Observational consequences of non-minimally coupled chaotic inflation

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where I want to take you ...

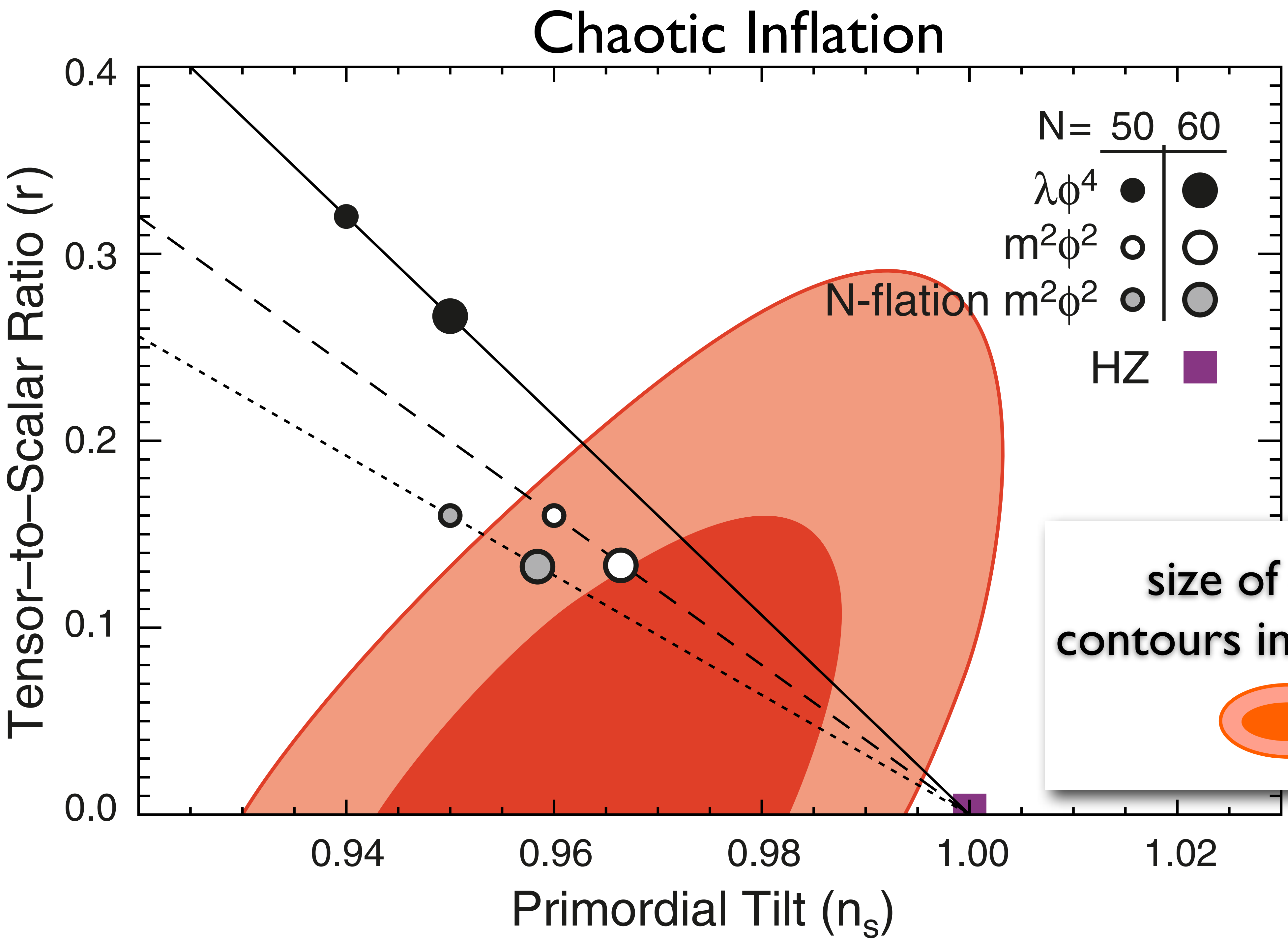
- the start:
  - inflation - short summary
  - why large-field inflation (Φ moves more than M_P)?

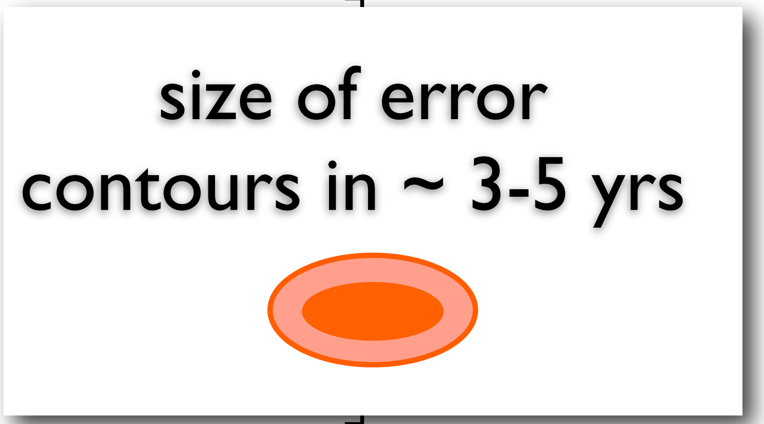
- review of general chaotic inflation in supergravity

- summary of Higgs inflation

- non-minimal chaotic inflation in supergravity
  - short-cutting into Jordan frame supergravity
  - scalar potential - 2 examples
  - parameter scan -- predictions

present status: $\text{WMAP 7yr} + \text{BAO} + H_0$





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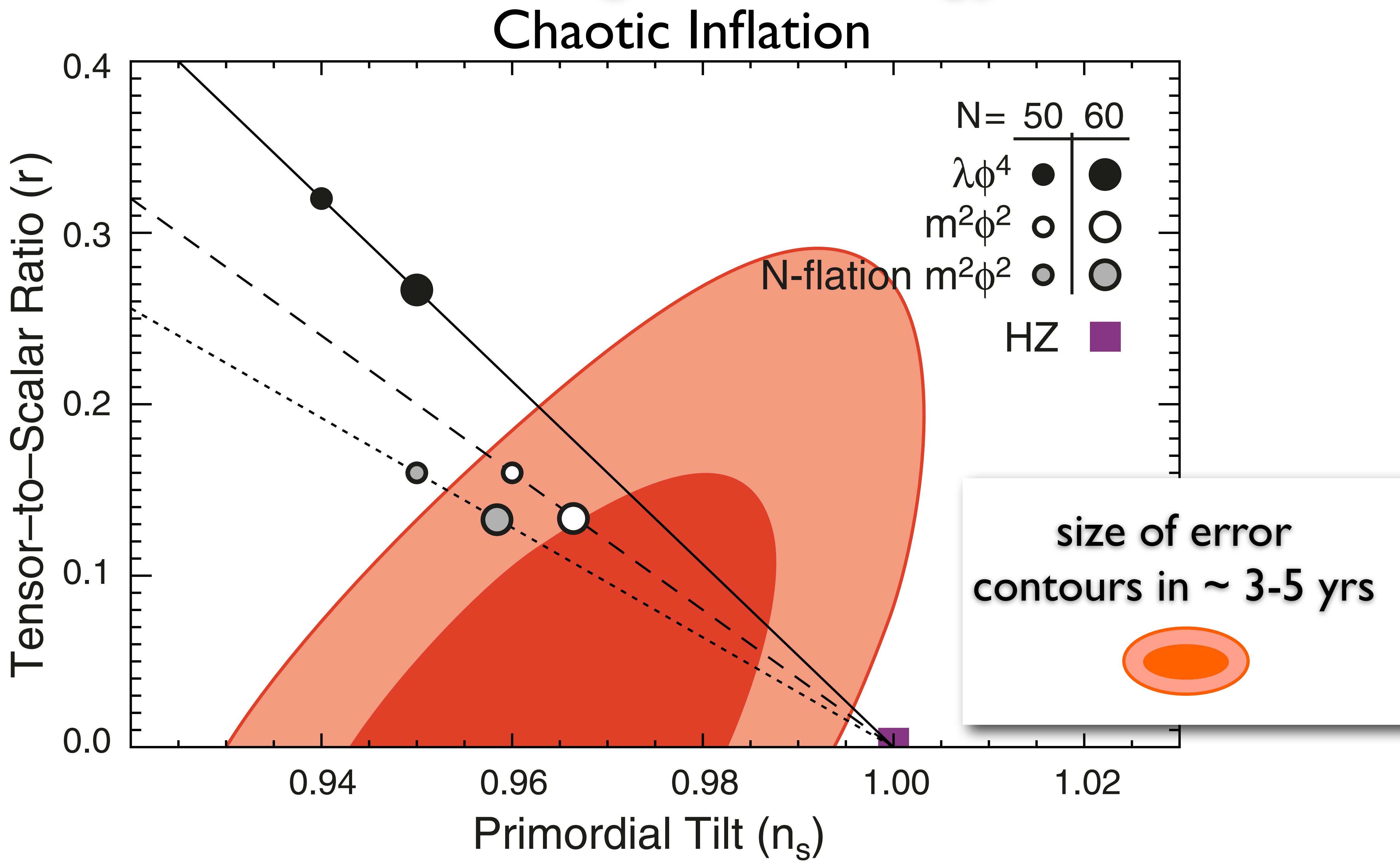
Planck & BICEP2 taking data, Keck Array ('10...)

