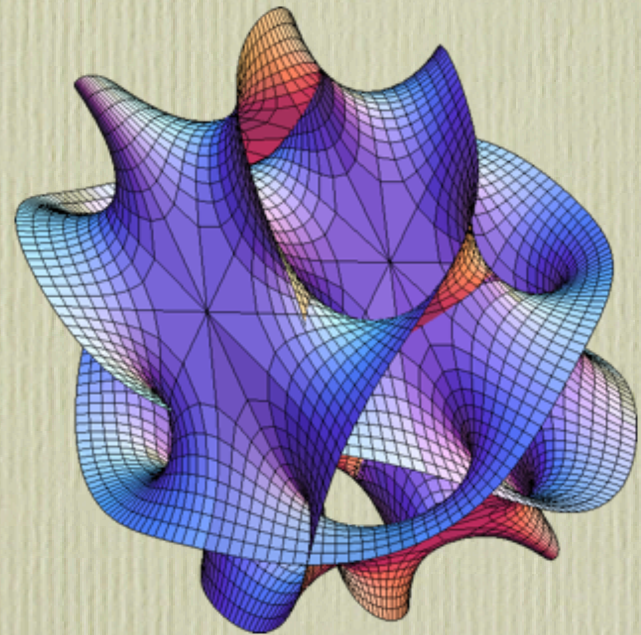
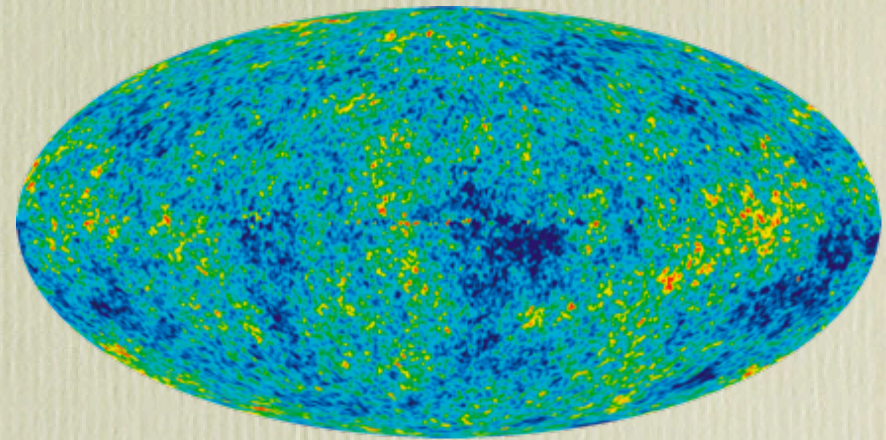
From Inflation to the Planck Scale





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where I want to take you ...

- (0) Short cosmological background:
  - why did & do we need inflation in the first place?
- (I) Slow-roll inflation
  - single-field slow-roll
  - small-field vs large-field inflation
  - The need for UV completion
- (II) How to test inflation - a short introduction to inflationary density perturbations
  - The gift of inflation - quantum fluctuations in (quasi) de Sitter space
  - shortcut to the curvature perturbation - seed of structure, power spectrum & ns
  - primordial gravity waves (tensors) & r

where I want to take you ...

- (III) UV-completion - Inflation in string theory
  - crash course in moduli stabilization - a *must* for inflation in string theory
  - good stringy inflatons: brane positions & axions
  - 2 examples:
    i) warped D3-brane inflation: prototype small-field
    ii) 5-brane axion monodromy: prototype large-field

From Inflation to the Planck Scale

Outline:

⓪ Short cosmological background – why did & do we need inflation in the first place?

Ⓘ Slow-roll inflation

- single-field slow-roll
- small-field vs. large-field inflation
- The need for UV completion

Ⓘⓘ How to test inflation – a short introduction to inflationary density perturbations

- The gift of inflation – quantum fluctuations of the inflaton in (quasi) de Sitter space
- shortcut to the curvature perturbation – seed of structure

– power spectrum – n_s, r – CMB

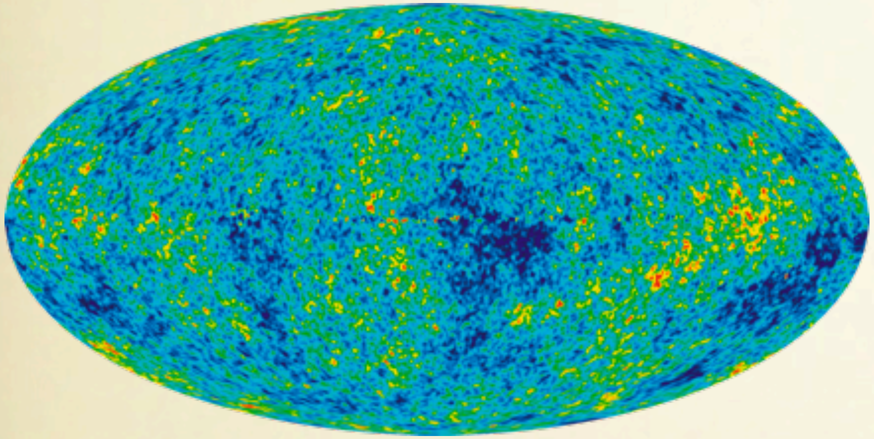
(III) UV completion – Inflation in string theory

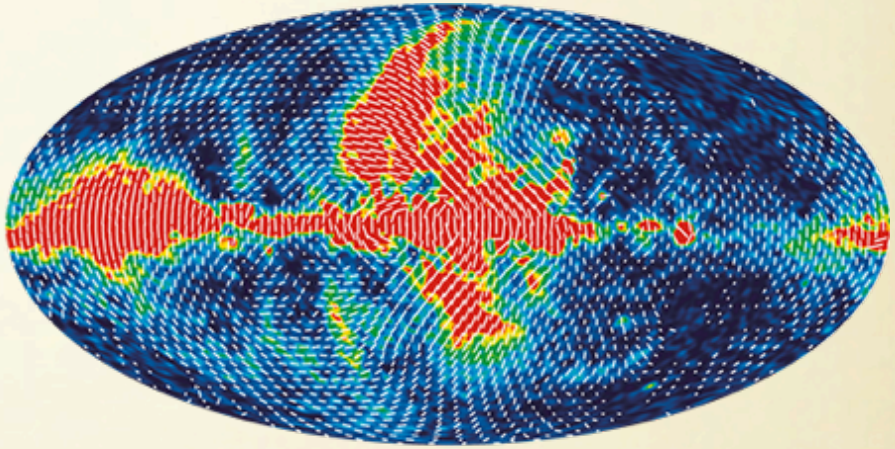
– crash course in moduli stabilization – a <u>must</u> for inflation

