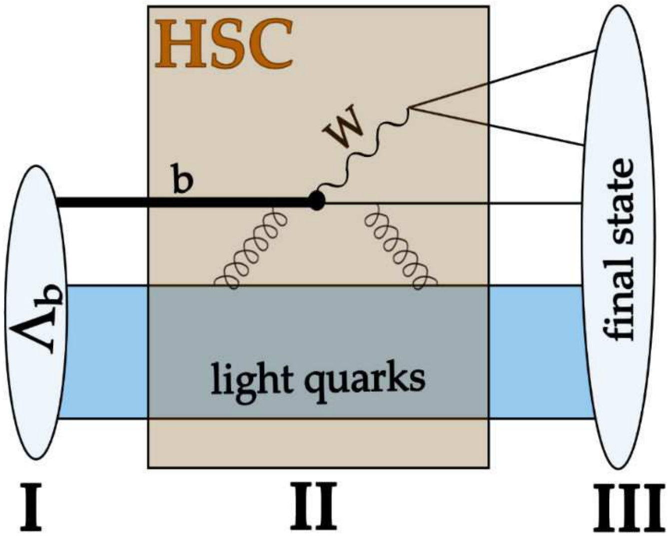

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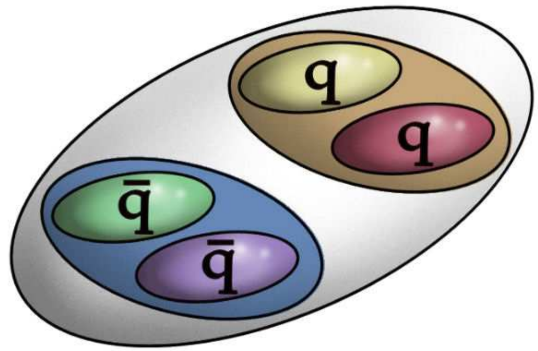


b-baryon light-cone distribution amplitudes and hidden-bottom tetraquarks

Christian Hambrock DESY, technische universität dortmund

Thursday, 03.03.2011

b-baryon LCDAs

Overview

b-baryon light-cone distribution amplitudes (LCDAs)

- Definition via non-local currents
- Calculation in QCD sum rules (QCDSR)
- Constrain model by QCDSR
- Conclusion & outlook

Introduction

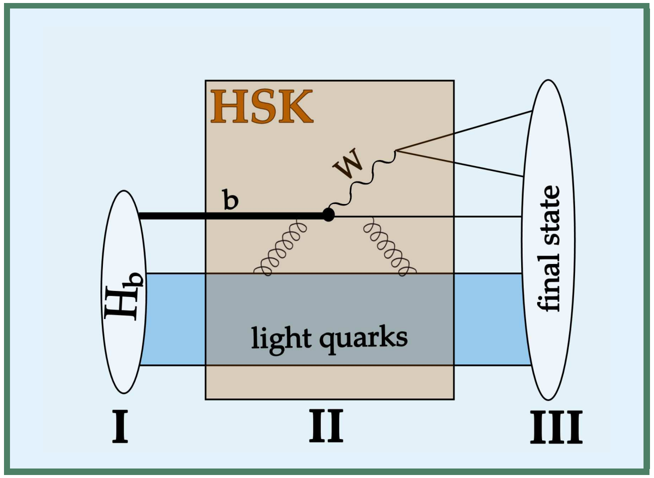
- Heavy baryons copiously produced at LHC
- Weak decays through FCNC

  $\Rightarrow$ BSM limits
- Calculating decays in Light-Cone Sum Rules (LCSRs)

  $\Rightarrow$ Non-perturbative input (parton distributions)

  $\Rightarrow$ Light-cone distribution amplitudes (LCDAs)
- First step: Λ_b LCDAs by [Braun, Ball and Gardi (2008)]
- New [A. Ali, C.H., A. Parkhomenko] :
  - All ground state b-baryons
  - Inclusion of s-quark

Framework of b-baryon LCDAs

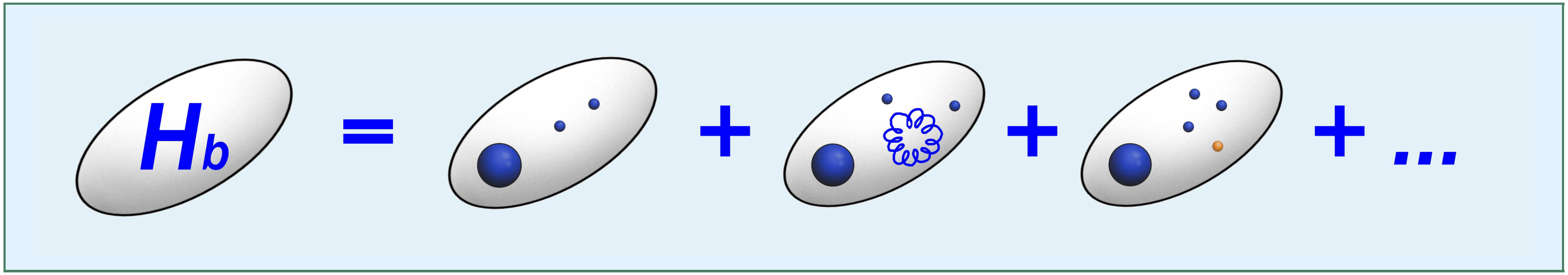


Energy scales:

- I bound initial state $\Lambda_{QCD} \approx 300\text{MeV}$
- II Hard Scattering Kernel (HSK) with $m_b \approx 4 - 5$ GeV
- III bound final state $\Lambda_{QCD} \approx 300\text{MeV}$

Very complicated $\Rightarrow$ 3 Simplifications

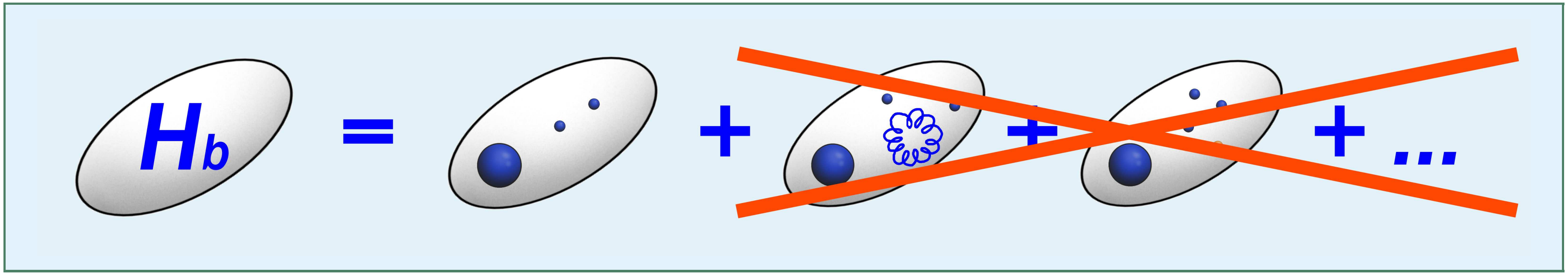
Simplification I: Valence quarks



- Fock basis

$$\langle \mathrm{f} | \, \mathrm{HSK} \, | H_b \rangle = \sum_I \langle \mathrm{f} | \, \mathrm{HSK} \, | P_I \rangle \langle P_I | \, H_b \rangle$$

Simplification I: Valence quarks



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- Valencequark approximation with $\langle P_0 | \,\widehat{=}\, \langle 0 |\, q_{1\alpha} q_{2\beta} b_\gamma$

$$\langle \mathrm{f} |\, \mathrm{HSK}\, | H_b \rangle \approx \langle \mathrm{f} |\, \mathrm{HSK}\, | P_0 \rangle \langle P_0 |\, H_b \rangle$$

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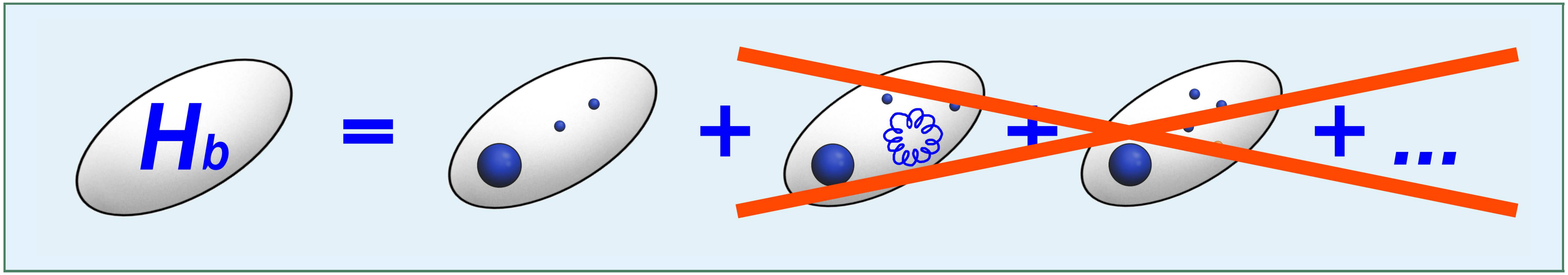

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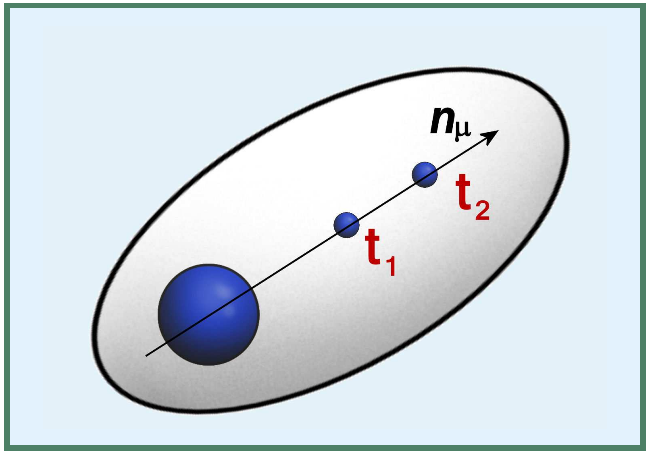
$$\langle \mathrm{f}|\,\mathrm{HSK}\,|H_b\rangle \approx \langle \mathrm{f}|\,\mathrm{HSK}\,|P_0\rangle\,\langle P_0|\,H_b\rangle$$

- LCDAs defined through 3-quark operators :

$$\langle P_0|\,H_b\rangle \mathrel{\widehat{=}} \langle 0|\, q_{1\alpha} q_{2\beta} b_\gamma\,|H_b\rangle$$

Simplification II: Light-cone

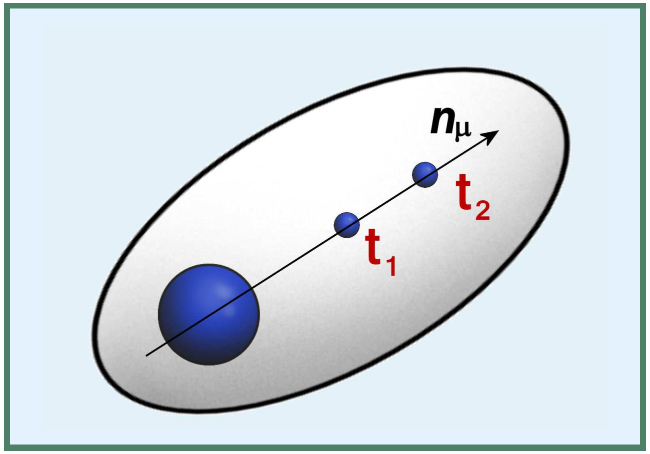


- Dominant contributions from fast moving light quarks

  ⇒ quarks move near light-cone

  ⇒ light-cone approximation (⇒ LCDA)

Simplification II: Light-cone



- Dominant contributions from fast moving light quarks

  $\Rightarrow$ quarks move near light-cone

  $\Rightarrow$ light-cone approximation ($\Rightarrow$ LCDA)

  $\Rightarrow$ reduced to $2\times$ one-dimensional problem

  $$\Rightarrow \langle P_0 |\, H_b \rangle \widehat{=} \langle 0 |\, q_{1\alpha}(t_1) q_{2\beta}(t_2) b_\gamma(x)\, | H_b \rangle$$

Simplification III: HQ limit

- LO expansion in $\frac{\Lambda_{QCD}}{m_b} \approx \frac{1}{10} \Rightarrow m_b \to \infty$ (heavy-quark-limit)

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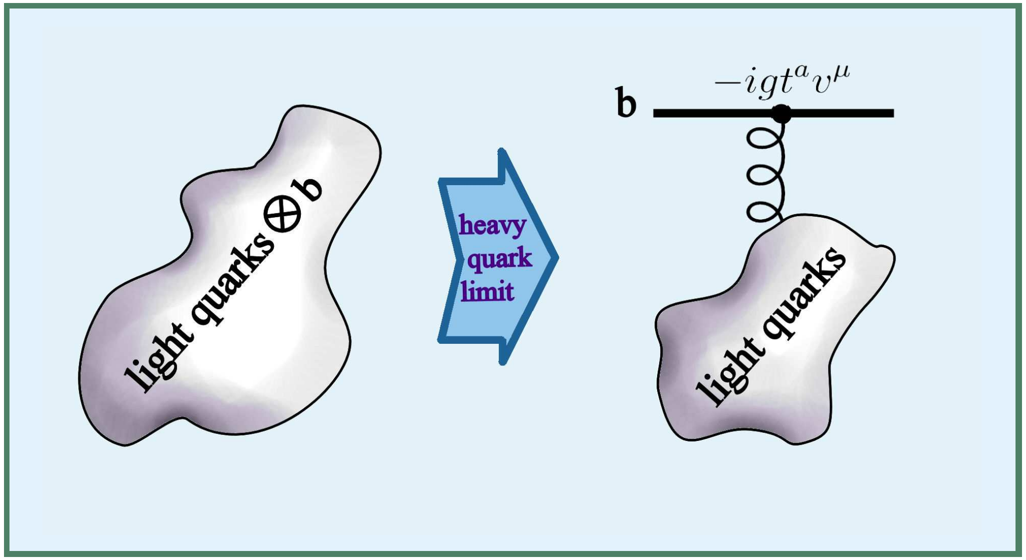


- b-quark $\longrightarrow$ onshell $\Rightarrow$ moving with v_μ
- No spin interactions with light quarks

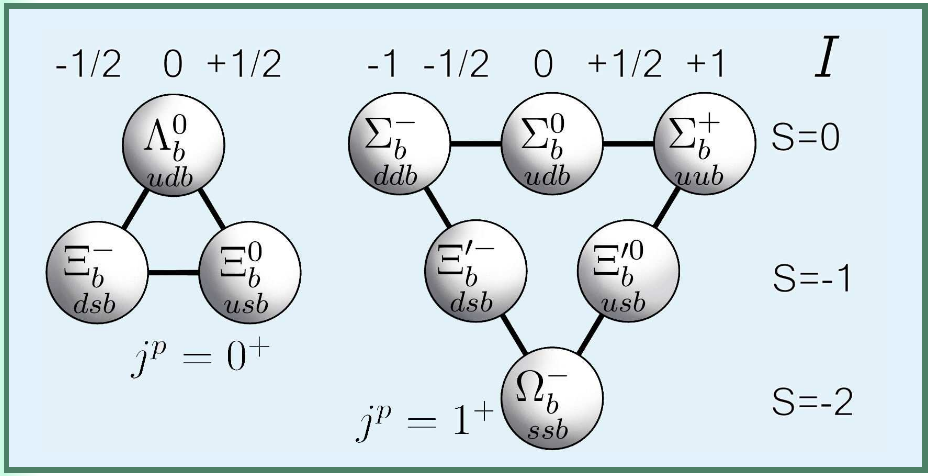
$$\Rightarrow \langle P_0 |\ H_b \rangle \mathrel{\widehat{=}} \langle 0 |\ q_{1\alpha}(t_1) q_{2\beta}(t_2)\ b(t \cdot v)\ | H_b \rangle \times \epsilon_\gamma$$

Multiplets in HQ limit

HQ-limit

$\Rightarrow$ Heavy quark spin decoupling

$\Rightarrow$ b-baryons classified by j^p of light quarks



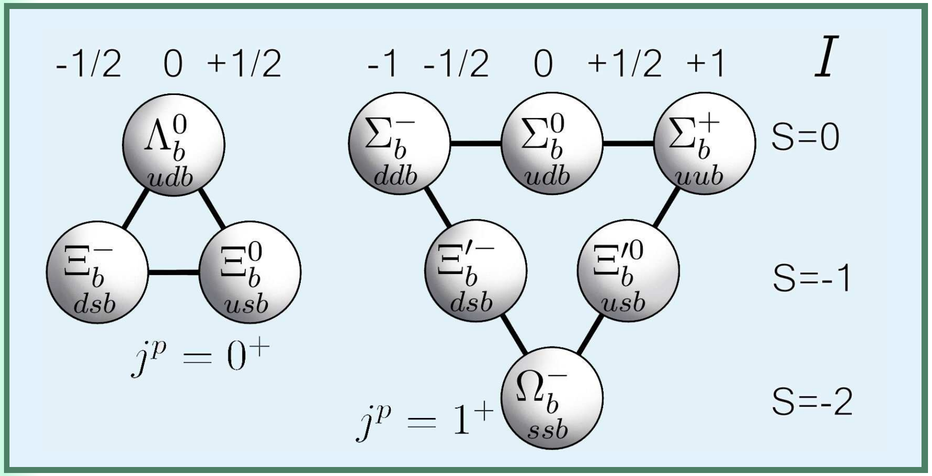
Baryon	J^P	Exp. mass [MeV]	HQET mass [MeV]
Λ_b	$1/2^+$	5620.2 ± 1.6	5637^{+68}_{-56}
Σ_b^+	$1/2^+$	5807.8 ± 2.7	5809^{+82}_{-76}
Σ_b^-	$1/2^+$	5815.2 ± 2.0	5809^{+82}_{-76}
Σ_b^{*+}	$3/2^+$	5829.0 ± 3.4	5835^{+82}_{-77}
Σ_b^{*-}	$3/2^+$	5836.4 ± 2.8	5835^{+82}_{-77}
Ξ_b^-	$1/2^+$	$\{ 5792.4 \pm 3.0$; 5790.9 ± 2.7	5780^{+73}_{-68}
Ξ_b'	$1/2^+$		5903^{+81}_{-79}
$\Xi_b'^*$	$3/2^+$		5903^{+81}_{-79}
Ω_b^-	$1/2^+$	$\{ 6165 \pm 16$; 6054.4 ± 6.9	6036 ± 81
Ω_b^*	$3/2^+$		6063^{+83}_{-82}

Multiplets in HQ limit

HQ-limit

- $\Rightarrow$ Heavy quark spin decoupling
  - $\Rightarrow$ b-baryons classified by j^p of light quarks

HQ spin effects $\approx$ 15 MeV



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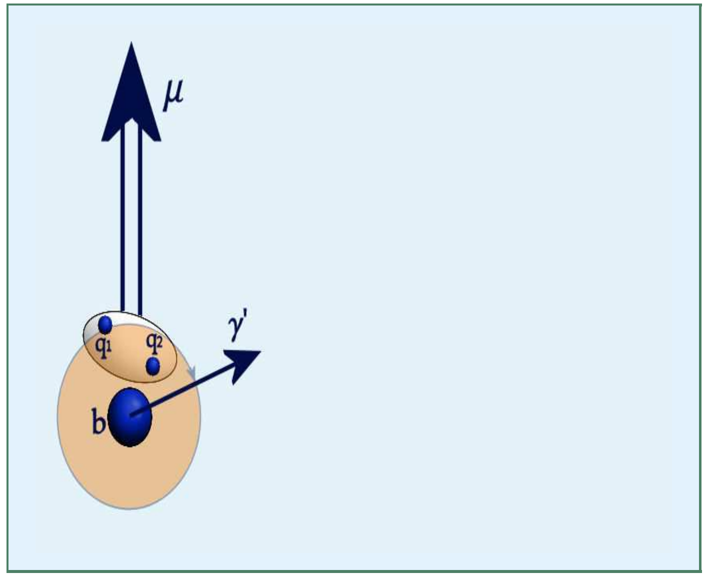
Restoring Lorentz invariance

- Currents contain spin $1/2$ and $3/2$:

$$\psi\varepsilon^{\mu}\epsilon_{\gamma} = \psi {R^{3/2\mu}}_{\gamma} + \psi {R^{1/2\mu}}_{\gamma}$$



