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THE HIGH ENERGY BUDGET ALLOCATIONS IN SHOCKS AND GAMMA RAY BURSTS

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ABSTRACT

The statistical distribution of energies among particles responsible for long gamma-ray burst (GRB) emission is analyzed in light of recent results of the *Fermi Observatory*. The all-sky flux, F_γ, recorded by the Gamma-Ray Burst Monitor (GBM) is shown, despite its larger energy range, to be not significantly larger than that reported by the Burst and Transient Explorer, suggesting a relatively small flux in the 3–30 MeV energy range. The present-day energy input rate in γ-rays recorded by the GBM from long GRBs is found, assuming star formation rates in the literature, to be $\dot{W}(0) = 0.5 F_\gamma H/c = 5 \times 10^{42}$ erg Mpc^{-3} yr^{-1}. The Large Area Telescope fluence, when observed, is about 5%–10% per decade of the total, in good agreement with the predictions of saturated, nonlinear shock acceleration. The high-energy component of long GRBs, as measured by *Fermi*, is found to contain only $\sim 10^{-2.5}$ of the energy needed to produce ultrahigh-energy cosmic rays (UHECRs) above 4 EeV, assuming the latter to be extragalactic, when various numerical factors are carefully included, if the cosmic-ray source spectrum has a spectral index of -2. The observed γ-ray fraction of the required UHECR energy is even smaller if the source spectrum is softer than E^{-2}. The AMANDA II limits rule out such a GRB origin for UHECRs if much more than 10^{-2} of the cosmic-ray energy goes into neutrinos that are within, and simultaneous with, the γ-ray beam. It is suggested that "orphan" neutrinos out of the γ-ray beam might be identifiable via orphan afterglow or other wide angle signatures of GRBs in lieu of coincidence with prompt γ-rays, and it is recommended that feasible single neutrino trigger criteria be established to search for such coincidences.

Key words: astroparticle physics – cosmic rays – gamma-ray burst: general

Online-only material: color figure

1. INTRODUCTION

The high energy range of the cosmic-ray (CR) spectrum is broken into three parts: the knee-to-ankle segment extends roughly from 10^{16} to 4×10^{18} eV, and the ultrahigh-energy CR (UHECR) above the ankle can be further segmented into those below the Greisen–Zatsepin–Kuzmin (GZK) cutoff at about 4×10^{19} eV and those above it. Those above the ankle are believed to be extragalactic, showing both a flattening of the slope in the observed flux and little anisotropy below the GZK cutoff, and some anisotropy toward the local supercluster above it. Both UHECR components require particle acceleration well beyond the energies at which γ-rays themselves are detected. The CRs beyond the ankle have been attributed to active galactic nuclei, but this is not the case for CRs in the range 10^{15}–10^{18} eV, whose origin remains a mystery. In this energy range, CRs are probably Galactic in origin and do not fill intergalactic space as the CRs above the ankle do.

Gamma-ray bursts (GRBs) have been considered by some authors (e.g., Levinson & Eichler 1993; Milgrom & Usov 1995; Waxman 1995; Vietri 1995) as sources of UHECRs. Levinson & Eichler (1993), while not taking a position on whether GRBs could account for UHECRs above the ankle (despite giving this possibility serious consideration), suggested that those just below are due to Galactic GRBs. Doubts that GRBs could supply the highest-energy CRs include the total energetics, discussed in this paper; adiabatic losses, which lower the maximum energy should the acceleration be in a compact region; and Galactic isotropy, which is discussed in a forthcoming paper. Later authors (Milgrom & Usov 1995; Vietri 1995; Waxman 1995) focused on the possible connection between GRBs and CRs above the GZK cutoff. GRBs have also been considered as sources of ultrahigh-energy (UHE) neutrinos (e.g., Eichler 1994; Waxman & Bahcall 1997; Eichler & Levinson 1999). The detection of UHECR and/or UHE neutrinos in association with GRBs would provide valuable information.

In recent years, there have been new and improved data. In particular, the Large Area Telescope (LAT) detector on the *Fermi Observatory* provides information about the energy in non-thermal tails of particle distributions, and the Gamma-Ray Burst Monitor (GBM), which can measure energies up to 30 MeV, provides a more reliable measurement of the GRB bolometric luminosities. AMANDA II has been operational for over 1000 days and has set limits on the neutrino output of GRBs. This paper takes another look at GRBs and UHECR energetics in the context of previous suggestions. In the steady state, the total demands of local power per unit cosmic volume on sources of UHECRs above the ankle, $\dot{W}_{\rm CR}$, can be written in terms of the measured all-sky UHECR flux in any specified energy range, $F_{\rm CR}$, as

$$\dot{W}_{\rm CR} \equiv (\tau_{\rm CR} H)^{-1} (F_{\rm CR} H/c)\, C_B = (\tau_{\rm CR} H)^{-1} \left(\frac{F_{\rm CR}}{F_{\rm GRB}}\right) \left(\frac{F_{\rm GRB} H/c}{\dot{W}_{\rm GRB}}\right) C_B \dot{W}_{\rm GRB}, \quad (1)$$

assuming that the UHECRs fill intergalactic space. Here, $\tau_{\rm CR}$ is the cosmic-ray energy loss time due to interactions with the microwave background. The quantity C_B is the bolometric correction assuming that the source produces a spectrum typical of familiar shock acceleration. The ratio $\frac{\dot{W}_{\rm CR}}{\dot{W}_{\rm GRB}}$ is thus given by a product of four separate dimensionless numbers. Below, we determine the values of $\frac{F_{\rm CR}}{F_{\rm GRB}}$, $\frac{F_{\rm GRB} H/c}{\dot{W}_{\rm GRB}(0)}$, $(\tau_{\rm CR} H)$, and C_B. We find that for CRs above the ankle, $\frac{F_{\rm CR}}{F_{\rm GRB}}$ is 4, $\frac{F_{\rm GRB} H/c}{\dot{W}_{\rm GRB}(0)}$ is about

2 for popular estimates of cosmic star formation rates (SFRs), $(\tau_{CR}H)^{-1}$ is between 5 and 8 for CR above the ankle and more for CR above the GZK cutoff, and C_B is between 4 and 20, depending on assumptions about the acceleration. Altogether, the value of $\frac{\dot{W}_{CR}}{\dot{W}_{GRB}}$ is of order $10^{2.5}$ to 10^3, despite the fact that each of the four factors may be modest.