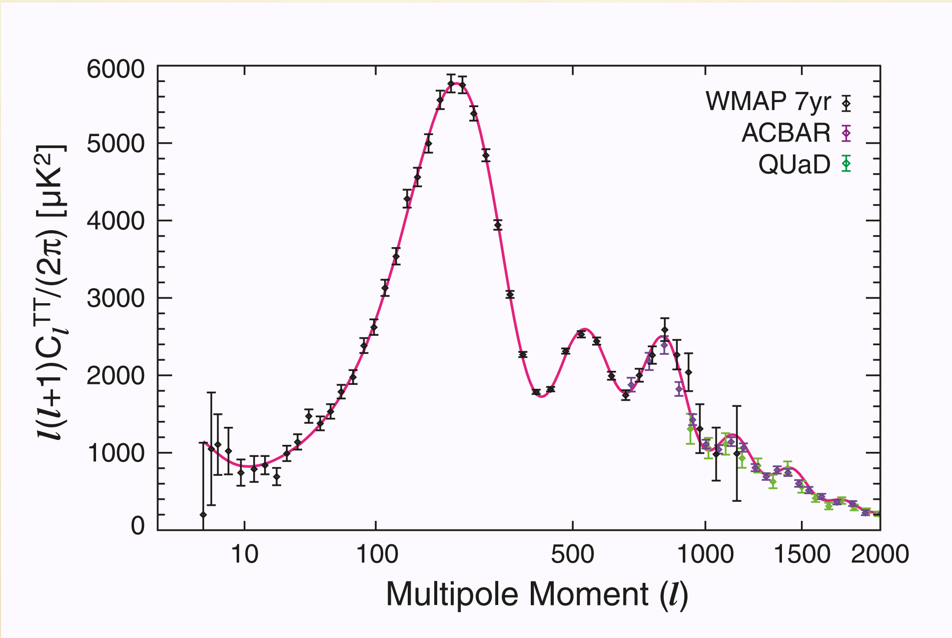
– good stringy inflatons: brane positions & axions

– 2 examples:
- warped D3-brane inflation
  → prototype small-field
- axion monodromy on 5-branes
  → prototype large-field

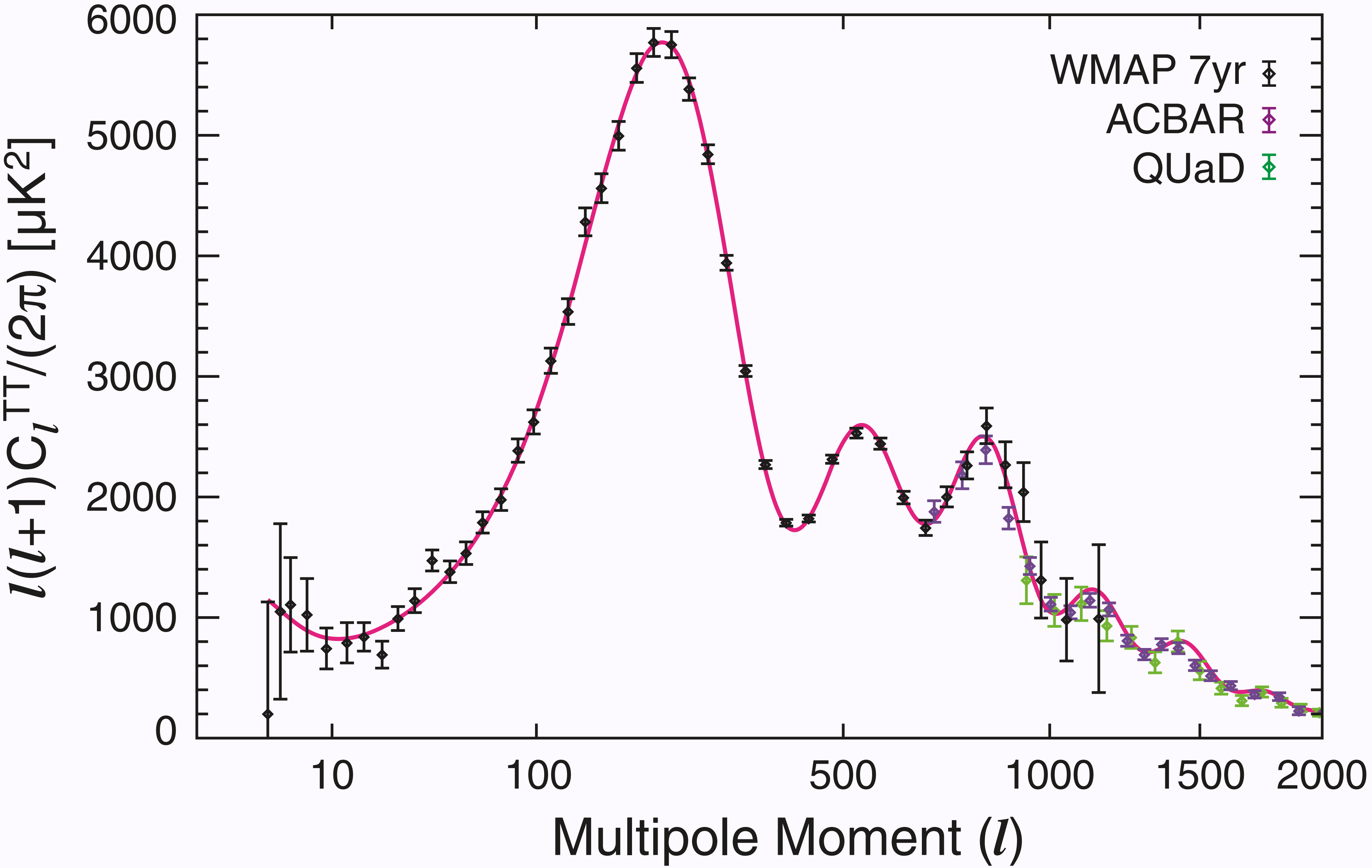
an age of precision cosmology ...







[NASA/WMAP Science Team]



$l(l+1)C_l^{TT}/(2\pi)$ [μK^2]
6000
5000
4000
3000
2000
1000
0
10
100
500
1000
1500
2000
Multipole Moment (l)
WMAP 7yr
ACBAR
QUaD

1

0. Why did & do we need inflation in the first place?

↳ Since Hubble, Friedmann & Co. we know the Universe expands, and well described by GR:

scale factor ('size of Universe')
$= a \Rightarrow \underbrace{\dot{a}}_{\text{speed of expansion}} = \underbrace{H}_{\text{Hubble parameter}} \cdot a$

redshift: $\frac{\lambda_0}{\lambda} = \frac{a_0}{a} = 1 + z$ — 0: today

2

↳ CMB: spatially isotropic
→ $\frac{\Delta T}{T} \lesssim 10^{-4}$ in all directions

& some evidence for spatial homogeneity

⇓

FRW space-time:

$$ds^2 = dt^2 - a^2(t) \cdot \left[dr^2 + f^2(r) \cdot d\Omega_2^2\right]$$

maximally symmetric 3-space

$$\frac{S^3,\ H^3,\ \mathbb{R}^3}{k = +1,\ -1,\ 0}$$

Einstein's equations

$$\Rightarrow H^2 = \frac{1}{3M_p^2}\rho - \frac{k}{a^2}$$

Friedmann equation

energy density: matter, radiation, ...

spatial curvature

ρ drives expansion ...

if ρ mostly:	c.c.	matter	radiation
$a(t)$	e^{Ht}	$t^{2/3}$	$\sqrt{t}$
$H(t)$	const.	$\sim \frac{1}{t}$	$\sim \frac{1}{t}$
p	$-\rho$	0	$\frac{1}{3}\rho$

inflation!

horizons:

light propagates as $ds^2 = 0$

$$\Rightarrow dr = \frac{dt}{a(t)} \quad \text{can see} \quad d = a_0 \int_0^{t_0} \frac{dt'}{a(t')}$$