Restoring Lorentz invariance

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$$\psi\varepsilon^{\mu}\epsilon_{\gamma} = \psi R^{3/2\mu}{}_{\gamma} + \psi R^{1/2\mu}{}_{\gamma}$$

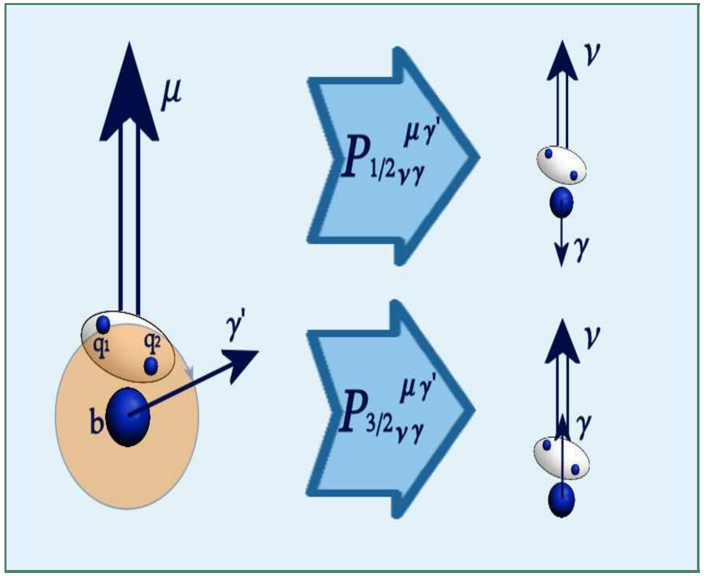
$\Rightarrow$ Define global projectors:

$$(P^{3/2})^{\mu}{}_{\nu\gamma}{}^{\gamma'} = \left[\delta^{\mu}_{\nu} - \frac{1}{3}(\gamma^{\mu} + v^{\mu})\gamma_{\nu}\right]_{\gamma}^{\gamma'}$$

$$(P^{1/2})^{\mu}{}_{\nu\gamma}{}^{\gamma'} = \left[\frac{1}{3}(\gamma^{\mu} + v^{\mu})\gamma_{\nu}\right]_{\gamma}^{\gamma'}$$

$P^2 = P$ and $P_{3/2} + P_{1/2} = \mathbb{1}$

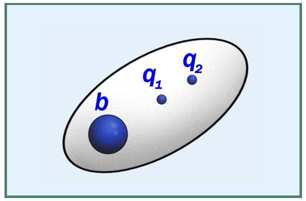
$$\Rightarrow \quad R^{I\mu}{}_{\gamma} = (P^{I})^{\mu}{}_{\nu\gamma}{}^{\gamma'}\varepsilon^{\nu}\epsilon_{\gamma'}$$



Non-local currents

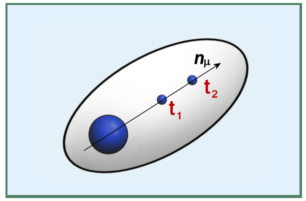
$$\langle P_I |\ H_b \rangle$$

$\Downarrow$ valence quark



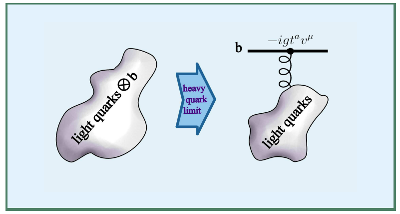
$$\langle 0 |\, q_{1\alpha} q_{2\beta} b_\gamma \,| H_b \rangle$$

$\Downarrow$ light cone



$$\langle 0 |\, q_{1\alpha}(t_1) q_{2\beta}(t_2) b_\gamma(x) \,| H_b \rangle$$

$\Downarrow$ HQ limit



$$\langle 0 |\, q_{1\alpha}(t_1) q_{2\beta}(t_2)\ b(t \cdot v) \,| H_b \rangle \times \epsilon_\gamma$$

$\Downarrow$ expand in $\Gamma_{\alpha\beta} = \gamma_{5\alpha\beta}, \gamma^\mu_{\alpha\beta} \ldots,\ \ t \longrightarrow 0$

$$\langle 0 |\, q_{1\alpha}(t_1) q_{2\beta}(t_2)\ b(0) \,| H_b \rangle \times \epsilon_\gamma = \sum_\Gamma \tilde{\psi}_\Gamma(t_1, t_2) \Gamma_{\alpha\beta} \times \epsilon_\gamma$$

$\Downarrow$ project $\Gamma_{\alpha\beta}$ in current

$$\langle 0 |\, [q_1(t_1) \Gamma q_2(t_2)]\ b(0) \,| H_b \rangle \times \epsilon_\gamma = \tilde{\psi}_\Gamma(t_1, t_2) \times \epsilon_\gamma \qquad j^p = 0^+$$

$$\langle 0 |\, [q_1(t_1) \Gamma^\mu q_2(t_2)]\ b(0) \,| H_b \rangle \times \epsilon_\gamma = \tilde{\psi}_\Gamma(t_1, t_2)\ \times \varepsilon^\mu\ \epsilon_\gamma \qquad j^p = 1^+$$

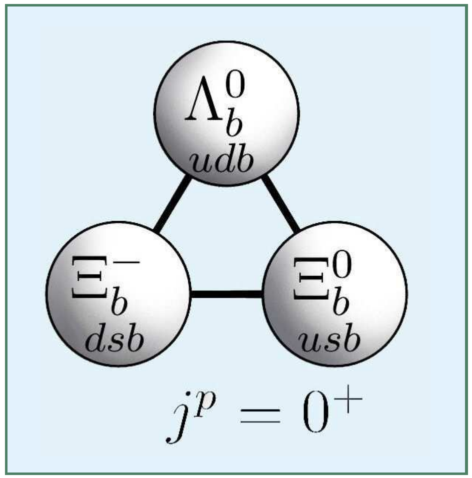
LCDAs $j^p = 0^+$

- 4 linear independent operators [Ball et al. (2008)] :

$$
\begin{array}{llllll}
\langle 0|[q_1(t_1) & \frac{1}{v_+} & C\gamma_5 \not{n} & q_2(t_2)]b(0)|H_b\rangle & = & f_H^{(2)}\,\Psi_2(t_1,t_2) \\
\langle 0|[q_1(t_1) & 1 & C\gamma_5 i\sigma_{\bar{n}n} & q_2(t_2)]b(0)|H_b\rangle & = & 2f_H^{(1)}\,\Psi_3^\sigma(t_1,t_2) \\
\langle 0|[q_1(t_1) & 1 & C\gamma_5 & q_2(t_2)]b(0)|H_b\rangle & = & f_H^{(1)}\,\Psi_3^s(t_1,t_2) \\
\langle 0|[q_1(t_1) & v_+ & C\gamma_5 \not{\bar{n}} & q_2(t_2)]b(0)|H_b\rangle & = & f_H^{(2)}\Psi_4(t_1,t_2)
\end{array}
$$

C – charge conjugation

$n^\mu,\ \bar{n}^\mu$ – light-like vectors $\quad n^2 = \bar{n}^2 = 0$



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\end{array}
$$

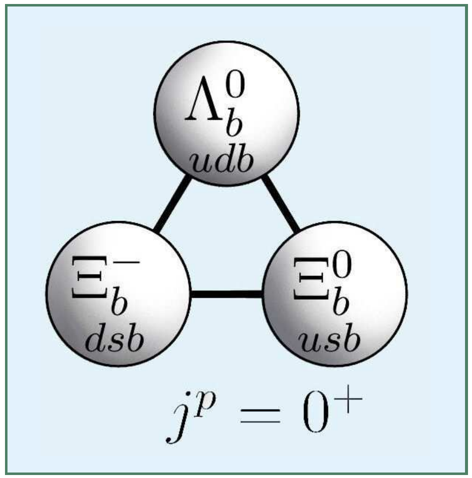
C – charge conjugation

n^μ, $\bar{n}^\mu$ – light-like vectors $\quad n^2 = \bar{n}^2 = 0$

- Frame $\vec{v} \parallel \hat{e}_z \quad v_+ < 1$:

$v_\mu = \frac{1}{2}(\frac{1}{v_+} n_\mu + v_+ \bar{n}_\mu) \Rightarrow$ “twist”

(analogue to heavy mesons) [Grozin, Neubert (1997)]



LCDAs $j^p = 1^+$ parallel

- Split $\varepsilon^\mu = \varepsilon_\parallel^\mu + \varepsilon_\perp^\mu$ in $\parallel$ and $\perp$ to $\hat{e}_z$

LCDAs $j^p = 1^+$ parallel

- Split $\varepsilon^\mu = \varepsilon^\mu_\parallel + \varepsilon^\mu_\perp$ in $\parallel$ and $\perp$ to $\hat{e}_z$
- 4 linear independent operators (longitudinal polarization)

[A. Ali, C.H., A. Parkhomenko] (compare vector mesons [Ball et al. (1998)]) :

$$
\begin{aligned}
\langle 0|[q_1(t_1) \quad \tfrac{1}{v_+} \quad w^\mu C \not n \quad q_2(t_2)]b(0)|H_b\rangle &= f_H^{(2)}\, \Psi_{\parallel 2}(t_1,t_2)\varepsilon^\mu_\parallel \\
\langle 0|[q_1(t_1) \quad 1 \quad w^\mu C i\sigma_{\bar n n} \quad q_2(t_2)]b(0)|H_b\rangle &= 2 f_H^{(1)}\, \Psi^\sigma_{\parallel 3}(t_1,t_2)\varepsilon^\mu_\parallel \\
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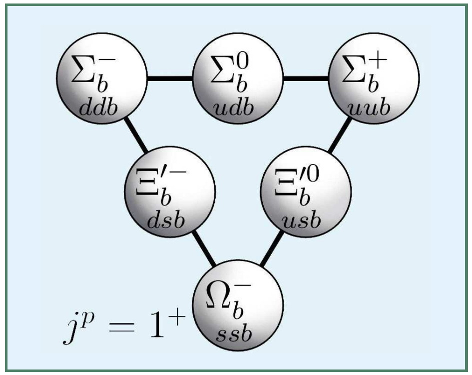
-\langle 0|[q_1(t_1) \quad v_+ \quad w^\mu C \not{\bar n} \quad q_2(t_2)]b(0)|H_b\rangle &= f_H^{(2)}\Psi_{\parallel 4}(t_1,t_2)\varepsilon^\mu_\parallel
\end{aligned}
$$

$w^\mu = (\bar n^\mu - n^\mu)/2$

C – charge conjugation

$n^\mu,\ \bar n^\mu$ – light-like vectors $\quad n^2 = \bar n^2 = 0$



- Frame: $v_\mu = \frac{1}{2}(\frac{1}{v_+} n_\mu + v_+ \bar n_\mu) \Rightarrow$ “twist”

LCDAs $j^p = 1^+$ transverse

- Split $\varepsilon^\mu = \varepsilon^\mu_\parallel + \varepsilon^\mu_\perp$ in $\parallel$ and $\perp$ to $\hat{e}_z$
- 4 linear independent operators (transverse polarization)

[A. Ali, C.H., A. Parkhomenko] (compare vector mesons [Ball et al. (1998)]):

$$\langle 0|[q_1(t_1) \quad \frac{1}{v_+} \quad Ci\sigma_{n\perp}{}^{\mu} \quad q_2(t_2)]b(0)|H_b\rangle \quad = \quad f_H^{(2)}\, \Psi_{\perp 2}(t_1, t_2)\varepsilon^\mu_\perp$$

$$\langle 0|[q_1(t_1) \quad 1 \quad C\gamma^\mu_\perp \quad q_2(t_2)]b(0)|H_b\rangle \quad = \quad 2f_H^{(1)}\, \Psi^{\sigma}{}_{\perp 3}(t_1, t_2)\varepsilon^\mu_\perp$$

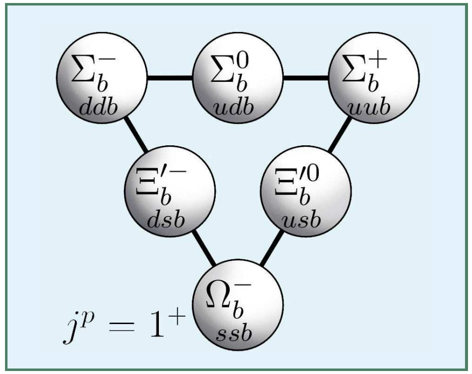
$$\langle 0|[q_1(t_1) \quad 1 \quad Ci\sigma_{\bar{n}n}\gamma^\mu_\perp \quad q_2(t_2)]b(0)|H_b\rangle \quad = \quad f_H^{(1)}\, \Psi^{s}{}_{\perp 3}(t_1, t_2)\varepsilon^\mu_\perp$$

$$\langle 0|[q_1(t_1) \quad v_+ \quad Ci\sigma_{\bar{n}\perp}{}^{\mu} \quad q_2(t_2)]b(0)|H_b\rangle \quad = \quad f_H^{(2)}\Psi_{\perp 4}(t_1, t_2)\varepsilon^\mu_\perp$$

$\gamma^\mu_\perp = \gamma^\mu - \left(\bar{\not{n}}\not{n} + \not{n}\bar{\not{n}}\right)/2$

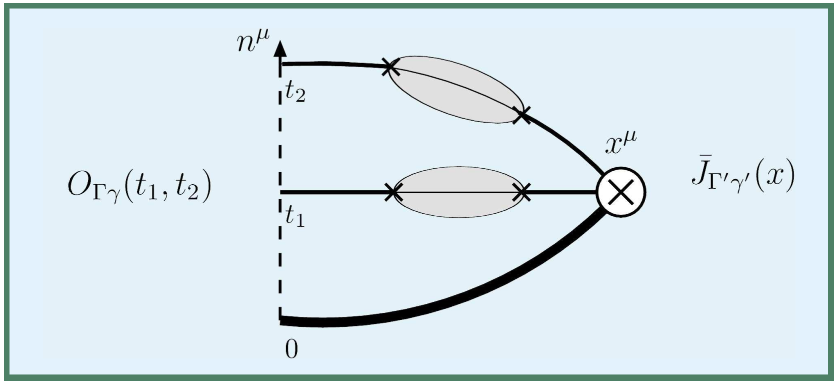
C – charge conjugation

$n^\mu,\ \bar{n}^\mu$ – light-like vectors $n^2 = \bar{n}^2 = 0$



- Frame: $v_\mu = \frac{1}{2}\left(\frac{1}{v_+}n_\mu + v_+\bar{n}_\mu\right) \Rightarrow$ "twist"

Correlation function

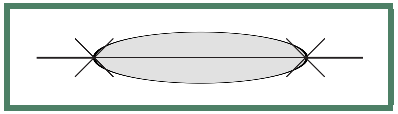


- Nonlocal vertices of different twist defined by

$$\tilde{\psi}_\Gamma(t_1, t_2) = \langle 0| \left[q_1(t_1)\Gamma q_2(t_2)\right] b(0) \left|H_b\right\rangle \equiv \boxed{\langle 0| O_\Gamma(t_1, t_2) \left|H_b\right\rangle}$$

- Local approximation for baryon

$$\left|H_b\right\rangle \approx \left[\bar{q}_1(x)\Gamma'\bar{q}_2(x)\right]\bar{b}(x) \left|0\right\rangle \equiv \boxed{\bar{J}_{\Gamma'}(x) \left|0\right\rangle}$$

- Quark propagators in QCD background  (OPE)

$\Rightarrow$ QCD-sum-rule calculation (first: 2 remarks)

Remark I: Local currents

- $t_i \to 0 \;\Rightarrow$ 2 operators survive
- $\psi^I(0,0) \equiv 1 \Rightarrow$ def. of decay constants:

$$
\begin{aligned}
\langle 0|\,[q_1(0) C\Gamma' q_2(0)]\, b(0)|H(v)\rangle &= f_H^{(1)} \\
\langle 0|\,[q_1(0) C\Gamma' \not{v} q_2(0)]\, b(0)|H(v)\rangle &= f_H^{(2)}
\end{aligned}
$$

$$
j^p = 0^+ : \qquad \Gamma' = \gamma_5
$$

$$
j^p = 1^+ : \qquad \Gamma' = \gamma_\parallel,\, \gamma_\perp
$$

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$$j^p = 0^+ : \qquad \Gamma' = \gamma_5$$

$$j^p = 1^+ : \qquad \Gamma' = \gamma_\parallel,\, \gamma_\perp$$

- Linear combination of local currents:

$$\bar{J}_{\Gamma'}(x) = \left[\bar{q}_2(x)\,[A + (1-A)\,\not{v}]\,\Gamma' C^T \bar{q}_1(x)\right]\bar{b}(x)$$

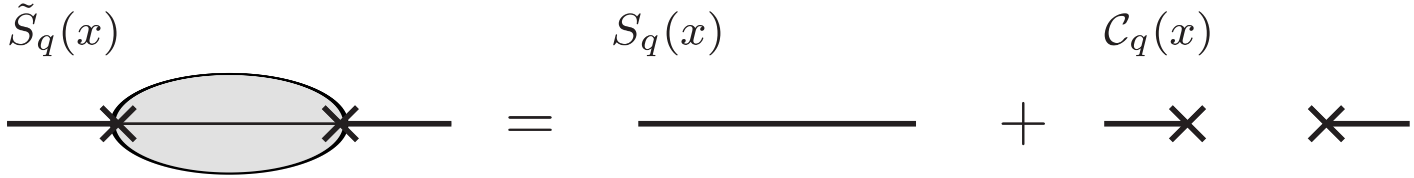
$\Rightarrow$ Variation in $A \in [0,1] \Rightarrow$ Errors , Result for $A = 1/2$

(supported by constituent quark model picture [Ball et al. (2008)])

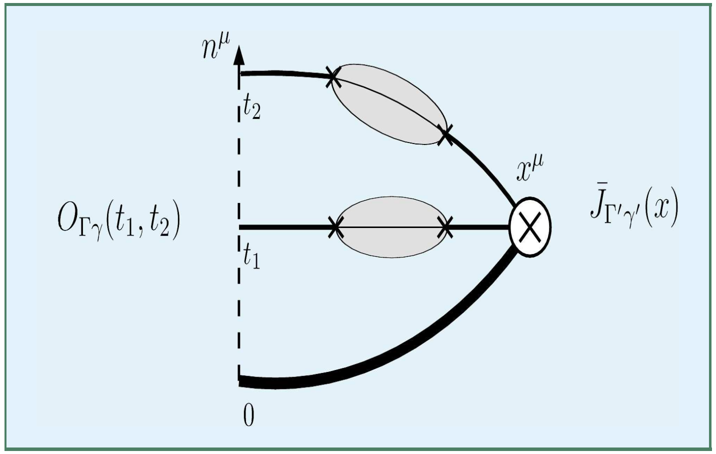
Problem: Errors large!