SPIDER, Clover, QUIET-II, EBEX, PolarBEAR ...

We live in the Golden Age of cosmology !



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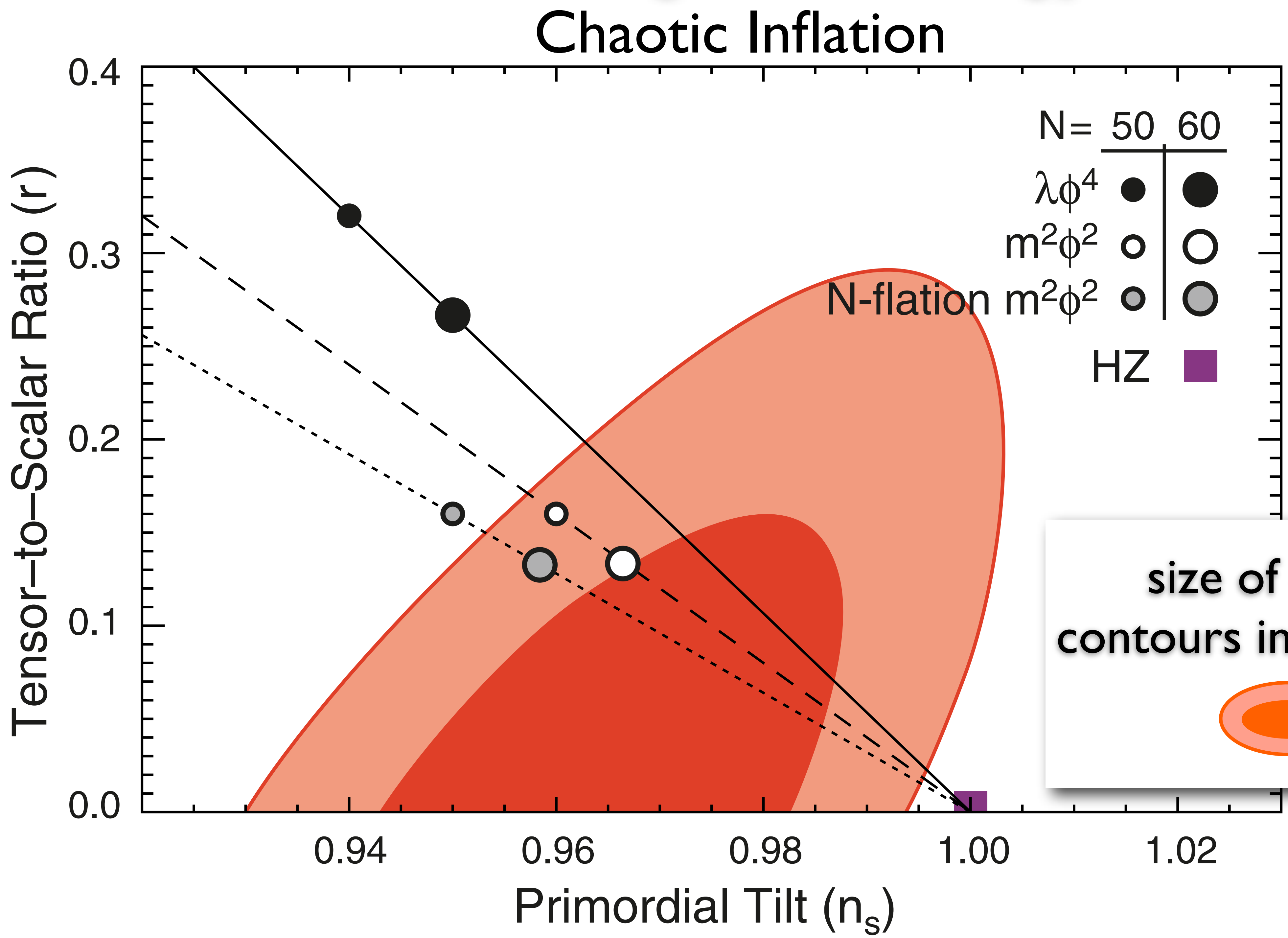

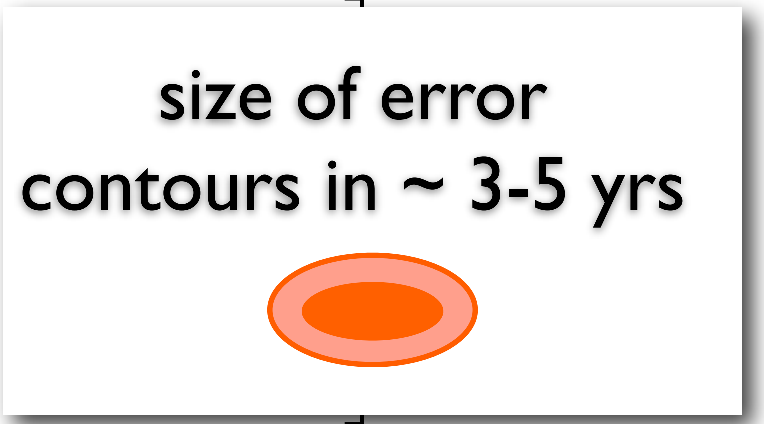

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scale factor grows exponentially : $a \sim e^{Ht}$ if : $\ddot{\phi} \ll \dot{\phi}$

$$\Rightarrow \quad \epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \quad , \quad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \simeq \frac{V''}{V} \ll 1$$

with the Hubble parameter $H^2 = \frac{\dot{a}^2}{a^2} \simeq const. \sim V$

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window to GUT scale & 'smoking gun': alternatives (e.g. ekpyrosis) have no tensors

- *but:* if field excursion sub-Planckian, no measurable gravity waves: [Lyth '97]

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_S} = 16\epsilon \leq 0.003 \left(\frac{50}{N_e}\right)^2 \left(\frac{\Delta\phi}{M_\mathrm{P}}\right)^2$$

why strings?

