(a_1 n) C \not{P}\not{n} u_j^\uparrow(a_2 n)]\not{n} d_k^\uparrow(a_3 n)|N^*(P)\rangle = \frac{\lambda_2^*}{12} P\cdot n \not{n} m_{N^*} u^\uparrow(P)\int[dx]e^{-iP\cdot n\sum x_i a_i}\Xi_4^{N^*}(x_i),$$

where $\Phi_4^{N^*,WW}(x_i)$ and $\Psi_4^{N^*,WW}(x_i)$ are the so-called Wandzura-Wilczek contributions, which can be expressed in terms of the leading-twist DA [23]. The two new normalization constants are given by the local matrix elements

$$\langle 0|\epsilon^{ijk}(u_i C\gamma_\mu u_j)\gamma_5\gamma^\mu d_k|N^*(P)\rangle = \lambda_1^* m_{N^*}\gamma_5 u(P),$$
$$\langle 0|\epsilon^{ijk}(u_i C\sigma_{\mu\nu} u_j)\gamma_5\sigma^{\mu\nu} d_k|N^*(P)\rangle = \lambda_2^* m_{N^*}\gamma_5 u(P).$$

The asymptotic distribution amplitudes (at very large scales) for the nucleon and N^* are the same:

$$\phi^{\rm as}(x_i) = 120 x_1 x_2 x_3, \qquad \Phi_4^{\rm as}(x_i) = 24 x_1 x_2,$$
$$\Phi_4^{WW,\rm as}(x_i) = 24 x_1 x_2[1 + \tfrac{2}{3}(1 - 5x_3)],$$
$$\Psi_4^{WW,\rm as}(x_i) = 24 x_1 x_3[1 + \tfrac{2}{3}(1 - 5x_2)],$$
$$\Xi_4(x_i) = 24 x_2 x_3, \qquad \Psi_4^{\rm as}(x_i) = 24 x_1 x_3.$$

Baryon states of different parity can be identified in a lattice calculation as those propagating forward or backward in (imaginary) time as long as their momentum $\vec{p}$ vanishes [17,18]. While $\vec{p} = \vec{0}$ is sufficient for the evaluation of the normalization constants, the higher moments of the DAs require $\vec{p} \neq \vec{0}$. In this case the signal in the negative parity channel is contaminated by a contribution of the $J^P = 1/2^+$ (nucleon) ground state. However, this effect seems to be quite small in our results: Replacing the parity projector $(1/2)(1 + \gamma_4)$ by $(1/2)[1 + (m_N/E_N)\gamma_4]$ [17] changes the first (second) moments of the N^* DAs by 1% (5%), which is well below the statistical error. In principle, there is still a contamination by the $J^P = 1/2^+$ $N^*(1440)$ (Roper) resonance, but for small momenta this effect is expected to be negligible [17]. Another issue is that in the physical spectrum there are two $J^P = 1/2^-$ resonances, $N^*(1535)$ and $N^*(1650)$, which cannot be distinguished by means of their small mass difference in our calculation. Because of the peculiar decay pattern of $N^*(1650)$, we expect, however, that this state has a much smaller coupling to the usual interpolating operators [18]. So our results can be identified with the contribution of $N^*(1535)$ alone. All of these questions certainly deserve a further study.

We have evaluated the required correlation functions on configurations generated by the QCDSF/DIK Collaborations with two flavors of clover fermions at the smallest lattice spacing a available ($a \simeq 0.07$ fm) corresponding to $\beta = 5.40$. Five values of the quark mass were considered; see Table II in [21], where further technical details can be found. In the case of the nucleon, we have observed in Ref. [21] that a somewhat coarser lattice led to compatible results, and we have taken the data from the finer lattice as our final numbers. To ensure the comparability of the results, we follow the same procedure also here. The results for the normalization constants f_{N^*}, λ_1^*, and λ_2^* and of the first few moments of the leading-twist DA of the $N^*(1535)$ resonance are compared to the analogous quantities for the nucleon [20] in Table I. It attracts attention that f_{N^*} is about 50% larger than f_N. This means that the wave function of the three quarks at the origin is larger in the $J^P = 1^-$ state than in the $J^P = 1^+$ state, which may be counterintuitive. The momentum fraction carried by the u quark with the same helicity as the baryon itself, φ^{100}, appears to be considerably larger for N^*, indicating that its DA is more asymmetric. These features suggest that the large asymmetry of the nucleon DA observed in QCD sum rule calculations [24–26] may be due to a contamination of the sum rules by states of opposite parity, which are difficult to separate in this approach.