Remark II: Propagators

Non-local condensates describe propagation in QCD background:



$$S_q(x) = \frac{i\not{x}}{2\pi^2 x^4} - \frac{m}{4\pi^2 x^2} \qquad \mathcal{C}_q(x) = \frac{1}{12}\langle \bar{q}(x) q(0)\rangle$$



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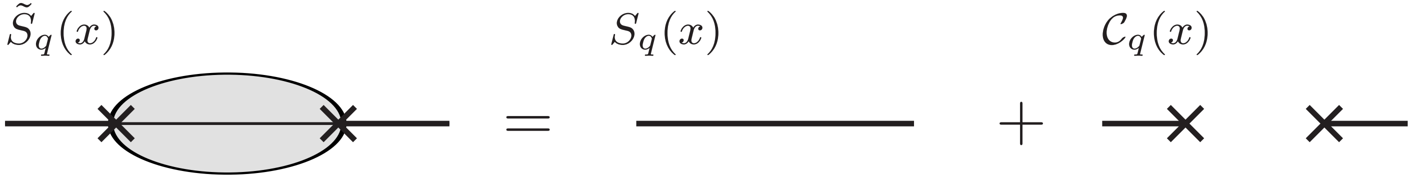
Non-local condensates describe propagation in QCD background:



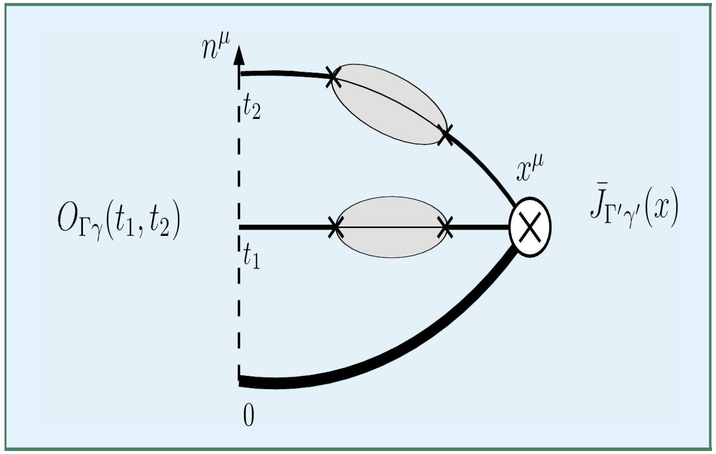
$$S_q(x) = \frac{i\not{x}}{2\pi^2 x^4} - \frac{m}{4\pi^2 x^2} \qquad \mathcal{C}_q(x) = \frac{1}{12}\langle \bar{q}(x) q(0) \rangle$$

Shape functions (OPE): [Braun et al., (1994, 2003)]

$$\mathcal{C}_q(x) = \langle \bar{q} q \rangle \int_0^\infty d\nu \, e^{\nu x^2/4} f(\nu)$$

$$f(\nu) = \frac{\lambda^{a-2}}{\Gamma(a-2)} \nu^{1-a} e^{-\lambda/\nu}$$

$$a = 3 + 4\lambda/m_0^2$$



$\langle \bar{q} q \rangle$ – local condensate, λ – correlation length,

m_0 – ratio quark-gluon and quark condensates

QCD sum rule calculation

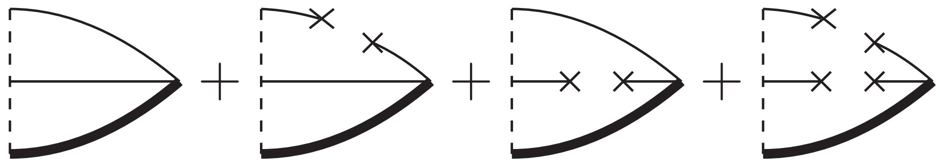
- Correlation function (by double Fourier transform):

$$\Pi_{\Gamma\Gamma'}(\omega_1,\omega_2;E) = i\int_{-\infty}^{\infty}\frac{dt_1\,dt_2}{(2\pi)^2}\,e^{i(\omega_1 t_1+\omega_2 t_2)}\int d^4x\,e^{-iE(vx)}\,\langle 0|\mathcal{O}^{\Gamma}(t_1,t_2)\,\bar{J}^{\Gamma'}(x)|0\rangle$$

also used: $\omega_1+\omega_2=\omega$: light quark total momentum

$u\omega=\omega_1,\ (1-u)\omega=\omega_2$ u: light quark momentum fraction

- Diagramatically:

$\Pi(\omega,u;E) =$ 

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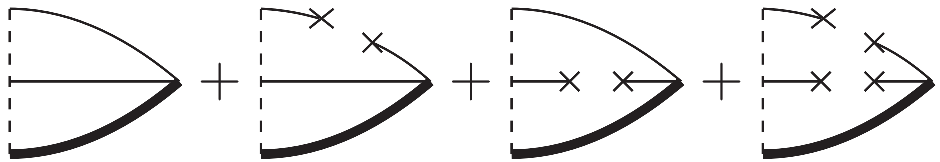


- Borel-Transform $\boldsymbol{B} \Rightarrow$ sum rule:

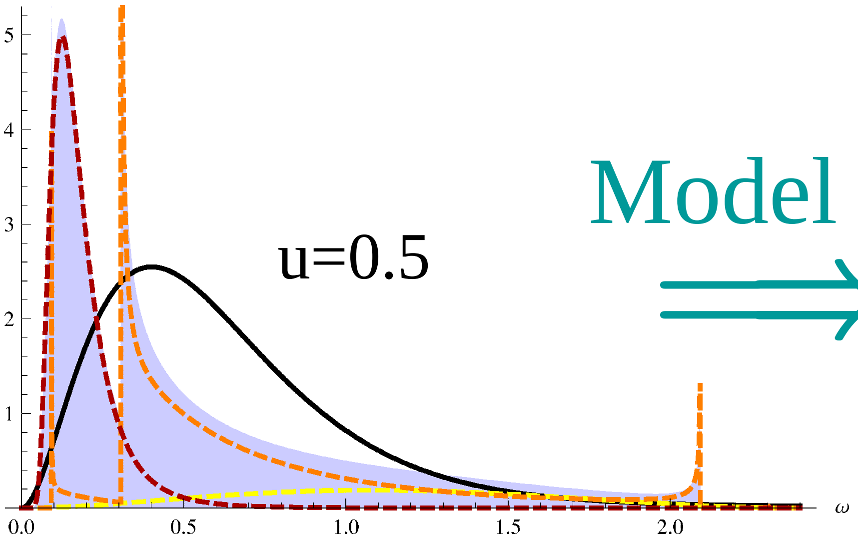
$$|f_H|^2\,\psi^{\Gamma}(\omega,u)\,e^{-\bar{\Lambda}_H/\tau} = \boldsymbol{B}[\Pi](\omega,u;\tau,s_0)$$

s_0 – momentum cutoff from quark-hadron duality

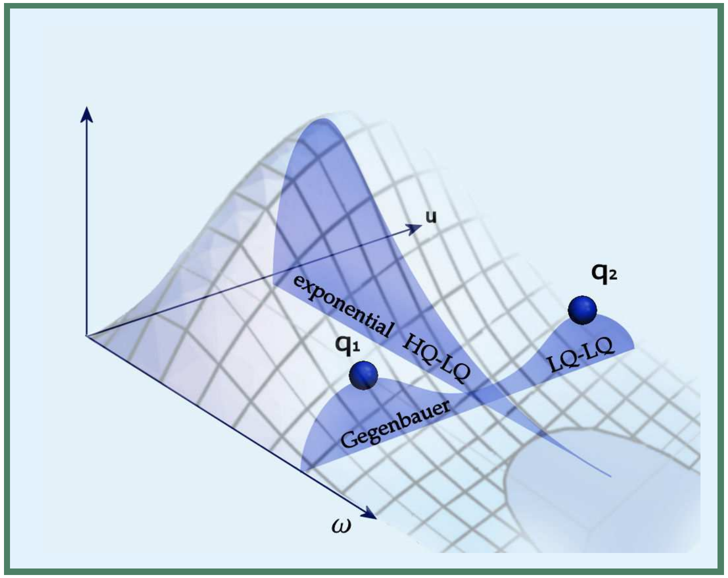
$\bar{\Lambda}_H = m_H - m_b$ – effective mass

QCDSR → Model

- Wrong asymptotic behavior of QCDSR



Model fits $\Longrightarrow$



- Model leading twist:

$$\tilde{\psi}_2(\omega, u) = \omega^2 u(1-u) \sum_{n=0}^{2} \frac{a_n}{{\epsilon_n}^4} C_n^{3/2}(2u-1)\, \mathrm{e}^{-\omega/\epsilon_n}$$

QCDSR $\rightarrow$ Model

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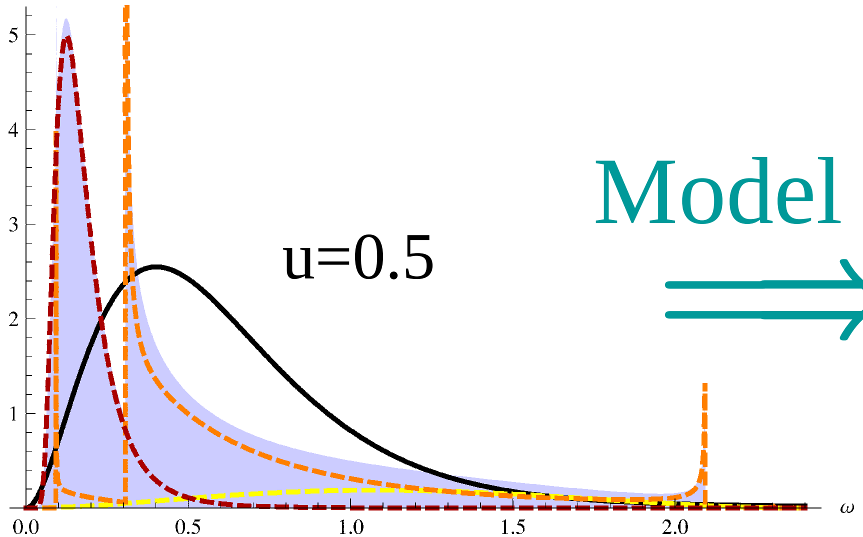

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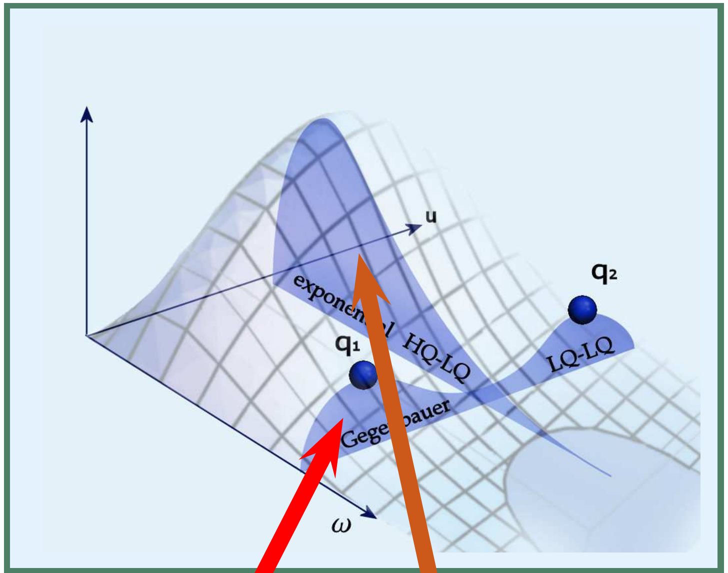

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light-light: Gegenbauer heavy-light: Exponential

Model functions

- Model functions of all twists

$$\tilde{\psi}_2(\omega,u) = \omega^2 u(1-u)\sum_{n=0}^{2}\frac{a_n^{(2)}}{\epsilon_n^{(2)\,4}}\,C_n^{3/2}(2u-1)\,\mathrm{e}^{-\omega/\epsilon_n^{(2)}}$$

$$\tilde{\psi}_{3s}(\omega,u) = \frac{\omega}{2}\sum_{n=0}^{2}\frac{a_n^{(3)}}{\epsilon_n^{(3)\,3}}\,C_n^{1/2}(2u-1)\,\mathrm{e}^{-\omega/\epsilon_n^{(3)}}$$

$$\tilde{\psi}_{3\sigma}(\omega,u) = \frac{\omega}{2}\sum_{n=0}^{3}\frac{b_n^{(3)}}{\eta_n^{(3)\,3}}\,C_n^{1/2}(2u-1)\,\mathrm{e}^{-\omega/\eta_n^{(3)}}$$

$$\tilde{\psi}_4(\omega,u) = \sum_{n=0}^{2}\frac{a_n^{(4)}}{\epsilon_n^{(4)\,2}}\,C_n^{1/2}(2u-1)\,\mathrm{e}^{-\omega/\epsilon_n^{(4)}}$$

light-light: Gegenbauer   heavy-light: Exponential

Numerical analysis

Estimates of the parameters entering the theoretical models for the heavy baryon LCDAs

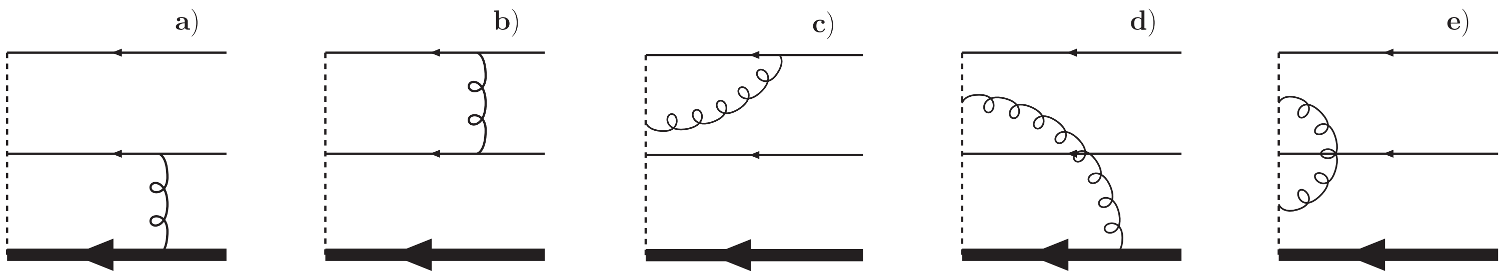
H_Q	t	$\varepsilon_0^{(t)}$	$\varepsilon_1^{(t)}$	$\varepsilon_2^{(t)}$	$a_1^{(t)}$	$a_2^{(t)}$
Λ_b	2	$0.201^{+0.143}_{-0.059}$	0	$0.551^{+\infty}_{-0.356}$	0	$0.391^{+0.279}_{-0.279}$
	3	$0.232^{+0.047}_{-0.056}$	0	$0.055^{+0.010}_{-0.020}$	0	$-0.161^{+0.108}_{-0.207}$
	4	$0.352^{+0.067}_{-0.083}$	0	$0.262^{+0.116}_{-0.132}$	0	$-0.541^{+0.173}_{-0.090}$
Ξ_b	2	$0.207^{+0.073}_{-0.063}$	$0.461^{+0.620}_{-0.284}$	$0.469^{+\infty}_{-0.559}$	$0.058^{+0.058}_{-0.034}$	$0.380^{+0.189}_{-0.319}$
	3	$0.218^{+0.043}_{-0.047}$	$0.877^{+0.820}_{-0.152}$	$0.049^{+0.005}_{-0.012}$	$0.037^{+0.032}_{-0.019}$	$-0.027^{+0.016}_{-0.027}$
	4	$0.378^{+0.065}_{-0.080}$	$2.291^{+\infty}_{-0.842}$	$0.286^{+0.130}_{-0.150}$	$0.039^{+0.030}_{-0.018}$	$-0.090^{+0.037}_{-0.021}$

H_Q	t	$\eta_1^{(t)}$	$\eta_2^{(t)}$	$\eta_3^{(t)}$	$b_2^{(t)}$	$b_3^{(t)}$
Λ_b	3	$0.324^{+0.054}_{-0.026}$	0	$0.633^{+0.06}_{-0.05}$	0	$-0.240^{+0.240}_{-0.147}$
Ξ_b	3	$0.218^{+0.043}_{-0.047}$	$0.877^{+0.820}_{-0.152}$	$0.049^{+0.005}_{-0.012}$	$0.037^{+0.032}_{-0.019}$	$-0.027^{+0.016}_{-0.027}$

Renormalization

- LO evolution equation for LCDAs

  $\Rightarrow$ Identify UV singularities of one-gluon-exchange diagrams:



- Transform to momentum space $\Rightarrow$ use light-cone Feynman rules

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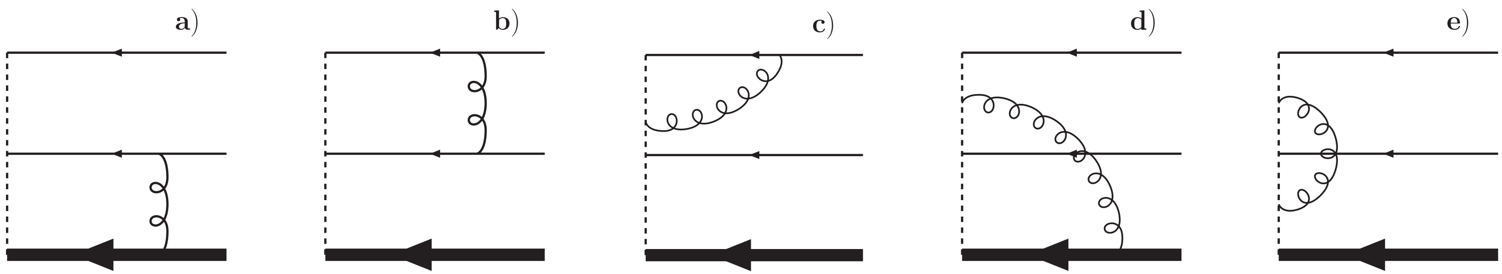


- Transform to momentum space ⇒ use light-cone Feynman rules
- At LO one quark always on-shell

  ⇒ Calculation adopted from:

  - B meson evolution [Lange, Neubert (2003)]
  - Light pseudoscalar meson evolution [Lepage, Brodsky (1980)]

Renormalization

- Renormalization of decay constants:

$$\langle 0| \left[q_1(0) C\Gamma' q_2(0)\right] b(0) |H(v)\rangle = f_H^{(1)}$$
$$\langle 0| \left[q_1(0) C\Gamma' \not{v} q_2(0)\right] b(0) |H(v)\rangle = f_H^{(2)}$$

- Scale dependence of the couplings (NLO order):

$$f_H^{(i)}(\mu) = f_H^{(i)}(\mu_0) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_1^{(i)}/\beta_0} \left[1 - \frac{\alpha_s(\mu_0) - \alpha_s(\mu)}{4\pi} \frac{\gamma_1^{(i)}}{\beta_0} \left(\frac{\gamma_2^{(i)}}{\gamma_1^{(i)}} - \frac{\beta_1}{\beta_0}\right)\right]$$

- NLO QCD sum rules at $\mu_0 = 1\ \mathrm{GeV}$ [Groote et all. (1997)] :

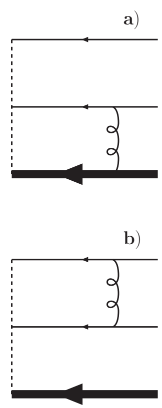
$$f_{\Lambda_b}^{(1)} \approx f_{\Lambda_b}^{(2)} \approx 0.030\ \mathrm{GeV}^3,\ f_{\Sigma_b}^{(1)} \approx f_{\Sigma_b}^{(2)} \approx 0.038\ \mathrm{GeV}^3$$

Evolution equation

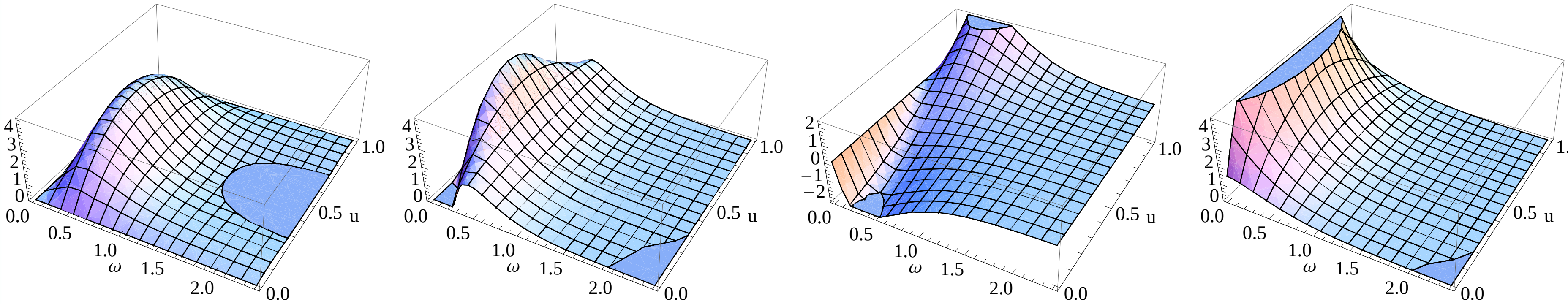
Evolution equation at LO :