- *large field* model of inflation, i.e. "chaotic inflation"

$$\Delta\phi > M_P \quad \Rightarrow \quad r > 0.01$$

- with control of ε & η over a super-Planckian field distance - avoid generic dim ≥ 6 operators:

$$\delta V \sim V(\phi)\frac{(\phi-\phi_0)^2}{M_{\rm P}^2}$$

need UV-complete theory: e.g. strings

- idea: arrange for approximate shift symmetry of ϕ , broken only by the inflaton potential itself [Linde '83]

chaotic inflation in supergravity ...
... you can see the shift symmetry ....

supergravity

- 4D limit of string theory often is a N=1 supergravity

scalars ϕ_i with fermion partners ψ_i $\longrightarrow$ chiral superfields Φ_i

graviton in a multiplet together with the gravitino Ψ_μ

- action in superspace:

$$\begin{aligned} S &= -\int d^4x d^2\theta 2\mathcal{E} \left[\frac{3}{8}(\bar{\mathcal{D}}^2 - 8R)e^{-\frac{1}{3}K(\bar{\Phi}_i,\Phi_i)} + W(\Phi_i)\right] + h.c. \\ &= -\int d^4x d^2\theta 2\mathcal{E} \left[3R - \frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)K(\bar{\Phi}_i,\Phi_i) + \ldots + W(\Phi_i)\right] + h.c. \end{aligned}$$

$$\Rightarrow \quad S \sim -\int d^4x\sqrt{-g}\left[R + \frac{\partial_2 K}{\partial\bar{\Phi}_i\partial\Phi_j}\partial_\mu\bar{\Phi}_i\partial^\mu\Phi_j - V(\Phi_i)\right] + fermions$$

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scalar potential in supergravity ...

- scalar potential $V(\Phi_i)$ determined by Kahler potential K and superpotential W:

$$V(\phi_i) \quad = \quad e^K \cdot \left[(K_{i\bar{\jmath}})^{-1} D_i W D_{\bar{\jmath}} \bar{W} - 3|W|^2 \right]$$

$$\text{with} \quad D_i W = \partial_i W + W \partial_i K$$

- canonical kinetic terms for:

$$K = \sum_i \bar{\Phi}_i \Phi_i \quad \text{or} \quad K = \sum_i \frac{1}{2} (\bar{\Phi}_i + \Phi_i)^2$$

chaotic inflation in supergravity ...

- the simplest model of chaotic inflation in supergravity [Kawaski, Yamaguchi, Yanagida, '00]

$$K = \frac{1}{2}(\bar{\Phi} + \Phi)^2 + \bar{S}S \qquad W = mS\Phi$$

- shift symmetry $\quad \Phi \rightarrow \Phi + i\,a$

- all fields except the inflaton Im(Φ) stabilized at zero VEV

$$\langle \mathrm{Re}\,\Phi \rangle = \langle S \rangle = 0 \qquad m_S^2 = m_{\mathrm{Re}\,\Phi}^2 = m^2$$

$$\langle W \rangle = 0 \quad , \quad V(\phi) = m^2\phi^2 \quad , \quad \phi \equiv \mathrm{Im}\,\Phi$$

general chaotic inflation in supergravity ...

- general chaotic inflation in supergravity [Kallosh & Linde & Rube '10] - just change

$$W = S f(\Phi)$$

- general large field inflaton potential

$$\langle \mathrm{Re}\, \Phi \rangle = \langle S \rangle = 0 \qquad\qquad V(\phi) = |f(\phi)|^2$$

- however, Re(Φ) , S sometimes tachyonic at zero VEV $\Rightarrow$ change K [Kawaski, Yamaguchi, Yanagida, '00 ; Lee '10]

$$K = \frac{1}{2}(\bar{\Phi} + \Phi)^2 + \bar{S}S - c_1(\bar{S}S)^2 - c_2\, \bar{S}S(\bar{\Phi} + \Phi)^2$$

general chaotic inflation in supergravity ...

- gives contributions to the mass of S and Re(Φ) for sufficient c_1 , $c_2 > 1$

$$(K_{\bar{S}S})^{-1} = 1 + 4c_1\bar{S}S + c_2(\mathrm{Re}\,\Phi)^2 + \dots$$

$$\Rightarrow \quad m_S^2 \sim -m^2 + c_1\mathcal{O}(1)|f(\phi)|^2$$

$$m_{\mathrm{Re}\,\Phi}^2 \sim -m^2 + c_2\mathcal{O}(1)|f(\phi)|^2$$

- so we get

$$V(\phi) = |f(\phi)|^2$$

with

$$\langle \mathrm{Re}\,\Phi \rangle = \langle S \rangle = \langle W \rangle = 0$$

$$m_S^2 \; , \; m_{\mathrm{Re}\,\Phi}^2 \gg |f(\phi)|^2 = V(\phi) \sim H^2$$

Higgs inflation ...

Higgs inflation ...[Shaposhnikov & Bezrukov, '06]

essential idea: consider a non-minimally coupled scalar field with Higgs potential - called Jordan frame gravity with frame function $\Omega^2(\phi)$

$$S = \int d^4x \sqrt{-g_J} \left[\Omega^2(\phi) R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V_J(\phi) \right] \quad , \quad V_J(\phi) = \lambda \left(\phi^2 - v^2 \right)^2$$

one can Weyl transform this into Einstein frame

$$g^J_{\mu\nu} \quad \rightarrow \quad g^E_{\mu\nu} = \Omega^2(\phi)\, g^J_{\mu\nu} \quad , \quad \Omega^2(\phi) = (1 + \xi\phi^2)$$

$$\Rightarrow \quad S = \int d^4x \sqrt{-g_E} \left[R + Z^{-1}(\phi) \partial_\mu \phi \partial^\mu \phi - V_E(\phi) \right]$$

$$\text{with} \qquad Z^{-1}(\phi) = \frac{1 + \xi\phi^2 + 6\xi^2\phi^2}{(1 + \xi\phi^2)^2}$$

Higgs inflation ...[Shaposhnikov & Bezrukov, '06]

the scalar potential in Einstein frame is

$$V_E(\phi) = \frac{1}{\Omega^4(\phi)} V_J(\phi) = \frac{\lambda}{4} \frac{(\phi^2 - v^2)^2}{(1 + \xi\phi^2)^2} \quad \longrightarrow \quad \frac{\lambda}{4\xi^2} = \text{const.}$$

$$\text{for } \phi \gg \frac{1}{\sqrt{\xi}} \gg v$$

$Z^{-1}(\phi)$ defines a canonically normalized scalar

$$\frac{d\varphi}{d\phi} = \sqrt{Z^{-1}(\phi)} \quad \Rightarrow \quad \phi \sim \frac{1}{\sqrt{\xi}} e^{\varphi/\sqrt{6}}$$

$$\Rightarrow \quad V_E(\varphi) \simeq \frac{\lambda}{4\xi^2} \left(1 - 2e^{-\frac{2}{\sqrt{6}}\varphi}\right) \quad \Rightarrow \quad \begin{cases} n_s = 1 - \frac{2}{N} = 0.967 \\ r = 16\epsilon = \frac{12}{N^2} = 0.003 \end{cases}$$

Higgs inflation ...[Shaposhnikov & Bezrukov, '06]

COBE normalization ($\delta\rho/\rho \sim 10^{-5}$ at $N = 60$ efolds)
can be satisfied for electro-weak parameters

$$\lambda = \mathcal{O}(1) \quad , \quad v \sim 100\,\mathrm{GeV} \quad \mathrm{IF} \quad \xi \sim 5 \times 10^4$$

quantum consistency still under discussion ...