2. PARTICLE SPECTRA IN GRBs

The average cosmic energy density in γ-rays from GRBs is a directly measurable quantity, independent of distracting uncertainties in the beaming angle, average energy, or the rate of GRBs. For a sample of hundreds of GRBs or more, and spectral coverage from 8 keV on upward, as we now have, we can measure the total all-sky flux in GRBs merely by summing over all events within a given time interval. Most of the photon energy in GRBs detected by *Fermi* is in the GBM range. We have considered all long ($T_{90} > 2$ seconds) bursts detected by the Fermi Gamma-Ray Burst Monitor from 2008 August until 2010 February (see Table 2 of Guetta et al. 2010). The fluences have been collected from the literature (mainly GCN circulars) and usually are given in the 8 keV–1 MeV energy range. There are ~20 GRBs with fluences given in the 50–300 keV energy range. For GRB 080916C, GRB 090902B, and GRB 100116A, the fluences are given up to 10 MeV. There exists the possibility of some additional unreported fluence outside the quoted range, but we presume the quoted range is selected so that the unreported fluence is relatively small. We have summed the reported GRB fluences and found the sum to be 3.0×10^{-3} erg cm^{-2}. Short bursts, which we do not include here, contributed another several percent. The GBM field of view (FOV) is roughly $\Omega_{GBM} = 2\pi$ steradians, and we can thus write the total all-sky flux as

$$F_\gamma = 4 \times 10^{-3} \left(\frac{2\pi \text{ sr}}{\Omega_{GBM}}\right) \text{erg cm}^{-2} \text{ yr}^{-1}. \quad (2)$$

This is below the 10–1000 keV flux for all GRBs that would have been derived from Burst and Transient Explorer (BATSE) data by a factor of about 1.25 (M. Schmidt 2002, private communication to V. S. Berezinsky; D. Eichler & D. Guetta, in preparation) to 2 (M. Gonzalez 2003, private communication to C. Dermer). In addition to differences in instrumental sensitivity and calibration, some discrepancy may be expected due to year-to-year fluctuations in the GRB flux, because it receives much of its contribution from the several brightest GRBs in any given year (see below). Although we continue to use the GBM number, we note that the error in this estimate may be as large as about 40% of the correct number.

The important conclusion is that the 3–30 MeV energy band, which was not covered by the BATSE, does not appear to contain enough flux to significantly affect our conclusions. It appears to be less than the flux below 3 MeV. This is consistent with the steep post-peak spectra, i.e., the large Band parameter β, that is seen for many of the bursts. As most of the GRB flux is in relatively bright GRBs, we can be reasonably confident that no significant flux is hidden in dim, nondetectable GRBs—unless they form a separate, as yet unidentified, class of events, rather than merely the low-luminosity extension of what has already been identified. There remains, however, the logical possibility that most GRBs are dominated by huge GRBs that are so infrequent that we do not yet have a fair sample of them.

Of the 205 long GRBs detected by the GBM before 2010 February, only ~ 12 long bursts were detected above 30 MeV by the LAT. We have estimated the fluences above 100 MeV of 10 GRBs detected by the LAT[5] to be between 5% and 30% of the total GBM fluence. The sum of the 100 MeV to 10 GeV fluences in these 10 LAT-detected GRBs is 1.34×10^{-4} erg cm^{-2}, which is about 13.5% of the total GBM fluence for the same GRBs, 1×10^{-3} erg cm^{-2}.

An additional nine GRBs were not detected by LAT despite being "LAT-eligible," i.e., within the LAT FOV and as bright as those that *were* detected by LAT. Their total GBM fluence was 4.5×10^{-4} erg cm^{-2}, and the upper limit to the LAT fluence for these GRBs was about 5% of the total. Thus, the overall emissivity in the 100 MeV to 10 GeV range is apparently below 13% of that in the GBM band, which is consistent with the view that GRBs typically (though not necessarily always) emit most of their photon energy in a quasi-thermal MeV peak (e.g., Levinson & Eichler 2000; Meszaros & Rees 2000; Ryde 2004; Ryde & Pe'er 2009). The total detected LAT fluence over 1.5 years corresponds[6] to an all-sky flux of

$$F_{LAT}/2 = (2.25 \pm 0.9) \times 10^{-4} \left(\frac{2.5 \text{ sr}}{\Omega_{LAT}}\right) \text{erg cm}^{-2} \text{ yr}^{-1}, \quad (3)$$

which is about 11% ± 4.5% of the *total* inferred all-sky GBM flux, cf. Equation (2). The error of ~40% $\sim 0.7/\sqrt{3}$ is estimated on the basis of the fact that more than 70% of the total is dominated by only three GRBs (090902B, 080916C, and 090926A). As we include two decades of energy in the above flux, the inferred flux per decade is $F_{LAT}/2$.