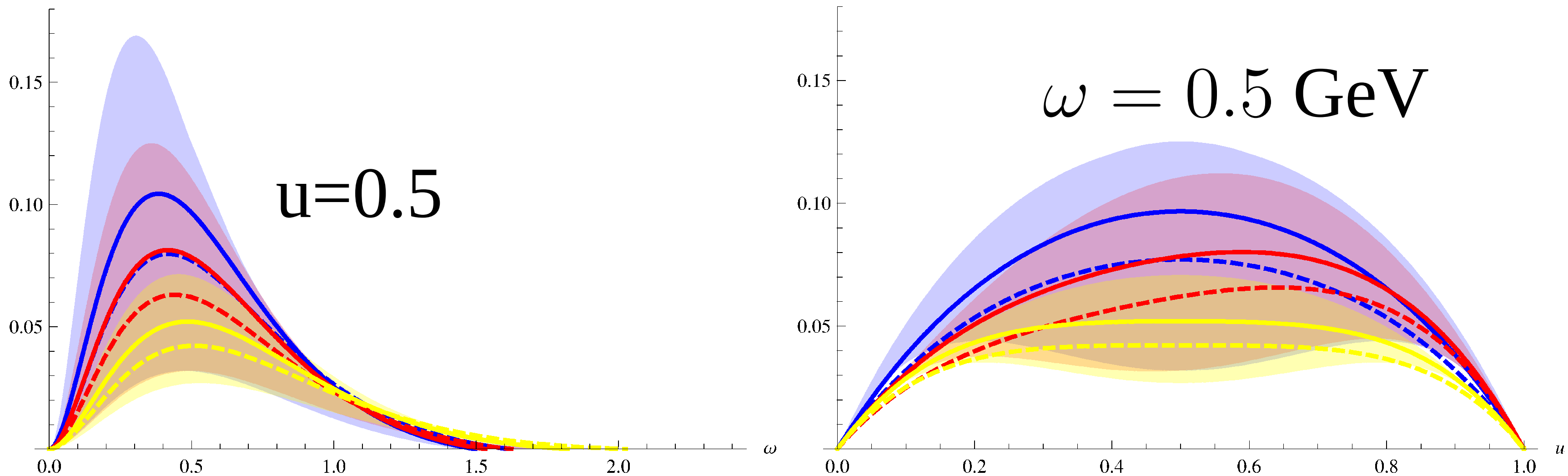
$$
\begin{aligned}
\mu\frac{d}{d\mu}\,\psi_2(\omega_1,\omega_2;\mu) \quad = \quad & -\frac{\alpha_s(\mu)}{2\pi}\,\frac{4}{3}\left\{\int_0^\infty d\omega_1'\,\gamma^{\rm LN}(\omega_1',\omega_1;\mu)\,\psi_2(\omega_1',\omega_2;\mu)\right.\\
& + \quad \int_0^\infty d\omega_2'\,\gamma^{\rm LN}(\omega_2',\omega_2;\mu)\,\psi_2(\omega_1,\omega_2';\mu)\\
& - \quad \left.\int_0^1 dv\,V(u,v)\,\psi_2(v\omega,\bar v\omega;\mu)+\frac{3}{2}\,\psi_2(\omega_1,\omega_2;\mu)\right\}
\end{aligned}
$$

- $3\psi_2/2$ from $f_H^{(2)}$ renormalization
- Kernel $\gamma^{\rm LN}(\omega',\omega;\mu)$ B-meson kernel
- $V(u,v)$ pseudoscalar mesons kernel



Distributions





- Twist-2 LCDAs of Σ (udb), Ξ (usb) and Ω (ssb) baryons
- Energy scales $\mu_0 = 1$ GeV (solid line) and $\mu = 2.5$ GeV (dashed line)
- Error bands $A \in [0, 1]$

Conclusion & outlook LCDAs

- LCDAs available for full ground state multiplet
- Inclusion of s-quark

  $\Rightarrow$ order 10% $SU(3)_F$ breaking effects

- Ambiguous local current $\Rightarrow$ Errors large

  $\Rightarrow$ Improved calculation & evolution beyond leading twist:
  Work in progress [A. Ali, A. Bharucha, C.H., W. Wang, Y. Wang]

  $\Rightarrow$ Improved calculations of form factors

⋮

BSM constraints

$[bq][\bar{b}\bar{q}]$-tetraquarks

Overview $[bq][\bar{b}\bar{q}]$-tetraquarks

- Based on "Tetraquark interpretation of $e^+e^- \to b\bar{b}$ data from the B-factory experiments Belle and BaBar" Ahmed Ali Tuesday, 01.02.2011

- Model for $\sigma(e^+e^- \to Y_b \to \Upsilon(1S)PP')$,
  $PP' = \pi^+\pi^-, K^+K^-, \eta\pi^0$
- Fit to $\Upsilon(1S)\pi^+\pi^-$ Belle spectra
  $\Rightarrow$ Testable predictions for $\Upsilon(1S)(K^+K^-, \eta\pi^0)$
- Consequence of model for tetraquark substructure
- Suggestion for wave function model

Exotic states: Motivation

Belle observations [A. Zupanc et al. (2009)] :

State	M (MeV)	Γ (MeV)	J^{PC}	Decay Modes	Production Modes	Also observed by
$Y_s(2175)$	2175 ± 8	58 ± 26	1^{--}	$\phi f_0(980)$	e^+e^- (ISR) $J/\psi \to \eta Y_s(2175)$	BaBar, BESII
$X(3872)$	3871.4 ± 0.6	< 2.3	1^{++}	$\pi^+\pi^- J/\psi$, $\gamma J/\psi$,$D\bar{D}^*$	$B \to KX(3872)$, $p\bar{p}$	BaBar CDF, D0,
$X(3915)$	3914 ± 4	28^{+12}_{-14}	$0/2^{++}$	$\omega J/\psi$	$\gamma\gamma \to X(3915)$	
$Z(3930)$	3929 ± 5	29 ± 10	2^{++}	$D\bar{D}$	$\gamma\gamma \to Z(3940)$	
$X(3940)$	3942 ± 9	37 ± 17	$0^{?+}$	$D\bar{D}^*$ (not $D\bar{D}$ or $\omega J/\psi$)	$e^+e^- \to J/\psi X(3940)$	
$Y(3940)$	3943 ± 17	87 ± 34	$?^{?+}$	$\omega J/\psi$ (not $D\bar{D}^*$)	$B \to KY(3940)$	BaBar
$Y(4008)$	4008^{+82}_{-49}	226^{+97}_{-80}	1^{--}	$\pi^+\pi^- J/\psi$	e^+e^- (ISR)	
$X(4160)$	4156 ± 29	139^{+113}_{-65}	$0^{?+}$	$D^*\bar{D}^*$ (not $D\bar{D}$)	$e^+e^- \to J/\psi X(4160)$	
$Y(4260)$	4264 ± 12	83 ± 22	1^{--}	$\pi^+\pi^- J/\psi$	e^+e^- (ISR)	BaBar, CLEO
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$Z(4250)$	4248^{+185}_{-45}	177^{+320}_{-72}	?	$\pi^\pm \chi_{c1}$	$B \to KZ^\pm(4250)$	
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$Y_b(10890)$	$10,890 \pm 3$	55 ± 9	1^{--}	$\pi^+\pi^- \Upsilon(1,2,3S)$	$e^+e^- \to Y_b$	

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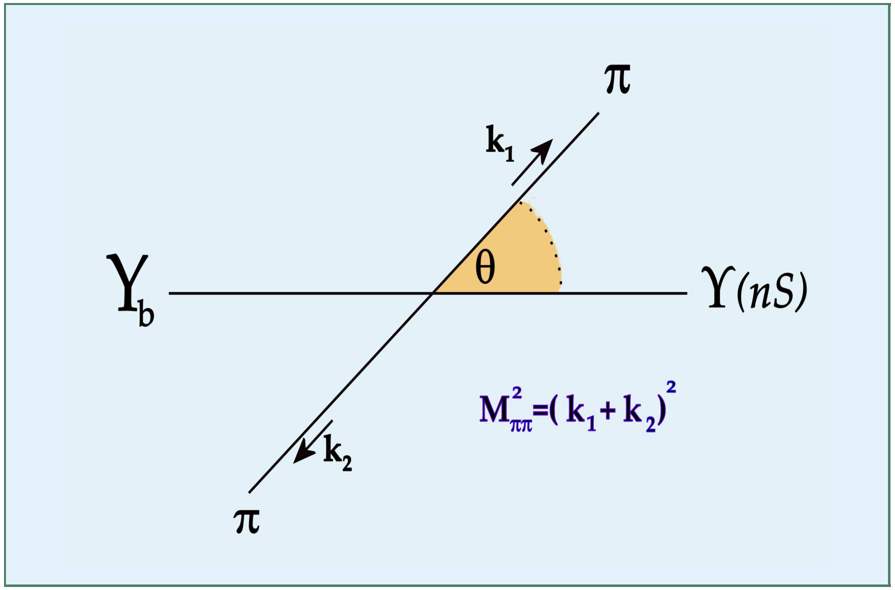
Our tetraquark candidate

[A. Ali, C. H. and J. Aslam, PRL 104]

[A. Ali, C. H. and S. Mishima, today online in PRL]

Enigmatic Belle data



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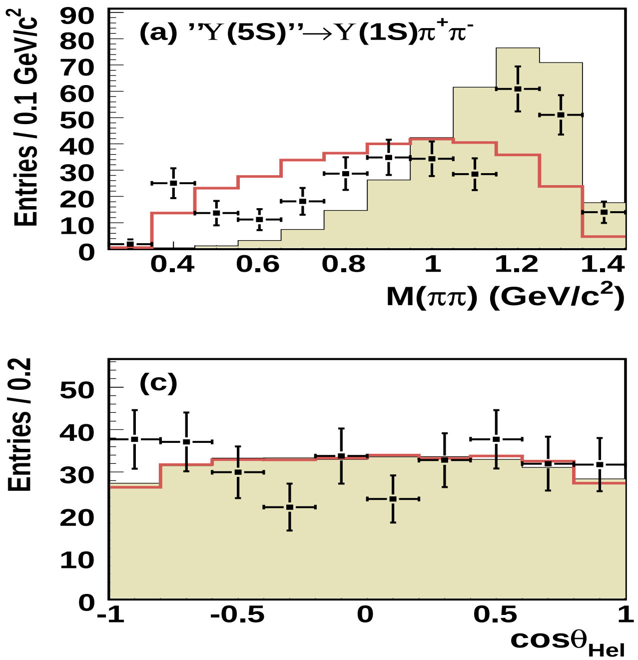

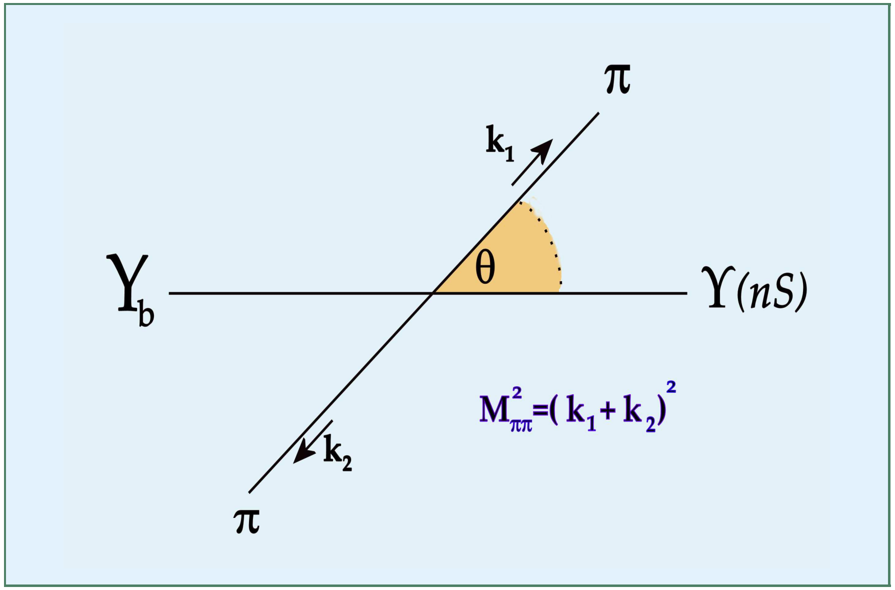

[K. F. Chen *et al.* [Belle Collaboration], Phys. Rev. Lett. **100**, 112001 (2008)]

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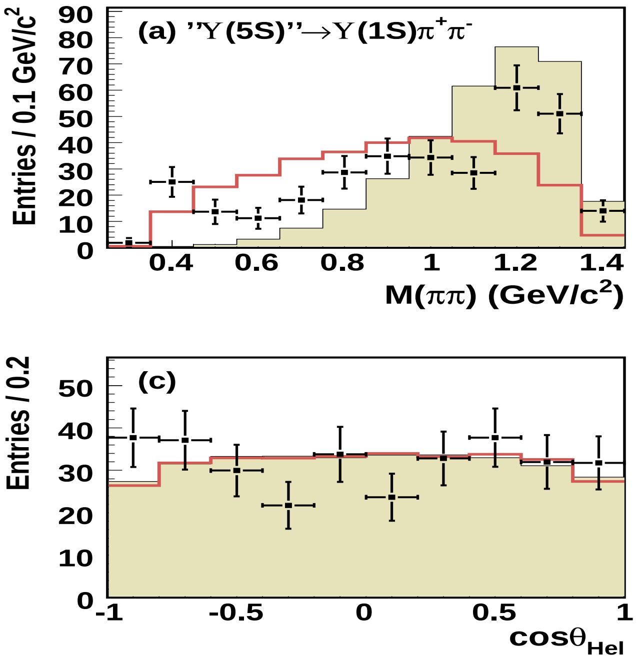

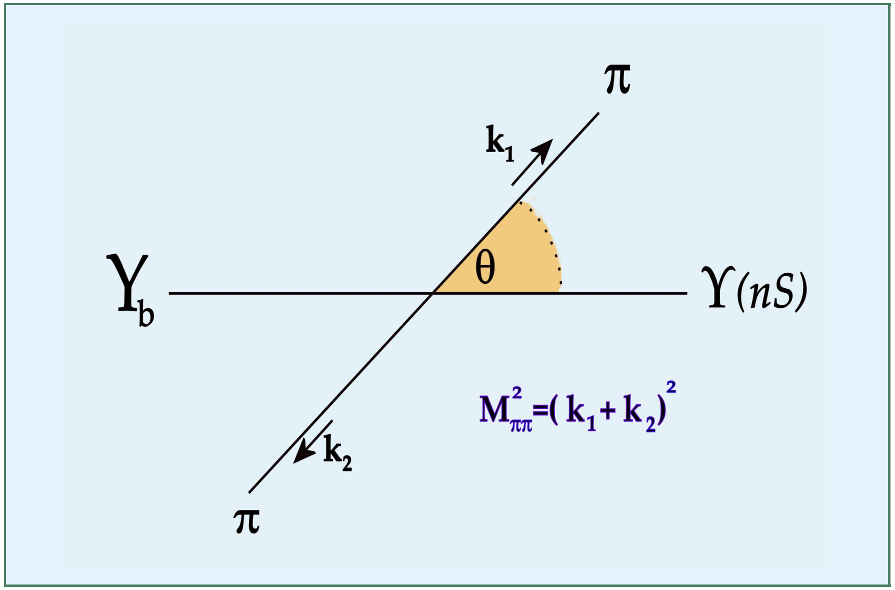

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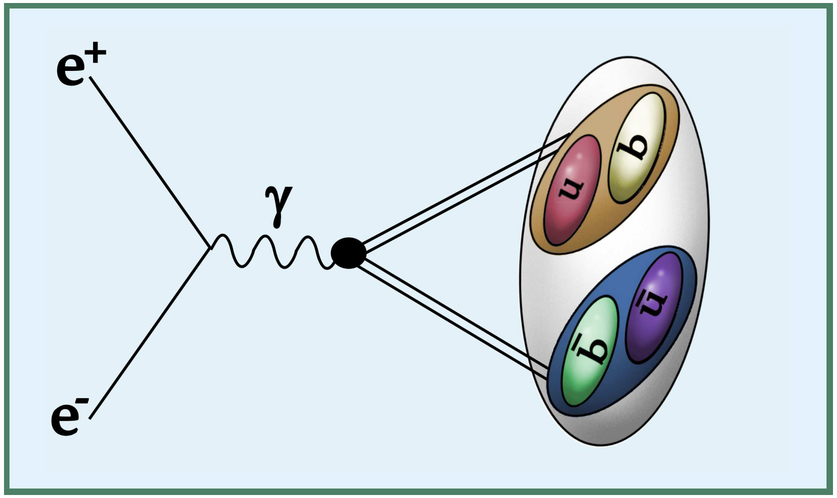
Differs by two orders of magnitude!!

Y_b production

- Van Royen-Weisskopf formula $\Rightarrow \Gamma(1^{--} \rightarrow e^+e^-)$

Assumption: Point-like diquarks

[A. Ali, C. H. and S. Mishima, today online in PRL]



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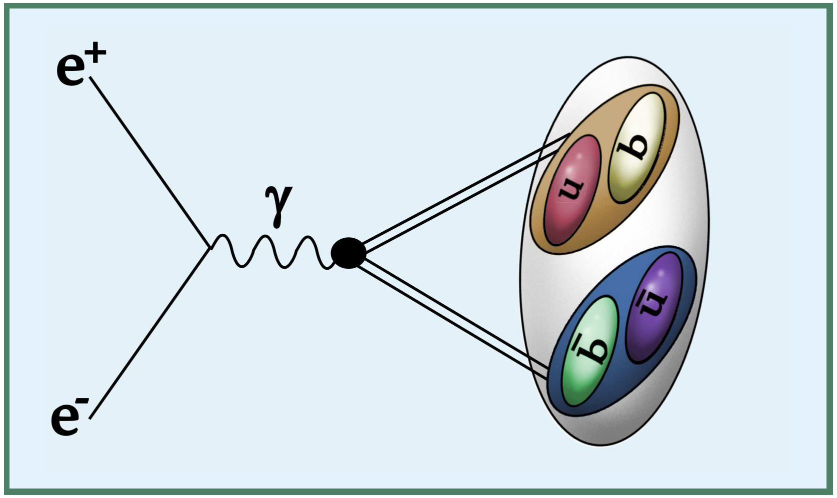

$$\Gamma_{ee}(Y_{[b,q]}) = \frac{24\alpha^2 Q^2_{[b,q]}}{M^4_{Y_{[b,q]}}} \kappa^2 \left| R^{(1)}_{11}(0) \right|^2$$

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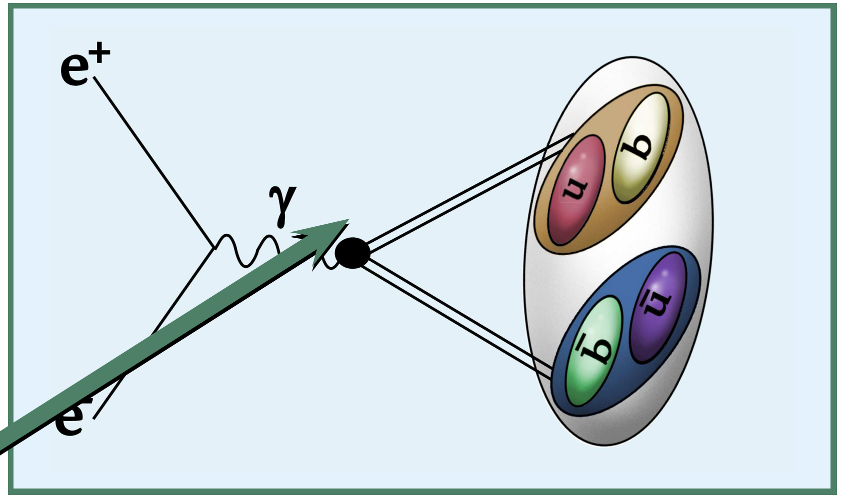

diquark charge

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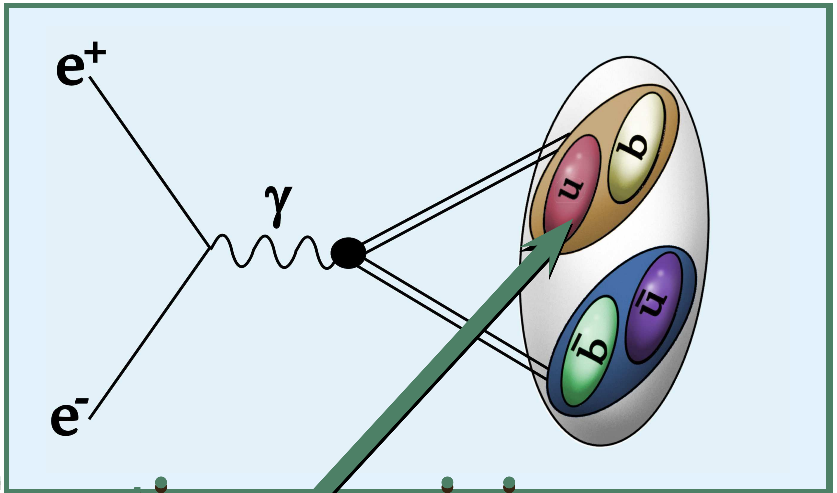

radial tetraquark wave function at origin

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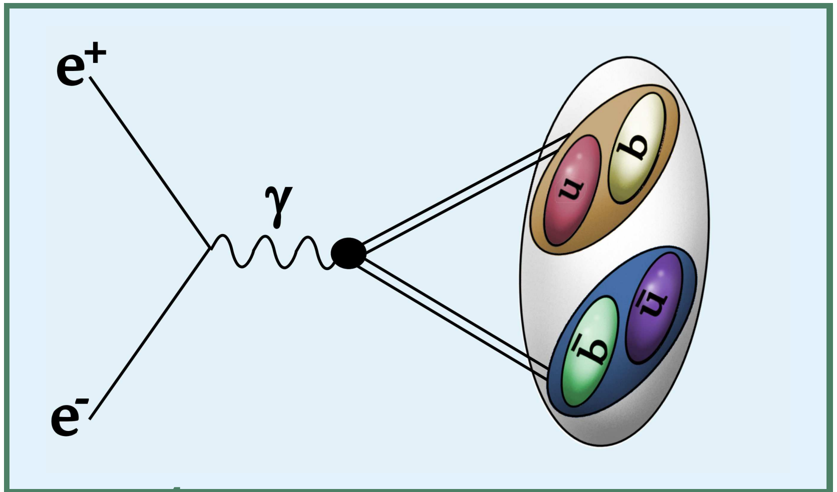

hadronic size parameter

$$\Gamma_{ee}(Y_{[b,q]}) = \frac{24\alpha^2 Q^2_{[b,q]}}{M^4_{Y_{[b,q]}}} \kappa^2 \left| R^{(1)}_{11}(0) \right|^2$$

