TaylUR 3, a multivariate arbitrary-order automatic differentiation package for Fortran 95

G.M. von Hippel [1]

NIC, DESY, Platanenallee 6, 15738 Zeuthen, Germany

Abstract

This new version of TaylUR is based on a completely new core, which now is able to compute the numerical values of all of a complex-valued function's partial derivatives up to an arbitrary order, including mixed partial derivatives.

Key words: automatic differentiation, higher derivatives, Fortran 95
PACS: 02.60.Jh, 02.30.Mv
1991 MSC: 41-04, 41A58, 65D25
Classification: 4.12 Other Numerical Methods, 4.14 Utility

NEW VERSION PROGRAM SUMMARY

Program Title: TaylUR
Program Version: 3.0
CPC Catalogue identifier: ADXR_v3_0
Licensing provisions: GPLv2 (see additional comments below)

No. of lines in distributed program: 6750
No. of bytes in distributed program: 19 162
Distribution format: tar.gz

Programming language: Fortran 95
Computer: Any computer with a conforming Fortran 95 compiler
Operating system: Any system with a conforming Fortran 95 compiler

CPC Catalogue identifier of previous version: ADXR_v2_0
Journal reference of previous version: Comput. Phys. Commun. **176** (2007) 710-711

Email address: georg.von.hippel@desy.de (G.M. von Hippel).
URL: http://www-zeuthen.desy.de/~hippel (G.M. von Hippel).
[1] Corresponding author

Nature of problem:
Problems that require potentially high orders of partial derivatives with respect to several variables or derivatives of complex-valued functions, such as e.g. momentum or mass expansions of Feynman diagrams in perturbative QFT, and which previous versions of TaylUR [1,2] cannot handle due to their lack of support for mixed partial derivatives.

Solution method:
Arithmetic operators and Fortran intrinsics are overloaded to act correctly on objects of a defined type `taylor`, which encodes a function along with its first few partial derivatives with respect to the user-defined independent variables. Derivatives of products and composite functions are computed using multivariate forms [3] of Leibniz's rule

$$D^\nu(fg) = \sum_{\mu\leq\nu} \frac{\nu!}{\mu!(\mu-\nu)!} D^\mu f D^{\mu-\nu} g$$

where $\nu = (\nu_1, \ldots, \nu_d)$, $|\nu| = \sum_{j=1}^d \nu_j$, $\nu! = \prod_{j=1}^d \nu_j!$, $D^\nu f = \frac{\partial^{|\nu|} f}{\partial^{\nu_1} x_1 \cdots \partial^{\nu_d} x_d}$, and $\mu < \nu$ iff either $|\mu| < |\nu|$ or $|\mu| = |\nu|$, $\mu_1 = \nu_1, \ldots \mu_k = \nu_k, \mu_{k+1} < \nu_{k+1}$ for some $k \in \{0, \ldots, d-1\}$, and of Fàa di Bruno's formula

$$D^\nu(f\circ g) = \sum_{p=1}^{|\nu|} (f^{(p)}\circ g) \sum_{s=1}^{|\nu|} \sum_{\{(k_1,\ldots,k_s;\lambda_1,\ldots,\lambda_s)\}} \frac{\nu!}{\prod_{j=1}^s k_j!\lambda_j!^{k_j}} \left(D^{\lambda_j} g\right)^{k_j}$$

where the sum is over

$$\{(k_1,\ldots,k_s;\lambda_1,\ldots,\lambda_s) \in \mathbb{Z}^{s(1+d)} : k_i > 0,\, \mathbf{0} < \lambda_1 < \ldots < \lambda_s, \sum_{i=1}^s k_i = p,\, \sum_{i=1}^s k_i\lambda_i = \nu\}.$$

An indexed storage system is used to store the higher-order derivative tensors in a one-dimensional array. The relevant indices $(k_1, \ldots, k_s; \lambda_1, \ldots, \lambda_s)$ and the weights occuring in the sums in Leibniz's and Fàa di Bruno's formula are precomputed at startup and stored in static arrays for later use.

Reasons for the new version:
The earlier version lacked support for mixed partial derivatives, but a number of projects of interest required them.