The *Fermi Gamma Ray Observatory* has a sufficiently large dynamic range of photon energy that it is beginning to provide a basis for comparison with theory of how energy in violent, non-thermal phenomena is distributed among radiative particles. Theory predicts power-law particle spectra whose index depends on the compression ratio of the shock. As the acceleration may be very efficient, the compression ratio may be affected by the particle acceleration itself, so that the problem can become nonlinear. Nonlinear theory of shock acceleration (Eichler 1979, 1984; Ellison & Eichler 1985) predicts that about half of the internal energy of the shocked fluid should be found in the energetic tail and the other half in thermal particles. The basic reason is that, because the energetic particle distribution must be injected from the thermal pool, some of the compression (about half as it turns out) must be saved for viscous heating of the latter,[7] otherwise the injection of fresh particles gets choked off. While the energetic tail receives of order 50% of the total, it is generally distributed over many decades, so that there is only of order $1/2N_D \lesssim 10\%$ per decade or less. This becomes significant in the interpretation of isolated energy ranges of high-energy emission. The energy in UHECRs is likely to be most of the total only if the spectrum is extremely hard—much harder than the conventionally applied models of shock acceleration—because the UHE range is so many decades above the low-energy cutoff.

The LAT fluence, when observed, appears to be only about 10%–20% of the total GBM fluence in the 100 MeV–10 GeV

5 These are 090902B, 080916C, 090926A, 090323, 090328, 080825C, 090217, 091003, 090626, and 091031. We included neither GRB 081215A, as it has a boresight angle of 86°, hence out of FOV of LAT, nor GRB100116A, as we did not find its LAT fluence.

6 The LAT field of view is ~2.5 sr.

7 Unless the phase velocity of the waves that couple the energetic particles to the fluid is negligible, in which case the thermal particles can get much less than half. Detailed calculations (Ellison & Eichler 1985) show that the phase velocity must, in this case, be extremely small, approximately two or more orders of magnitude below the shock velocity.

range, or about 5%–10% per LAT decade, which is perhaps less than expected from certain early synchrotron emission models. The 100 MeV–10 GeV fluences observed by LAT may or may not be a good proxy for the total output of UHECRs and/or UHE neutrinos. As opposed to the GBM flux, which may represent a thermal pool, the LAT fluxes measure the non-thermal tail of the energy distribution in the GRB primary charged particles, which, if hadronic, is the part that could contribute to the UHECR and UHE neutrino fluxes. However, there still remain the uncertainties of the electron-to-proton ratio, the microscopic details of the injection process, and the optical depth at which the particle acceleration takes place. It is possible that degradation of high-energy particles and/or photons also contributes to the thermal component. The point nevertheless remains that maximally efficient shock acceleration over a large range of high energy is generally limited from above by the large number of decades in the overall energy range of the acceleration process, to at most 5%–10% per decade of particle energy, so it is consistent with the spectra of the LAT sources without the need to invoke pair opacity. This necessitates a considerable bolometric correction, of order $1/2N_D \lesssim 10\%$, when comparing the total GRB flux with cosmic-ray energies in a given energy decade.

Were pair opacity the only uncertainty, we might reasonably say that the LAT flux is a lower limit and the GBM flux is an upper limit, but this upper limit would include three decades in the GBM range, plus N_p, the number of decades in the range subject to pair opacity. As the total GBM flux is only 20 times the LAT flux per decade, the upper and lower limits so obtained would be rather close together. In our view, the main uncertainties in using the LAT fluence as a proxy for the UHECR output are the uncertainty in the electron-to-proton ratio and the possibility that only electrons contribute to the LAT γ-rays.

Relativistic shocks can accelerate charged particles, and in the case of efficient stochastic scattering a spectral index of −2.3 is generally predicted (Kirk & Schneider 1987; Bednarz & Ostrowski 1998). Because the energy spectrum is convergent at high energy, it need not affect the compression ratio. It follows that only a small fraction $[\sim E_{\rm UHE}/E_{\rm th}]^{-0.3}$ of the relativistic shock's energy budget can be apportioned to the highest decade of the energy range. We conclude that the measured LAT fluxes are consistent with maximally efficient shock acceleration, if the GBM is interpreted as a thermal particle peak or as a catch basin for pair cascade end products. They are also consistent with the $E^{-2.3}$ spectra expected from relativistic shocks, since they lie nearly 3 orders of magnitude above the thermal peak and contain about 10% of the total flux.

Another important energy reservoir for GRBs is the kinetic energy of the blast, $E_{\rm KE}$, which is the ultimate source of shock energy for producing UHECRs. As the proton-to-electron efficiency ratio ϵ_p/ϵ_e, the proton to prompt-gamma-ray efficiency ratio $\epsilon_p/\epsilon_\gamma$, and the blast energy to prompt γ-ray efficiency ratio $\epsilon_{\rm KE}/\epsilon_\gamma$ could each conceivably be large, no constraint can rigorously be placed on the proton energy budget from γ-rays, so the latter could come from inverse Comptonizing electrons. Afterglow calorimetry models (Frail et al. 1997; Waxman et al. 1998; Freedman & Waxman 2001) typically put $\epsilon_{\rm KE}/\epsilon_\gamma \sim 1$ (but see Eichler & Waxman 2005 and below). Also, Eichler & Jontof-Hutter (2005) argue on the basis of correlation between $\epsilon_{\rm KE}/\epsilon_\gamma$ (as derived from afterglow calorimetry) and spectral peak that $\epsilon_{\rm KE}/\epsilon_\gamma$ is in fact nearly an order of magnitude below previous estimates, and this is supported by the detailed analysis by Vergani et al. (2008) of GRB 060418. Simulations of relativistic shocks typically give $\epsilon_p/\epsilon_e \sim 1$ (Spitkovsky 2008), but, as shocks responsible for UHECRs are more likely to be non-relativistic, this result may not apply. The possibility that $E_{\rm KE}$ is much larger than the γ-ray output E_γ, necessary to make the GRB–UHECR connection, is not consistent with many claims that $E_{\rm KE} \lesssim E_\gamma$, but we cannot rule it out, as there remains the possibility that the injection of electrons into the diffusive shock acceleration process is of low efficiency (Eichler & Waxman 2005).