- κ measure for deviation from bb-onia

Y_b production

- Van Royen-Weisskopf formula $\Rightarrow \boldsymbol{\Gamma(1^{--} \to e^+ e^-)}$

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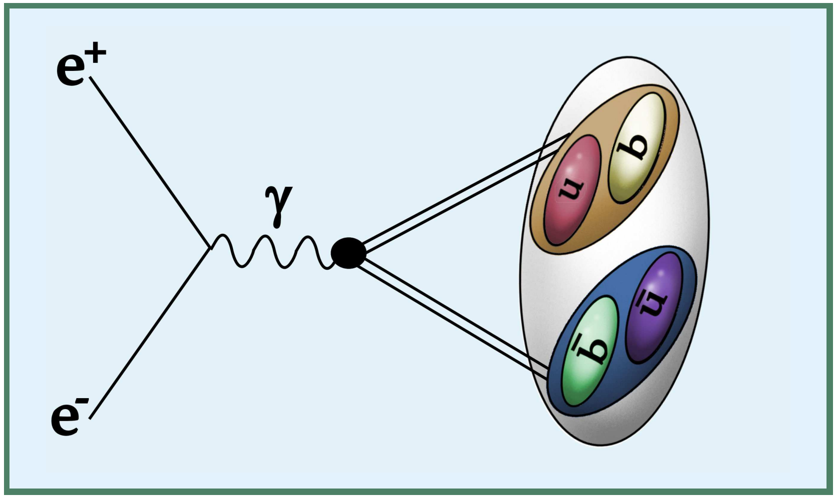

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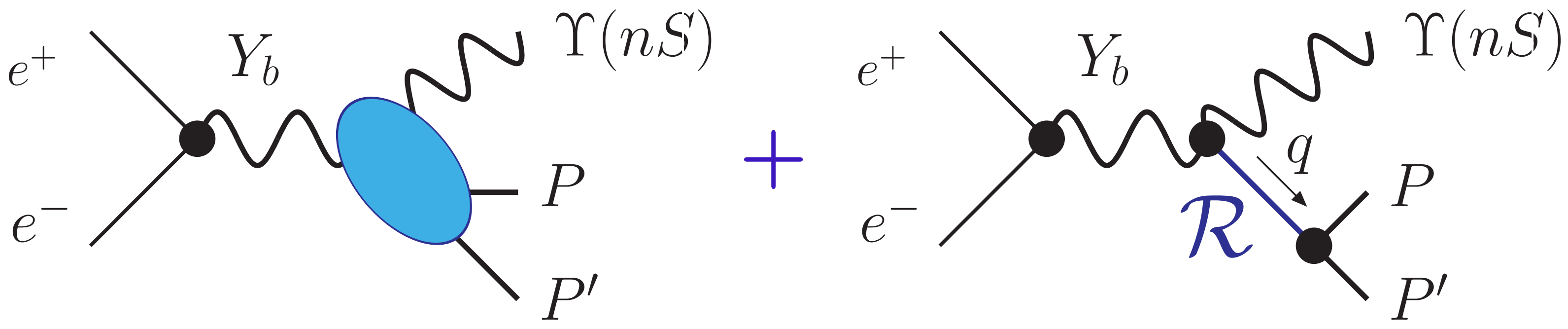
- κ measure for deviation from bb-onia
- Production ratio: $\Gamma_{Y_{[b,u]}} / \Gamma_{Y_{[b,d]}} = 1/4$

Types of contributions

Continuum | Resonance



- Breit-Wigner shape for resonance:

$$\frac{1}{(q^2 - M^2) + iM\Gamma}$$

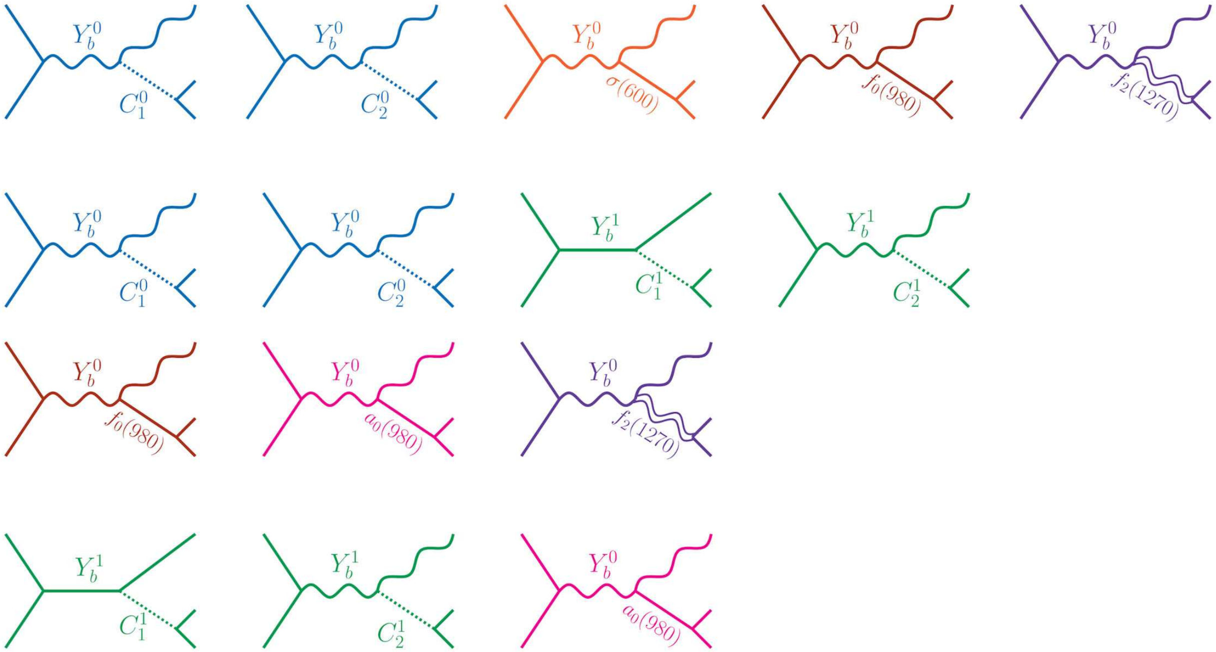
$q^2 \equiv M^2_{PP'} \Rightarrow$ Resonances show in $M_{PP'}$ spectrum ($\sigma(600)$, $f_0(980)$, $a_0(980)$ and $f_2(1270)$)

Couplings: Overview

$\pi^+\pi^-$:

K^+K^-:

$\eta\pi^0$:



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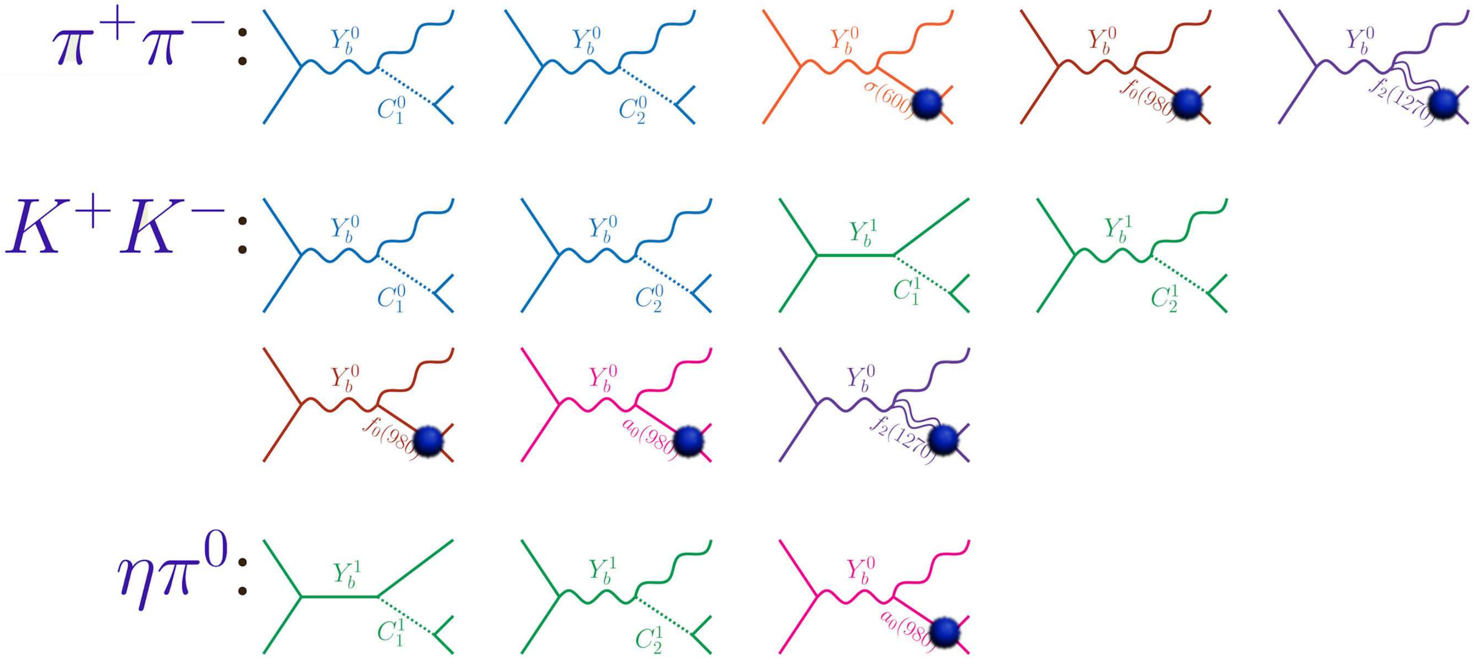
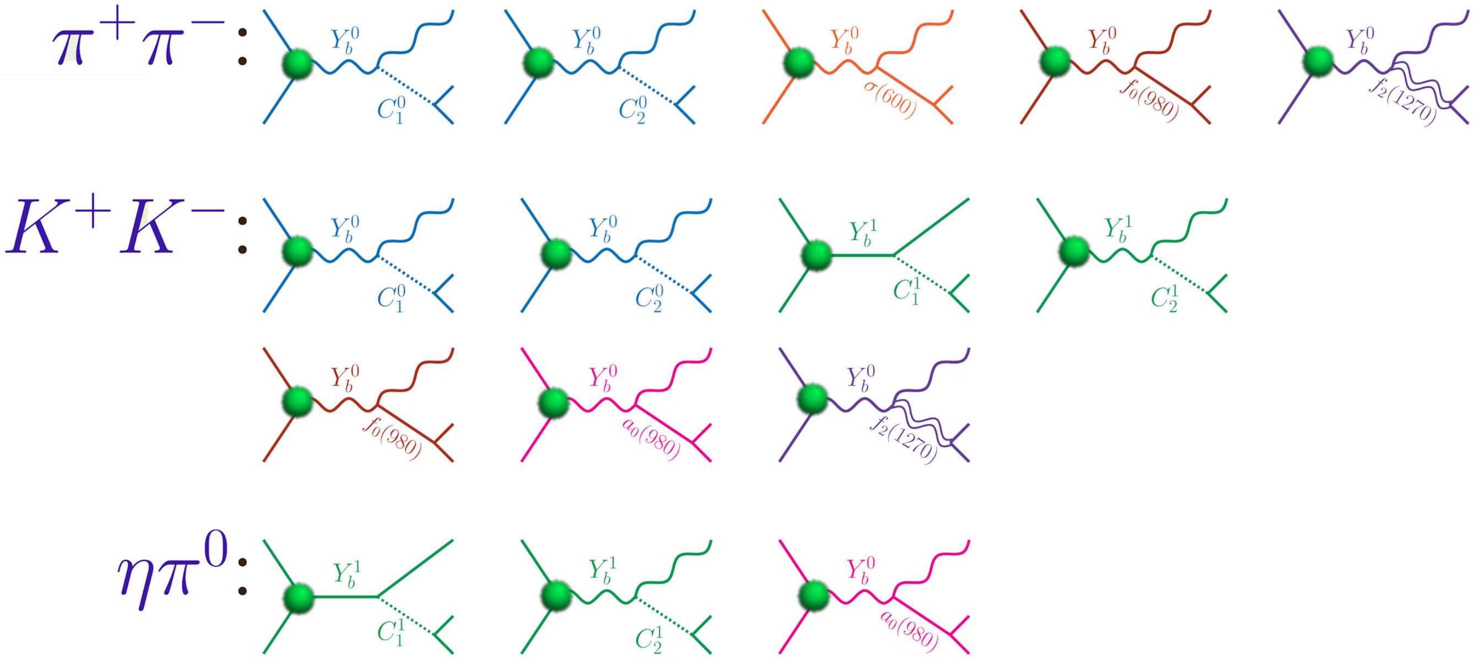


- known : Experimental input

Couplings: Overview



- known : Experimental input
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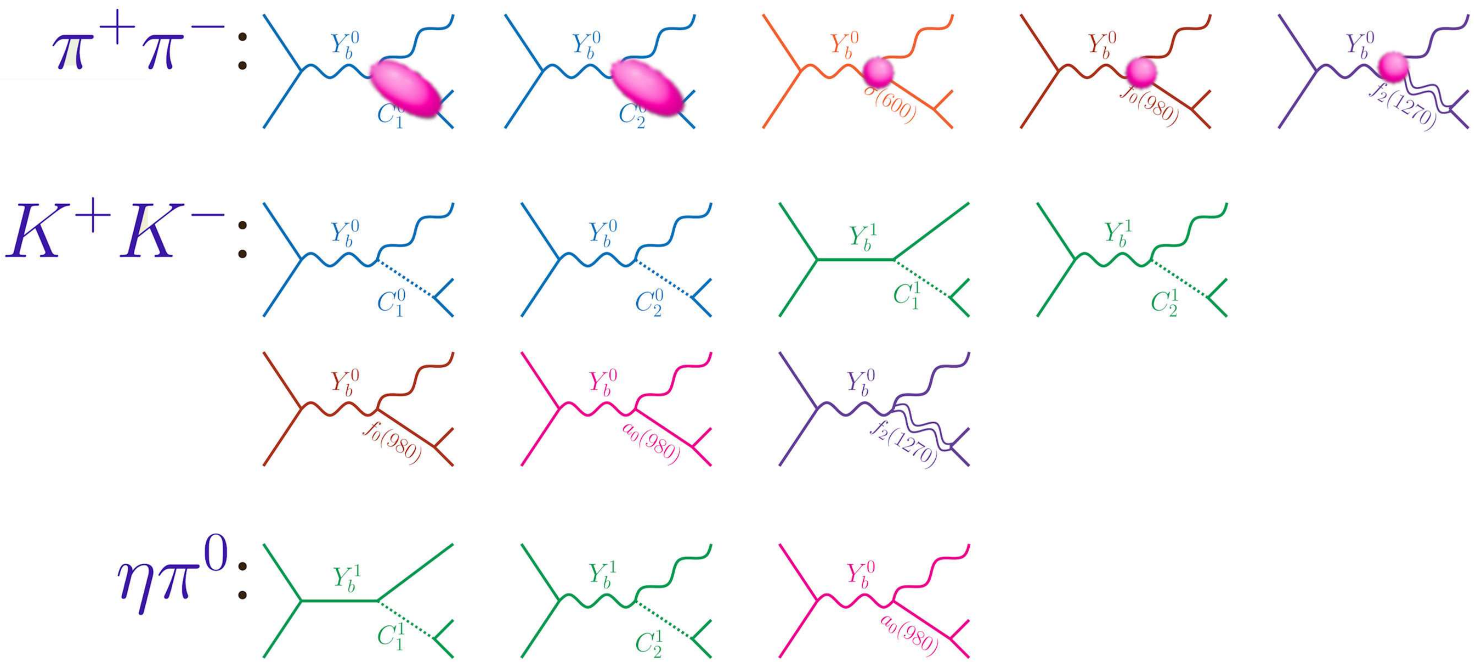

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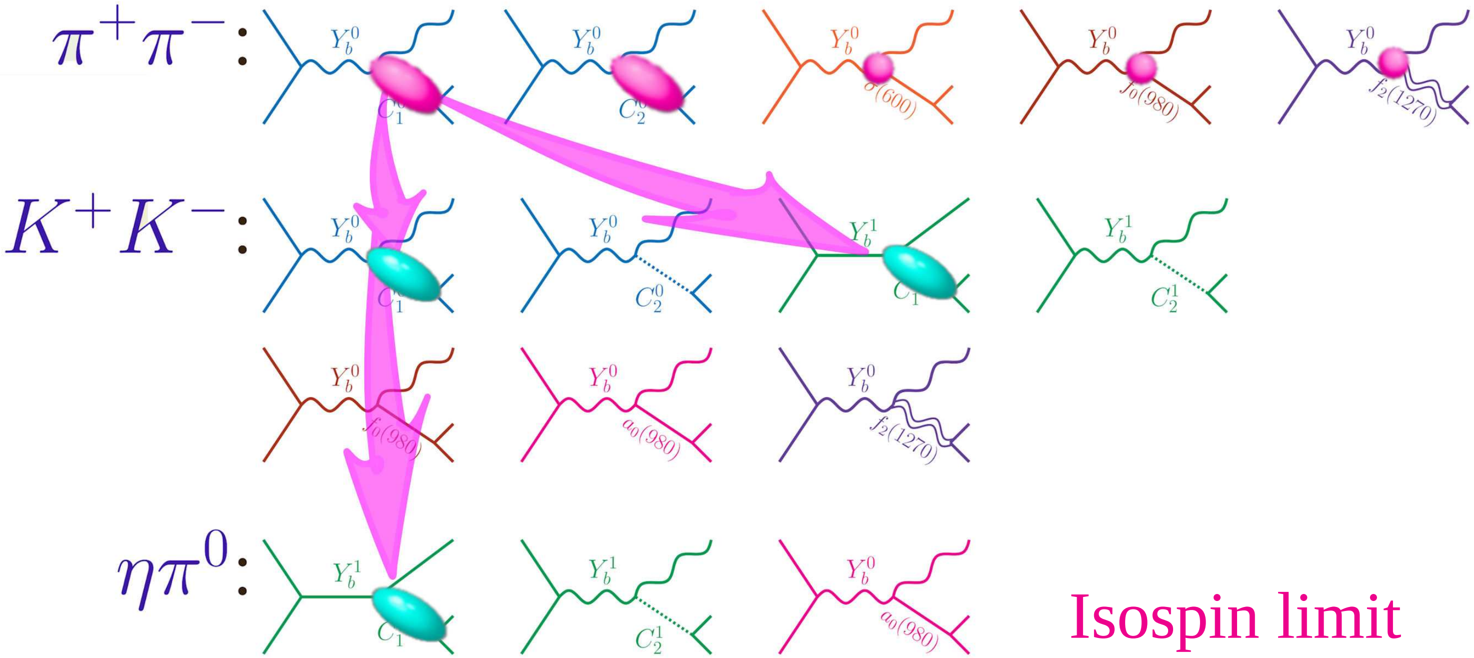
- unknown : Fit parameters

$$A',\ B',\ g'_{Y_b^0\Upsilon(1S)\sigma}(=g'_{Y_b^0\Upsilon(1S)f_0}),\ g'_{Y_b^0\Upsilon(1S)f_2},\ \varphi_\sigma,\ \varphi_{f_0},\ \varphi_{f_2}$$

