2- AND 3-LOOP HEAVY FLAVOR CORRECTIONS TO TRANSVERSITY*

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We calculate the two- and three-loop massive operator matrix element (OME) contributing to the heavy flavor Wilson coefficients of transversity. We obtain the complete result for the two-loop OME and compute the first thirteen Mellin moments at three-loop order. As a by-product of the calculation, the moments $N = 1$ to 13 of the complete two-loop and the T_F -part of the three-loop transversity anomalous dimension are obtained.

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1. Framework

The transversity distribution belongs to the three twist-2 parton distribution functions (PDFs), together with those for unpolarized and polarized deep-inelastic scattering. It is a flavor non-singlet, chiral-odd distribution and can be measured in semi-inclusive deep-inelastic scattering (SIDIS) and via the polarized Drell–Yan process (for a review see Ref. [1]). Different experiments perform transversity measurements at the moment, *cf.* Refs [2]. Recently, a first phenomenological parameterization has been given for the transversity up- and down-quark distributions in Ref. [3], the moments of which are in qualitative agreement with first lattice calculations [4].

The scattering cross-section for semi-inclusive deeply inelastic charged lepton–nucleon scattering $lN \rightarrow l'h + X$ is given by

$$\frac{d^3\sigma^{\text{SIDIS}}}{dx dy dz} = \frac{4\pi\alpha_{\text{em}}^2 s}{Q^4} \sum_{a=q,\bar{q}} e_a^2 x \left\{ \frac{1}{2} \left[1 + (1-y)^2 \right] F_a(x, Q^2) D_a(z, Q^2) \right. \\ \left. - (1-y) |S_{\perp}| |S_{h\perp}| \cos(\phi_S + \phi_{S_h}) \Delta_T F_a(x, Q^2) \Delta_T D_a(z, Q^2) \right\}, \quad (1)$$

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after the $\mathbf{P}_{h\perp}$ -integration has been performed, [1]. We consider, for definiteness, only scattering cross-sections free of $\mathbf{k}_\perp$ -effects to refer to twist-2 quantities. x and y denote the Bjorken variables, z the fragmentation variable, $q^2 = -Q^2$ the space-like 4-momentum transfer, α_{em} the fine structure constant, e_a the quark charge, and s the c.m.s. energy squared. $\mathbf{S}_\perp$ and $\mathbf{S}_{h\perp}$ are the transverse spin vectors of the incoming nucleon N and the measured hadron h . $F_a(z, Q^2)$, $\Delta_{\text{T}}F_a(z, Q^2)$ and $D_a(z, Q^2)$, $\Delta_{\text{T}}D_a(z, Q^2)$ denote the unpolarized and transversity structure- and fragmentation functions, respectively. The angles ϕ_{S, S_h} are measured in the plane transverse to the γ^*N axis between the x -axis and the respective vector. In process (1) the spin of the *transversely* polarized hadron h has to be measured.

The transversity distribution may also be measured in the transversely polarized Drell–Yan processes. In Mellin space the scattering cross-section is given by [5]

$$\frac{d\Delta_{\text{T}}\sigma^{\text{DY}}}{d\phi} = \frac{\alpha_{\text{em}}^2}{9s} \cos(2\phi) \Delta_{\text{T}}H(N, M^2) \Delta_{\text{T}}C_q^{\text{DY}}(N, M^2), \quad (2)$$

where N denotes the Mellin variable and ϕ is the azimuthal angle of one of the final state leptons $l^\pm$ relative to the axis defined by the transverse polarizations.

$$\Delta_{\text{T}}H(N, Q^2) = \sum_q e_q^2 [\Delta_{\text{T}}q_1(N, Q^2) \Delta_{\text{T}}\bar{q}_2(N, Q^2) + \Delta_{\text{T}}\bar{q}_1(N, Q^2) \Delta_{\text{T}}q_2(N, Q^2)]$$

is a combination of transversity parton distributions for the incoming light (anti-)quarks, and $\Delta_{\text{T}}C_q^{\text{DY}}(N, M^2)$ denotes the Wilson coefficient of the Drell–Yan process, with M^2 the invariant mass of the produced lepton pair.

