New results for $t\bar{t}$ production at hadron colliders

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We present new theoretical predictions for the $t\bar{t}$ production cross section at NNLO at the Tevatron and the LHC. We discuss the scale uncertainty and the errors due to the parton distribution functions (PDFs). For the LHC, we present a fit formula for the pair production cross section as a function of the center of mass energy and we provide predictions for the pair production cross section of a hypothetical heavy fourth generation quark t'.

1 $t\bar{t}$ - Production at Tevatron and LHC

The experimental measurements of the top mass m_t and the $t\bar{t}$ production cross section have reached a relative accuracy of 0.75% [2] and 9% [3], respectively. Therefore it is necessary to provide improved theoretical predictions for the total cross section at the Tevatron and the LHC in perturbative QCD.

The total hadronic cross section for $t\bar{t}$ production depends on the top mass m_t, the center of mass energy $s = E^2$, the factorisation scale μ_f, the renormalization scale μ_r, and the PDF set and it is given by

$$\sigma(s, m_t, \mu_r, \mu_f) = \sum_{i,j=g,q,\bar{q}} f_{i/p}(\mu_f^2) \otimes f_{j/p}(\mu_f^2) \otimes \hat{\sigma}(m_t, \mu_r, \mu_f), \qquad (1)$$

where $f_{i/p}$ are the proton PDFs. In the following, we discuss these dependencies.

We have updated the results from Refs. [4, 5] as follows. To obtain more reliable estimates of the scale uncertainty we have used the full dependence on μ_r *and* μ_f. We have performed a consistent singlet - octet - decomposition when matching our NNLO threshold expansion at NLO. Further corrections (electroweak contributions [6, 7, 8], QCD bound state effects near threshold [9], and new parton channels qq, $\bar{q}\bar{q}$, and $q_i\bar{q}_j$ for unlike quarks opening at NNLO [10, 11]) are generally small and have been estimated.

As a new result we have studied the dependence of the $t\bar{t}$ production cross section on the definition of the mass parameter. We have used the $\overline{\text{MS}}$ mass scheme as an alternative mass description exploiting the conversion relation between the pole mass m_t and the $\overline{\text{MS}}$ mass $m(\mu_r)$ [12]. We find that the convergence of the perturbation expansion through NNLO is improved using the $\overline{\text{MS}}$ mass. This expansion has a considerably reduced scale dependence even at NLO. The NLO and NNLO corrections in the $\overline{\text{MS}}$ scheme are much smaller than the corresponding corrections in the pole mass scheme. Therefore, we find good properties of convergence of the perturbation series. From the measured $t\bar{t}$ cross section at the Tevatron [13] we derive a $\overline{\text{MS}}$ mass $\overline{m} = 160.0^{+3.3}_{-3.2}\,\text{GeV}$, which corresponds to a pole mass of $168.9^{+3.5}_{-3.4}\,\text{GeV}$. More details of this analysis are presented in [14]. Throughout this article,

*Talk given by U. Langenfeld at the XVII International Workshop on Deep-Inelastic Scattering and Related Subjects, DIS 2009, 26-30 April 2009, Madrid, see Ref. [1]

we have chosen the PDF set CTEQ6.6 [15]. In Ref. [14], results for the PDF set MSTW NNLO 2008 [16] can be found. The top mass is the pole mass and is set to the most recent value $m_t = 173\,\mathrm{GeV}$ [2] if not otherwise stated.

We have analysed the dependence of the cross section on the renormalisation *and* factorisation scale. In Fig. 1, we display the result for the Tevatron and the LHC. At the Tevatron, the gradient is nearly parallel to the diagonal, and we find errors of -5% at $(\mu_f, \mu_r) = (m_t/2, m_t/2)$ and $+3\%$ at $(2m_t, 2m_t)$. Likewise for the LHC, the scale uncertainty is about 1% at about $(2m_t, m_t/2)$ and -4% at $(2m_t, 2m_t)$. Note that in the case of the LHC, the cross section is not a monotonically decreasing function if $\mu_r = \mu_f$ as it is in the case of the Tevatron, see Ref. [14] for details.

