Higgs Boson Production via Vector-Boson Fusion at Next-to-Next-to-Leading Order in QCD

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We present the total cross sections at next-to-next-to-leading order in the strong coupling for Higgs boson production via weak-boson fusion. Our results are obtained via the structure function approach, which builds upon the approximate, though very accurate, factorization of the QCD corrections between the two quark lines. The theoretical uncertainty on the total cross sections at the LHC from higher order corrections and the parton distribution uncertainties are estimated at the 2% level each for a wide range of Higgs boson masses.

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One of the main aims for the now-running LHC collider machine is to elucidate the mechanism of electroweak symmetry breaking, and, in particular, to determine whether a standard model Higgs boson exists or not.

Among the various production mechanisms for the Higgs boson, vector-boson fusion (VBF) offers certainly one of the most promising and interesting signals [1–3]. The corresponding cross section is second in size outnumbered only by the gluon-gluon fusion process, it decreases rather mildly with the Higgs boson mass m_H and it is proportional to the tree-level Higgs coupling to the vector bosons, W, Z. Moreover, it provides such an experimentally clean signature with the presence of at least two jets in the forward or backward rapidity region that a rich variety of decay modes can be searched for, opening the access to the very difficult measurements of the Higgs couplings [4]. Given the importance and the experimental prospects for searches of VBF signals it is an urgent task to provide the corresponding theory predictions with the best possible precision including quantum corrections.

Higgs production in VBF is a pure electroweak process at leading order (LO), see Fig. 1. However, at a hadron collider the QCD radiative corrections are typically sizable, and they have first been computed for the total cross section now almost two decades ago in the so-called structure function approach [5]. More recently, the differential cross section at next-to-leading order (NLO) accuracy in QCD has become available [6] along with its implementation in a Monte Carlo event generator [7], and also the full set of combined NLO QCD and electroweak corrections are now known [8]. The typical accuracy of the current QCD predictions can be estimated in the 5%–10% range.

In this Letter we briefly present the results of the computation of the dominant contributions to VBF at next-to-next-to-leading order (NNLO) accuracy in QCD, which give rise to dramatic reductions of the theoretical uncertainties. To that end, we are using the structure function approach. As we will argue in the following, this method, although not truly exact at NNLO, includes the bulk of the radiative corrections so that the remaining contributions, which are both, parametrically small and kinematically suppressed, can be safely neglected.

The first point to be addressed is to which extent VBF is a well-defined process by itself: interference effects with other processes occur, possibly at higher orders in the strong and/or electroweak coupling (α_s respectively $\alpha_{\rm EW}$), whose size sets the target accuracy to which VBF as such can possibly be determined. For example, VBF processes interfere already at LO with the so-called Higgs associated production, leading to a Higgs boson and two-jet final state, i.e., $pp \to HV^{(*)} \to Hjj$, an effect which is, however, at the per mil level [8]. At higher orders, interference with gluon-gluon fusion processes can occur starting at $\alpha_{\rm EW}^2\alpha_s^2$, but they are also found to be well below the percent level [9,10]. It seems therefore conceivable that Higgs production through VBF can be defined in perturbation theory up to an ambiguity of not much better than 1%, which sets the target precision for theoretical predictions.

The structure function approach is based on the observation that to a very good approximation the VBF process can be described as a double deep-inelastic scattering process (DIS), see Fig. 1, where two (virtual) vector bosons V_i (independently) emitted from the hadronic initial states fuse into a Higgs boson. This approximation builds on

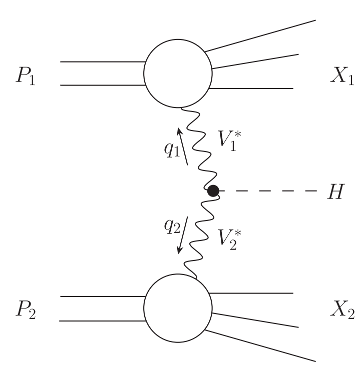

FIG. 1. Higgs production via the VBF process.