Summary of revisions:
The internal representation of a `taylor` object has changed to a one-dimensional array which contains the partial derivatives in ascending order, and in lexicographic order of the corresponding multiindex within the same order. The necessary mappings between multiindices and indices into the `taylor` objects' internal array are

computed at startup.
To support the change to a genuinely multivariate `taylor` type, the `DERIVATIVE` function is now implemented via an interface that accepts both the older format `derivative(f,mu,n)`$= \partial_\mu^n f$ and also a new format `derivative(f,mu(:))`$= D^\mu f$ that allows access to mixed partial derivatives. Another related extension to the functionality of the module is the `HESSIAN` function that returns the Hessian matrix of second derivatives of its argument.
Since the calculation of all mixed partial derivatives can be very costly, and in many cases only some subset is actually needed, a masking facility has been added. Calling the subroutine `DEACTIVATE_DERIVATIVE` with a multiindex as an argument will deactivate the calculation of the partial derivative belonging to that multiindex, and of all partial derivatives it can feed into. Similarly, calling the subroutine `ACTIVATE_DERIVATIVE` will activate the calculation of the partial derivative belonging to its argument, and of all partial derivatives that can feed into it.
Moreoever, it is possible to turn off the computation of mixed derivatives altogether by setting `Diagonal_taylors` to `.TRUE.`. It should be noted that any change of `Diagonal_taylors` or `Taylor_order` invalidates all existing `taylor` objects.
To aid the better integration of TaylUR into the HPSrc library [4], routines `SET_DERIVATIVE` and `SET_ALL_DERIVATIVES` are provided as a means of manually constructing a `taylor` object with given derivatives.

Restrictions:
Memory and CPU time constraints may restrict the number of variables and Taylor expansion order that can be achieved. Loss of numerical accuracy due to cancellation may become an issue at very high orders.

Unusual features:
These are the same as in previous versions, but are enumerated again here for clarity.
The complex conjugation operation assumes all independent variables to be real.
The functions `REAL` and `AIMAG` do *not* convert to real type, but return an result of type `taylor` (with the real/imaginary part of each derivative taken) instead. The user-defined functions `VALUE`, `REALVALUE` and `IMAGVALUE`, which return the value of a `taylor` object as as a complex number, and the real and imaginary part of this value, respectively, as a real number are also provided.
Fortran 95 intrinsics that are defined only for arguments of real type (`ACOS`, `AINT`, `ANINT`, `ASIN`, `ATAN`, `ATAN2`, `CEILING`, `DIM`, `FLOOR`, `INT`, `LOG10`, `MAX`, `MAXLOC`, `MAXVAL`, `MIN`, `MINLOC`, `MINVAL`, `MOD`, `MODULO`, `NINT`, `SIGN`) will silently take the real part of `taylor`-valued arguments unless the module variable `Real_args_warn` is set to `.TRUE.`, in which case they will return a quiet NaN value (if supported by the compiler) when called with a taylor argument whose imaginary part exceeds the module variable `Real_args_tol`.
In those cases where the derivative of a function becomes undefined at certain points (as for `ABS`, `AINT`, `ANINT`, `MAX`, `MIN`, `MOD`, and `MODULO`), while the value is well defined, the derivative fields will be filled with quiet NaN values (if supported by the compiler).

Additional comments:
This version of TaylUR is released under the second version of the GNU General Public License (GPLv2). Therefore anyone is free to use or modify the code for their own calculations. As part of the licensing, it is requested that any publications including results from the use of TaylUR or any modification derived from it cite refs. [1,2] as well as this paper. Finally, users are also requested to communicate to the author details of such publications, as well as of any bugs found or of required or useful modifications made or desired by them.

Running time:
The running time of TaylUR operations grows rapidly with both the number of variables and the Taylor expansion order. Judicious use of the masking facility to drop unneeded higher derivatives can lead to significant accelerations, as can activation of the `Diagonal_taylors` variable whenever mixed partial derivatives are not needed.

Acknowledgments:
The author thanks Alistair Hart for helpful comments and suggestions. This work is supported by the Deutsche Forschungsgemeinschaft in the SFB/TR 09.

References:

[1] G. M. von Hippel, TaylUR, an arbitrary-order diagonal automatic differentiation package for Fortran 95, Comput. Phys. Commun. **174** (2006) 569 [physics/0506222].

[2] G. M. von Hippel, New version announcement for TaylUR, an arbitrary-order diagonal automatic differentiation package for Fortran 95, Comput. Phys. Commun. **176** (2007) 710 [arXiv:0704.0274].

[3] G. M. Constantine, T. H. Savits, A multivariate Faa di Bruno formula with applications, Trans. Amer. Math. Soc. **348** (1996) 2:503.

[4] A. Hart, G. M. von Hippel, R. R. Horgan, E. H. Müller, Automated generation of lattice QCD Feynman rules, Comput. Phys. Commun. **180** (2009) 2698 [arXiv:0904.0375].