Like in the case of unpolarized and polarized deep-inelastic processes transversity receives heavy flavor corrections in higher orders in QCD. These are given by the corresponding heavy flavor Wilson coefficients. As for other non-singlet quantities [6, 7], these corrections start at $O(a_s^2)$, with $a_s = \alpha_s/(4\pi)$. In SIDIS one can tag $Q\bar{Q}$ -production in the same way as in the deep-inelastic process, [8]. A measurement is possible in high luminosity experiments. In the Drell–Yan process, on the other hand, heavy flavor contributions emerge inclusively since there the final-state l^+l^- -pairs are measured in the first place. The calculation of the heavy quark Wilson coefficients for $Q^2 \gg m^2$ proceeds in the same way as in unpolarized and polarized deep-inelastic scattering [6, 7, 9, 10].

The complete Wilson coefficients for transversity can be decomposed into a light- and a heavy quark contribution

$$C_q^{\text{TR}}\left(x, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right) = C_q^{\text{TR, light}}\left(x, \frac{Q^2}{\mu^2}\right) + H_q^{\text{TR}}\left(x, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right). \quad (3)$$

As shown in [6], the heavy quark Wilson coefficient for hard processes factorizes into the light quark Wilson coefficients and the massive operator matrix element $A_{qq,Q}^{\text{TR}}$ at large enough scales $Q^2 \gg m^2$. We apply this to the heavy flavor Wilson coefficient for transversity¹

$$\begin{aligned} H_q^{\text{TR}} \left(x, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2} \right) &= C_q^{\text{TR,light}} \left(x, \frac{Q^2}{\mu^2} \right) \otimes A_{qq,Q}^{\text{TR}} \left(x, \frac{m^2}{\mu^2} \right) \\ &= a_s^2 \left[\Delta_{\text{T}} A_{qq,Q}^{(2),\text{NS,TR}} + \Delta_{\text{T}} \hat{C}_q^{(2)}(N_f) \right] \\ &\quad + a_s^3 \left[\Delta_{\text{T}} A_{qq,Q}^{(3),\text{NS,TR}}(N_f + 1) + \hat{C}_q^{(3)}(N_f) \right. \\ &\quad \left. + \Delta_{\text{T}} A_{qq,Q}^{(2),\text{NS,TR}} \otimes \Delta_{\text{T}} C_q^{(1)} \right]. \end{aligned} \quad (4)$$

The aim of this article is to present a computation of the renormalized two- and three-loop heavy-flavor operator matrix elements contributing to transversity. Details of the calculation are given in Ref. [11]. The operator matrix element $\langle q | O^{\text{NS,TR}} | q \rangle$ is given by a two-point Green's function containing a closed loop of a heavy quark Q and external massless quarks q . The local operator is given by [12]

$$O_{F,a;\mu\mu_1\dots\mu_n}^{\text{NS,TR}} = i^n \mathbf{S} \left[\bar{\psi} \gamma_5 \sigma_{\mu\mu_1} D_{\mu_2} \dots D_{\mu_n} \frac{\lambda_a}{2} \psi \right] - \text{trace terms}. \quad (5)$$

Here, $\sigma_{\mu\nu} = (i/2) [\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu]$, $\mathbf{S}$ denotes symmetrization of the Lorentz indices and D_μ is the covariant derivative. The Green's function in $D = 4 + \varepsilon$ dimensions obeys the following vector decomposition

$$\begin{aligned} \hat{G}_{\mu,q,Q}^{ij,\text{TR,NS}} &= J_N \left\langle q | O_{F,a;\mu\mu_1\dots\mu_n}^{\text{NS,TR}} | q \right\rangle = \delta_{ij} \left\{ \Delta_\rho \sigma^{\mu\rho} \hat{A}_{qq,Q}^{\text{TR,NS}} \left(\frac{\hat{m}^2}{\mu^2}, \varepsilon, N \right) \right. \\ &\quad \left. + c_1 \Delta^\mu + c_2 p^\mu + c_3 \gamma^\mu \not{p} + c_4 \not{p} \Delta^\mu + c_5 \not{p} p^\mu \right\} (\Delta \cdot p)^{N-1} \end{aligned} \quad (6)$$

contracting the OME with a source term $J_N = \Delta^{\mu_1} \dots \Delta^{\mu_N}$, with $\Delta^2 = 0$ and p the parton momentum. It determines the un-renormalized massive OME

$$\begin{aligned} \hat{A}_{qq,Q}^{\text{TR,NS}} \left(\frac{\hat{m}^2}{\mu^2}, \varepsilon, N \right) &= \frac{-i \delta^{ij}}{4N_c (\Delta \cdot p)^{N+1} (D-2)} \left\{ \text{Tr} \left[\not{p} p^\mu \hat{G}_{\mu,q,Q}^{ij,\text{TR,NS}} \right] \right. \\ &\quad \left. - \Delta \cdot p \text{Tr} \left[p^\mu \hat{G}_{\mu,q,Q}^{ij,\text{TR,NS}} \right] + i \Delta \cdot p \text{Tr} \left[p^\mu \hat{G}_{\mu,q,Q}^{ij,\text{TR,NS}} \right] \right\} \end{aligned} \quad (7)$$

¹ We use the notation $\hat{f}(N_f) = f(N_f + 1) - f(N_f)$.