In Figure 1, we adopt the SFR

$$R(z) = R(0)\frac{46\exp(3.4z)}{\exp(3.8z)+45}\,F(z,\Omega_M,\Omega_\Lambda), \tag{4}$$

where

$$F(z,\Omega_M,\Omega_\Lambda) = [\Omega_M(1+z)^3+\Omega_\Lambda]^{1/2}/(1+z)^{3/2}, \tag{5}$$

with $\Omega_\Lambda = 0.7$, $\Omega_M = 0.3$ (Porciani & Madau 2001), and, for comparison, that of Hopkins & Beacom (2006), which gives very similar results. Assuming that the GRB energy input rate $\dot{W}(z)$ scales as $R(z)$, we plot the expected energy contribution per unit time of GRB to the present cosmic energy density $\dot{W}(z)/(1+z)$ as a function of redshift. In the second frame, we calculate the cumulative contribution at redshift less than z,

$$\eta(z) \equiv \frac{\int_0^z[\dot{W}(z)/(1+z)][dt/dz]dz}{\int_0^{z(T)}[\dot{W}(z)/(1+z)][dt/dz]dz} = \frac{\int_0^z[\dot{W}_\gamma(z)/(1+z)][dt/dz]dz}{F_\gamma/c}, \tag{6}$$

where $T = 13$ Gyr. Both SFRs give a present-day value for $-d\eta/dt$ of

$$-\dot{\eta} = (d\eta/dz)(-dz/dt) \approx 0.5H \approx 0.04\ {\rm Gyr}^{-1}. \tag{7}$$

The present-day energy input from GRBs in the GBM range is thus given by

$$\dot{W}_\gamma(0) = -\dot{\eta}F_\gamma/c = 0.5F_\gamma H/c = 5\times10^{42}\ {\rm erg\,Mpc^{-3}\,yr^{-1}}, \tag{8}$$

which may be somewhat below some previous estimates. Such a difference would appear if GRBs with known afterglows are systematically brighter than those with unknown afterglows, for it is the former group that is the basis for estimating the typical isotropic equivalent energies of GRBs. This is connected to the claim (Nakar & Piran 2005; Band & Preece 2005) that GRBs with unknown redshifts are less energetic on average, even if placed at a very high redshift, than those of known redshifts, which are reported to lie on the Amati relation. On the other hand, M. Schmidt (2002, private communication to V. S. Berezinsky) obtains $\dot{W}_\gamma(0) = 6\times10^{42}\ {\rm erg\,Mpc^{-3}\,yr^{-1}}$, in good agreement with our value.

We now compare the total flux in GRB radiation to the flux per decade in UHECRs. The all-sky differential number flux $df(E)/dE$ in UHECRs is given by

$$E^3 df(E)/dE = 4\pi\times4\times10^{27}\left(\frac{E}{10^{19}\ {\rm eV}}\right)^{0.3}\ {\rm eV^2\,cm^{-2}\,yr^{-1}} \tag{9}$$

(Abraham et al. 2010a), in the range 4–40 EeV. This translates to an all-sky energy flux $F_{[4,40]}$ over the interval $4\times10^{18}\ {\rm eV} \leqslant$

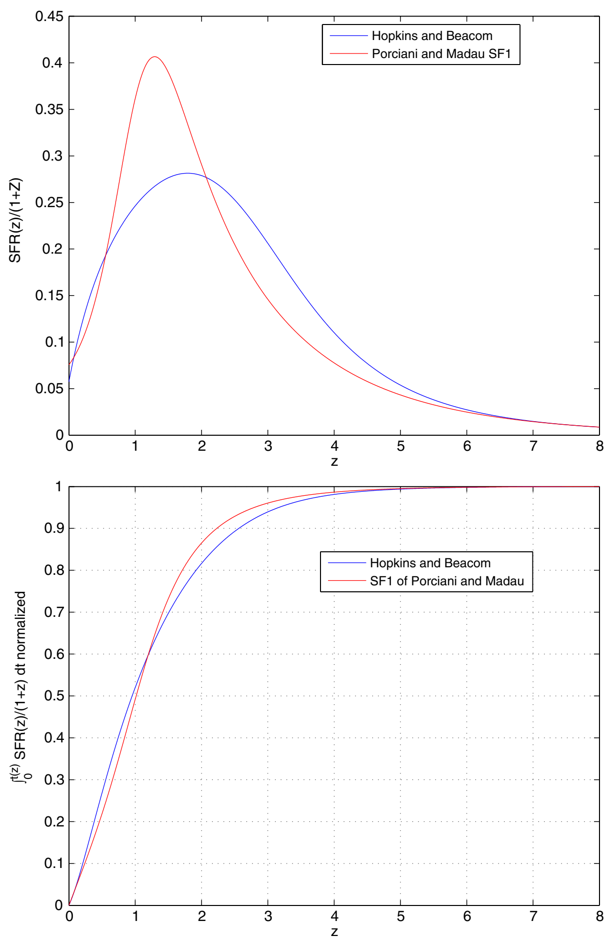

Figure 1. Upper panel: differential contribution to the overall GRB fluence as a function of the redshift. Lower panel: integrated contribution to the overall GRB fluence as a function of the redshift. The assumed star formation rates are discussed in the text.