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the absence (or smallness) of the QCD interference between the two inclusive final states X_1 and X_2. In that case the total cross section is given as a product of the matrix element $\mathcal{M}^{\mu\rho}$ for VBF, i.e., $V_1^\mu V_2^\rho \to H$, and of the DIS hadronic tensor $W_{\mu\nu}$, the latter being expressed in terms of the standard DIS structure functions $F_i(x, Q^2)$ with $i = 1$, 2, 3:

$$\begin{aligned} d\sigma = & \frac{1}{2S} 2G_F^2 M_{V_1}^2 M_{V_2}^2 \frac{1}{(Q_1^2 + M_{V_1}^2)^2} \frac{1}{(Q_2^2 + M_{V_2}^2)^2} \\ & \times W_{\mu\nu}(x_1, Q_1^2) \mathcal{M}^{\mu\rho} \mathcal{M}^{*\nu\sigma} W_{\rho\sigma}(x_2, Q_2^2) \\ & \times \frac{d^3 P_{X_1}}{(2\pi)^3 2E_{X_1}} \frac{d^3 P_{X_2}}{(2\pi)^3 2E_{X_2}} ds_1 ds_2 \frac{d^3 P_H}{(2\pi)^3 2E_H} \\ & \times (2\pi)^4 \delta^4(P_1 + P_2 - P_{X_1} - P_{X_2} - P_H). \end{aligned} \quad (1)$$

Here $Q_i^2 = -q_i^2$, $x_i = Q_i^2/(2P_i q_i)$ are the usual DIS variables, $s_i = (P_i + q_i)^2$ are the invariant masses of the ith proton remnant, and M_{V_i} denote the vector-boson masses, see Fig. 1. G_F is Fermi's constant and at a given center-of-mass energy $\sqrt{S}$ of the collider the three-particle phase space is given by the second line in Eq. (1).

The factorization underlying Eq. (1) does not hold exactly already at LO, because interference can occur either between identical final state quarks (i.e., $uu \to Huu$) or between processes where either a W or a Z can be exchanged (i.e., $ud \to Hud$). However, at LO, they can be easily computed and have been included in our results. On the other hand, simple arguments of kinematics (based on the behavior of the propagators in the matrix element [11]) show that such contributions are heavily suppressed and contribute to the total cross section well below the percent level, a fact that has been confirmed by complete calculation even through NLO [8]. Apart from these interference effects, the factorization of Eq. (1) is still exact at NLO, so that the DIS structure functions at NLO [12] can be employed. This is due to color conservation: QCD corrections to the upper quark line, Figs. 2(a) and 2(b), are independent from those of the lower line, i.e., $\mathrm{Tr}(t^a) = 0$ for generators t^a of the color $SU(N_c)$ gauge group.

At NNLO due to the possibility of linking the upper and lower quark lines with two colored particles the factorization breaks down. However, it is easy to see that such class of corrections, Figs. 3(a) and 3(b), are infrared and ultraviolet finite, gauge invariant and suppressed [13] both kinematically and parametrically by a factor $1/N_c^2$. The dominant contributions, Figs. 2(c) and 2(d) can therefore be included in the structure functions at NNLO [14–17]. At order $\alpha_{\mathrm{EW}}^3 \alpha_s^2$ another class of diagrams arises, Figs. 3(c) and 3(d), which contributes significantly to associated Higgs production, $pp \to VH$, see [18,19]. While a full computation of these diagrams in VBF is in progress and will be presented elsewhere [20], a first conservative estimate can be easily obtained in the $m_b \to 0$, $m_t \to \infty$ limit, where the triangle diagrams dominate. In this limit, the contribution to the total cross section is well below the percent level and therefore can be neglected. At order $\alpha_{\mathrm{EW}}^3 \alpha_s^2$, also di-jet amplitudes with a single quark line arise where the Higgs boson is emitted from a virtual weak-boson loop. This class of diagrams is gauge invariant but not infrared safe and as it is not a VBF process, it is not included in our calculation. Its contribution to typical VBF final states with a Higgs boson and two jets has been found to be negligible [21].

We now turn to the discussion of the results. For the sake of illustration we consider only a $\sqrt{S} = 7$ TeV LHC, keeping in mind that the conclusions presented here are qualitatively the same for a $\sqrt{S} = 14$ TeV LHC, and also for Tevatron, see [20]. Our reference parton distribution functions (PDFs) set is Ref. [22] (MSTW) and the electroweak parameters (G_F, M_Z, M_W, $\sin^2\theta_W$) are set to their respective Particle Data Group values [23].

Figure 4 presents the cross section as a function of the Higgs boson mass at LO, NLO, and NNLO in QCD, together with the uncertainties coming from (uncalculated) higher orders. These are estimated by an independent variation of the factorization and renormalization scales in the range μ_R, $\mu_F = \xi_{R,F} Q$ with $\xi_{R,F} \in [1/4, 4]$, where Q is the virtuality of the vector boson probing the corresponding structure function to which we apply a technical

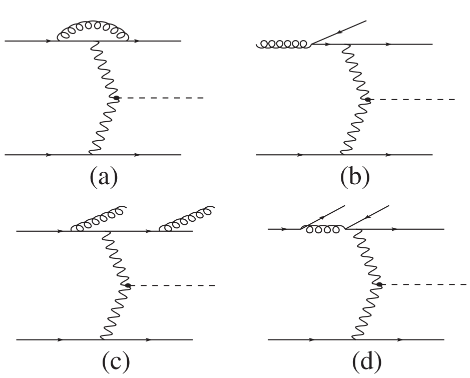

FIG. 2. Representative Feynman diagrams of processes included in the structure function approach.