A total of 129 diagrams contribute, which were generated using **QGRAF** [13]. These were projected onto $\hat{A}_{qq,Q}^{\text{TR,NS}}$, *cf.* [9], using codes written in **FORM** [14]. After tensor reduction, the loop integrals are of the tadpole-type, since the single external quark is on-shell and massless. The integrals were then evaluated using **MATAD** [15]. The renormalization of the OMEs is described in Ref. [9].

After mass- and charge renormalization one obtains the massive OMEs in the on-mass-shell scheme, *cf.* [9],

$$\Delta_{\text{T}} A_{qq,Q}^{(2),\text{NS},\overline{\text{MS}}} = \frac{\beta_{0,Q} \gamma_{qq}^{(0),\text{TR}}}{4} \ln^2\left(\frac{m^2}{\mu^2}\right) + \frac{\hat{\gamma}_{qq}^{(1),\text{TR}}}{2} \ln\left(\frac{m^2}{\mu^2}\right) + a_{qq,Q}^{(2),\text{TR}} - \frac{\beta_{0,Q} \gamma_{qq}^{(0),\text{TR}}}{4} \zeta_2, \quad (8)$$

$$\begin{aligned} \Delta_{\text{T}} A_{qq,Q}^{(3),\text{NS},\overline{\text{MS}}} = & -\frac{\gamma_{qq}^{(0),\text{TR}} \beta_{0,Q}}{6} (\beta_0 + 2\beta_{0,Q}) \ln^3\left(\frac{m^2}{\mu^2}\right) + \frac{1}{4} \left\{ 2\gamma_{qq}^{(1),\text{TR}} \beta_{0,Q} \right. \\ & \left. - 2\hat{\gamma}_{qq}^{(1),\text{TR}} (\beta_0 + \beta_{0,Q}) + \beta_{1,Q} \gamma_{qq}^{(0),\text{TR}} \right\} \ln^2\left(\frac{m^2}{\mu^2}\right) + \frac{1}{2} \left\{ \hat{\gamma}_{qq}^{(2),\text{TR}} \right. \\ & \left. - \left(4a_{qq,Q}^{(2),\text{TR}} - \zeta_2 \beta_{0,Q} \gamma_{qq}^{(0),\text{TR}} \right) (\beta_0 + \beta_{0,Q}) + \gamma_{qq}^{(0),\text{TR}} \beta_{1,Q} \right\} \ln\left(\frac{m^2}{\mu^2}\right) \\ & + 4\bar{a}_{qq,Q}^{(2),\text{TR}} (\beta_0 + \beta_{0,Q}) - \gamma_{qq}^{(0)} \beta_{1,Q}^{(2)} - \frac{\gamma_{qq}^{(0),\text{TR}} \beta_{0,Q} \zeta_3}{6} - \frac{\gamma_{qq}^{(1),\text{TR}} \beta_{0,Q} \zeta_2}{4} \\ & + 2\delta m_1^{(1)} \beta_{0,Q} \gamma_{qq}^{(0),\text{TR}} + \delta m_1^{(0)} \hat{\gamma}_{qq}^{(1),\text{TR}} + 2\delta m_1^{(-1)} a_{qq,Q}^{(2),\text{TR}} + a_{qq,Q}^{(3),\text{TR}} \quad (9) \end{aligned}$$

at 2- and 3-loops. Here, ζ_k denotes the Riemann ζ -function and $\gamma_{qq}^{(l),\text{TR}}$ are the transversity anomalous dimensions for $l = 0, 1, 2$ in LO [16], NLO [5, 17], and NNLO [18]. For the other quantities we refer to [9]. The new terms being calculated are $a_{qq,Q}^{(2),\text{TR}}(N)$, $\bar{a}_{qq,Q}^{(2),\text{TR}}(N)$ and $a_{qq,Q}^{(3),\text{TR}}(N)$, and for the higher values of N , $\hat{\gamma}_{qq}^{(2),\text{TR}}(N)$.