[Lerner & McDonald '09, Barbon & Espinosa '09, Burgess, Lee & Trott '10, Hertzberg '10, Giudice & Lee '10, more ...]

- SM Higgs has couples with would-be Goldstone bosons: gravity induces UV strong coupling scale

$$\mathcal{L}_E \supset (\partial\vec{\phi})^2 + \frac{\xi^2}{M_P^2}(\vec{\phi}\cdot\partial\vec{\phi})^2 + \ldots \quad \Rightarrow \quad \Lambda_{UV} \sim M_P/\xi \ll M_P/\sqrt{\xi} \sim \phi_{end}$$

- transformation to Einstein frame does not help: "unitary gauge" - strong coupling problem shifted into gauge field sector of the SM; exception: singlet

non-minimally coupled inflation in supergravity ...

non-minimal inflation in supergravity ...

[Ferrara, Kallosh, Linde, Marrani & Van Proeyen '10]

we can port non-minimal inflation into supergravity by identifying

$$\Omega^2(\bar{\Phi}, \Phi) = -3e^{-\frac{1}{3}K(\bar{\Phi},\Phi)}$$

$$-\int d^4x d^2\theta\, \mathcal{E}\, \frac{3}{4}(\bar{\mathcal{D}}^2 - 8R)e^{-\frac{1}{3}K(\bar{\Phi},\Phi)} \quad \rightarrow \quad \int d^4x d^2\theta\, \mathcal{E}\, \frac{1}{4}\Omega^2(\bar{\Phi}, \Phi)(\bar{\mathcal{D}}^2 - 8R)$$

$$\rightarrow \quad \int d^4x \sqrt{-g_J}\Omega^2(\phi)R$$

canonical kinetic terms in Jordan frame

$$\Rightarrow \quad \Omega^2(\Phi) = -3 + \bar{\Phi}\Phi + \big(J(\Phi) + h.c.\big)$$

non-minimal inflation in supergravity ...

[Ferrara, Kallosh, Linde, Marrani & Van Proeyen '10]

2 cases

$$J(\Phi) = 0 \quad \Rightarrow \quad \Omega^2(\phi) R \to \frac{1}{6}\phi^2 R \quad \text{"conformal coupling"}$$

$$J(\Phi) = -\frac{3}{4}\chi\,\Phi^2 \quad \Rightarrow \quad \Omega^2(\phi) R \to \left(1+\xi(\mathrm{Re}\,\Phi)^2\right) R \quad \text{"non} - \text{minimal coupling"}$$

$$\text{with} \quad \chi = \pm\frac{2}{3}(1+6\xi)$$

we can rewrite

$$\Omega^2 = -3+\bar\Phi\Phi+\left(J(\Phi)+h.c.\right) = -3-\frac{1}{4}\left(1+\frac{3}{2}\chi\right)(\bar\Phi-\Phi)^2+\frac{1}{4}\left(1-\frac{3}{2}\chi\right)(\bar\Phi+\Phi)^2$$

$$\chi = \pm\frac{2}{3} \leftrightarrow \xi = 0 \quad \text{recovers minimal coupling and shift symmetry !}$$

non-minimal chaotic inflation in supergravity ...

[Kallosh, Linde & Rube '10 ; Linde, Noorbala & AW '11]

we can now go to Einstein frame, and forget about Jordan frame by taking

$$\begin{aligned} K &= -3\ln\left(-\Omega^2/3\right) = -3\ln\left[1 - \frac{1}{3}\bar{\Phi}\Phi + \frac{1}{4}\chi\left(\bar{\Phi}^2 + \Phi^2\right)\right] \\ &= -3\ln\left[1 + \frac{1}{12}\left(1 + \frac{3}{2}\chi\right)(\bar{\Phi} - \Phi)^2 - \frac{1}{12}\left(1 - \frac{3}{2}\chi\right)(\bar{\Phi} + \Phi)^2\right] \end{aligned}$$

if we now add back *S* and the quartic terms in *K*

$$K = -3\ln\left[1 - \frac{1}{3}\bar{\Phi}\Phi - \frac{1}{3}\bar{S}S + \frac{1}{4}\chi\left(\bar{\Phi}^2 + \Phi^2\right) + c_1(\bar{S}S)^2\right]$$

then with $W = S\, f(\Phi)$ we are back with generalized

non-minimal chaotic inflation in supergravity !

non-minimal chaotic inflation in supergravity ...

[Linde, Noorbala & AW '11]

look at 2 examples, get n_s and r as function of v , ξ

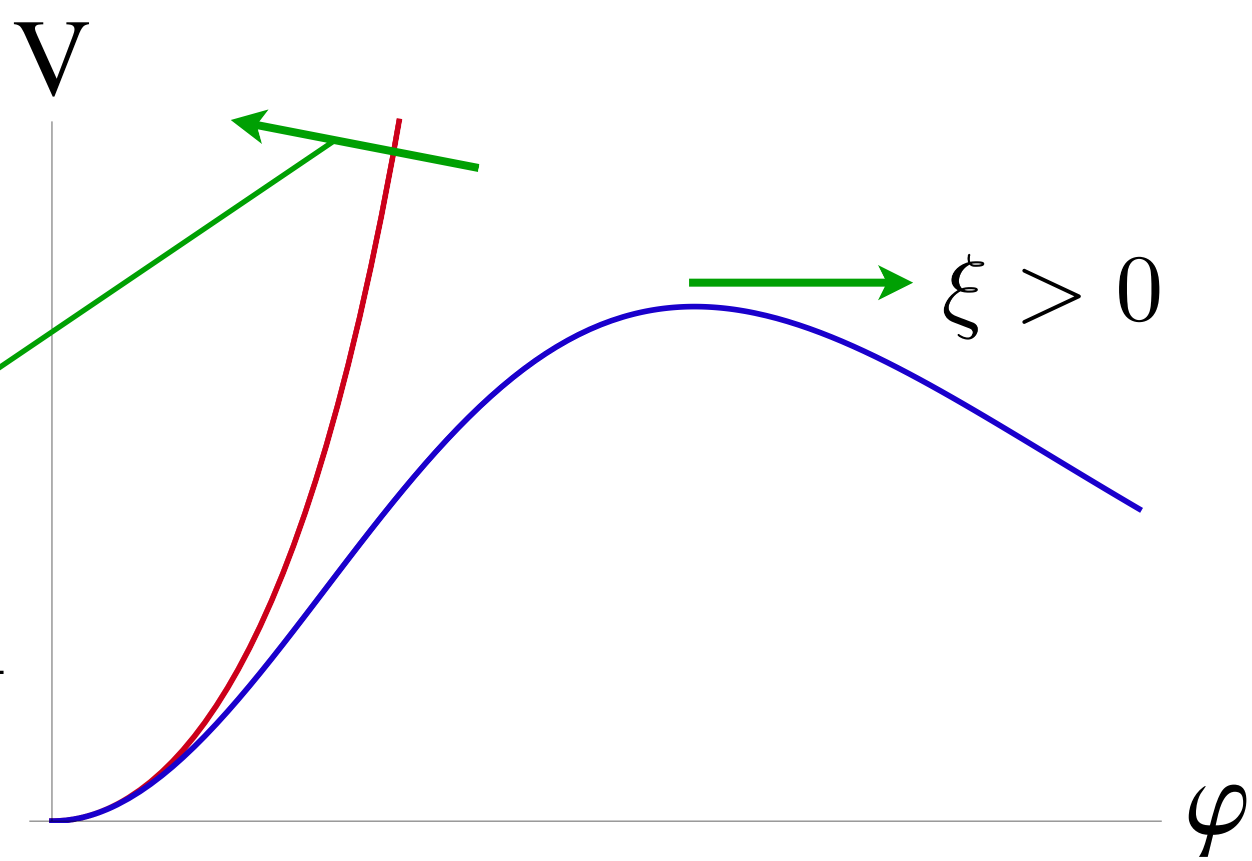
$$f(\Phi) = m\Phi \quad \rightarrow \quad V_E(\phi) = \frac{m^2\phi^2}{(1+\xi\phi^2)^2}$$