In Figs. 2 and 3, we show the mass dependence of the total hadronic cross section for both colliders including the scale uncertainty for $\mu_r = \mu_f \equiv \mu = m_t/2$ and $\mu = 2m_t$.

The pure PDF error $\Delta\mathcal{O}$ is given by

$$\Delta\mathcal{O} = \sqrt{\frac{1}{2} \sum_{k=1,n_{\mathrm{PDF}}} \left(\sigma_{k+} - \sigma_{k-}\right)^2}, \tag{2}$$

where $\Delta\mathcal{O}$ is determined from the variation of the cross section with respect to the parameters of the PDF fit. The PDF errors are added linearly. The result is presented in Fig. 4 for the Tevatron and in Fig. 5 for the LHC. We show for both colliders the NLO and NNLO cross sections together with their error bands. This demonstrates the shrinking of the total error for the NNLO cross section.

Having discussed scale uncertainty and PDF error, we present our prediction for the cross section at the Tevatron and the LHC. To obtain a more conservative error bound, we calculate the contribution of the scale uncertainty as

$$\min_{\mu_r,\mu_f \in [m_t/2, 2m_t]} \sigma(\mu_r, \mu_f) \leq \sigma(\mu_r, \mu_f) \leq \max_{\mu_r,\mu_f \in [m_t/2, 2m_t]} \sigma(\mu_r, \mu_f). \tag{3}$$

For $t\bar{t}$ production at the Tevatron, this definition changes nothing, but for $t\bar{t}$ production at the LHC, the upper bound is shifted to larger values by a few per cent. See also Fig. 1 and the corresponding discussion. For the CTEQ6.6 PDF set and $m_t = 173\,\mathrm{GeV}$ (pole mass), we arrive at

$$\sigma(p\bar{p} \to t\bar{t}) = 7.34^{+0.23}_{-0.38}\,\mathrm{pb}\ @\ \text{Tevatron},$$

$$\sigma(pp \to t\bar{t}) = 874^{+14}_{-33}\,\mathrm{pb}\ @\ \text{LHC 14 TeV}.$$

For the LHC, we have calculated the total hadronic cross section as a function of the center of mass energy E for a value of $m_t = 172.5\,\mathrm{GeV}$ as used in ATLAS studies, see Fig. 6. We parametrize the result using the ansatz

$$\sigma(E,\mu) = a + bx + cx^2 + dx \log\left(\frac{E}{\sqrt{s}}\right) + ex \log^2\left(\frac{E}{\sqrt{s}}\right) + fx^2 \log\left(\frac{E}{\sqrt{s}}\right) + gx^2 \log^2\left(\frac{E}{\sqrt{s}}\right) \tag{4}$$

with $x = E/\mathrm{GeV}$ and $\sqrt{s} = 14\,\mathrm{TeV}$. The numerical values for the coefficients $a, \ldots, g$ can be found in Tab. 1 for $\mu = m_t$, $m_t/2$, and $2m_t$. The fit is valid for $3\,\mathrm{TeV} \leq E \leq 14\,\mathrm{TeV}$ and has an accuracy of better than 0.05% within this range. This ansatz is justified by general limits for cross sections at high energies and is consistent with unitarity. Parametrisations of the total cross section as a function of m_t can be found for various scenarios in Ref. [14].

	$a[\times 10]$	$b[\times 10^{-1}]$	$c[\times 10^{-5}]$	$d[\times 10^{-1}]$	$e[\times 10^{-2}]$	$f[\times 10^{-5}]$	$g[\times 10^{-6}]$
$\sigma(\mu = m_t)$	3.42553	−5.12699	4.09683	−2.46892	−3.93892	−1.75175	2.02029
$\sigma(\mu = m_t/2)$	3.20912	−4.85885	3.90541	−2.33781	−3.71957	−1.65930	1.88132
$\sigma(\mu = 2m_t)$	3.31748	−4.76706	3.82310	−2.31392	−3.73054	−1.60468	1.74661

Table 1: Numerical values of the coefficients (in pb) of Eq. 4 for $m_t = 172.5\,\text{GeV}$ and the PDF set CTEQ6.6.