2. Results

2.1. Massive operator matrix elements

At $O(a_s^2)$ the massive operator matrix elements for transversity $\Delta_{\text{T}} A_{qq,Q}^{(2),\text{NS},\overline{\text{MS}}}$ are obtained for general values of N , *cf.* Eq. (8). The un-renormalized OME is computed to $O(\varepsilon)$ to also extract the function $\bar{a}_{qq,Q}^{(2),\text{TR}}(N)$. The new terms at 2-loops are $a_{qq,Q}^{(2),\text{TR}}$ and $\bar{a}_{qq,Q}^{(2),\text{TR}}$, *cf.* Eqs (8), (9):

$$a_{qq,Q}^{\text{TR},(2)}(N) = C_F T_F \left\{ -\frac{8}{3} S_3 + \frac{40}{9} S_2 - \left[\frac{224}{27} + \frac{8}{3} \zeta_2 \right] S_1 + 2\zeta_2 + \frac{(24 + 73N + 73N^2)}{18N(1+N)} \right\}, \quad (10)$$

$$\bar{a}_{qq,Q}^{\text{TR},(2)}(N) = C_F T_F \left\{ - \left[\frac{656}{81} + \frac{20}{9} \zeta_2 + \frac{8}{9} \zeta_3 \right] S_1 + \left[\frac{112}{27} + \frac{4}{3} \zeta_2 \right] S_2 - \frac{20}{9} S_3 \right. \\ \left. + \frac{4}{3} S_4 + \frac{1}{6} \zeta_2 + \frac{2}{3} \zeta_3 + \frac{(-144 - 48N + 757N^2 + 1034N^3 + 517N^4)}{216N^2(1+N)^2} \right\}, \quad (11)$$

with $S_k \equiv S_k(N)$ denoting the single harmonic sums.

At $O(a_s^3)$ the moments $N = 1$ to 13 were computed for the massive OME, as *e.g.*

$$\Delta_{\text{T}} A_{qq,Q}^{(3),\text{NS},\overline{\text{MS}}}(13) = C_F T_F \left\{ \left[\frac{1751446}{110565} C_A - \frac{7005784}{1216215} T_F (N_f + 2) \right] \ln^3 \left(\frac{m^2}{\mu^2} \right) \right. \\ - \left[\frac{20032048197492631}{2193567563187000} C_F + \frac{137401473299}{8027019000} C_A + \frac{93611152819}{3652293645} T_F \right] \ln^2 \left(\frac{m^2}{\mu^2} \right) \\ + \left[\left(\frac{1705832327329042449983}{263491335690022440000} + \frac{7005784}{45045} \zeta_3 \right) C_F + \left(\frac{3385454488248191237}{65807026895610000} \right. \right. \\ \left. \left. - \frac{7005784}{45045} \zeta_3 \right) C_A - \frac{458114791076413771}{6580702689561000} N_f T_F - \frac{217179304}{3648645} T_F \right] \ln \left(\frac{m^2}{\mu^2} \right) \\ - \left(\frac{7005784}{135135} B_4 - \frac{3502892}{15015} \zeta_4 + \frac{81735983092}{243486243} \zeta_3 - \frac{55376278299522733837425052493}{122080805651901196900800000} \right) C_F \\ + \left(\frac{3502892}{135135} B_4 - \frac{3502892}{15015} \zeta_4 + \frac{4061479439}{12162150} \zeta_3 - \frac{3486896974743882556775647}{12935029206601101600000} \right) C_A \\ - \left(\frac{279922752632160355860697}{3557133031815302940000} - \frac{56046272}{1216215} \zeta_3 \right) T_F N_f \\ \left. + \left(\frac{291526550302760070155303}{7114266063630605880000} - \frac{14011568}{173745} \zeta_3 \right) T_F \right\}, \quad (12)$$

where

$$B_4 = -4\zeta_2 \ln^2(2) + \frac{2}{3} \ln^4(2) - \frac{13}{2} \zeta_4 + 16\text{Li}_4\left(\frac{1}{2}\right).$$

Like for the massive OMEs in case of unpolarized deep-inelastic scattering the structure of $\Delta_{\text{T}} A_{qq,Q}^{(3),\text{NS},\overline{\text{MS}}}(N)$ is widely known for general values of N , except for the finite part $a_{qq,Q}^{(3),\text{NS},\text{TR}}$ and the 3-loop anomalous dimension $\hat{\gamma}_{qq}^{(2),\text{TR}}(N)$. One notices the cancellation of all ζ_2 terms in $\Delta_{\text{T}} A_{qq,Q}^{(3),\text{NS},\overline{\text{MS}}}(N)$ after renormalization.