$$f(\Phi) = \frac{\lambda}{4}(\Phi^2 - v^2) \quad \rightarrow \quad V_E(\phi) = \frac{\frac{\lambda^2}{4}(\phi^2 - v^2)^2}{(1+\xi\phi^2)^2}$$

quadratic case:

$\xi < 0$, $|\xi|\phi^2 < 1$

singularity at $\phi = 1/\sqrt{|\xi|}$



non-minimal chaotic inflation in supergravity ...

[Linde, Noorbala & AW '11]

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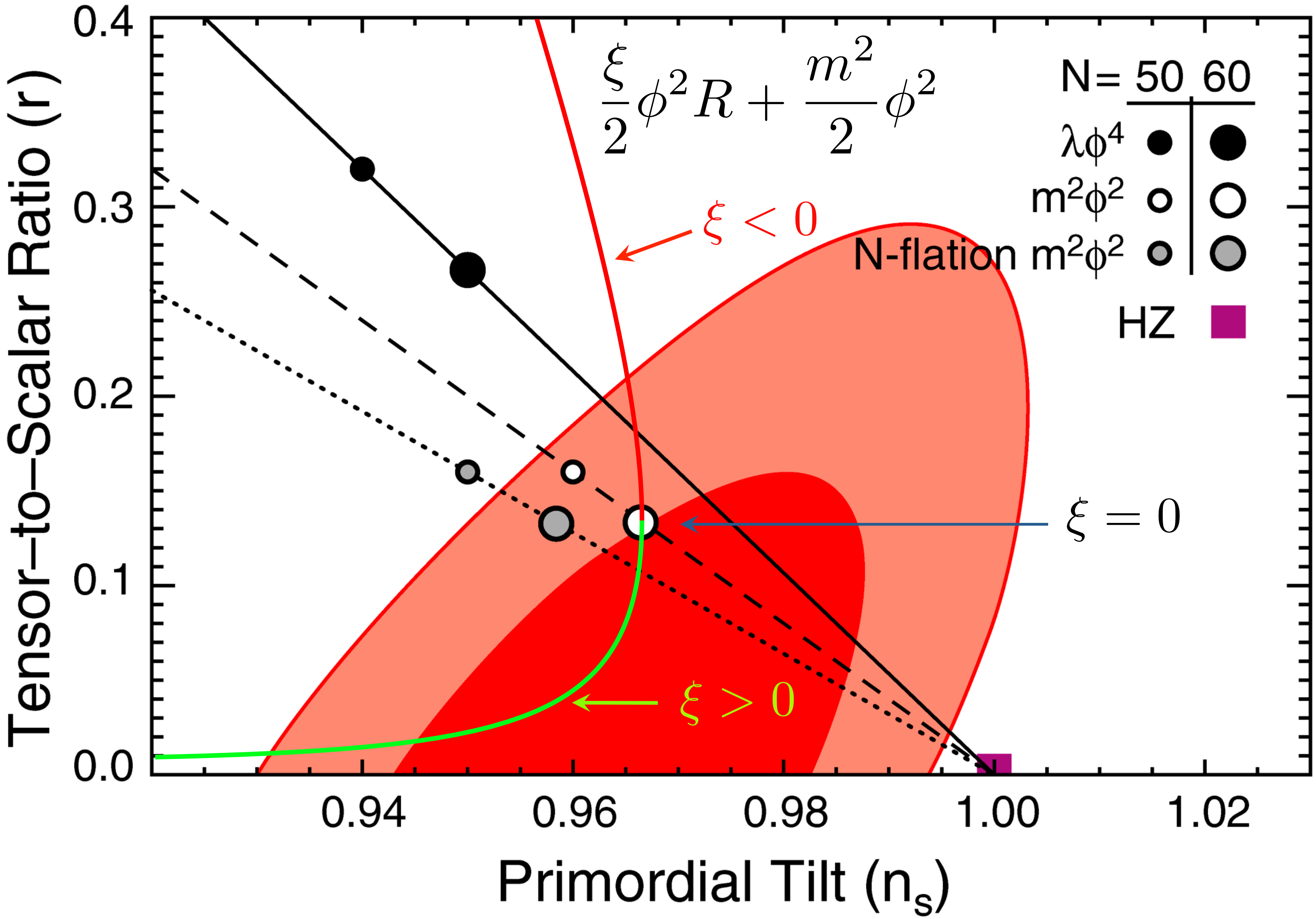
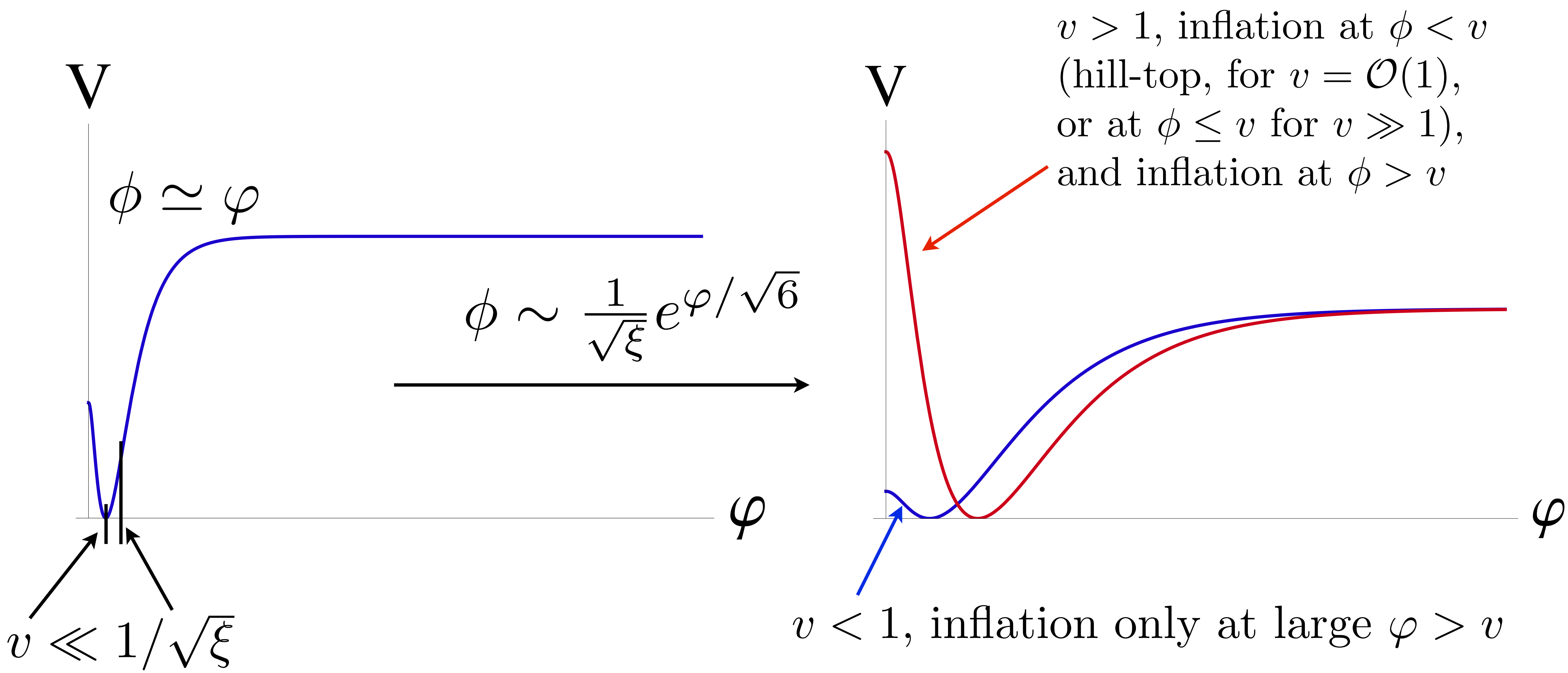