Couplings: Overview



Isospin limit

- known : Experimental input
- known : Our Van Royen-Weisskopf formula
- unknown : Fit parameters $\Rightarrow$ ● known

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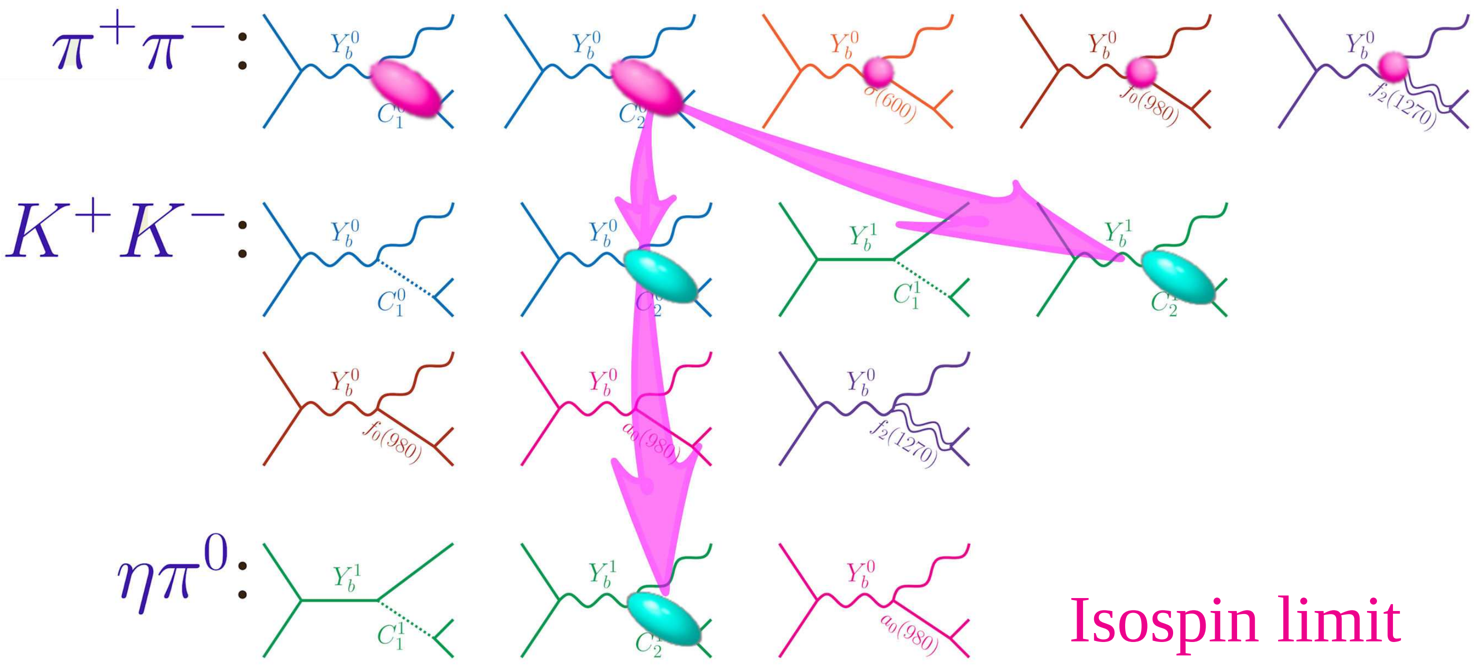
Couplings: Overview



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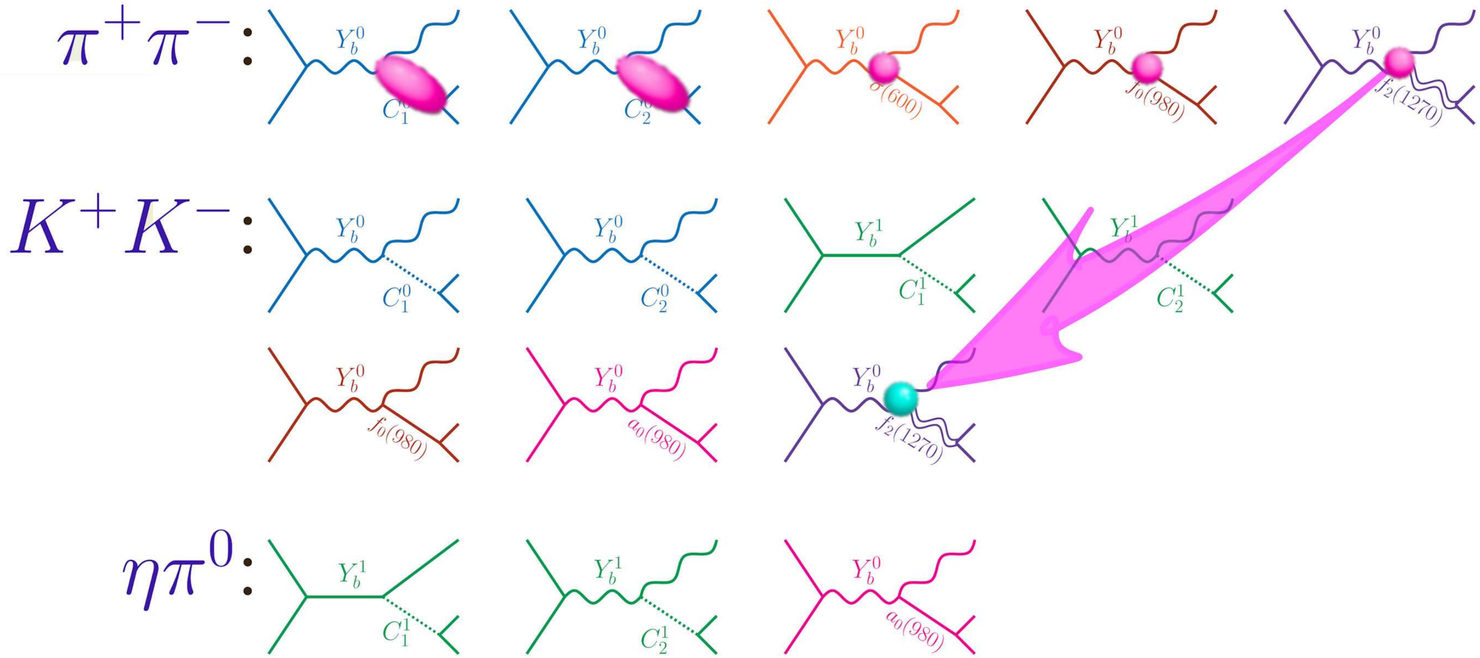
Couplings: Overview



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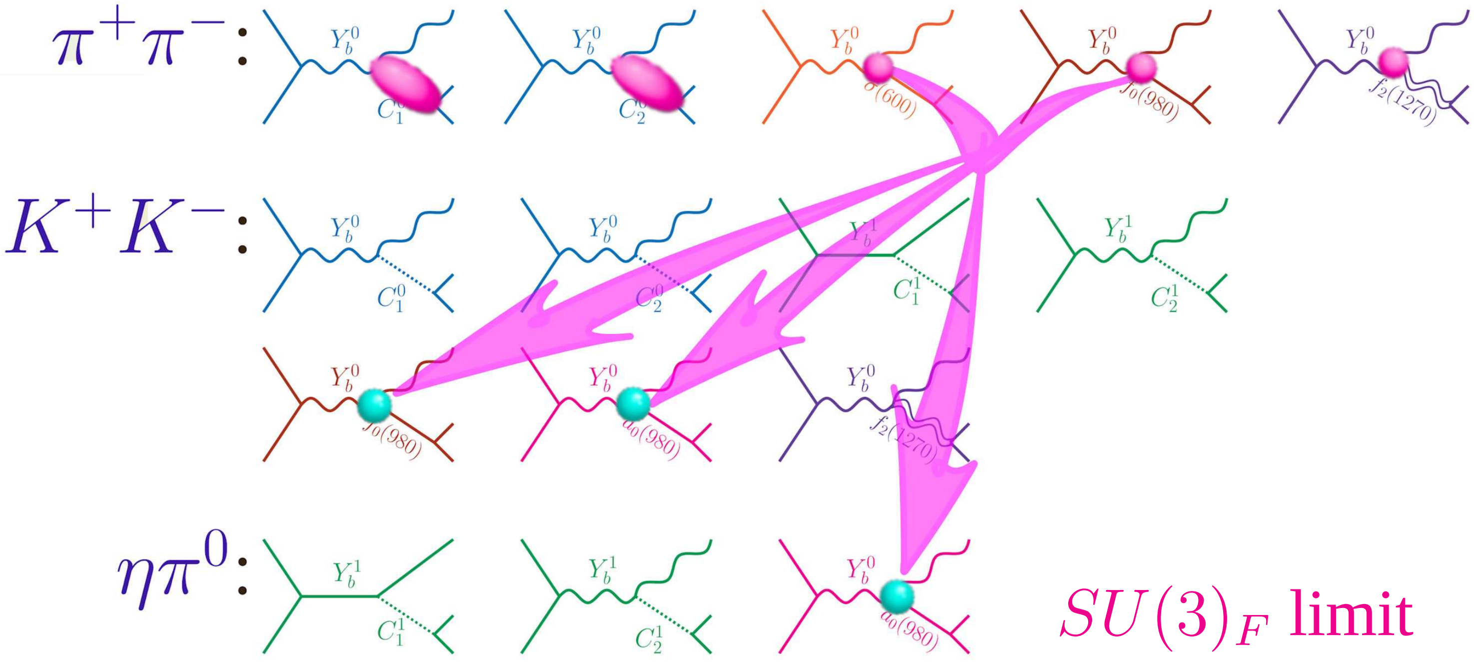
A', B', $g'_{Y_b^0 \Upsilon(1S)\sigma} (= g'_{Y_b^0 \Upsilon(1S) f_0})$, $g'_{Y_b^0 \Upsilon(1S) f_2}$, φ_σ, φ_{f_0}, φ_{f_2}

Couplings: Overview

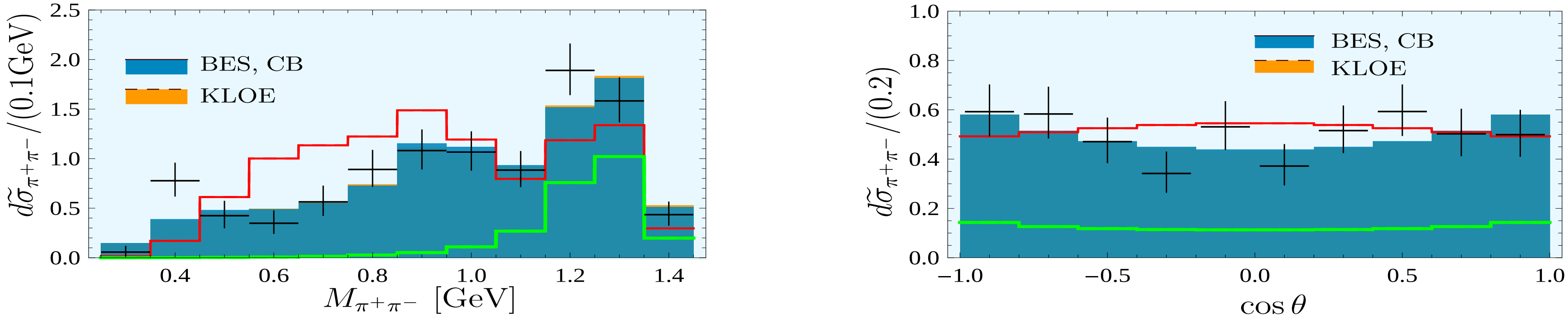


$SU(3)_F$ limit

- known : Experimental input
- known : Our Van Royen-Weisskopf formula
- unknown : Fit parameters $\Rightarrow$ ● known

$A',\ B',\ g'_{Y_b^0\Upsilon(1S)\sigma}(=g'_{Y_b^0\Upsilon(1S)f_0}),\ g'_{Y_b^0\Upsilon(1S)f_2},\ \varphi_\sigma,\ \varphi_{f_0},\ \varphi_{f_2}$

Fit to $\sigma(e^+e^- \to Y_b \to \Upsilon(1S)\pi^+\pi^-)$



- Fit results:

	A'	B'	$g'_{Y_b^0\Upsilon(1S)f_0}$	$g'_{Y_b^0\Upsilon(1S)f_2}$	φ_σ	φ_{f_0}	φ_{f_2}
BES, CB	0.000079	−0.00020	0.318	0.439	0.36	−2.76	−0.46

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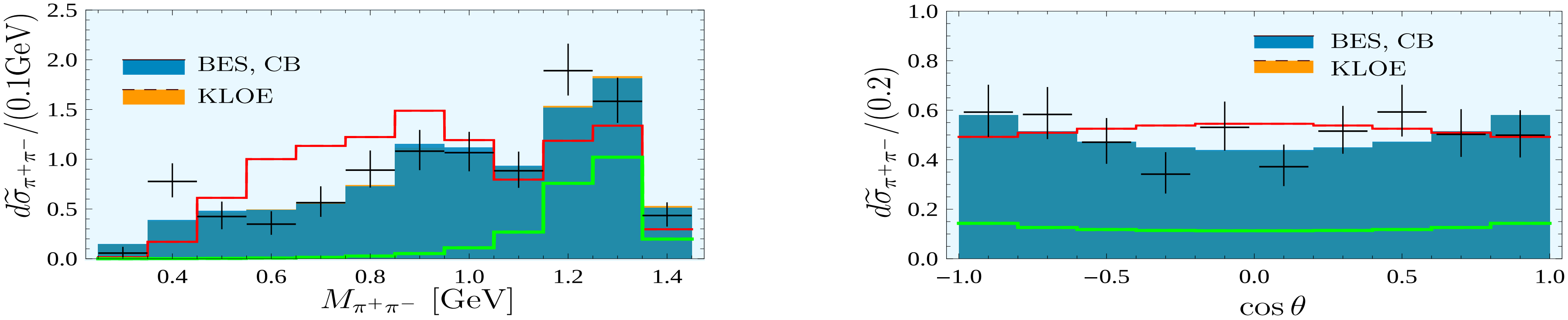

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- Stable for BES, CB and KLOE input

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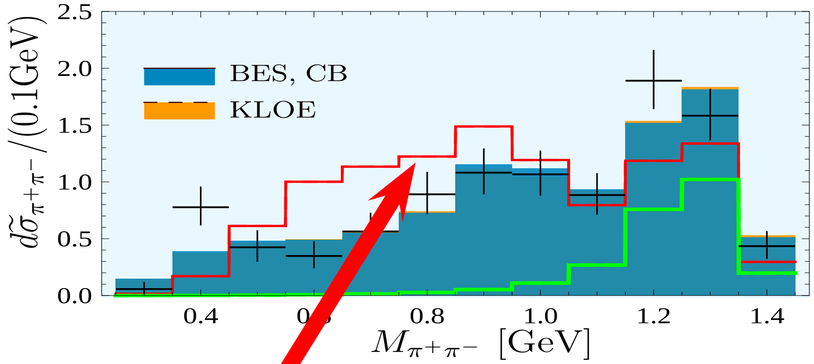

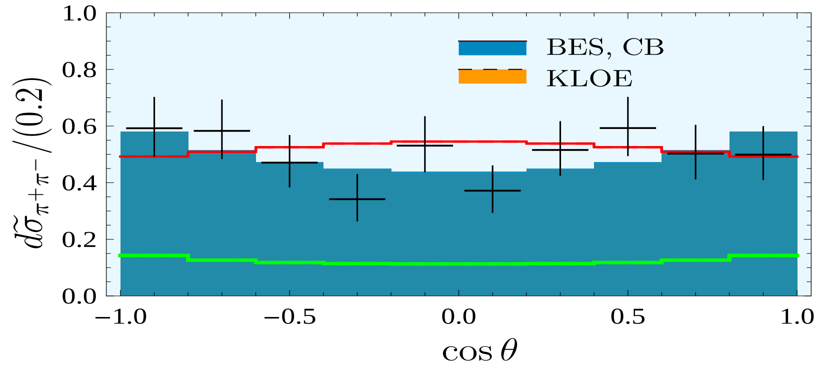

resonance contribution

- Fit results:

	A'	B'	$g'_{Y_b^0\Upsilon(1S)f_0}$	$g'_{Y_b^0\Upsilon(1S)f_2}$	φ_σ	φ_{f_0}	φ_{f_2}
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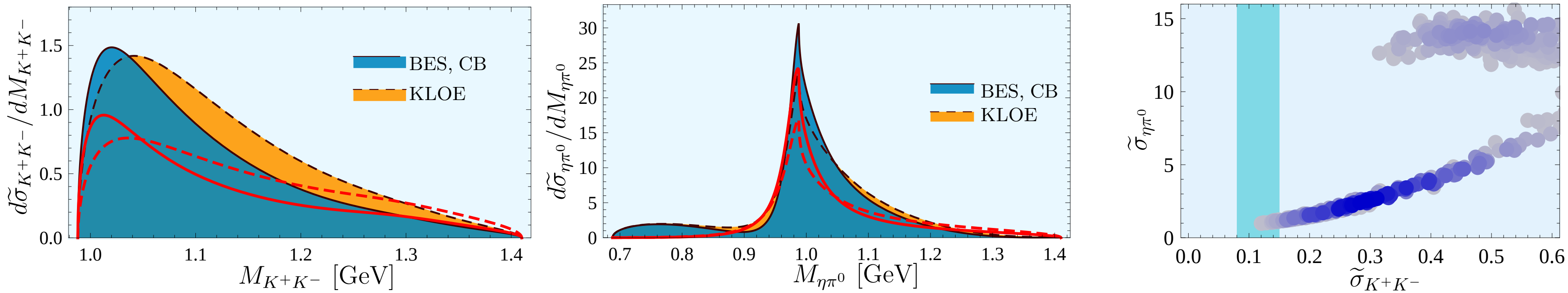
- $\chi^2/\text{d.o.f.} = 21.5/15 \Rightarrow$ Good agreement with data
- Stable for BES, CB and KLOE input
- Clear resonance dominance!