2.2. Anomalous dimensions

The transversity anomalous dimension is given by

$$\gamma_{qq}^{\text{TR}}(N, a_s) = \sum_{i=1}^{\infty} a_s^i \gamma_{qq}^{(i),\text{TR}}(N). \quad (13)$$

From Eq. (9) one may determine the complete 2-loop anomalous dimension [5, 17] and the T_F -part of the 3-loop anomalous dimension [18]. We agree with the results given in [5, 17] and confirm the T_F -contributions for the moments $N = 1$ to 8 given in Ref. [18]. Furthermore, we newly obtain $\hat{\gamma}_{qq}^{(2),\text{TR}} = \gamma_{qq}^{(2),\text{TR}}(N_f + 1) - \gamma_{qq}^{(2),\text{TR}}(N_f)$ for $N = 9$ to 13, as *e.g.*

$$\begin{aligned} \hat{\gamma}_{qq}^{(2),\text{TR}}(N = 13) = & -C_F T_F \left[\frac{36713319015407141570017}{131745667845011220000} C_F - \frac{14011568}{45045} \right. \\ & \times (C_F - C_A) \zeta_3 + \frac{66409807459266571}{3290351344780500} T_F (1 + 2N_f) + \left. \frac{6571493644375020121}{65807026895610000} C_A \right]. \quad (14) \end{aligned}$$

2.3. A remark on the Soffer bound

If the Soffer inequality [19]

$$|\Delta_T f(x, Q^2)| \leq \frac{1}{2} [f(x, Q^2) + \Delta f(x, Q^2)] \quad (15)$$

holds for the non-perturbative PDF in Eq. (15) one may check its generalization from $f_i \rightarrow F_i$ for the corresponding structure functions. This includes the non-singlet evolution operator (Eq. (6), Ref. [20]) and the heavy flavor Wilson coefficient. At perturbative scales, it holds for the evolution operator [11], generalizing a NLO result from [5] to the moments $N = 1$ to 13 at NNLO. For the heavy quark Wilson coefficient in SIDIS we only know the massive OME so far. As shown in Ref. [11], a final conclusion can only be drawn knowing the yet undetermined massless Wilson coefficients. The difference $[A_{qq,Q}^V - A_{qq,Q}^{\text{TR}}](x)$ of the massive OMEs, shows a sign change to negative values for Q^2/m^2 in the physical range. For large scales $Q^2/m^2 \gg 1$ positive values are obtained.

REFERENCES

- [1] V. Barone, A. Drago, P.G. Ratcliffe, *Phys. Rep.* **359**, 1 (2002) [[hep-ph/0104283](#)].
- [2] A. Airapetian *et al.* [HERMES Collaboration], *Phys. Rev. Lett.* **94**, 012002 (2005) [[hep-ex/0408013](#)]; A. Airapetian *et al.* [HERMES Collaboration], *J. High Energy Phys.* **0806**, 017 (2008) [[arXiv:0803.2367 \[hep-ex\]](#)]; M. Alekseev *et al.* [COMPASS Collaboration], *Phys. Lett.* **B673**, 127 (2009) [[arXiv:0802.2160 \[hep-ex\]](#)]; V.Y. Alexakhin *et al.* [COMPASS Collaboration], *Phys. Rev. Lett.* **94**, 202002 (2005) [[hep-ex/0503002](#)]; COMPASS Collaboration, private communication; M.F. Lutz, *et al.*, [The PANDA Collaboration], [arXiv:0903.3905 \[hep-ex\]](#); A. Afanasev *et al.*, [hep-ph/0703288](#).