"Hubble radius"

today: $\sim 4 \cdot 10^{10}$ ly, CMB

$$\sim t_0 \sim \frac{1}{H_0}$$

matter or radiation

CMB: photon decoupling at $3000\,K \simeq 0.3\,eV \rightarrow z = 1100$

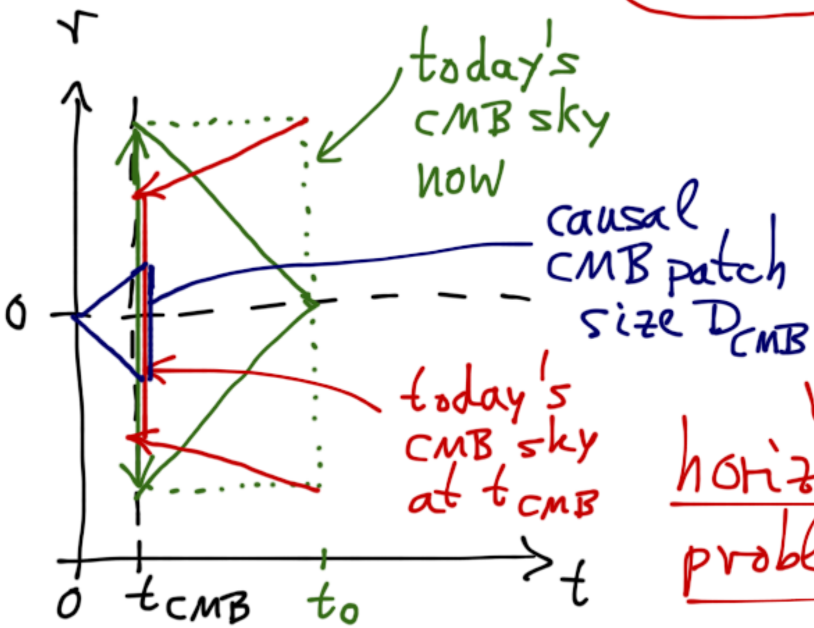
$\rightarrow 3K$ today

$\boxed{t_{CMB} \simeq 400.000\ yr}$

to-day's CMB sky was smaller at t_{CMB} by:

$$\frac{a(t_{CMB})}{a_0} = \left(\frac{t_{CMB}}{t_0}\right)^{2/3} = \frac{1}{1+z} = 10^{-3}$$

so $4\cdot 10^7\, ly$ then. But causal patch at t_{CMB} only $\boxed{D_{CMB} \simeq 4\cdot 10^5\, ly}$.



horizon problem

an idea: try getting $\ddot{a} > 0$

$\Rightarrow$ $a(t)$ grows faster than t

$\Rightarrow$ today's CMB sky was smaller than causal patch at some early time

solves horizon problem

$\Downarrow$

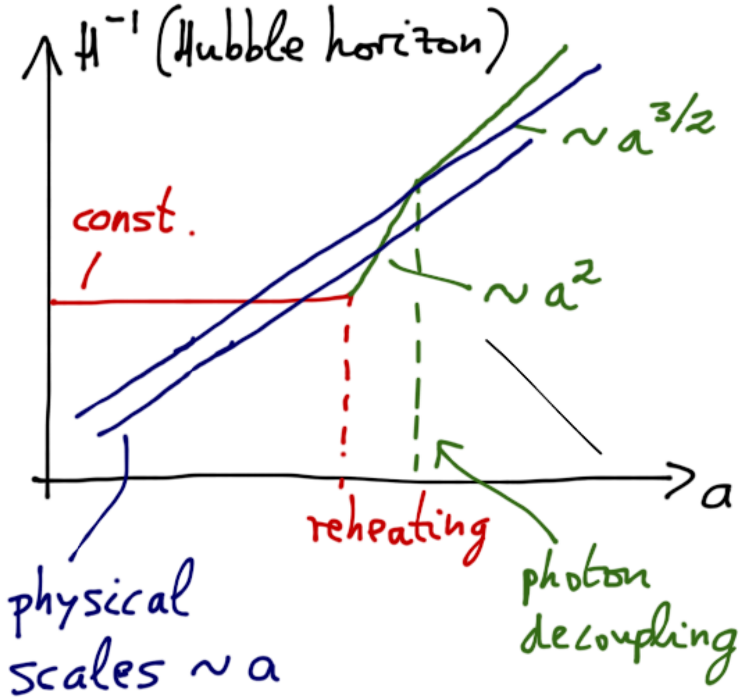
try a finite epoch of a quasi-cosmological constant - a large one:

$$\Rightarrow \rho_\Lambda = \text{const.} = -p_\Lambda$$

$$\Rightarrow H_\Lambda^2 = \frac{\rho_\Lambda}{3M_P^2} = \text{const.} = \frac{\dot{a}^2}{a^2}$$

$$\Rightarrow \boxed{a \sim e^{H_\Lambda \cdot t}}$$

inflation



↘ modes left the horizon during inflation & re-entered afterwards → were in causal contact early on

→ solves horizon problem & some others (flatness, ...)

how to do this?

historically 1^{st}: Guth '80

→ supercool Universe into a meta-stable state of positive vacuum energy → problems with exit & reheating

↓

(I) slow-roll inflation (Linde '82, Steinhardt)

1 scalar field ϕ

action:

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)\right]$$

scalar potential

$\downarrow \delta S/\delta g^{\mu\nu}$

$$T_{\mu\nu}: \ \rho = \frac{1}{2}(\partial\phi)^2 + V, \ p = \frac{1}{2}(\partial\phi)^2 - V$$

consider : $\vec{\nabla}\phi = 0$, only $\dot{\phi}$

redshifts fast, if $a \sim e^{H\cdot t}$

then if : $\boxed{\dot{\phi}^2 << V}$ $\Rightarrow p = -\rho$

motion dominated by potential energy

$$\begin{cases} a \sim e^{H\cdot t} \\ H \simeq \text{const.} \end{cases}$$

need this for $N_e \simeq H\cdot t \overset{!}{\simeq} 60$ efolds to solve the horizon etc. problems ...

can ensure this, if slow-roll:

e.o.m. for ϕ :

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi} \equiv -V'$$

slow-roll : $|\ddot{\phi}| << |3H\dot{\phi}|, |V'|$

$\Rightarrow 3H\dot{\phi} = -V'$ slow-roll e.o.m. (*)

then i) : $p \simeq -\rho$

$$\Rightarrow 1 >> \frac{\dot{\phi}^2}{V} = \frac{V'^2}{9VH^2} , H^2 \simeq \frac{V}{3}$$

$$= \frac{1}{3}\left(\frac{V'}{V}\right)^2 \equiv \frac{2}{3}\epsilon , \quad \boxed{\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2}$$

$\Rightarrow$ $\boxed{\epsilon << 1 \text{ ensures } p \simeq -\rho .}$

<u>1st slow-roll condition</u>

ensure slow-roll for long time:

↳ maintain: $\ddot{\varphi} << 3H\dot{\varphi}$

from (*) $\Rightarrow$ $\dot{\varphi}^2 = \frac{V'^2}{3V}$

$\Rightarrow$ $\dot{\varphi}\ddot{\varphi} = \frac{1}{2}\left(\frac{V'^2}{3V}\right)' \cdot \dot{\varphi}$

$\Rightarrow$ $\ddot{\varphi} = V' \cdot \left(\frac{1}{3}\frac{V''}{V} - \frac{V'^2}{V^2}\right)$

define: $\boxed{\eta \equiv \frac{V''}{V}}$

$\Rightarrow$ $\frac{\ddot{\varphi}}{3H\dot{\varphi}} = 2\epsilon - \frac{1}{3}\eta << 1$

implies: $\boxed{\eta << 1}$ if $\epsilon << 1$

2nd slow-roll condition

if:

$\epsilon, \eta << 1$

then:

$$\epsilon \simeq \epsilon_H = -\frac{\dot{H}}{H^2} = 1 - \frac{1}{H^2}\frac{\ddot{a}}{a}$$

$$\eta \simeq \eta_H = \frac{\dot{\epsilon}_H}{\epsilon_H H}$$

$\Rightarrow$ $\epsilon << 1$ implies $\epsilon_H << 1$

$\Rightarrow$ $\ddot{a} > 0$

consistent.