Predictions for $\Upsilon(1S)(K^+K^-, \eta\pi^0)$

Fit determines couplings $\Longrightarrow$ predictions for spectra:



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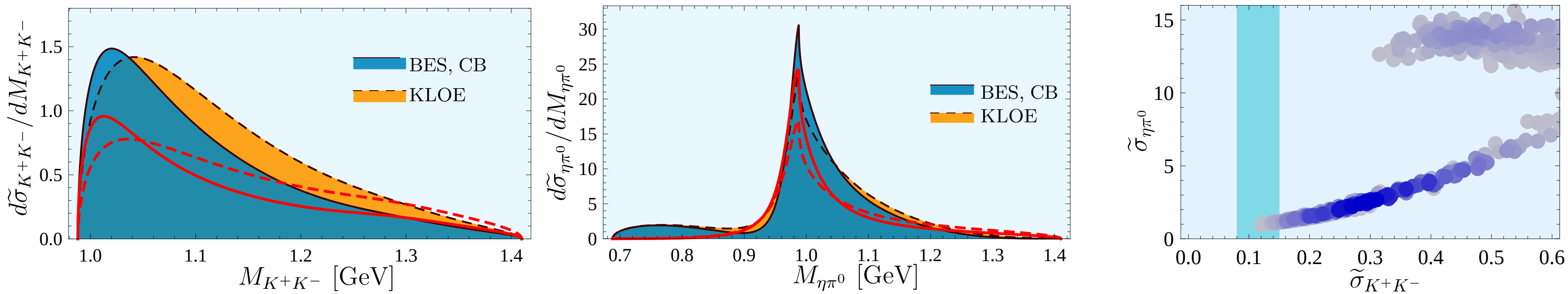

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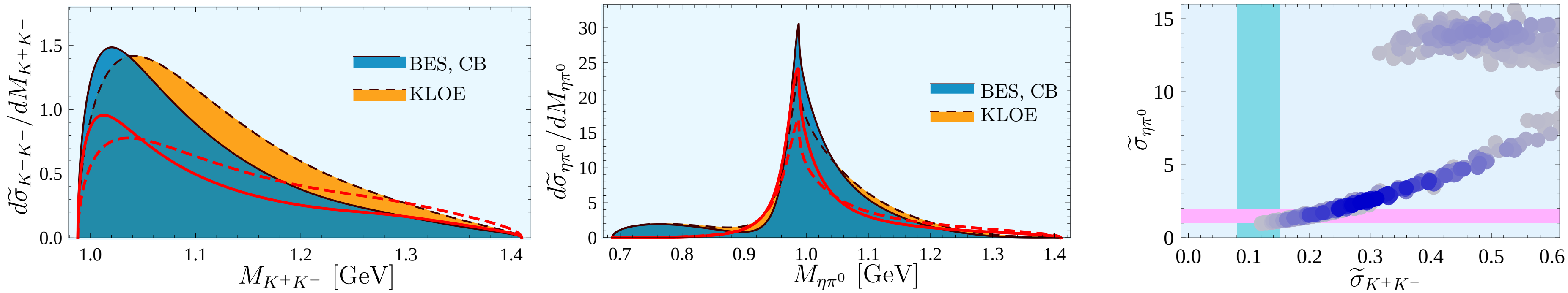

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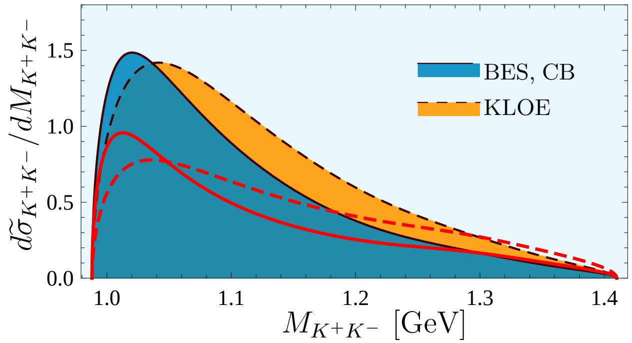

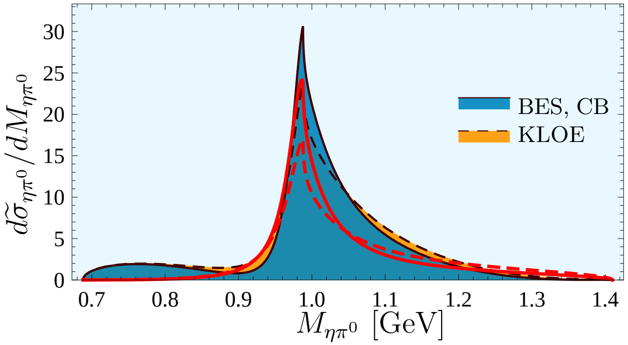

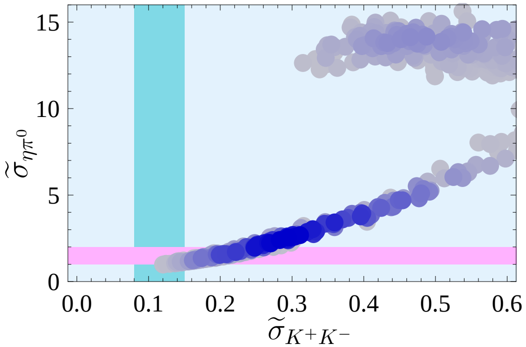

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  $\Rightarrow$ Characteristic shape

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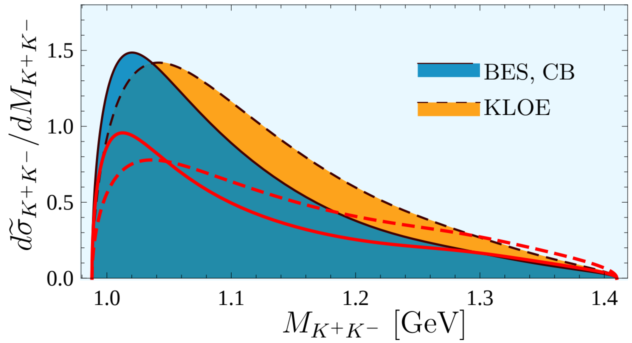

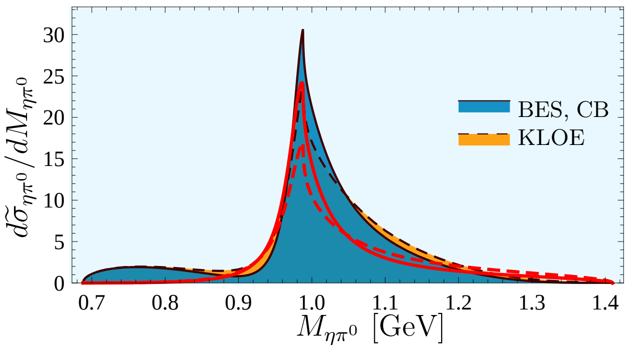

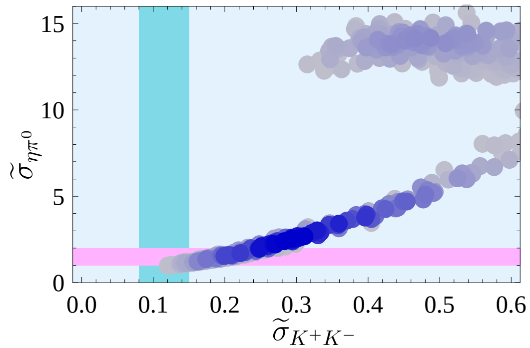

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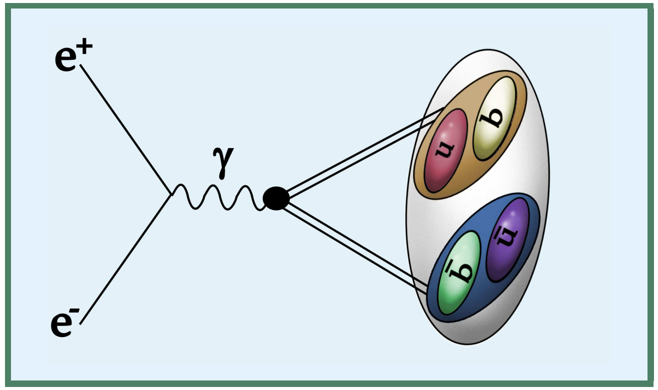
  $\Rightarrow$ **Excellent tests**

Further predictions



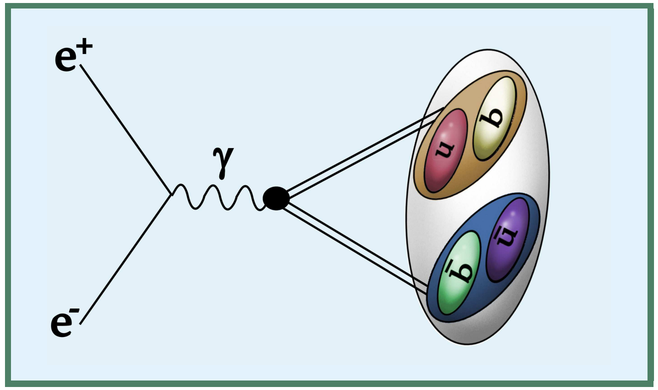
Further predictions

- Production of $K\bar{K} \propto Q^2_{[bq]}$

$$\Rightarrow \frac{\sigma_{\Upsilon(1S)K^+K^-}}{\sigma_{\Upsilon(1S)K^0\bar{K}^0}} = \frac{Q^2_{[bu]}}{Q^2_{[bd]}} = \frac{1}{4}$$

flavor eigenstate diquark charge:

$$Q_{[bd]} = -2/3 \qquad Q_{[bu]} = +1/3$$



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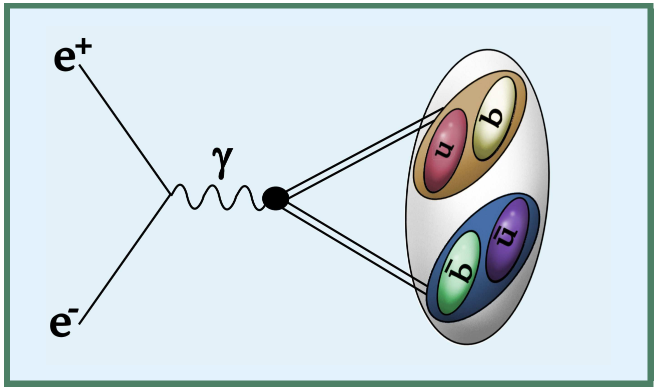

- $B\bar{B}$ produced by $\Upsilon(5S) + Y_b^{(1)}$

$$\Rightarrow \frac{\sigma_{B^+B^-}}{\sigma_{B^0\bar{B}^0}} \approx 1 - \frac{0.2}{\kappa^2+0.27}$$

$\Rightarrow B^0\bar{B}^0$ 10% enhanced vs B^+B^-

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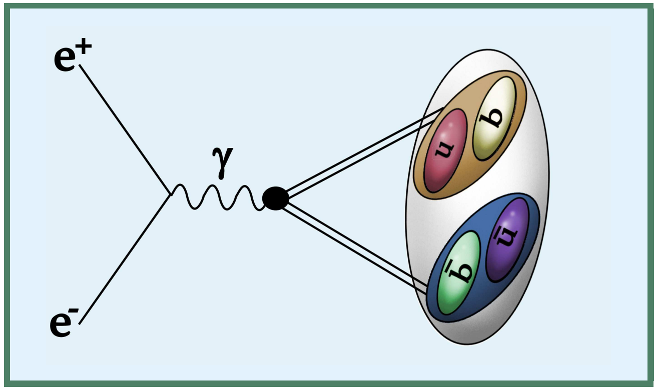

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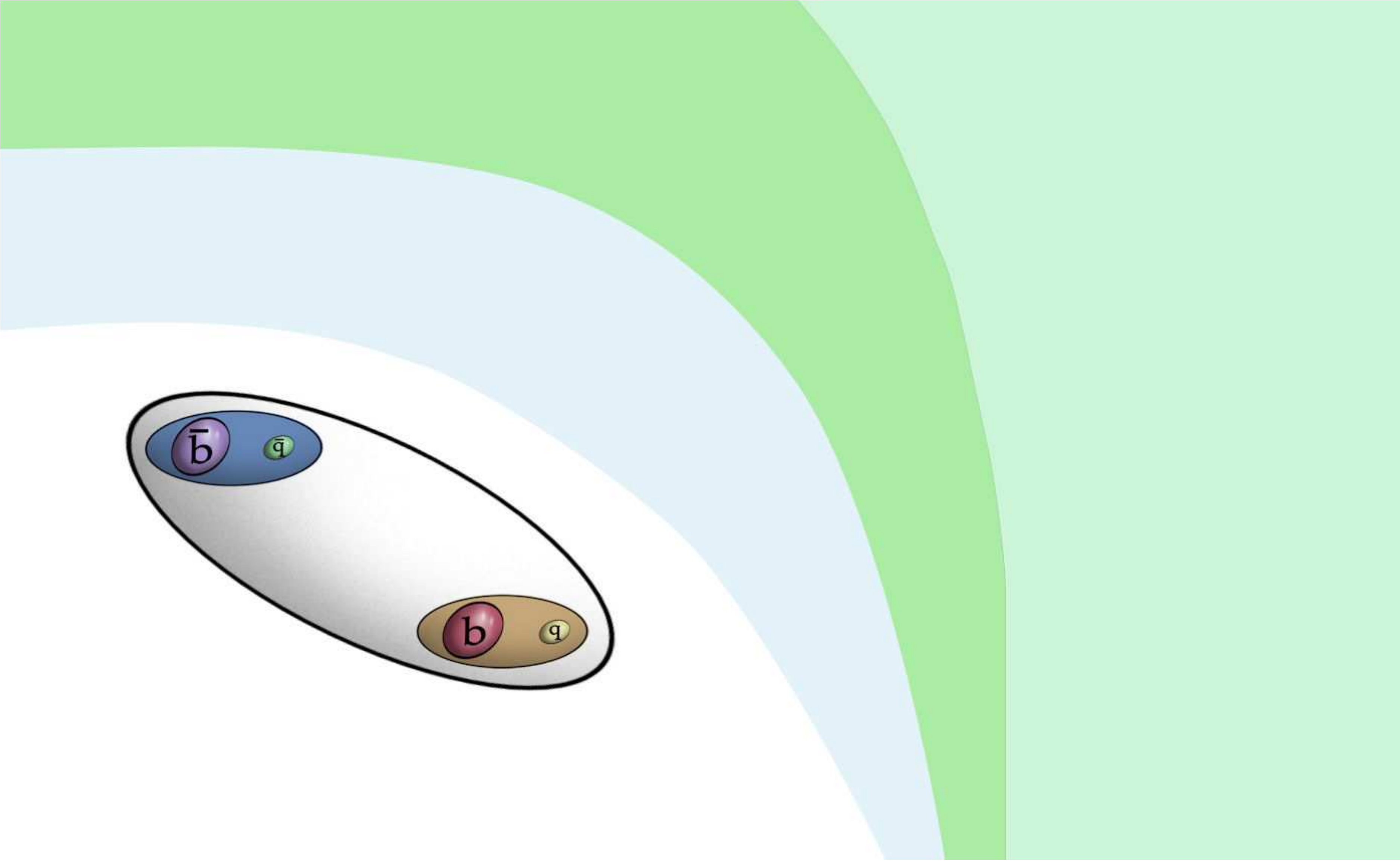
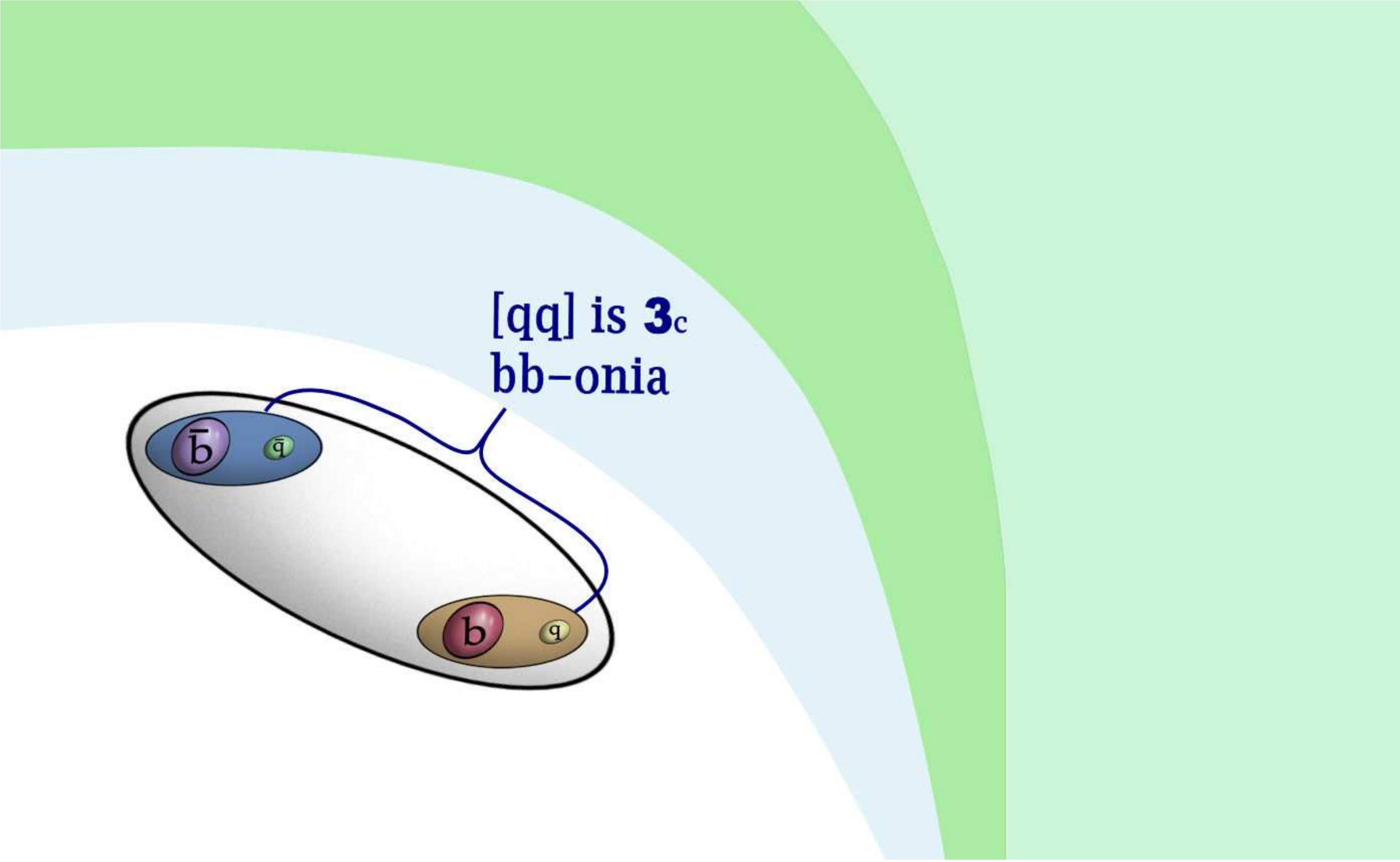
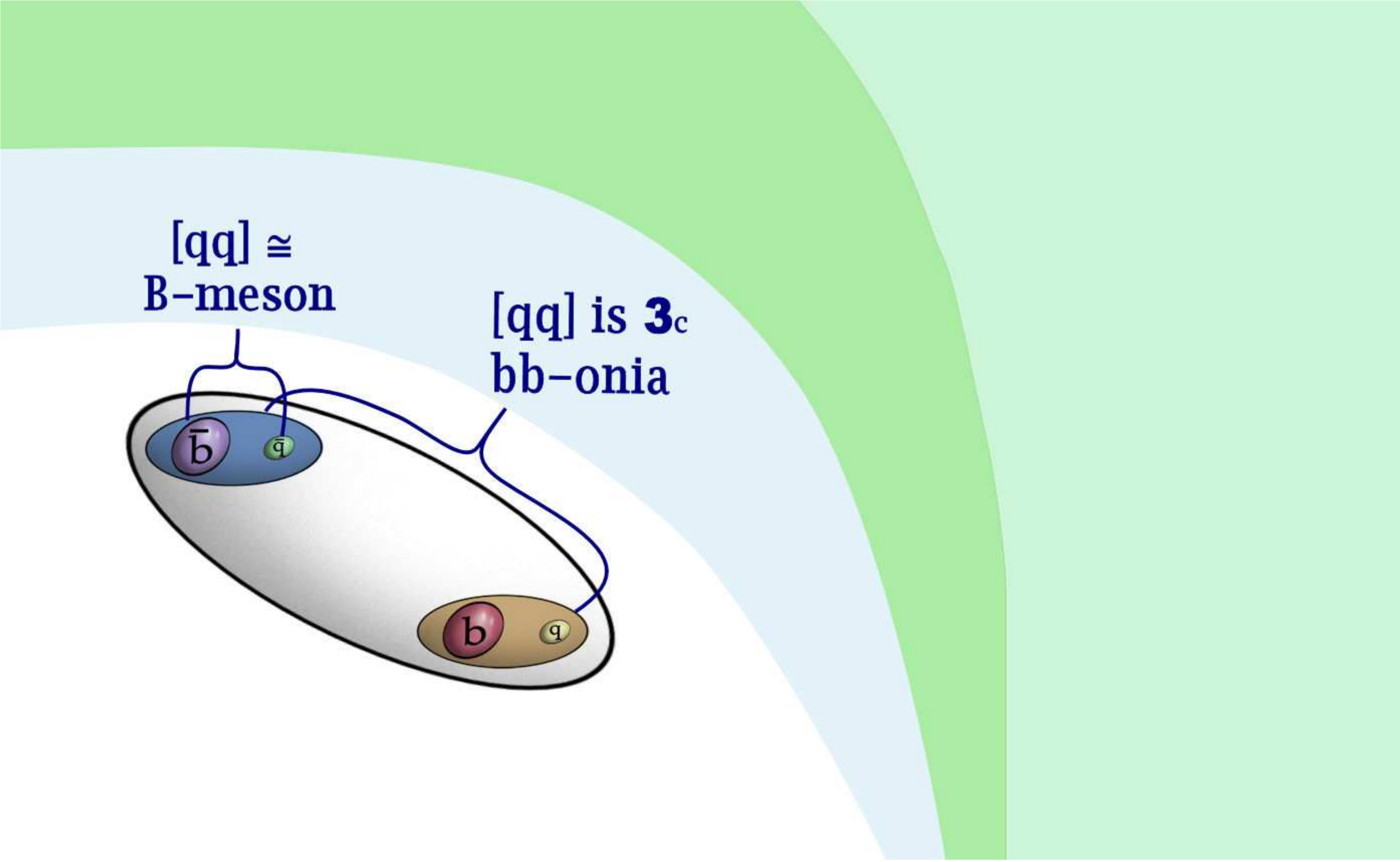
Distinct for tetraquarks with pointlike diquarks