The calculated moments can be used to model the N^* leading-twist DA as an expansion in orthogonal polynomials corresponding to the contributions of multiplicatively renormalizable operators (in leading order); see [20]. The comparison of such models for N and N^*, obtained using the polynomial expansion to second order and the central values of the lattice parameters, is shown in Fig. 1.

Helicity amplitudes from LCSRs.—The matrix element of the electromagnetic current $j_\nu^{\rm em}$ between spin-1/2 states of opposite parity can be parametrized in terms of two independent form factors, which can be chosen as

TABLE I. The normalization constants (in units of 10^{-3} GeV^2) and moments of the DAs obtained from QCDSF/DIK configurations at the lattice spacing $a \simeq 0.07$ fm for the nucleon (N) and $N^\star(1535)$ at $\mu^2_{\overline{\rm MS}} = 1$ GeV^2. The first error is statistical, and the second error represents the uncertainty due to the chiral extrapolation and renormalization. The systematic error should be considered with caution.

	Asympt.	N	$N^\star(1535)$
f_N		3.234(63)(86)	4.544(117)(223)
$-\lambda_1$		35.57(65)(136)	37.55(101)(768)
λ_2		70.02(128)(268)	191.9(44)(391)
φ^{100}	$\frac{1}{3} \simeq 0.333$	0.3999(37)(139)	0.4765(33)(155)
φ^{010}	$\frac{1}{3} \simeq 0.333$	0.2986(11)(52)	0.2523(20)(32)
φ^{001}	$\frac{1}{3} \simeq 0.333$	0.3015(32)(106)	0.2712(41)(136)
φ^{200}	$\frac{1}{7} \simeq 0.143$	0.1816(64)(212)	0.2274(89)(307)
φ^{020}	$\frac{1}{7} \simeq 0.143$	0.1281(32)(106)	0.0915(45)(224)
φ^{002}	$\frac{1}{7} \simeq 0.143$	0.1311(113)(382)	0.1034(160)(584)
φ^{011}	$\frac{2}{21} \simeq 0.095$	0.0613(89)(319)	0.0398(132)(497)
φ^{101}	$\frac{2}{21} \simeq 0.095$	0.1091(41)(152)	0.1281(56)(131)
φ^{110}	$\frac{2}{21} \simeq 0.095$	0.1092(67)(219)	0.1210(101)(304)

$$\langle N^*(P')|j_\nu^{\rm em}|N(P)\rangle = \bar{u}_{N^*}(P')\gamma_5\Gamma_\nu u_N(P),$$
$$\Gamma_\nu = \frac{G_1(q^2)}{m_N^2}(\not{q}q_\nu - q^2\gamma_\nu) - i\frac{G_2(q^2)}{m_N}\sigma_{\nu\rho}q^\rho, \tag{2}$$

where $q = P' - P$ is the momentum transfer. The helicity amplitudes $A_{1/2}(Q^2)$ and $S_{1/2}(Q^2)$ for the electroproduction of $N^*(1535)$ can be expressed in terms of the form factors [27]:

$$A_{1/2} = eB[Q^2G_1(Q^2) + m_N(m_{N^*} - m_N)G_2(Q^2)],$$
$$S_{1/2} = \frac{e}{\sqrt{2}}BC[(m_N - m_{N^*})G_1(Q^2) + m_N G_2(Q^2)].$$

Here e is the elementary charge, and B and C are kinematic factors defined as

$$B = \sqrt{\frac{Q^2 + (m_{N*} + m_N)^2}{2m_N^5(m_{N^*}^2 - m_N^2)}}, \qquad C = \sqrt{1 + \frac{(Q^2 - m_{N^*}^2 + m_N^2)^2}{4Q^2 m_{N^*}^2}}.$$

The LCSRs are derived from the correlation function $\int dx e^{-iqx}\langle N^*(P)|T\{\eta(0)j_\mu^{\rm em}(x)\}|0\rangle$, where η is a suitable operator with nucleon quantum numbers, e.g., the Ioffe current [28]. Making use of the duality of QCD quark-gluon and hadronic degrees of freedom through dispersion relations, one can write a representation for the form factors appearing in (2) in terms of the DAs of N^*. In leading order, the sum rules for $Q^2G_1(Q^2)/(m_N m_{N^*})$ and $-2G_2(Q^2)$ have the same functional form as the similar sum rules [12,13] for the Dirac and Pauli electromagnetic form factors of the proton, with the replacement $m_N \to m_{N^*}$ in the light-cone expansion part, and different DAs.