(A color version of this figure is available in the online journal.)

$E \leqslant 4 \times 10^{19}$ eV of

$$F_{[4,40]} = \int_{4\,\mathrm{EeV}}^{40\,\mathrm{EeV}} dE\ E\ f(E)$$
$$= 1.7 \times 10^{-2}\ \mathrm{erg\,cm^{-2}\,yr^{-1}} = 4.0 \times F_\gamma. \quad (10)$$

The horizon for UHECRs in the 4–40 EeV range can be estimated by noting that the logarithmic energy-loss rate, $\tau_{\mathrm{CR}}^{-1} + H$ in that range, mostly due to pair production from cosmic microwave background photons (e.g., Dermer & Menon 2009), is given by

$$\frac{\dot{E}}{E} = -(1+z)^3(c/\mathrm{Gpc})\frac{0.74 + 1.8\ \ln(y/2)}{\sqrt{y}}$$
$$+ \frac{d\ln(1+z)}{dt} \sim -4.2\,(1+z)^3\,H - H. \quad (11)$$

Here, H is the Hubble constant, tentatively assumed to be constant at small z, $y \equiv E$ in EeV, and we for convenience conservatively approximate the coefficient of loss due to pair production, $[0.74 + 1.8\ \ln(y/2)]/y^{1/2}$, as 1.0 at $E \geqslant 4$ EeV. The second term on the right-hand side represents loss due to cosmic expansion. Substituting $d\ln(1+z)/dt$ for H into Equation (9) and solving shows that a UHECR injected at $E(z)$ at redshift (1+z) has a present-day energy E_o of

$$E_o = (1+z)^{-1}\ \exp(-1.4\,[(1+z)^3 - 1])E(z). \quad (12)$$

The horizon, defined to be the value of z at which $(1+z)E_o/E(z) = 1/e$, is thus at a redshift of $z_h = 0.2$, and the energy attenuation beyond this horizon is steep.

Assuming the source spectrum $S(E, z)$ to be constant in time, to be multiplied by the star formation rate $R(z)$, the total number per comoving volume of CRs at the present time $N(E_o)$ is given by

$$N(E_o)dE_o = \int_0^\infty S[E(z)]\,dE(z)\,R(z)\,dt \quad (13)$$
$$= S(E_o)\,R(0)\int_0^\infty \frac{S[E(z)]\,dE(z)\,R(z)}{S[E_o]\,R(0)\,(1+z)\,H}\,dz.$$

Assuming $S(E) \propto E^{-\alpha}$, then

$$\frac{S[E(z)]dE(z)}{S(E_o)dE_o} = (1+z)^{-\alpha+1}\exp\left(1.4(1-\alpha)[(1+z)^3 - 1]\right). \quad (14)$$

The present number density of CR at energy E_o is therefore given by

$$N(E_o) = S(E_o)R(0)\int_0^\infty \frac{(1+z)^{-\alpha}}{H}$$
$$\times \exp\left(1.4(1-\alpha)[(1+z)^3 - 1]\right)\frac{R(z)}{R(0)}dz. \quad (15)$$

Evaluating the integral numerically for $\alpha = 2.4$, we find

$$S(E_o)R(0) = 6.45N(E_o)H \quad (16)$$

and similarly $\dot{W}(0) \approx 6.45FH/c \approx 2.5 \times 10^{44}\ \mathrm{erg\,Mpc^{-3}\,yr^{-1}}$. Uncertainty in α, $2.0 \leqslant \alpha \leqslant 2.7$, introduces about a 20% uncertainty in this estimate. However, the observed CR spectral index between 4 and 40 EeV, −2.7, suggests that the slope cannot be much flatter, because the pair creation loss time is nearly energy independent.

Above 4×10^{19} eV, in the trans-GZK regime, the flux, to within the Auger error bars, is

$$E^3\,df(E)/dE = 4\pi \times 5 \times 10^{27}\left(\frac{E}{3 \times 10^{19}\ \mathrm{eV}}\right)^{-1.2}\mathrm{eV^2\,cm^{-2}\,yr^{-1}}. \quad (17)$$

This implies an energy flux

$$F_{>\mathrm{GZK}} = 1.1 \times 10^{-3}\ \mathrm{erg\,cm^{-2}\,yr^{-1}}. \quad (18)$$

This flux may be supplied only by recent ($z \ll 1$) GRBs, if any, because the sources must come from within $c\,t_{\mathrm{GZK}} \sim$ 200 Mpc or so (e.g., Olinto et al. 2010), so the contributing GRBs therefore must have occurred within the past 6.18×10^8 yr, i.e., at $z \lesssim 0.05$. The horizon length above 40 EeV is very sensitive to energy and therefore to the exact energy calibration of the Auger detector. For the purposes of a rough estimate, we write the average horizon length as (200 Mpc) l_{200}. Using the above SFR, we roughly estimate the fraction of this recent contribution to the total to be $\eta(z[t_{\mathrm{GZK}}]) \approx t_{\mathrm{GZK}}\dot{\eta} = 0.025\,l_{200}$ and that the present rate of trans-GZK CR production is $F_{>\mathrm{GZK}}/c\eta(0.05) = 4.4 \times 10^{-2}/l_{200}c\ \ \mathrm{erg\,cm^{-2}\,yr^{-1}} = 1.3\,l_{200}^{-1} \times 10^{44}\ \mathrm{erg\,Mpc^{-3}\,yr^{-1}}$. This can be compared to other estimates, which range from

$0.45(\alpha - 1) \times 10^{44}$ erg Mpc^{-3} yr^{-1} above 30 EeV per unit internal in ln E (Katz et al. 2009) to 1.67×10^{44} erg Mpc^{-3} yr^{-1} (Berezinsky 2008), if the energy calibration is set to optimize the fit to the observed GZK cutoff. The uncertainty lies in the instrumental energy calibration and in the location of the high energy cutoff.