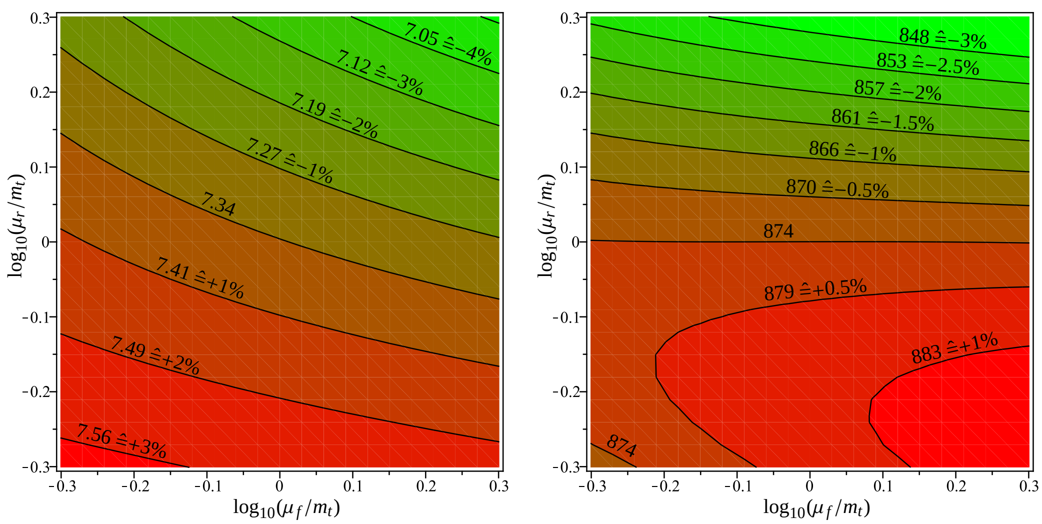

Figure 1: Contour lines of the total hadronic cross section from the independent variation of renormalization and factorization scale μ_r and μ_f for the Tevatron with $\sqrt{s} = 1.96\,\text{TeV}$ (left) and the LHC with $\sqrt{s} = 14\,\text{TeV}$ (right) with CTEQ6.6[15]. The range of μ_r and μ_f corresponds to $\mu_r, \mu_f \in [m_t/2, 2m_t]$.

2 Predictions for $t'\bar{t}'$ Production at Tevatron and LHC

We briefly present theoretical predictions for the pair production cross section of a hypothetical heavy fourth generation quark t' at the Tevatron and the LHC. In this calculation we have set the number of light flavours to $n_f = 6$. As one can see in Figs. 7 and 8 the cross section decreases very rapidly with increasing t' mass. At the Tevatron, we predict for a 200 GeV t' quark a cross section of $\sigma(p\bar{p} \to t'\bar{t}') = 3.3 \pm 0.3\,\text{pb}$ and for $m_{t'} = 500\,\text{GeV}$, we predict $\sigma(p\bar{p} \to t'\bar{t}') = 1.3^{+0.2}_{-0.4}\,\text{fb}$. Scale uncertainty and PDF error contribute roughly equal parts to the total error. At the LHC, we can test higher $m_{t'}$ masses. We predict for $m_{t'} = 500\,\text{GeV}$ a cross section of $\sigma(pp \to t'\bar{t}') = 4.0^{+0.5}_{-0.6}\,\text{pb}$ and for $m_{t'} = 2000\,\text{GeV}$, we have $\sigma(pp \to t'\bar{t}') = 0.27^{+0.8}_{-0.9}\,\text{fb}$. At the LHC, the PDF error is much larger than the scale uncertainty. Most $t'\bar{t}'$ pairs are produced via the gg channel, the PDF error of the gluon PDF is large in the relevant kinematic region, i.e. at high x. The hadronic cross sections for $t'\bar{t}'$ production including the total error bands are presented in Fig. 7 for the Tevatron and in Fig. 8 for the LHC.