ϵ_H, η_H 'physical' Hubble slow-roll parameters can do much more:

- multiple fields
- higher derivatives
- non-trivial kinetic terms

but here simple single-field slow-roll will suffice.

General story here:

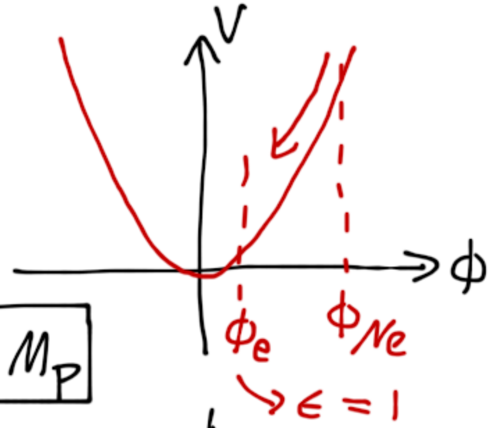
- need a $V(\phi)$ that satisfies $\epsilon, \eta \ll 1$ at some ϕ_{N_e},
- $\epsilon, \eta < 1$ for $N_e \simeq 60$ efolds at least, then $\epsilon > 1$ must be reached at some ϕ_e → slow-roll ends, ϕ oscillates → reheating, FRW

2 classes of $V(\phi)$:

i) large-field models

examples: $V(\phi) \sim \phi^p$, $p \geq 2$

e.g. $V(\phi) = \frac{m^2}{2}\phi^2$



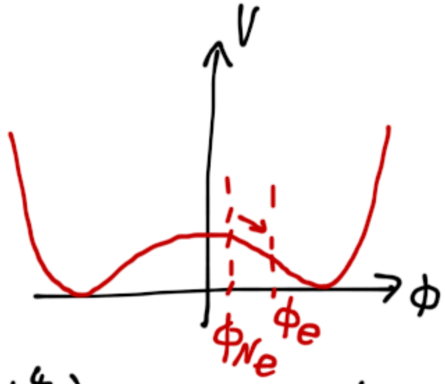
ϕ_e: $\epsilon(\phi_e) = 1$

$$\epsilon = \frac{p^2}{2\phi^2} \Rightarrow \boxed{\phi_e \sim M_P}$$

$$N_e = \int_{t_e}^{t_{N_e}} H\,dt = \int_{\phi_e}^{\phi_{N_e}} \frac{H}{\dot\phi}\,d\phi = \int_{\phi_e}^{\phi_{N_e}} \frac{d\phi}{\sqrt{2\epsilon}} \simeq \frac{\phi_{N_e}^2}{2p} \Rightarrow \boxed{\phi_{N_e} = \sqrt{2p N_e} \gg M_P}$$

ii) small-field models

example: hill-top in $V(\phi)=\lambda(\phi^2-v^2)^2$ and $\phi_{N_e} \ll v$



$\downarrow$

$$V(\phi) = V_0\left(1-\frac{2}{v^2}\phi^2+\frac{\phi^4}{v^4}\right), \quad V_0=\lambda v^4$$

$$\Rightarrow \epsilon = \frac{8\phi^2}{v^4}\left(1-\frac{\phi^2}{v^2}\right), \quad \phi \ll v$$

$$\Rightarrow \phi_e \simeq \frac{1}{2\sqrt{2}}\left(\frac{v}{M_p}\right)^2 \cdot M_p$$

if $v \lesssim M_p$ $\rightarrow$ small-field

$\hookrightarrow$ if a 2nd field ends slow-roll by 'waterfall' into a minimum

$\rightarrow$ 'hybrid' inflation ...

Why $\phi \sim M_p$ as discriminator?

$\rightarrow$ so far this was classical

$\rightarrow$ consider dim-6 operators correcting V ...

$\Rightarrow$ generically we get among them:

$$\Delta V_6 \sim V \cdot \frac{\phi^2}{M_p^2}$$

$$\Rightarrow \Delta\eta = \frac{\Delta V_6''}{V} \sim \mathcal{O}(1)$$

$$\Rightarrow \Delta\epsilon \sim \left(\frac{\phi}{M_p}\right)^2 \rightarrow 1 \text{ at } \phi \sim M_p$$

$\rightarrow$ destroys slow-roll: "eta-problem"

2 ways out :

- keep $\phi \ll M_p$ "small-field" and look for UV-theory to enumerate finite # of dim-6 corrections – and tune
- find a symmetry, that forbids all dim-6 terms → then $\phi \gg M_p$ possible...

$\rightsquigarrow$ example: $V(\phi) \sim \phi^p$ + gravity

graviton vertex $\sim T_{\mu\nu} \sim V, V''$

$\Rightarrow \Delta V \sim V^2, V''V$ not $V \cdot \frac{\phi^2}{M_p^2}$

→ shift symmetry of gravity...

→ $\boxed{\text{Clearly need UV completion!}}$

(II) How to test inflation – density fluctuations from inflation

i) The gift of inflation – (near) scale-invariant quantum fluctuations in (near) de Sitter space:

$\rightsquigarrow$ slow-roll inflation → long phase of quasi-exponential expansion $a \sim e^{H \cdot t}$ with slowly varying $H \sim \sqrt{V(\phi)}$

approximate space-time by exact exponential expansion – de Sitter (dS) space:

$$ds^2 = dt^2 - a^2(t)\cdot d\vec{x}_3^{\,2} = a^2(t)\cdot\left(d\eta^2 - d\vec{x}_3^{\,2}\right)$$

$$a(t) = e^{H\cdot t}, \quad H = \sqrt{\frac{\Lambda}{3}} = \text{const.}$$

$$d\eta = \frac{dt}{a} \iff \eta = -\frac{1}{aH}$$

'conformal time'

scalar field e.o.m. in conformal time for free scalar field:

$$S = \int d^4x \sqrt{-g}\, g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$$

$$g^{\mu\nu} = \frac{1}{a^2}\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1\end{pmatrix}$$

$$\sqrt{-g} = a^4$$

$$= \int d^4x\, \frac{1}{\eta^2 H^2}\left[\phi'^2 - (\vec{\nabla}\phi)^2\right]$$

$$(\ )' \equiv \frac{\partial}{\partial\eta}(\ )$$

$$\phi(\vec{r},t) = \int \frac{d^3k}{(2\pi)^3}\,\phi_k\, e^{i\vec{k}\vec{r}}$$

$$\Rightarrow \quad S_k = \int d\eta \, \frac{1}{\eta^2 H^2} \left(\phi_k'^2 - k^2 \cdot \phi_k^2 \right)$$

$\Rightarrow$ e.o.m. for scalar field fluctuations ϕ_k:

$$\frac{\partial}{\partial \eta} \left(\frac{1}{\eta^2 H^2} \frac{\partial}{\partial \eta} \phi_k \right) + \frac{k^2}{\eta^2 H^2} \phi_k = 0$$

$$\Rightarrow \quad \phi_k'' - \frac{2}{\eta} \phi_k + k^2 \phi_k = 0$$

dS e.o.m.