If most of the trans-GZK CRs are heavy nuclei, as suggested by recent analysis of AUGER data (Abraham et al. 2010b), then their production is limited to an even smaller look-back time (Stecker & Salamon 1999), and the UHECR production demanded of individual GRB would be raised accordingly. This issue is somewhat controversial at present, and we have conservatively taken the UHECR to be protons for the sake of this discussion.

Finally, we turn to the bolometric correction C_B. The forward shocks of relativistic blast waves that produce GRB afterglow, if they have a spectral index of -2.3 from some minimum energy $E_{\rm th}$ through some ultrahigh-energy $E_{\rm UHE}$, would require a bolometric correction of $C_B = [E_{\rm UHE}/E_{\rm th}]^{0.3}$. If we take $\log_{10}[E_{\rm UHE}/E_{\rm th}]$ to be 5, as in Dermer (2010), we would get $C_B \approx 10$. (Dermer takes a relativistic spectral index of 2.2, in which case $C_B \approx 5$, illustrating how sensitive C_B is on the spectral index.) The less energy-demanding source of the UHE primaries is generally non-relativistic shocks, which can have flatter spectra (or possibly radiative or cosmic-ray-lossy shocks, which do not conserve energy from upstream to downstream). These could, for example, be internal shocks in a GRB baryonic outflow (Levinson & Eichler 1993; Rees & Meszaros 1994), or the subrelativistic end phase of GRB blast waves. Here, the minimum energy is at most $\sim E_{\rm max}/\Gamma_S m_p c^2$, where Γ_S is the shock Lorentz factor, and the bolometric correction for any given decade of energy, e.g., [4, 40] EeV, is about $[\log_{10} E_{\rm max} - \log_{10} E_{\rm th}] \sim [11 - \log_{10}(\Gamma_S)] \sim 8$.

We also consider the energy per decade inferred from the *Fermi* LAT detections in the energy decades 100 MeV to 10 GeV as the basis for a representative estimate of the amount of the energy per decade that could be expected for UHECRs and UHE neutrinos at even higher energies. The GBM flux, which spans several orders of magnitude of photon energy and which is an order of magnitude higher than the LAT flux, may tentatively be interpreted as a measure of the total energetics of GRBs, as it can be associated with thermal photons and/or pair cascade end products that emerge from a photosphere. For example, the GBM flux, which appears to have a quasi-thermal spectrum peaking around 1 MeV, may be the catch basin of all of the radiation that was converted into pairs. For GRBs unseen by LAT, this can be as much as all the fireball energy. In cases where the GRB is detected by LAT, the observed GBM-to-LAT flux ratio is consistent with the assumption that shock acceleration could put at most of order $1/2N_D$ of its energy budget at any given decade well beyond the thermal peak of cosmic-ray energy. We tentatively make the conventional best-case assumption for shock acceleration (tentatively leaving aside the extreme case of negligible phase velocity (Ellison & Eichler 1985)) that the spectral index is -2.0, which represents a spectral distribution of equal energy per decade of individual particle energy.

Altogether, assuming $C_B = 8$ and combining it with $\frac{F_{\rm UHECR}}{F_{\rm GRB}} = 4$, $\frac{F_{\rm GRB} H/c}{\dot{W}_{\rm GRB}(0)} = 2$, and $(\tau_{\rm CR} H)^{-1} \sim 6.5$, we would obtain $\frac{\dot{W}_{\rm UHE}(0)}{\dot{W}_{\rm GRB}(0)} = 416$. If we replace $C_B \sim 8$ with the correction that would obtain using the LAT proxy for energy per decade, $F_{\rm GBM}/[F_{\rm LAT}/2] \sim 20$, we would obtain $\frac{\dot{W}_{\rm UHE}(0)}{\dot{W}_{\rm GRB}(0)} = 1000$.

It may be possible to circumvent much of the bolometric correction by assuming that only the highest energy CRs escape the shock at the acceleration site, while the rest are recycled by adiabatic expansion. This might give a source spectrum that is much harder than the post-shock spectrum. However, this does not seem to be the case for supernova remnants (SNRs), if they are the source of low-energy CR, as the Galactic energy budget in CR is only about 3% of the energy budget of SNR. Rather, it would seem that adiabatic losses of UHECRs (which we have not included) would exacerbate the bolometric correction. There are ways in the literature to get a very hard spectrum[8] and this is an interesting issue. But the inferred value for $\frac{\dot{W}_{\rm UHE}(0)}{\dot{W}_{\rm GRB}(0)}$ is quite large even without any bolometric correction.

3. UHE NEUTRINOS

We now briefly consider UHE neutrino detectability from GRBs. Muon neutrino collection for IceCube is most efficient per unit energy in the range $1 \leqslant E_\nu \leqslant 100$ erg (Gaisser 2010), where the effective collecting area is about 0.3 to $\sim 10^2$ m^2. In what follows, we consider the contribution in these two decades, assuming that the muon–neutrino cross section is $1(E/{\rm erg}){\rm m}^2$ in this region, which is accurately averaged over the energy range $1 \leqslant E_\nu \leqslant 100$ erg, and to within a factor of 2 at any given energy within. The minimum flux to produce 1 count per year is therefore (1–2) $\times 10^{-4}$ erg cm^{-2} yr^{-1}. At high energies, the expected ratio of photon energy to neutrino (plus anti-neutrino) energy is about 4:5, because neutral pions are about half as numerous as charged ones and put all their energy into two photons whereas the charged pions put only about half their energy into a muon neutrino/antineutrino pair and divide the other half about equally between two neutrinos and a charged lepton, the latter of which probably radiates its energy into photons. Assuming oscillations thoroughly mix the neutrino energy among the three neutrino types, the ratio of muon neutrino energy to photon energy is then about 5:12. The measured LAT flux, if from hadrons or their collision products, combined with the hypothesis of a flat energetic hadron spectrum would suggest a flux of $\sim 2[\Omega_{\rm IceCube}/4\pi][\Omega_{\rm LAT}/(2.5\,{\rm sr})]^{-1}$ counts per year.