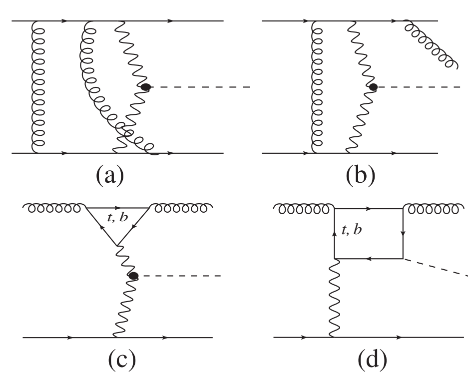

FIG. 3. Representative Feynman diagrams of processes not included in the structure function approach.

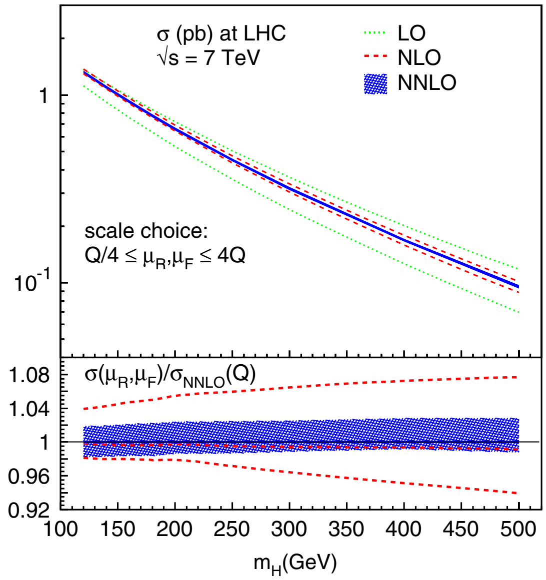

FIG. 4 (color online). The total cross section at LO, NLO, and NNLO as a function of m_H for a $\sqrt{S} = 7$ TeV LHC employing the MSTW PDF set [22]. The uncertainty bands are obtained by scale variation as explained in the text.

cutoff of 1 GeV. The lower inlay of Fig. 4 zooms in on the relative variations normalized to the NNLO cross section at μ_R, $\mu_F = Q$, so that the exceptionally good convergence of the perturbation series can be appreciated. For NNLO this is at the 2% level and in principle, could be pushed even further within the structure function approach by incorporating the available hard corrections at order α_s^3 [24–26]. Numbers for our best estimate, i.e., NNLO in QCD, are presented in Table I.

The most natural choice μ_R, $\mu_F = \xi_{R,F} Q$ as a reference scale is also supported by kinematics arguments, i.e., the observation that the average gauge boson virtuality in VBF amounts only to $\langle Q \rangle \simeq 20$ GeV for a $\sqrt{S} = 7$ TeV LHC. Of course, other scale choices, e.g., μ_R, $\mu_F \in [m_H/4, 4m_H]$, are equally valid. However, they typically exhibit a much poorer convergence of the perturbative expansion and lead to sizable deviations in the lower order

TABLE I. Cross sections (pb) at a $\sqrt{S} = 7$ TeV LHC with the uncertainty due to independent scale variations μ_R, $\mu_F \in [Q/4, 4Q]$ at LO, NLO, and NNLO in QCD as obtained with the MSTW PDF sets [22].

	$\sqrt{S} = 7$ TeV		
Higgs mass	LO	NLO	NNLO
120	$1.235^{+0.131}_{-0.116}$	$1.320^{+0.054}_{-0.022}$	$1.324^{+0.025}_{-0.024}$
160	$0.857^{+0.121}_{-0.099}$	$0.915^{+0.046}_{-0.016}$	$0.918^{+0.019}_{-0.015}$
200	$0.614^{+0.106}_{-0.082}$	$0.655^{+0.038}_{-0.012}$	$0.658^{+0.015}_{-0.010}$
300	$0.295^{+0.070}_{-0.049}$	$0.314^{+0.022}_{-0.010}$	$0.316^{+0.008}_{-0.004}$
400	$0.156^{+0.045}_{-0.030}$	$0.166^{+0.013}_{-0.007}$	$0.167^{+0.005}_{-0.001}$

predictions, especially for heavy Higgs bosons (e.g., a 7% difference for $m_H = 400$ GeV at NLO). Only at NNLO, both the central values and the uncertainty band for the latter choice agree within the 2% level with those in Table I. This clearly demonstrates the markedly improved scale stability of our NNLO predictions.