non-minimal chaotic inflation in supergravity ...

[Linde, Noorbala & AW '11]

quartic case I, II: $\phi < v$ (I) and $\phi > v$ (II), with

$\xi > 0$, like Higgs inflation, but we scan both ξ and v



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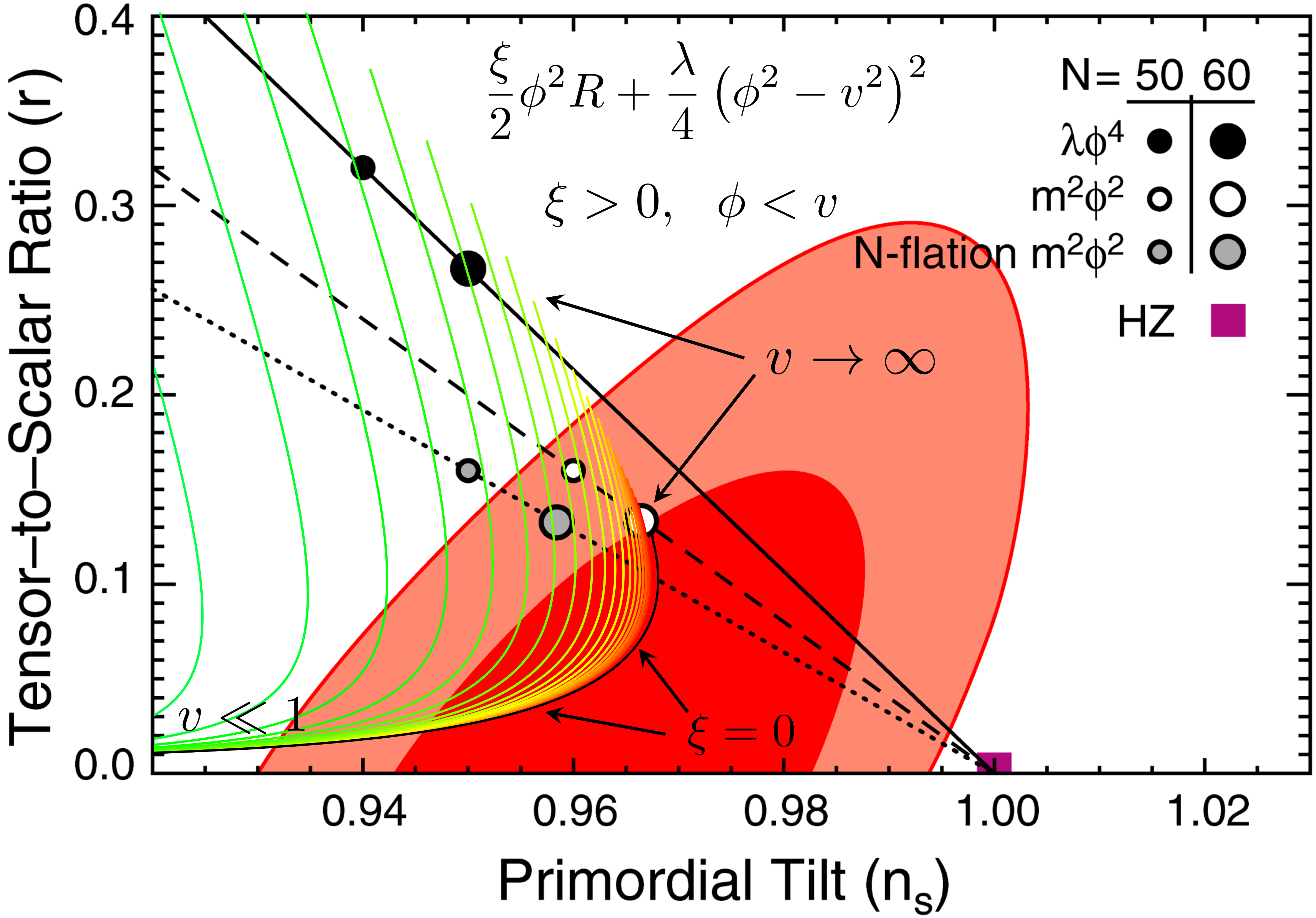
[Linde, Noorbala & AW '11]