[8] Although there has been no observational evidence for this, it is mathematically possible for shocks to dissipate most of their energy into CRs of the highest energy that escape the system (Eichler 1983; Ellison & Eichler 1985). The escaping CRs have the compression-enhancing effect of radiative losses, and the spectrum is much harder than E^{-2}. This does not, however, lessen the energy requirement above 4 EeV, which still challenges the hypothesis of a GRB origin for UHECRs. Other "ultrahard" acceleration scenarios exist for baryonic acceleration in GRBs which could produce much harder spectra than shock acceleration. In the neutron pick-up scenario (Eichler & Levinson 1999; Levinson & Eichler 2003), neutrons that leak out of the surrounding matter into the fireball are picked up by a high-Γ Poynting flux when a charged particle within the high-Γ flow collides with them. After half a gyration or isotropization in the high-Γ flow they end up with $\sim\Gamma^2 mc^2$ of energy. A collisional avalanche ensues in which each collision between a charged particle and a neutron produces further charged particles at $\sim\Gamma^2 mc^2$. Most of the dissipation of the flow is through high-energy hadronic collisions, and much of the energy loss is in the form of UHE neutrinos. Pick-up of ex-neutrons (and/or other ex-neutrals) can accelerate hadrons by a factor of Γ^2 as long as it does not exceed $2\sigma^{2/3}$ (Eichler 2003), where $\sigma\, mc^2/Ze$ is the maximum potential drop across the MHD flow (Michel 1983). For GRB central engines with magnetic fields in excess of $\sigma \geqslant 10^{15}$, ultrahigh energies fall within this limit. The maximum Lorentz factor of GRB fireballs is still unknown. Repeated charge conversion can lead to multiple encounters of this sort and accelerate particles up to the limit, potentially leading to a hard spectrum (Derishev et al. 2003). Similarly, multiple encounters with independent cells in relativistic turbulence of Lorentz factor γ can achieve similar repetition of amplification by γ^2 (C. Dermer 2010, private communication).

Here, $\Omega_{\rm IceCube}/4\pi$, the fraction of sky available for neutrino detection, may be close to unity in the context of GRBs, where there is so small a noise problem from atmospheric muons.

The number of neutrino events per year allowed in IceCube from GRBs is limited by the lack of neutrino events in AMANDA correlating with GRBs. AMANDA II has had ∼3.8 years of live observing time from 2000 through 2007 and has an effective collection area of about 1/20 of IceCube 80 (De Young 2008). Despite the differences in the spectral response of IceCube and AMANDA, and the varying availability of the contemporaneous GRB monitoring needed to eliminate blind searches, the failure to identify a single neutrino event in AMANDA associated with known GRB (Achterberg et al. 2008) limits (to modest confidence) the expected GRB contribution to the IceCube detection rate to ∼5 events per year, unless the neutrinos are (1) out of the γ-ray beam (Eichler & Levinson 1999) or (2) dominated by large episodic events more than a year or two apart. (A $(1 - e^{-n})$ confidence limit would be n times higher.)

This limit corresponds to a UHE neutrino flux per energy decade of less than 10^{-3} erg cm^{-2} yr^{-1}. This is less than 5×10^{-3} of the cumulative energy density, $F_{[4,40]}/\eta(0.2)$, that must have been put into UHECRs in the 4–40 EeV range to maintain the observed $F_{[4,40]}$, and about 2×10^{-2} of that needed to maintain the observed trans-GZK flux, $F_{>{\rm GZK}}/\eta(z_{\rm GZK})$. This would then rule out models in which GRBs produce the UHECRs in regions of moderate to high optical depth to hadronic interactions and subsequent photons, unless the UHECR energy spectrum is harder than E^{-2} in the 10^{15}–10^{20} eV range.

Note that this limit does not apply to "orphan" neutrinos from obscured or otherwise unseen GRBs (Eichler & Levinson 1999) as there is no associated GRB to which to "pin" the neutrino burst. Collimation of fireballs within the envelope of host stars, now believed to almost certainly take place, would naturally lead to wider neutrino beams than the γ-ray beams if there is proton acceleration at collimation shocks. Similarly, shocks in an accompanying baryonic outflow (Levinson & Eichler 1993) may produce a wide neutrino beam while accompanying γ-rays are stopped. There may eventually be "γ-quiet" ways to confirm that a UHE neutrino is associated with a GRB (Eichler & Levinson 1999; Guetta & Eichler 2010). For example, an orphan photon afterglow is detectable within several days, and, at $E_\nu \geqslant$ TeV, the atmospheric background within the IceCube angular resolution, less than 1 per square degree per year, is unlikely to cause confusion.