In Fig. 5 the dependence on the parton distributions and their errors is studied, which estimates the uncertainty of the total cross section due to the nonperturbative parton dynamics inside the proton. To this aim we employ the MSTW 68% confidence level PDF sets [22] through NNLO and compare also with the central predictions obtained with the other available PDF sets based on complete NNLO QCD predictions, i.e., ABKM [27] and JR09VF [28]. The results are consistent and show that an almost constant 2% PDF uncertainty at NNLO can be associated to the cross sections for a wide range of Higgs boson masses, which is slightly reduced compared to the NLO value of 2.5%. The PDFs are dominantly sampled at an average $\langle x \rangle \simeq 0.05$ at a $\sqrt{S} = 7$ TeV LHC, that is in a region where further nonperturbative corrections such as higher-twist effects are negligible; see, e.g., Ref. [29].

Let us next turn to a discussion of the systematics of our approach. In Table II we present an alternative study of the perturbative series; namely, we consider the effects of "improving" the α_s and α_s^2 expansions of the cross section, in terms of expansions of the structure functions at NLO and NNLO which enter as a square. The NLO^2 are obtained by keeping the α_s terms in both structure functions, i.e., including formally α_s^2 terms in the cross section and using the NLO PDFs, while the NNLO^2 results perform the same procedure at one higher order in α_s. A comparison of the numbers in Tables I and II shows that the different higher order approximations agree extremely well within the respective errors bands. Implicitly, our findings also demonstrate that the impact of the NNLO QCD corrections at the central value, i.e., μ_R, $\mu_F = Q$, is

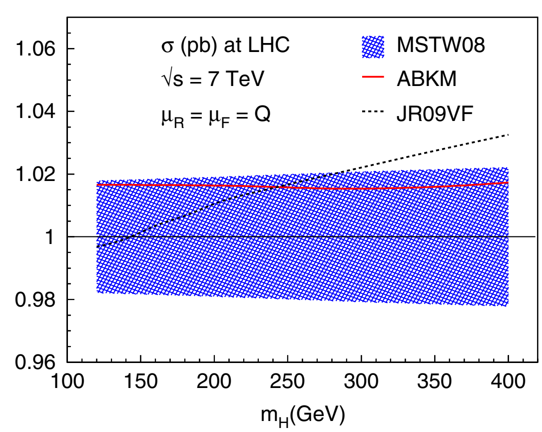

FIG. 5 (color online). The PDF uncertainty of the total cross section at NNLO as a function of m_H at a $\sqrt{S} = 7$ TeV LHC for the 68% C.L. MSTW PDF set [22]. For ABKM [27] and JR09VF [28] the ratio of the central value is plotted.

TABLE II. Cross sections (pb) at a $\sqrt{S} = 7$ TeV LHC with the same parameters as Table I. The approximations NLO^2 (NNLO^2) employ the complete NLO (NNLO) expressions for the structure functions both on the upper and lower quark lines, cf. Fig. 1.

	$\sqrt{S} = 7$ TeV	
Higgs mass	NLO^2	NNLO^2
120	$1.319^{+0.054}_{-0.020}$	$1.323^{+0.024}_{-0.022}$
160	$0.914^{+0.047}_{-0.014}$	$0.918^{+0.020}_{-0.015}$
200	$0.654^{+0.039}_{-0.011}$	$0.657^{+0.016}_{-0.011}$
300	$0.313^{+0.024}_{-0.009}$	$0.315^{+0.009}_{-0.005}$
400	$0.166^{+0.014}_{-0.006}$	$0.167^{+0.005}_{-0.002}$

relatively small, and in fact smaller than 1%, as we have explicitly checked. Altogether this gives further evidence that nonfactorizable contributions at NNLO which are not accounted for in the structure function approach and can very conservatively be estimated to amount to less than 10% of the NNLO factorizable ones, are completely negligible.

To summarize, we have presented the first computation of the VBF cross section at NNLO in QCD. The inclusion of higher order corrections stabilizes the results at the 2% level against arbitrary variations of the renormalization and factorization scales, indicating an extremely well-behaved perturbative expansion. PDF uncertainties are estimated at the 2% level as well, uniform over a wide range of Higgs boson masses. Our results motivate the calculation of the differential cross sections distributions at NNLO via an exclusive method such as that proposed in Ref. [30]. Moreover, our approach can be used to provide cross-section predictions to NNLO accuracy for any weak-boson fusion process leading to a weakly interacting n-particle final state X, i.e., $V^* V'^* \to X$. Work in the latter direction is in progress.

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