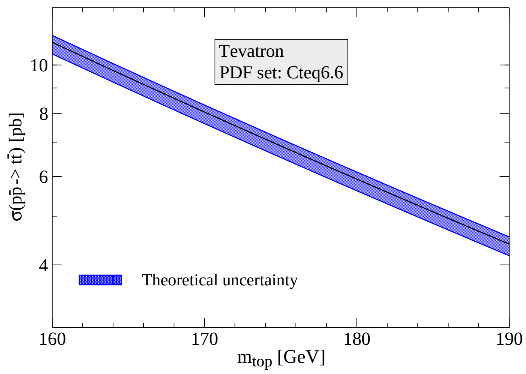

Figure 2: NNLO $t\bar{t}$ production cross section at the Tevatron. The blue band indicates the scale uncertainty.

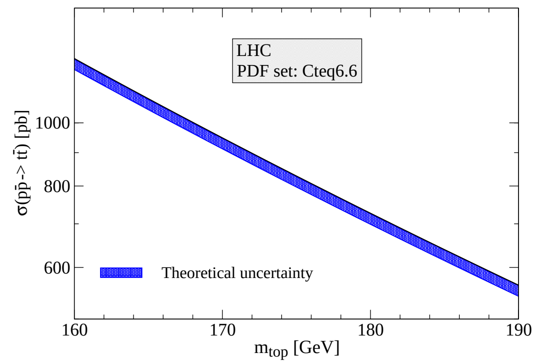

Figure 3: NNLO $t\bar{t}$ production cross section at the LHC. The blue band indicates the scale uncertainty.

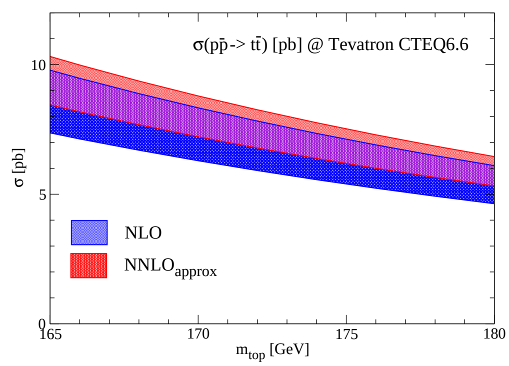

Figure 4: Combined scale uncertainty and PDF error for $t\bar{t}$ production at NLO (blue band) and NNLO (red band) at the Tevatron.

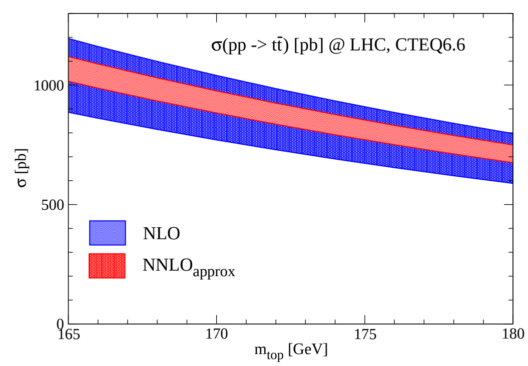

Figure 5: Combined scale uncertainty and PDF error for $t\bar{t}$ production at NLO (blue band) and NNLO (red band) at the LHC.

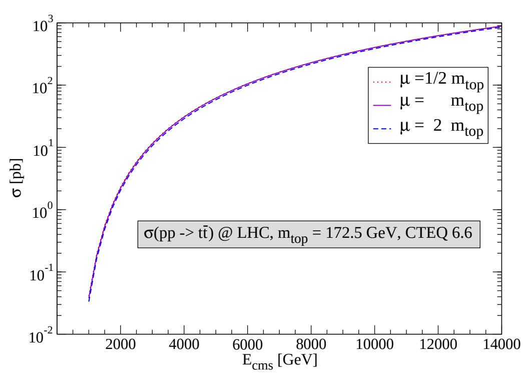

Figure 6: $t\bar{t}$ production at the LHC as a function of the center of mass energy E for $m_t = 172.5\,\text{GeV}$ and for three different scales $\mu = 1/2m_t, m_t, 2m_t$, see Eq. (4) and Tab. 1 for the parametrisation.