quartic case I: $\xi > 0 \ , \phi < v$



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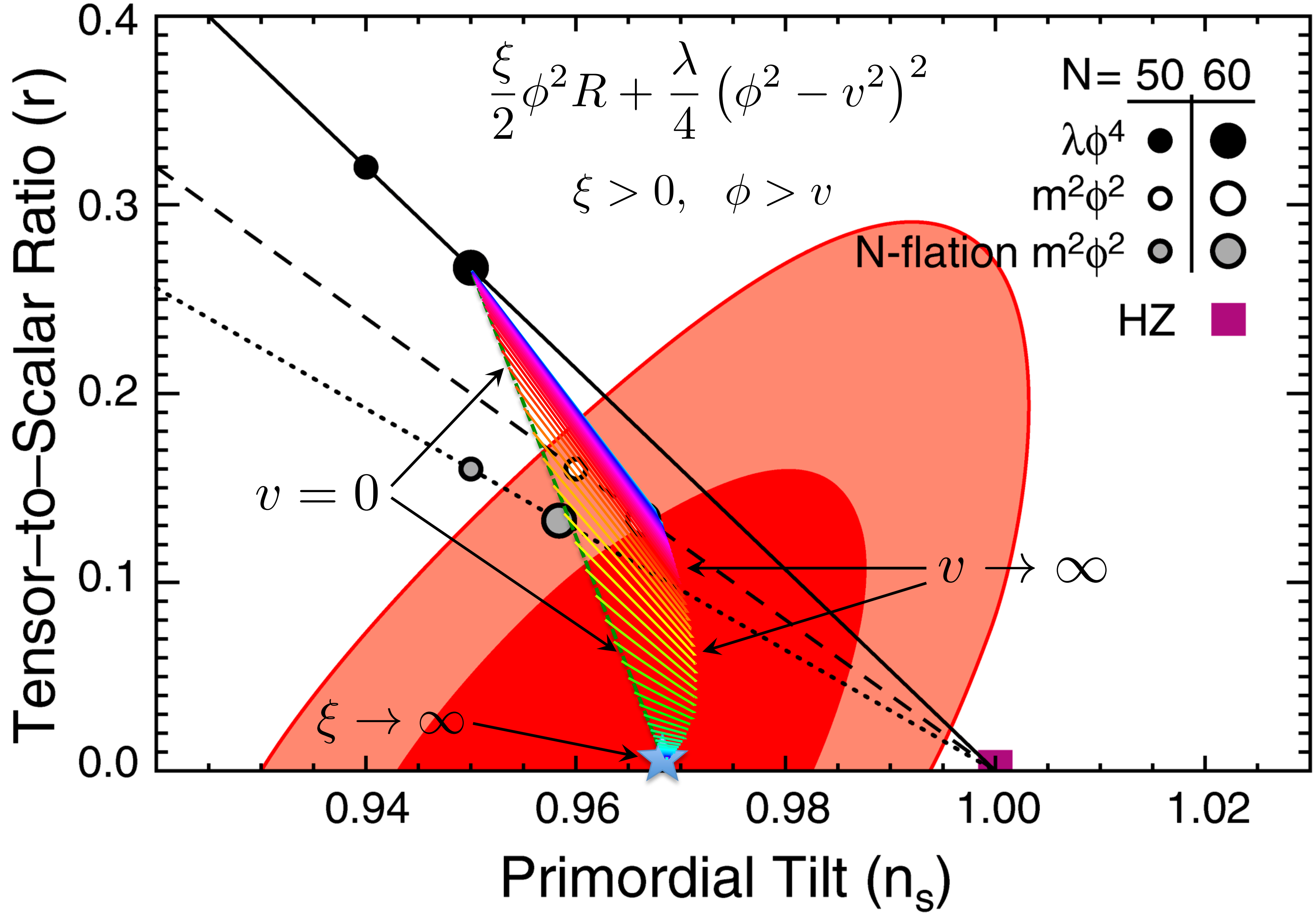
quartic case II: $\xi > 0 \;,\; \phi > v$



non-minimal chaotic inflation in supergravity ...

[Linde, Noorbala & AW '11]

quartic case III: $\phi < v$ and $v < \phi < 1/\sqrt{|\xi|}$, with

$\xi < 0$, again singularity at $\phi = 1/\sqrt{|\xi|}$

unexpected coincidence - take: $v^2|\xi| < 1$, but $v^2|\xi| \to 1$

then in terms of the correct canonical inflaton:

$$V \simeq \frac{\lambda}{4\xi^2}\left(1 - 2e^{-\frac{2}{\sqrt{6}}\tilde{\varphi}}\right)$$

where $\tilde{\varphi} = \varphi_0(v) - \varphi$ and again

$$\begin{cases} n_s = 1 - \frac{2}{N} = 0.967 \\ r = 16\epsilon = \frac{12}{N^2} = 0.003 \end{cases}$$ for $|\xi|v^2 \to 1$

mirror-Higgs inflation

non-minimal chaotic inflation in supergravity ...

[Linde, Noorbala & AW '11]

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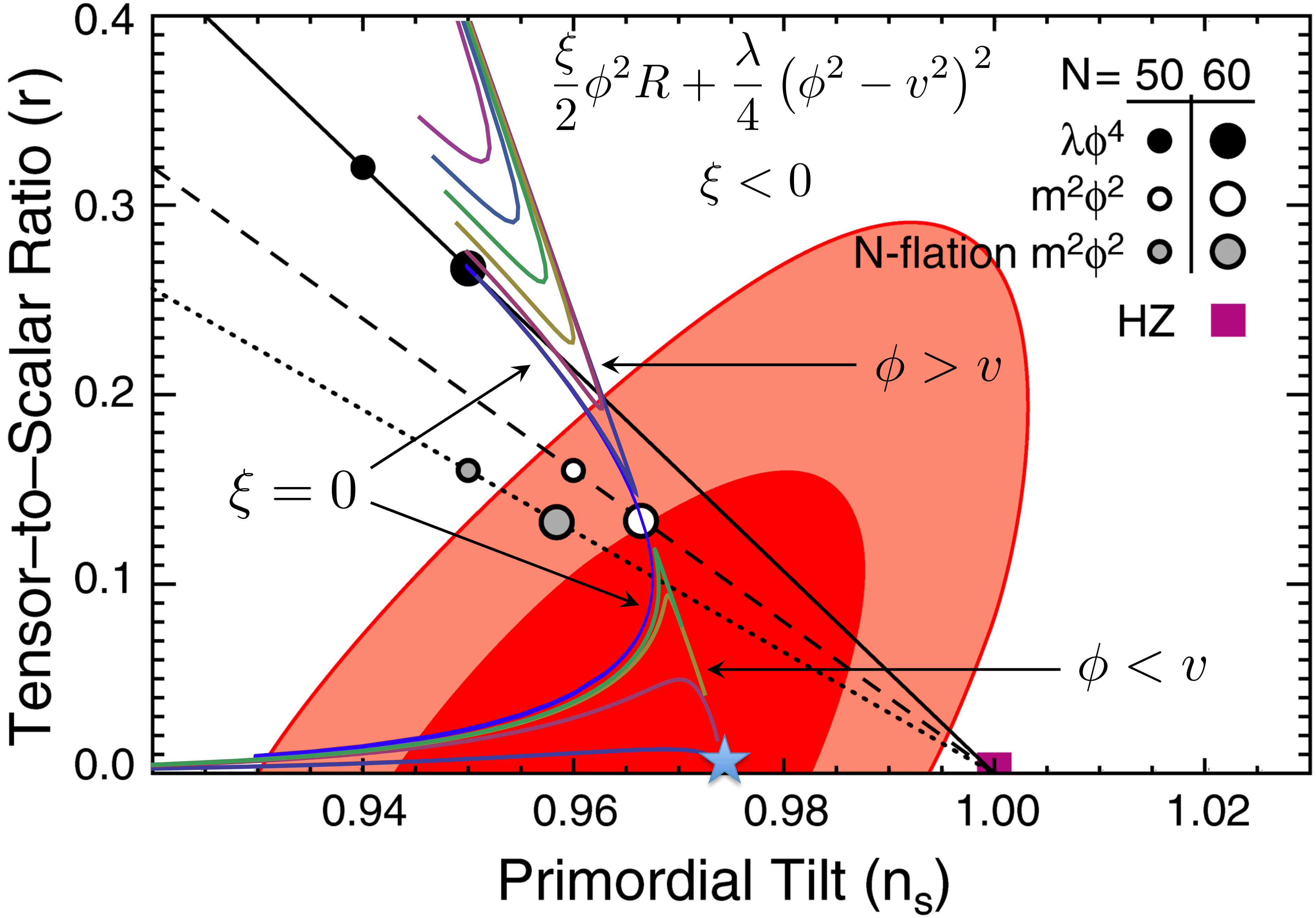