- [3] M. Anselmino *et al.*, [arXiv:0807.0173 \[hep-ph\]](#).
- [4] S. Aoki, M. Doui, T. Hatsuda, Y. Kuramashi, *Phys. Rev.* **D56**, 433 (1997) [[hep-lat/9608115](#)]; M. Göckeler *et al.*, *Nucl. Phys. Proc. Suppl.* **53**, 315 (1997) [[hep-lat/9609039](#)]; A. Ali Khan *et al.*, *Nucl. Phys. Proc. Suppl.* **140**, 408 (2005) [[hep-lat/0409161](#)]; D. Dolgov *et al.* [LHPC Collaboration and TXL Collaboration], *Phys. Rev.* **D66**, 034506 (2002) [[hep-lat/0201021](#)]; M. Diehl *et al.* [QCDSF Collaboration and UKQCD Collaboration], [hep-ph/0511032](#); M. Göckeler *et al.* [QCDSF Collaboration and UKQCD Collaboration], *Phys. Rev. Lett.* **98**, 222001 (2007) [[hep-lat/0612032](#)]; D. Renner, private communication.
- [5] W. Vogelsang, *Phys. Rev.* **D57**, 1886 (1998) [[hep-ph/9706511](#)].
- [6] M. Buza, Y. Matiounine, J. Smith, R. Migneron, W.L. van Neerven, *Nucl. Phys.* **B472**, 611 (1996) [[hep-ph/9601302](#)]; M. Buza, Y. Matiounine, J. Smith, W.L. van Neerven, *Nucl. Phys.* **B485**, 420 (1997) [[hep-ph/9608342](#)].
- [7] I. Bierenbaum, J. Blümlein, S. Klein, *Nucl. Phys.* **B780**, 40 (2007) [[hep-ph/0703285](#)]; I. Bierenbaum, J. Blümlein, S. Klein, C. Schneider, *Nucl. Phys.* **B803**, 1 (2008) [[arXiv:0803.0273 \[hep-ph\]](#)].
- [8] S. Alekhin, J. Blümlein, S. Klein, S. Moch, [arXiv:0908.2766 \[hep-ph\]](#).
- [9] I. Bierenbaum, J. Blümlein, S. Klein, *Nucl. Phys.* **B820**, 417 (2009) [[arXiv:0904.3563 \[hep-ph\]](#)]; [arXiv:0907.2615 \[hep-ph\]](#).
- [10] M. Buza, Y. Matiounine, J. Smith, W.L. van Neerven, *Eur. Phys. J.* **C1**, 301 (1998) [[hep-ph/9612398](#)]; J. Blümlein, A. De Freitas, W.L. van Neerven, S. Klein, *Nucl. Phys.* **B755**, 272 (2006) [[hep-ph/0608024](#)]; I. Bierenbaum, J. Blümlein, S. Klein, *Phys. Lett.* **B672**, 401 (2009) [[arXiv:0901.0669 \[hep-ph\]](#)]; [arXiv:0706.2738 \[hep-ph\]](#).
- [11] J. Blümlein, S. Klein, B. Tödtli, [arXiv:0909.1547 \[hep-ph\]](#).
- [12] B. Geyer, D. Robaschik, E. Wiczorek, *Fortsch. Phys.* **27**, 75 (1979).
- [13] P. Nogueira, *J. Comput. Phys.* **105**, 279 (1993).
- [14] J.A.M. Vermaseren, [math-ph/0010025](#).
- [15] M. Steinhauser, *Comput. Phys. Commun.* **134**, 335 (2001) [[hep-ph/0009029](#)].
- [16] F. Baldracchini, N.S. Craigie, V. Roberto, M. Socolovsky, *Fortsch. Phys.* **30**, 505 (1981); M.A. Shifman, M.I. Vysotsky, *Nucl. Phys.* **B186**, 475 (1981); A.P. Bukhvostov, G.V. Frolov, L.N. Lipatov, E.A. Kuraev, *Nucl. Phys.* **B258**, 601 (1985); J. Blümlein, *Eur. Phys. J.* **C20**, 683 (2001) [[hep-ph/0104099](#)]; A. Mukherjee, D. Chakrabarti, *Phys. Lett.* **B506**, 283 (2001) [[hep-ph/0102003](#)].
- [17] A. Hayashigaki, Y. Kanazawa, Y. Koike, *Phys. Rev.* **D56**, 7350 (1997) [[hep-ph/9707208](#)]; S. Kumano, M. Miyama, *Phys. Rev.* **D56**, 2504 (1997) [[hep-ph/9706420](#)].
- [18] J.A. Gracey, *Nucl. Phys.* **B662**, 247 (2003) [[hep-ph/0304113](#)]; *Nucl. Phys.* **B667**, 242 (2003) [[hep-ph/0306163](#)]; *J. High Energy Phys.* **0610**, 040 (2006) [[hep-ph/0609231](#)]; *Phys. Lett.* **B643**, 374 (2006) [[hep-ph/0611071](#)].

- [19] J. Soffer, *Phys. Rev. Lett.* **74**, 1292 (1995) [[hep-ph/9409254](#)].
- [20] J. Blümlein, H. Böttcher, A. Guffanti, *Nucl. Phys.* **B774**, 182 (2007) [[hep-ph/0607200](#)].
- [21] Slides: <http://prac.us.edu.pl/us2009/talks/Toedtli.pdf>