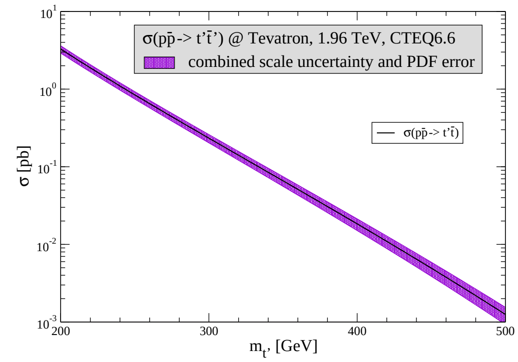

Figure 7: Pair production cross section for a hypothetical heavy fourth generation quark at the Tevatron. The violet band indicates the combined scale uncertainty and PDF error.

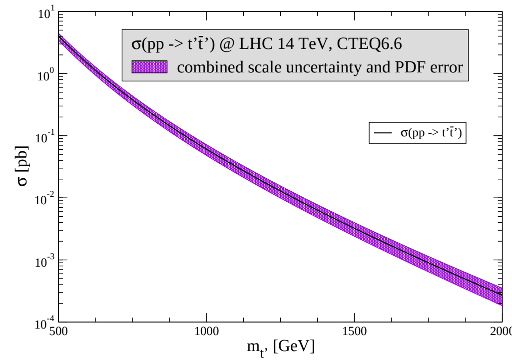

Figure 8: Pair production cross section for a hypothetical heavy fourth generation quark at the LHC. The violet band indicates the combined scale uncertainty and PDF error.

Acknowledgments

This work is supported by the Helmholtz Gemeinschaft under contract VH-NG-105 and by the Deutsche Forschungsgemeinschaft under contract SFB/TR 9. P.U. acknowledges the support of the Initiative and Networking Fund of the Helmholtz Gemeinschaft, contract HA-101 ("Physics at the Terascale").

References

[1] Slides:
`http://indico.cern.ch/contributionDisplay.py?contribId=143&sessionId=5&confId=53294`

[2] [Tevatron Electroweak Working Group and CDF and D0 Collaboration], arXiv:0903.2503 [hep-ex].

[3] A. Lister [CDF and D0 Collaborations], arXiv:0810.3350 [hep-ex].

[4] S. Moch and P. Uwer, Phys. Rev. D **78** (2008) 034003 arXiv:0804.1476 [hep-ph].

[5] S. Moch and P. Uwer, Nucl. Phys. Proc. Suppl. **183** (2008) 75 arXiv:0807.2794 [hep-ph].

[6] W. Beenakker, A. Denner, W. Hollik, R. Mertig, T. Sack and D. Wackeroth, Nucl. Phys. B **411** (1994) 343.

[7] W. Bernreuther, M. Fücker and Z. G. Si, Phys. Rev. D **74**, 113005 (2006) arXiv:hep-ph/0610334.

[8] J. H. Kühn, A. Scharf and P. Uwer, Eur. Phys. J. C **51**, 37 (2007) arXiv:hep-ph/0610335.

[9] Y. Kiyo, J. H. Kühn, S. Moch, M. Steinhauser and P. Uwer, Eur. Phys. J. C **60**, 375 (2009) arXiv:0812.0919 [hep-ph].

[10] S. Dittmaier, P. Uwer and S. Weinzierl, Phys. Rev. Lett. **98**, 262002 (2007) arXiv:hep-ph/0703120.

[11] S. Dittmaier, P. Uwer and S. Weinzierl, Eur. Phys. J. C **59**, 625 (2009) arXiv:0810.0452 [hep-ph].

[12] N. Gray, D. J. Broadhurst, W. Grafe and K. Schilcher, Z. Phys. C **48**, 673 (1990).

[13] V. M. Abazov *et al.* [D0 Collaboration], arXiv:0903.5525 [hep-ex].

[14] U. Langenfeld, S. Moch and P. Uwer, arXiv:0906.5273 [hep-ph].

[15] P. M. Nadolsky *et al.*, Phys. Rev. D **78** (2008) 013004 arXiv:0802.0007 [hep-ph].

[16] A. D. Martin, W. J. Stirling, R. S. Thorne and G. Watt, arXiv:0901.0002 [hep-ph].