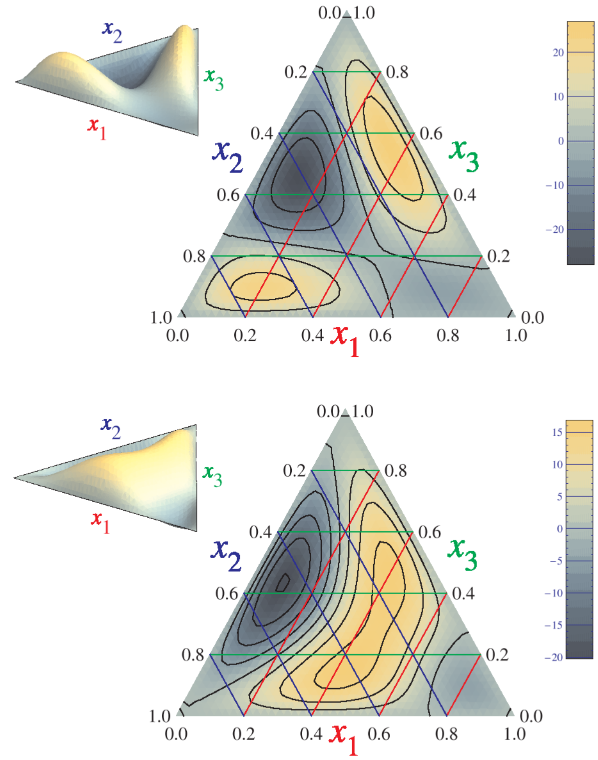

FIG. 1 (color online). Barycentric plot of the distribution amplitudes for nucleon (up) and $N^\star(1535)$ (down) at $\mu_{\overline{\rm MS}} = 1$ GeV using the central values of the lattice results. The lines of constant x_1, x_2, and x_3 are parallel to the sides of the triangle labeled by x_2, x_3, and x_1, respectively.

In the present calculation we used a model for the leading-twist DA including first order corrections in the polynomial expansion, asymptotic expressions for the "genuine" twist-4 DAs, and the corresponding Wandzura-Wilczek corrections up to twist 6 as given in Ref. [22]. The results are shown in Fig. 2. The shaded areas correspond to the uncertainty in the lattice values as given in Table I. In the region $Q^2 > m_{N^*}^2 \sim 2$ GeV^2, where the light-cone expansion may be expected to converge, the results appear to be in good agreement with the experimental data [29–33].

Discussion and conclusions.—In this work we suggest to calculate transition form factors for nucleon resonances at intermediate momentum transfer, combining the constraints on DAs from a lattice calculation with LCSRs. This approach seems to be especially promising for $N^*(1535)$, the parity partner of the nucleon, because of the relative ease to separate the states of opposite parity on the lattice. Calculations with smaller pion masses and on larger lattices would remove a major source of uncertainties which is due to the chiral extrapolation, although crossing the threshold for the decay $N^* \to N\pi$ may prove to be nontrivial. Future calculations will also include dynamical strange quarks, although we do not expect this effect to be significant.

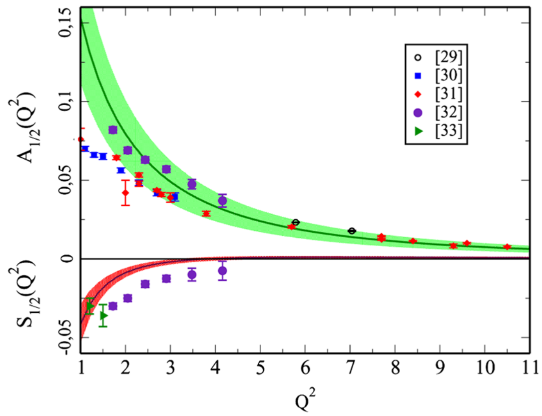

FIG. 2 (color online). The LCSR calculation for the helicity amplitudes $A_{1/2}(Q^2)$ and $S_{1/2}(Q^2)$ for the electroproduction of the $N^*(1535)$ resonance using the lattice results from Table I for the lowest moments of the $N^*(1535)$ DAs. The curves are obtained using the central values of the lattice parameters, and the shaded areas show the corresponding uncertainty.

In order to match the accuracy of the lattice results, the LCSR calculations of baryon form factors will have to be advanced to include radiative corrections in next-to-leading order, as it has become standard for meson decays. For the first effort in this direction, see [34]. In addition, one needs a technique for the resummation of "kinematic" corrections to the sum rules that are due to the large masses of the resonances.

The ultimate accuracy of our approach is limited by the intrinsic uncertainty of the duality approximation which is central in the LCSR framework. Based on the large experience from similar calculations of electromagnetic form factors and heavy meson decays, this irreducible uncertainty can be of the order of 20% in a broad range of momentum transfers starting from $Q^2 \sim m_{N^*}^2$, where the operator product expansion for the relevant correlation functions becomes applicable.

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