As noted early on (Eichler 1994), *individual GRBs* are unlikely to produce more than one neutrino count unless they are exceptionally energetic to the extent of being rare, and this is important for *prospective* identification of HE muon/neutrino events with GRBs. The brightest individual LAT fluence was 4.3×10^{-5} erg cm^{-2},[9] which would correspond to an expected neutrino fluence of about 0.2 by the above estimation procedure, and this is consistent with the assumption that several percent of GRB-related neutrinos would arrive paired, which is important for follow-up searches for paired neutrinos (Franckowiak et al. 2009). However, the 0.1 to 0.2 probability applies only to the four brightest examples. The average for the 19 LAT-eligible long GRBs, on the other hand, was limited from above by 8×10^{-6} erg cm^{-2}, which would imply a pairing fraction less than 1/19. The number of expected neutrino pairs from GRBs, if LAT fluences are used as a basis for estimate, is thus less than 1 yr^{-1}, though obviously uncertain. Because neutrino pairs from GRBs are (under reasonable assumptions) probably rare, as discussed above, we therefore suggest that single HE and UHE neutrino counts in IceCube be subjected to (a) follow-up searches for "sibling" orphan afterglows, for example, with robotic telescopes and/or (b) high-energy cuts. The selection criteria for choosing a tractable set of single events to be followed up is a non-trivial matter, but a high-energy threshold seems to be one obvious approach.

The 2004 December 27 flare from the soft γ-ray repeater SGR 1806-20, which, at a fluence of about 1 erg cm^{-2}, produced the equivalent of 250 years of GRB prompt-emission fluence, did not register in AMANDA II, and the limit this sets on neutrino fluence to γ-ray fluence ratio is 3×10^{-3} (Achterberg et al. 2006). Present-day understanding of GRBs, which may (or may not) be sufficiently similar to giant SGR outbursts to make the analogy, is thus compatible with (but does not imply) a signal of less than 10^{-1} UHE neutrino events in IceCube per year from GRBs.

4. SUMMARY AND DISCUSSION

We find that the prompt emission and total afterglow emission from GRBs produce a cosmic energy density that is 2–3 orders of magnitude below that ascribed to processes that yield UHECRs. This estimate assumes that UHECRs are generated with an E^{-2} spectrum. For even a slightly steeper spectrum, the difference between the implied UHECR energy input and the GRB photon energy input is even larger. It applies to the net production of UHECRs after adiabatic losses. Such losses are to some extent inevitable for shock acceleration, which relies on compression of trapped particles, but the degree to which they affect the UHE output has not, to our knowledge, been calculated.

The total LAT flux from GRBs, if it is of hadronic origin and if GRBs generate, in UHE neutrinos in the 1–100 TeV range, the same energy per decade as what LAT detects in high-energy γ-rays, implies a neutrino rate of about two events per year in IceCube. Based on the fact that the overall GBM flux is mostly from the brightest GRB, there is a good chance that these neutrinos would mostly correlate with the small minority of bright GRBs rather than one of the much larger number of dim GRBs.

Fermi can directly observe γ-rays from GRBs up to about 100 GeV beyond which an undetected additional component from, e.g., inverse Compton scattering could hide. Direct limits on TeV band emission (Aharonian et al. 2009) from GRBs are not constraining on account of severe absorption by pair production in intergalactic space, but the pair production will feed an electromagnetic cascade that transports the radiation energy into the 10–100 GeV band, where it would appear as extragalactic background. Recent *Fermi* measurements of that background indicate that the intensity in the 10–100 GeV band is about 5×10^{-10} erg cm^{-2} s^{-1} sr^{-1} (Abdo et al. 2010), or an all-sky flux of 2×10^{-1} erg cm^{-2} yr^{-1}. This can be compared to the implied full-sky flux of $\sim 10^{-1}$ erg cm^{-2} yr^{-1} that is implied by Equation (4) when the entire energy in an E^{-2} spectrum from 100 GeV to trans-GZK energies is cascaded into the 10–100 GeV band. This means that, whatever the origin of UHECRs, a significant fraction of them *may* have undergone high-energy collisions. This admits, for *non-GRB* scenarios of a UHECR origin, the "thick target" scenario, that much or most of the UHE particle acceleration in the universe is buried at large optical depths and that the UHE electromagnetic radiation we observe, even in a particular episodic event, is just the part that

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stands out at modest optical depth. The diffuse 10–100 GeV γ-ray background would, by itself, admit much larger UHE neutrino fluxes, up to ∼250 IceCube events per year, even if the associated photons escape the acceleration site.

Although we have attempted careful estimates of what is known, the overwhelming uncertainty in the above discussion is the spectral index of the CRs which we have taken as E^{-2} for reference. The UHECR and neutrino outputs, whose relevant energies are much higher than those of the γ-rays observed so far, are so sensitive to the spectral index as to render relatively minor the other considerations that we can discuss with some degree of knowledge.

The other chief uncertainty is the optical depth at which hadronic particle acceleration takes place. If most of it is at high optical depth, the UHE neutrino signal should be relatively strong. However, the AMANDA limits on GRB-associated UHE neutrinos are relevant for any optical depth.

The main conclusions to be drawn are as follows.

1. Identification of UHECRs in association with GRBs would mean that GRBs put more than ∼50 times as much energy into UHECRs above the ankle in the UHECR spectrum as into all γ-rays. If the spectral index of accelerated particles is -2, then they put about $10^{2.5}$ to 10^3 times as much energy into CRs over the full spectral range. In view of the large energy requirements for the latter possibility, such a result, should it come to pass, could be interpreted as evidence for ultrahard UHE hadronic acceleration scenarios within the GRB, as well as a huge baryonic blast wave component.
2. The hypothesis of a GRB origin for UHECRs by processes occurring over a range of optical depths that include moderate to high optical depth for the UHECR would need to confront the non-detection by AMANDA II of any GRB-associated neutrinos.
3. Prospective searches for supernovae or GRB signatures with single neutrino events in IceCube are motivated by the chance that doublet events will be too rare.

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