solutions:

$$\phi_k = c_1 \, \frac{1 - ik\eta}{\sqrt{2k^3}} e^{ik\eta} + c_2 \cdot (h.c.)$$

now: the past is $t \to 0$, $a \to 0$ $\Rightarrow \eta \to \infty$

$\rightsquigarrow$ then the ϕ_k was short wavelength $\ll H^{-1}$

$\Rightarrow$ should be Minkowski mode

$$\phi_k \xrightarrow[\eta \to \infty]{} \frac{i}{a\sqrt{2k}} e^{ik\eta}$$

$\Rightarrow$ choose: $c_1 = H$, $c_2 = 0$

'Bunch-Davies vacuum'

$$\Rightarrow \phi_k = H \cdot \frac{1 - ik\eta}{\sqrt{2k^3}} e^{ik\eta}$$

quantize:

$$\phi = \int \frac{d^3k}{(2\pi)^3}\left(a_{\vec k}\,\phi_k\, e^{i\vec k\vec r} + a^+_{\vec k}\,\phi_k^*\, e^{-i\vec k\vec r}\right)$$

$$\left[a_{\vec k},\, a^+_{\vec k'}\right] = (2\pi)^3\,\delta^{(3)}(\vec k-\vec k')$$

compute 2-point function:

$$\langle\phi^2\rangle = \langle 0|\phi(x)\,\phi(x')|0\rangle$$

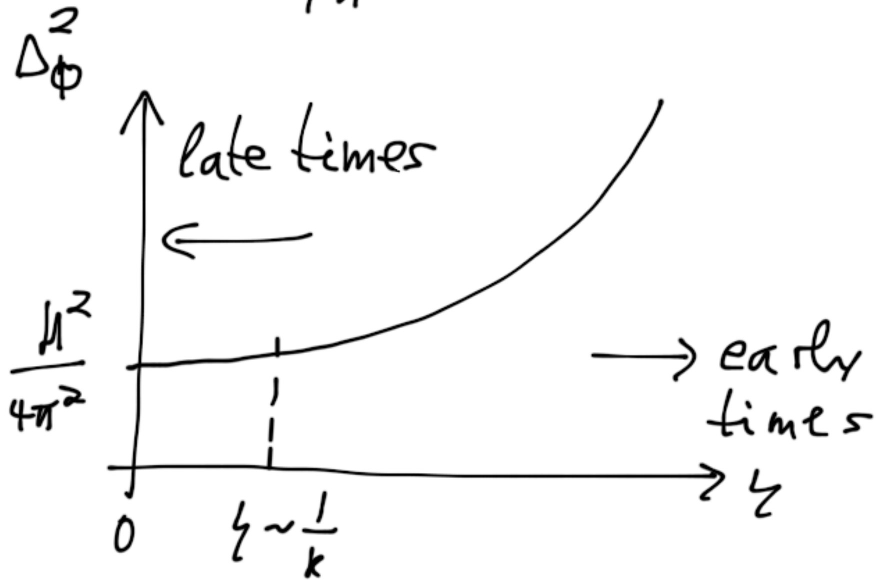
$$= \int\frac{d^3k}{(2\pi)^3}\int\frac{d^3k'}{(2\pi)^3}\,\phi_k\,\phi^*_{k'}\, e^{i(\vec k\vec r - \vec k'\vec r')}\left[a_{\vec k},\, a^+_{\vec k'}\right]$$

$$= \int \frac{d^3k}{(2\pi)^3}\cdot|\phi_k|^2\cdot e^{i\vec k(\vec r-\vec r')}$$

$$= \int d\ln k\cdot \underbrace{\frac{|\phi_k|^2\cdot k^3}{2\cdot\pi^2}}_{\equiv\, \Delta^2_\phi}\cdot\frac{\sin\left(|\vec k|\cdot|\vec r-\vec r'|\right)}{|\vec k|\cdot|\vec r-\vec r'|}$$

'power spectrum of 2-point function of ϕ'

$$\Rightarrow \Delta^2_\phi = \frac{H^2}{4\pi^2}\cdot\left(1+k^2\eta^2\right)$$



crucial: Δ_ϕ^2 freezes at late times, when it becomes superhorizon → and so does any other quantity which is quasi-scalar...

slow-roll vs. exact dS: corrections are $\mathcal{O}(\epsilon,\eta)$

$$\Rightarrow \Delta_\phi^2 \Big|_{\underbrace{k\sim\frac{1}{\eta}=aH}_{\text{'at horizon crossing'}}} \simeq \frac{H^2}{4\pi^2}$$

how does the metric fluctuate with ϕ? ⇝ need fluctuations of gravitational potential to seed $\frac{\delta\rho}{\rho}$ 'density perturbations'

...

guidance: Schwarzschild metric of gravitating mass

$$ds^2 = (1-2\zeta)dt^2 - \frac{dr^2}{1-2\zeta} - d\vec{x}_2^2$$

at weak fields:

$$ds^2 = (1-2\zeta)dt^2 - (1+2\zeta)dr^2 - \ldots$$

(full argument long ↔ gauge inv. of GR)

now, in slow-roll :

inflaton jump $\delta\phi$

$\Rightarrow$ need $\delta N = H \cdot \delta t = \frac{H}{\dot\phi} \cdot \delta\phi$

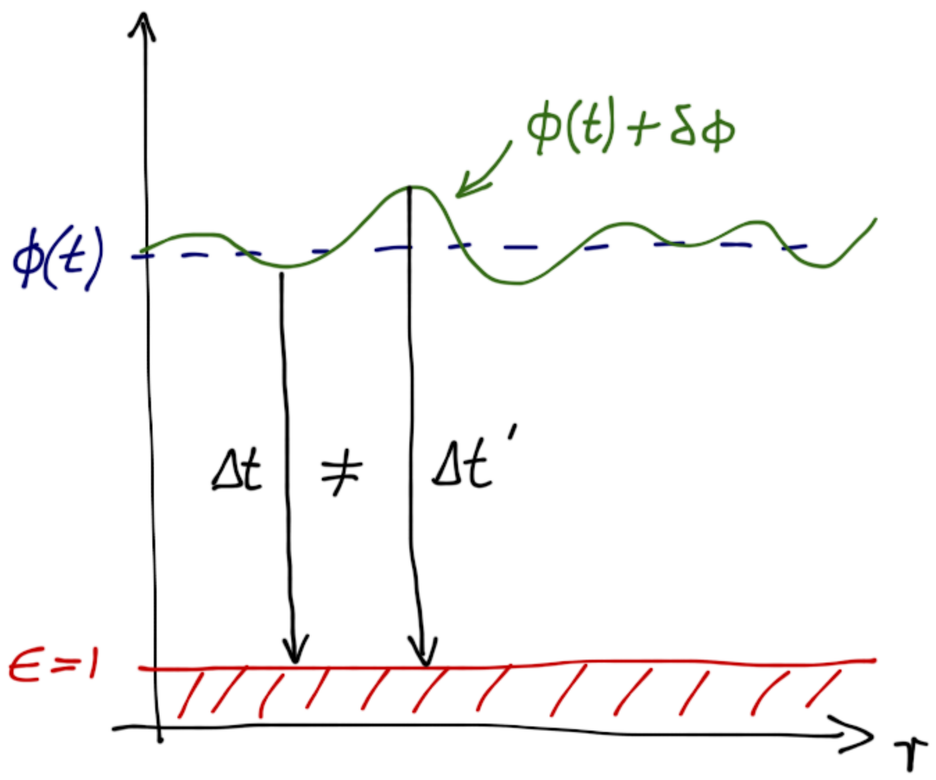
more/less e-folds to reach reheating

$$\Rightarrow \quad ds^2 = dt^2 - e^{2H\cdot t} \cdot d\vec{x}_3^{\,2}$$

$$\longrightarrow \quad ds^2 = dt^2 - e^{2H(t+\delta t)} \cdot d\vec{x}_3^{\,2}$$

$$= dt^2 - e^{2Ht} \cdot (1 + 2\cdot\delta N) \cdot d\vec{x}_3^{\,2}$$

we can display this graphically :