Models for tetraquark wave functions


b̄
q̄
b
q



- 1. Diquarks comparably small $\mathbf{3}_{SU(3)_C} \Rightarrow$ Model from bb-onia



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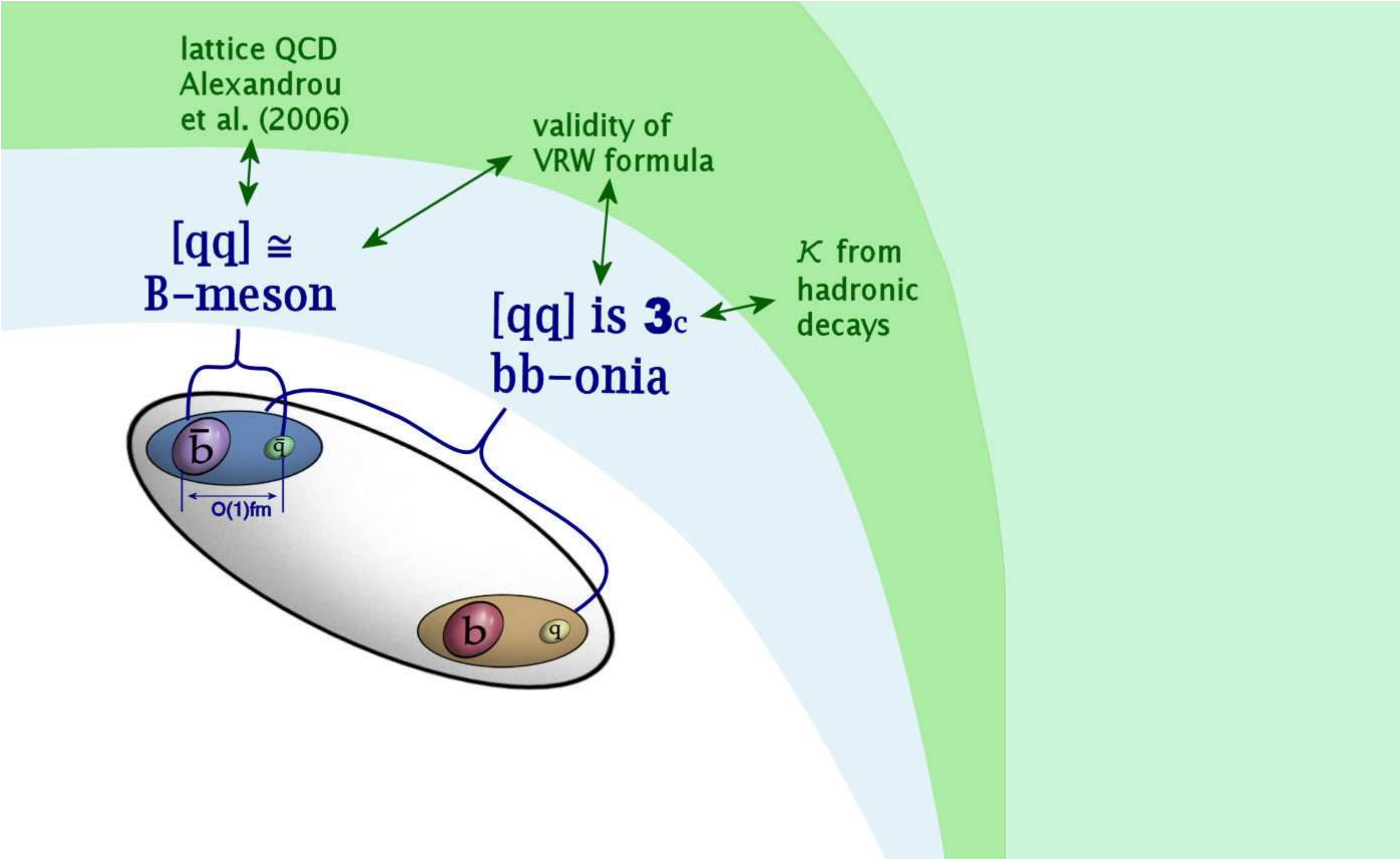

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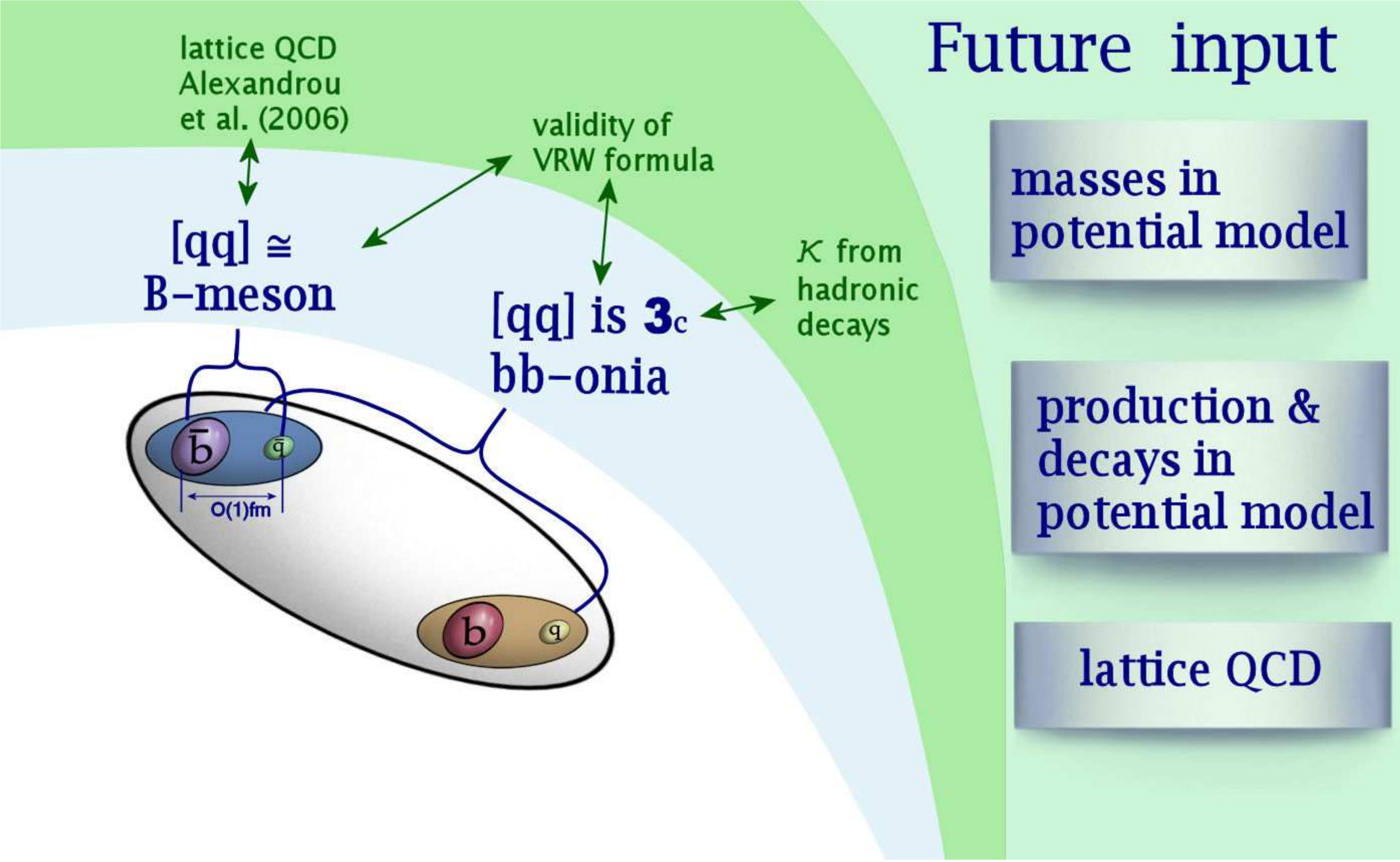

- 1. Diquarks comparably small $\mathbf{3}_{SU(3)_C} \Rightarrow$ Model from bb-onia
- 2. Diquark heavy-light $\Rightarrow$ Model from B-mesons
- How much can the simple model tell about tetraquarks?

Conclusion $[bq][\bar{b}\bar{q}]$-tetraquarks

- $Y_b^{(1)} \to \Upsilon(nS)\pi^+\pi^-$ decays are Zweig allowed
  $\Rightarrow$ Explanation of large observed decay widths
- Coupling to light quark resonances ($\sigma(600)$, $f_0(980)$, $f_2(1270)$)
  $\Rightarrow$ Explanation of shape of invariant mass spectrum
- Crucial tests
  - $\Rightarrow$ Tetraquarks confirmed?
    - $\Rightarrow$ If so: Information about parton distributions
      - $\Rightarrow$ Can the distributions be modeled?
- Tetraquarks: Growing interest from lattice QCD!

Outlook $[bq][\bar{b}\bar{q}]$-tetraquarks

- Future projects underway in combination with lattice QCD

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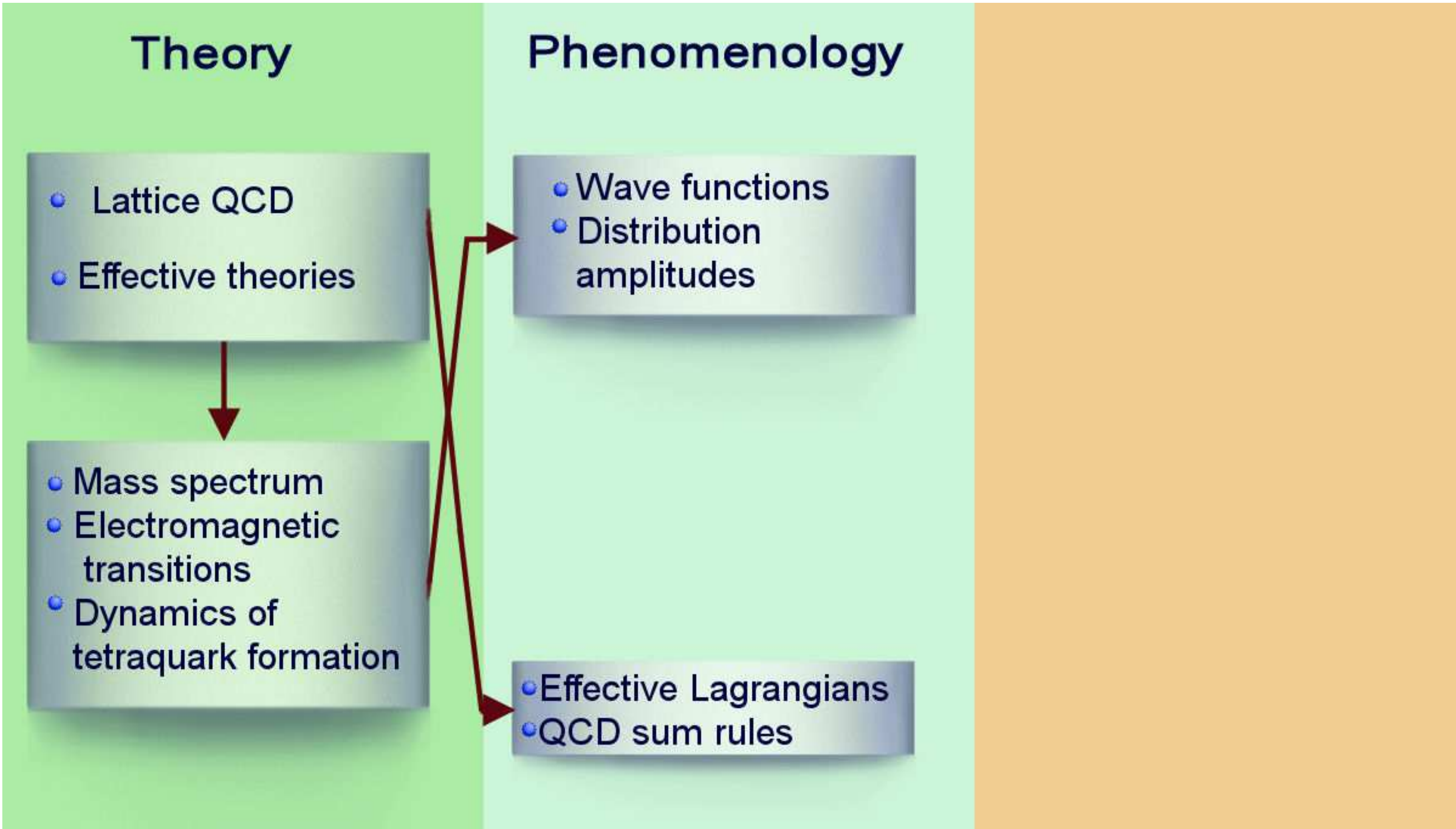

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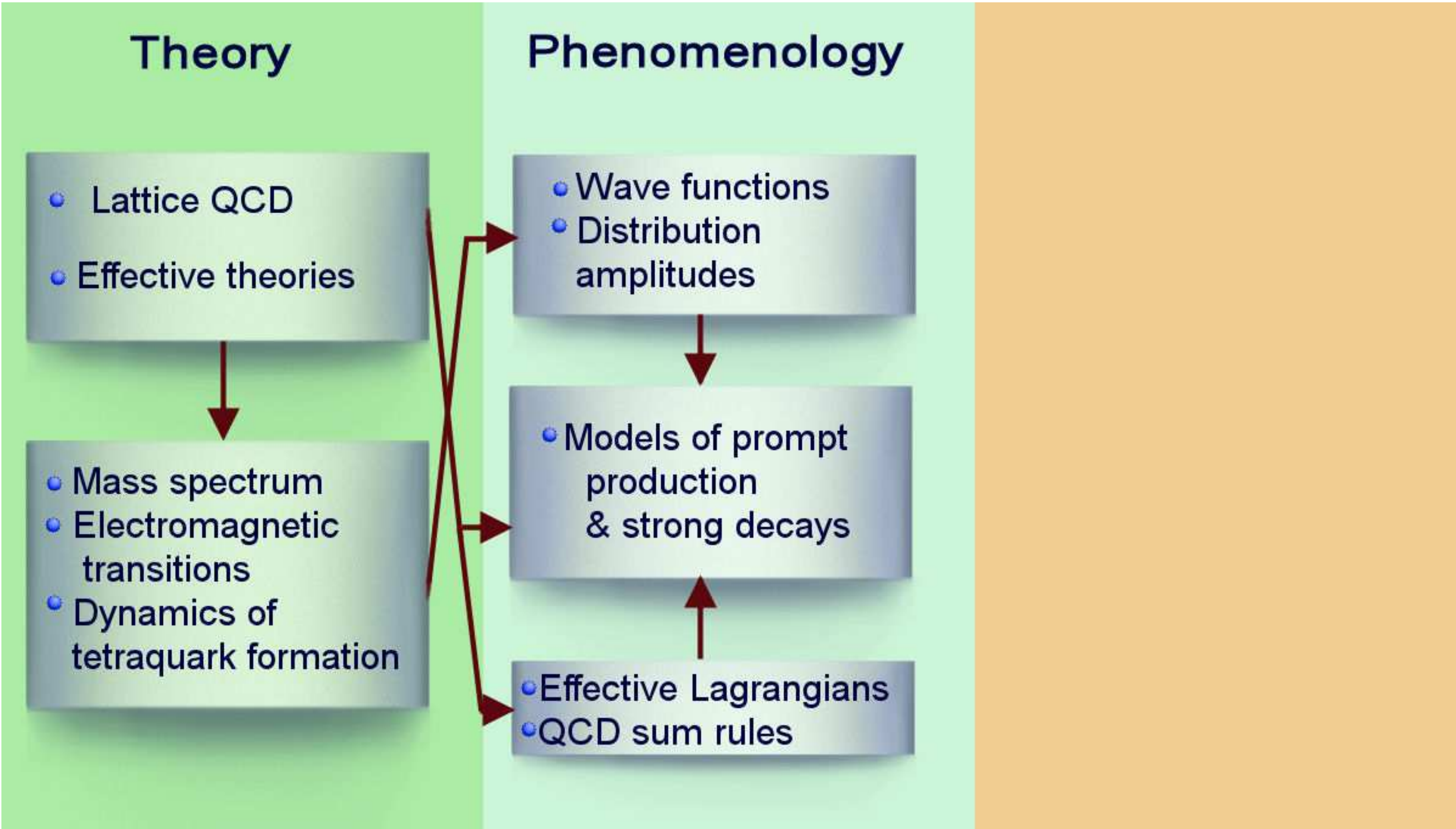

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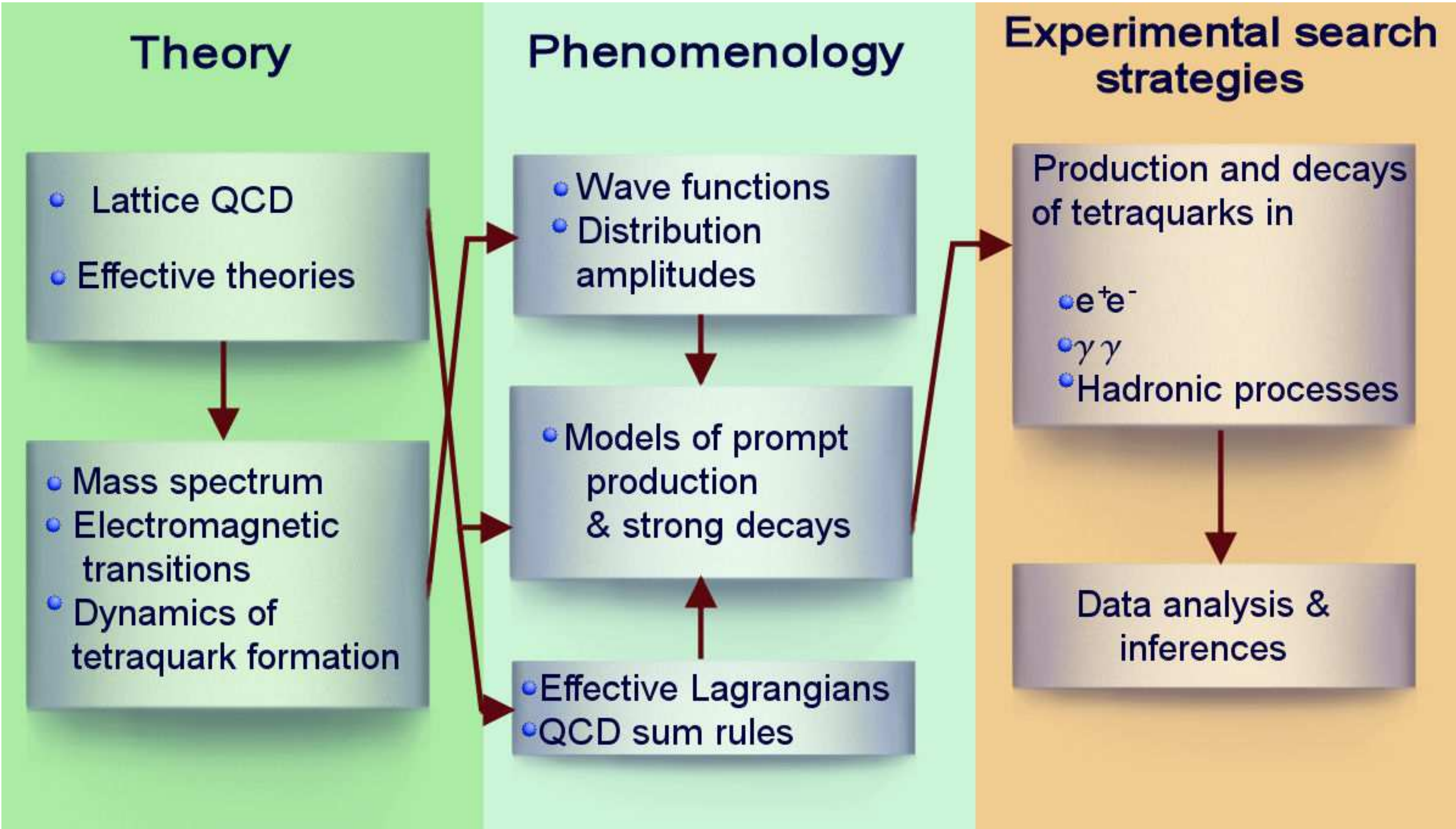

thank you!