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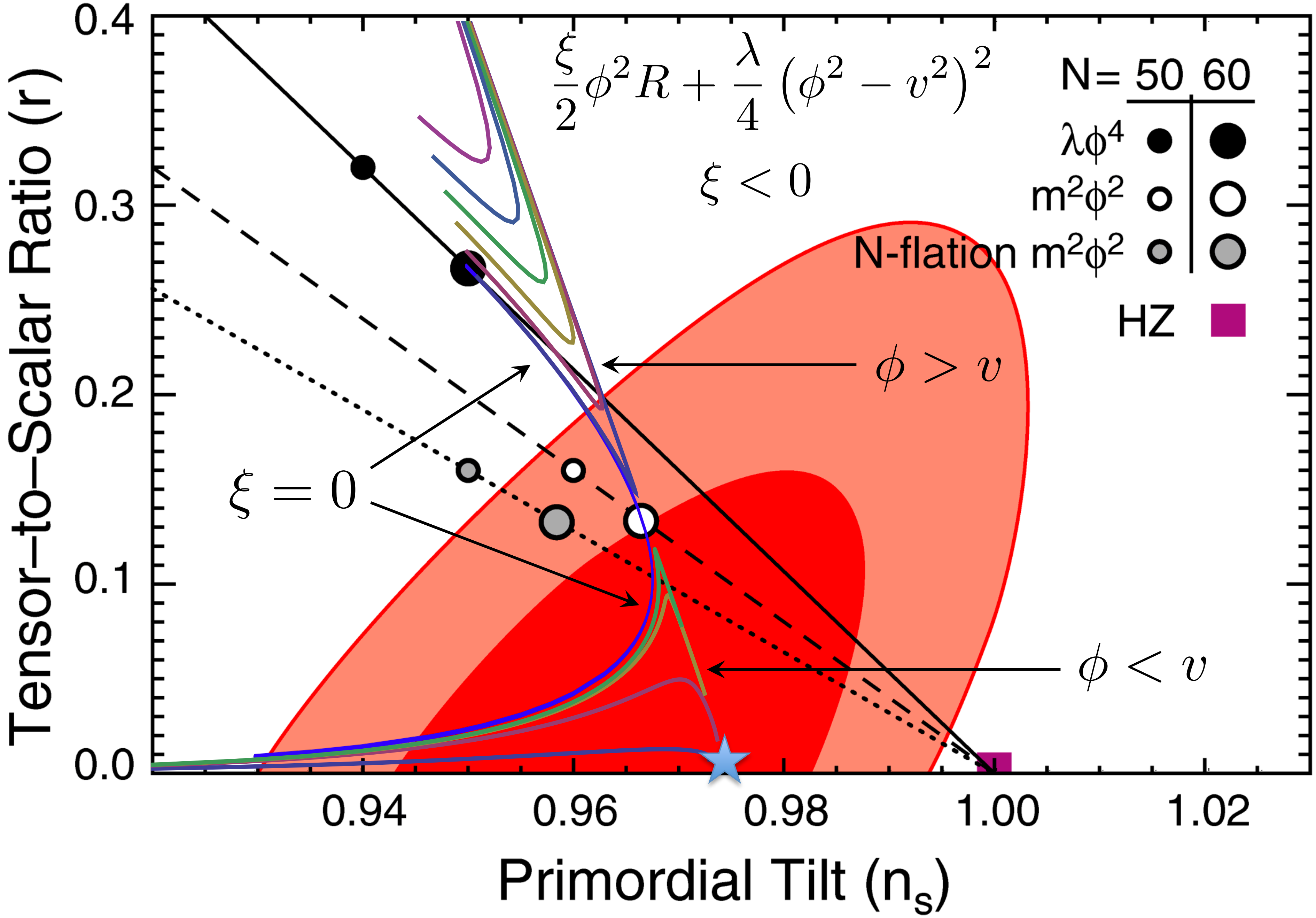

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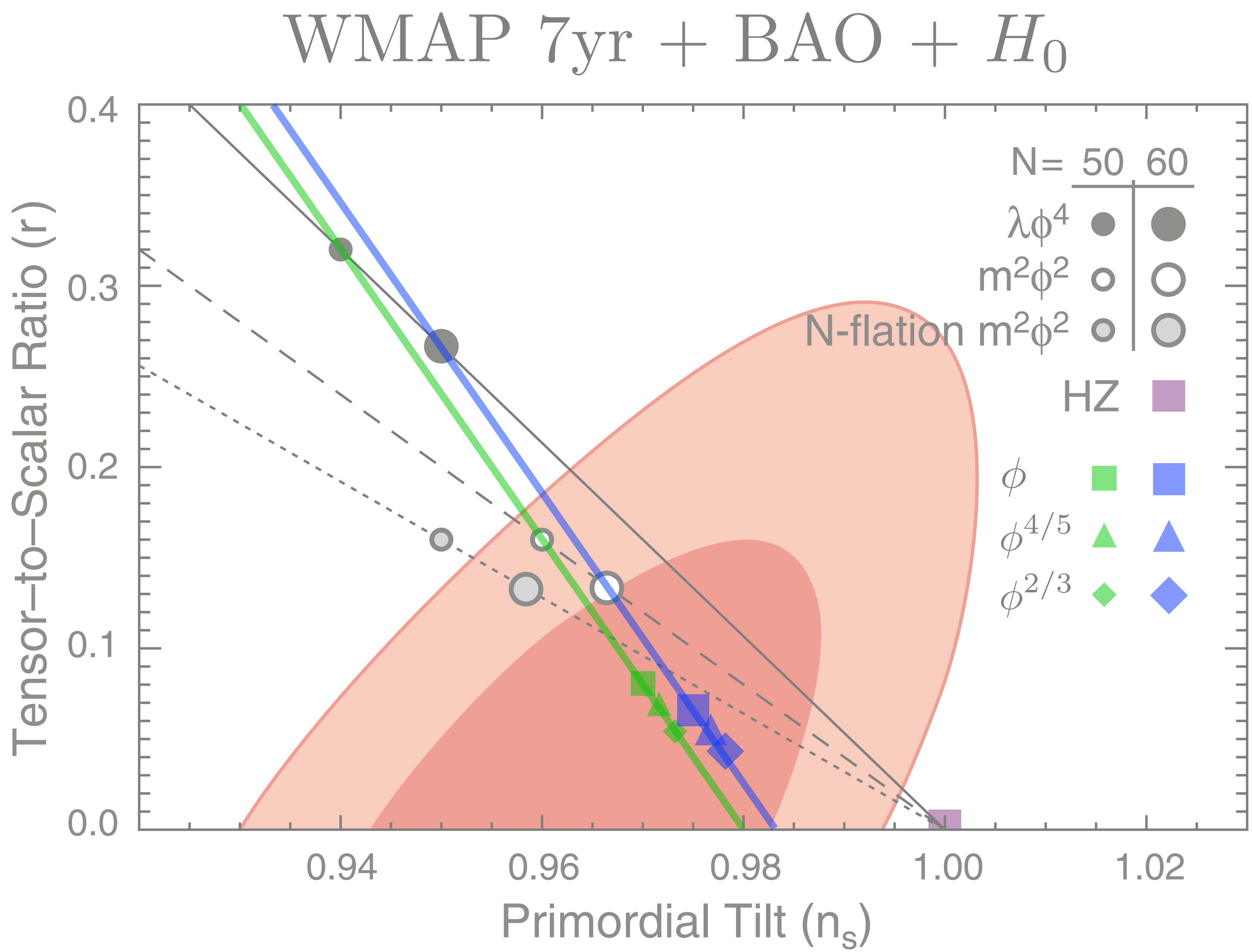

summary ...

- non-minimally coupled inflation (a la Higgs inflation) can be embedded in a general class of chaotic inflation models in supergravity based on a shift symmetry; this includes large-field inflation with the observational boon of gravity waves

- a non-minimal coupling provides a modification to the scalar potential, without explicitly adding terms

- scanning the non-minimal coupling can render a *quartic* potential *consistent* with observational data, and cover significant parts of the patch of the (n_S,r)-plane allowed by the WMAP 7-year data