compare:

$$\zeta = \delta N = \frac{H}{\dot\phi} \cdot \delta\phi$$

'curvature perturbation' induced by inflaton fluctuation $\delta\phi$

$$\rightsquigarrow \quad \zeta^2 = \frac{H^2}{\dot\phi^2} \delta\phi^2$$

$$\Rightarrow \Delta_\zeta^2 = \frac{H^2}{\dot\phi^2} \Delta_\phi^2 = \frac{H^4}{4\pi^2 \dot\phi^2}$$

in slow-roll: $\dot\phi = -\frac{V'}{3H}$

$$\Rightarrow \Delta_\zeta^2 = \frac{1}{12\pi^2} \cdot \frac{V^3}{V'^2} = \frac{1}{24\pi^2} \cdot \frac{V}{\epsilon}$$

can show:

$$\frac{\delta\rho}{\rho} = \frac{2}{5}\sqrt{\Delta_\zeta^2} \rightarrow \frac{\Delta T}{T} \text{ of CMB}$$

$$\boxed{\begin{array}{l} \text{CMB:} \\ \frac{\Delta T}{T} \sim 10^{-5} \\ \text{measures } \frac{V}{\epsilon} \end{array}}$$

in exact dS:

$$\Delta_\phi^2 = \text{const.}$$

in slow-roll can parametrize:

$$\Delta_\zeta^2(k) = \Delta_\zeta^2(k_0) \cdot \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$n_s - 1$: spectral tilt

expand:

$$\ln \Delta_\zeta^2(k) = \ln \Delta_\zeta^2(k_0) + \frac{d \ln \Delta_\zeta^2(k_0)}{d \ln k} \cdot \ln\frac{k}{k_0} + \dots$$

$$\Rightarrow n_s - 1 = \left.\frac{d \ln \Delta_\zeta^2}{d \ln k}\right|_{k = k_0 = aH}$$

relation between comoving wave number k and physical wave number $k_{phys.}$:

$$k = \frac{k_{phys.}}{a} = k_{phys.} \cdot e^{-N}$$

$$\Rightarrow d \ln k = - dN$$

$$\Rightarrow n_s - 1 = \left.\frac{d \ln \Delta_\zeta^2}{dN}\right|_{N \simeq 60}$$

$$\frac{d \ln \Delta_\zeta^2}{dN} = \frac{1}{\Delta_\zeta^2} \cdot \frac{d \Delta_\zeta^2}{dN}$$

$$= 12\pi^2 \cdot \frac{V'^2}{V^3} \cdot \frac{d\phi}{dN} \cdot \frac{d}{d\phi}\left(\frac{1}{12\pi^2} \frac{V^3}{V'^2}\right)$$

$$\frac{d\phi}{dN} = \dot{\phi} \cdot \frac{dt}{dN} = \frac{\dot{\phi}}{H} = -\frac{V'}{3H^2}$$

$$= -\frac{V'}{V}$$

$$= -\frac{V'^3}{V^4} \cdot \left(3 \frac{V^2}{V'} - 2 \frac{V^3}{V'^3} V''\right)$$

$$= -6 \cdot \left(\frac{1}{2} \frac{V'^2}{V^2}\right) + 2 \cdot \frac{V''}{V} = -6\epsilon + 2\eta$$

$$\Rightarrow n_s = 1 - 6\epsilon + 2\eta$$

WMAP: $n_s = 0.963 \pm 0.013$, 1σ

PLANCK: $\Delta n_s = 0.005$ at 2σ

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inflation also seeds primordial gravitational waves:

$\rightsquigarrow$ tensor perturbations of the dS metric

$$ds^2 = (1-2\zeta)dt^2 - \left[(1+2\zeta)\delta_{ij} + h_{ij}\right]dx^i dx^j$$

tensor perturbations

$$S = \int d^4x \frac{1}{\eta^2 H^2}\left(h_{ij}' h^{ij\,\prime} - (\vec{\nabla} h_{ij})^2\right)$$

Fourier decompose:

$$h_{ij} = \int \frac{d^3k}{(2\pi)^3} e_{ij} h_k e^{i\vec{k}\vec{r}}$$

polarization tensor

$\Downarrow$

e.o.m.:

$$h_k'' - \frac{2}{\eta} h_k + k^2 \cdot h_k = 0$$

$$\Rightarrow \quad h_k = H \cdot \frac{1 - ik\eta}{\sqrt{2k^3}} e^{ik\eta}$$

$\Rightarrow$ quantization of h_k as for ϕ_k

$$\Rightarrow \quad \Delta_h^2 = \frac{|h_k^2| \cdot k^3}{2 \cdot \pi^2} = \frac{H^2}{4\pi^2}$$

no further 'translation factor' unlike Δ_ζ^2 – h_k is already a metric perturbations ...

define tensor-to-scalar ratio r:

$$r = \frac{\Delta_h^2}{\Delta_\zeta^2} = \frac{\dot\phi^2}{H^2} = 2\epsilon$$

... doing a better job on normalization:

$$\boxed{r = 16\epsilon}$$

example: $V \sim \phi^p$

$$n_s = 1 - \frac{2+p}{2N_e} \simeq 0.97$$

$$r = \frac{4p}{N_e} = 0.13 \quad \text{for } p = 2.$$

measuring r determines ϵ, and via $\delta\rho/\rho$ from $\Delta T/T$ the scale of inflation V!

2nd significance of r:

compute $N_e = \int H dt = \int \frac{d\varphi}{\sqrt{2\epsilon}}$

$$\Rightarrow \quad N_e \simeq \frac{\Delta\phi}{M_p} \cdot \frac{1}{\sqrt{2\epsilon}}$$

$$\Leftrightarrow \quad r = 16\epsilon \simeq \frac{8}{N_e^2} \cdot \left(\frac{\Delta\phi}{M_p}\right)^2$$

$$\Rightarrow \quad \boxed{r \simeq 0.003 \cdot \left(\frac{50}{N_e}\right)^2 \cdot \left(\frac{\Delta\phi}{M_p}\right)^2}$$

'Lyth bound'

$\rightsquigarrow$ $r \sim 0.01$ corresponds to boundary between large-field and small-field inflation.

how to measure r ?

i) Δ_h^2 converts into $\frac{\Delta T}{T}$ at large angular scales $> 10^0$

$\rightarrow$ WMAP bound, $r \lesssim 0.2$

ii) B-mode polarization of CMB:

$\rightarrow$ look for curl-like pattern of polarization vectors around cold spot in CMB:

 instead of  or 

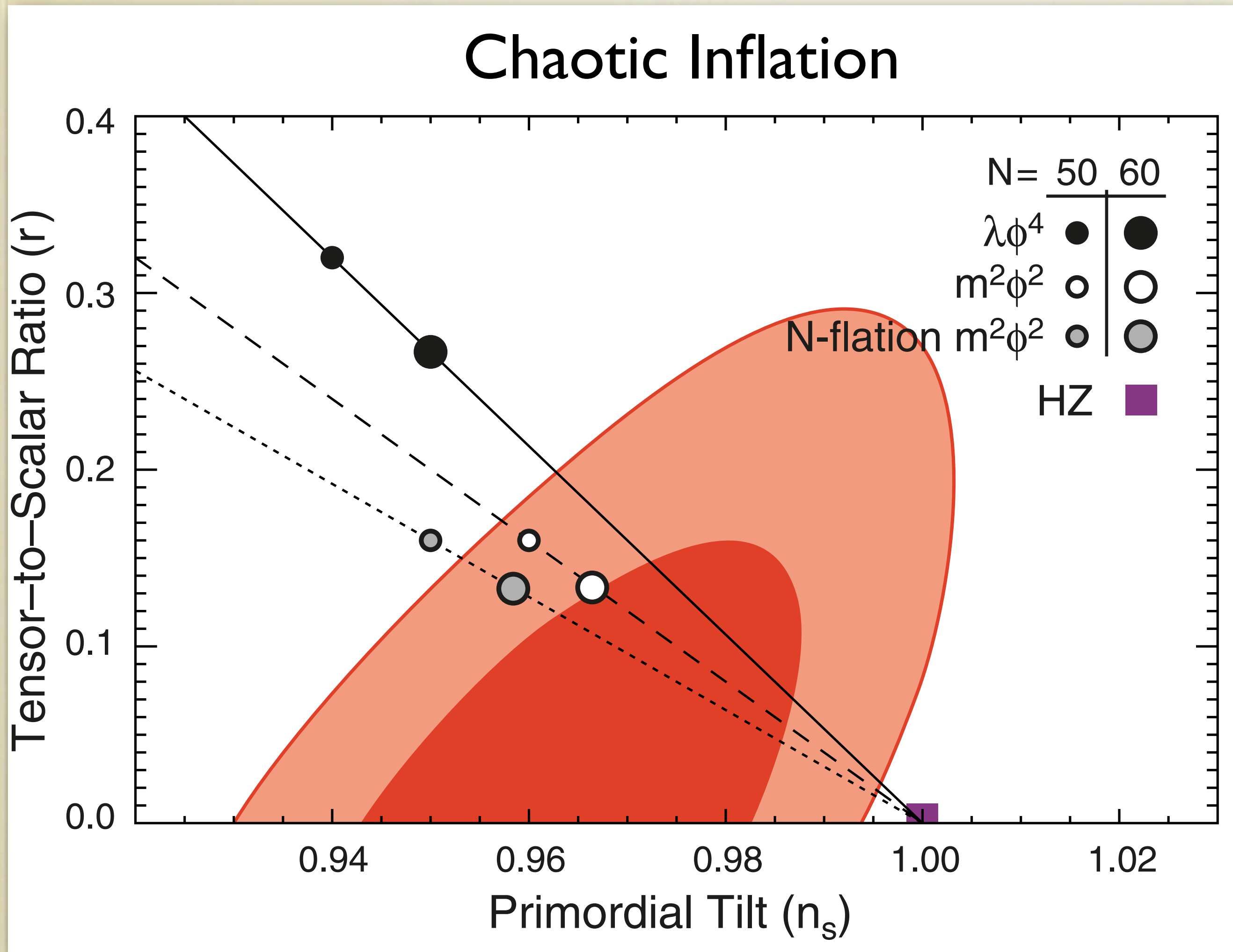
$\rightsquigarrow$ PLANCK : $r \lesssim 0.03 \ldots 0.05$ (extended run, 2.5 yr)

similar range for ground-based: QUIET, Keck array, Spider ...

by 2013-2014 : 3 yrs !

$\rightsquigarrow$ observational reach on r $\hat{=}$ small-field / large-field boundary ...

present status: $\mathrm{WMAP\ 7yr + BAO} + H_0$



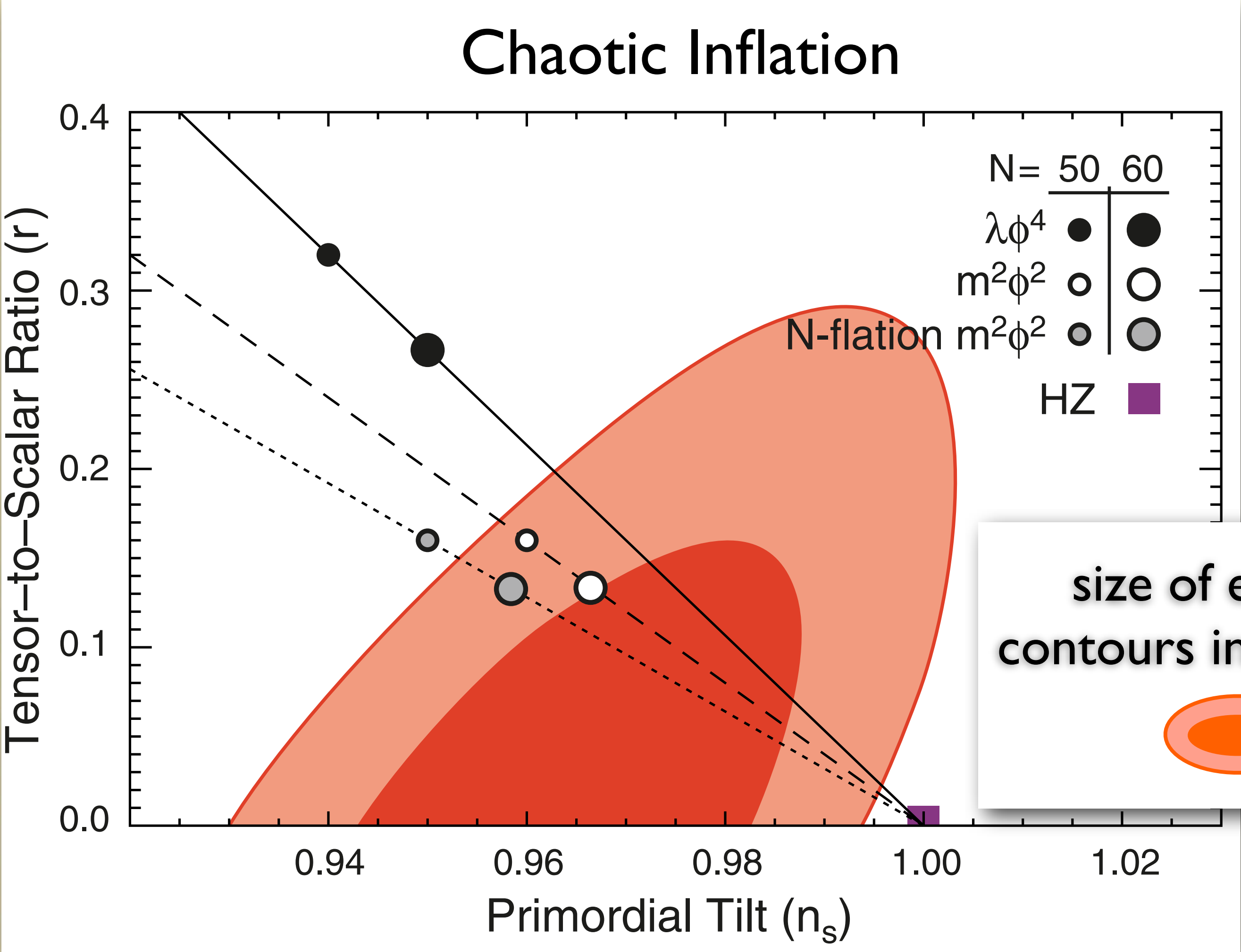
$$n_s = 0.963 \pm 0.012\ (68\%)$$

$$r < 0.24\ (95\%)$$

$$-10 < f_{NL}^{local} < 74$$

$$-214 < f_{NL}^{equil} < 266$$

We live in the Golden Age of cosmology !



$$n_s = 0.963 \pm 0.012\ (68\%)$$

$$r < 0.24\ (95\%)$$

$$-10 < f_{NL}^{local} < 74$$

$$-214 < f_{NL}^{equil} < 266$$

size of error
contours in ~ 5 yrs

expect <u>dramatic</u> improvement in next 5 yrs:

Planck & **BICEP2** taking data, Keck Array ('10...)

SPIDER, Clover, QUIET-II, EBEX, PolarBEAR ...

III UV completion – inflation in string theory

basics of string theory:

↳ 1-dimensional extended objects

↳ open: or closed:

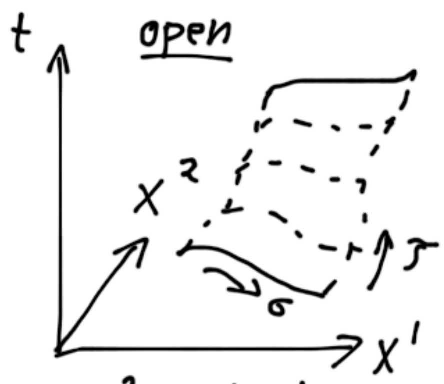
with tension $\sim \frac{1}{\alpha'}$, $\alpha' = \ell_s^2$

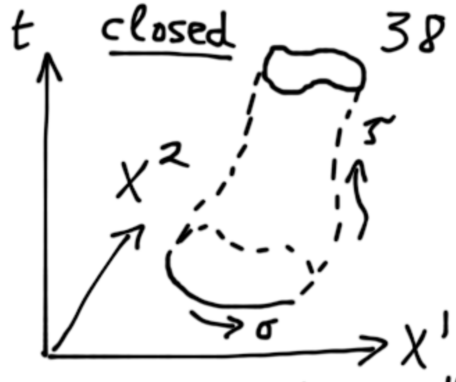
so they vibrate ...

↳ action:

$$S = \frac{1}{\alpha'}\int d^2\sigma \sqrt{-\gamma} = \text{"area of world sheet"}$$

induced metric on world sheet





$d^2\sigma = d\tau\, d\sigma$ X^μ : embedding "target" space-time

equivalent action:

$$S = \frac{1}{\alpha'}\int d^2\sigma \sqrt{-\gamma}\,\gamma^{\alpha\beta} g^{\mu\nu}\partial_\alpha X_\mu \partial_\beta X_\nu$$

$$\text{with} \begin{cases} \alpha, \beta = \tau, \sigma \\ \mu, \nu = 0, \dots, D-1 \end{cases}$$

solutions: standing wave spectrum

→ closed: independent left- & right-moving modes

→ closed: left- & right movers tied by boundary conditions

(open string) D-brane inflation ...

- Brane-Antibrane Dvali & Tye; Alexander; Dvali, Shafi & Solganik;
  Burgess, Majumdar, Nolte, Quevedo, Rajesh & Zhang.
- D3-D7. Dasgupta, Herdeiro, Hirano & Kallosh; Hsu, Kallosh & Prokushkin;
  Hsu & Kallosh; Aspinwall & Kallosh; Haack, Kallosh, Krause, Linde,
  Lüst & Zagermann.
- ➥ warped brane-antibrane Kachru, Kallosh, Linde, Maldacena, McAllister & Trivedi;
  Firouzjahi & Tye; Burgess, Cline, Stoica & Quevedo;
  Iizuka & Trivedi; Krause & Pajer; Baumann, Dymarsky, Klebanov,
  McAllister & Steinhardt; Baumann, Dymarsky, Kachru, Klebanov &
  McAllister;
- DBI. Silverstein & Tong; Alishahiha, Silverstein & Tong; Chen; Chen;
  Shiu & Underwood; Leblond & Shandera,...

closed string moduli inflation ...

- Racetrack Blanco-Pillado, Burgess, Cline, Escoda, Gomez-Reino, Kallosh,
  Linde & Quevedo; Linde & AW;
- Kähler moduli AW; Conlon & Quevedo.
- Roulette Bond, Kofman, Prokushkin, Vaudrevange.
- N-flation Dimopoulos, Kachru, McGreevy, Wacker; Easther & McAllister; Grimm;
- ➥ High-Fibre Cicoli, Burgess & Quevedo
- ➥ axion monodromy Silverstein & AW; McAllister, Silverstein & AW; Flauger, McAllister, Pajer, AW &
  Xu; Berg, Pajer & Sjors; Hannestad, Haugbolle, Jarnhus & Sloth; Flauger &
  Pajer; Dong, Horn, Silverstein & AW;

quantization: modes $\to$ creation & annihilation operators ...

$\downarrow$

massless states:

open: $\alpha^i_{-1}|0\rangle$, $i=2,\dots,D-1$; $m^2 = \frac{D-26}{\alpha'}$

$\nwarrow$ SO(1,D-1) vector: gauge field A_μ

closed: $\alpha^i_{-1}(L)\alpha^j_{-1}(R)|0\rangle$, $m^2 = \frac{D-26}{\alpha'}$

$\nwarrow$ SO(1,D-1) 2-tensor:

$g_{\mu\nu} \to$ gravity

$B_{\mu\nu} \to$ antisymm. 2-form potential

$\phi \to$ dilaton scalar

$\rightsquigarrow$ $\underline{D=26}$

add supersymmetry:

$\to$ removes tachyon of $|0\rangle$

$\to$ 16 X^μ become fermions $\Rightarrow$ $\boxed{D=10}$

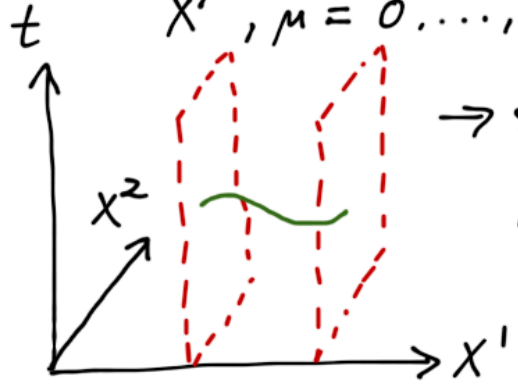
<u>crucial feature:</u> not only strings ...

$\rightsquigarrow$ turn X^{D-1} into S^1 with radius R

$\rightsquigarrow$ now shrink $R \to 0$

$\rightsquigarrow$ there is a T-dual string theory with dual coordinate X'^{D-1} on dual S^1 with radius $1/R$ — <u>and</u> strings are <u>fixed at point</u> on dual S^1:

$\to$ can only move on hyperplane X^μ, $\mu = 0,\dots,p=D-2$



$\to$ strings <u>end</u> on a '<u>D_p-brane</u>'!

<u>our world is 3+1 dimensional</u> down to lengths $\sim \text{TeV}^{-1} \sim 10^{-19}\,m$

→ however, $l_s = \sqrt{\alpha'} \sim L_p \sim 10^{-35}\,m$

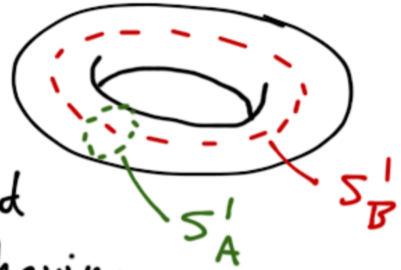
→ <u>compactify</u> 6 of 9 spatial dimensions

→ if you want 4d $\mathcal{N}=1$ SUSY: compact 6D-space is <u>Calabi-Yau</u>

<u>CY_3</u>: 6D space with specific form of metric & pairing of the 6 X^i into 3 complex coord. z^a such that Ricci curvature $R_{ab} = 0$ ('Ricci-flat')

<u>produces problem:</u> CY_3 have sub spaces, like cheese holes...

think of 2-torus:



→ has 2 non-shrinkable S^1-s "cheese-holes" related to its topology of having a <u>'handle'</u>...

↝ CY_3 has <u>non-trivial 2- and 3-spheres</u> within ↔ topology of CY_3:

↝ size of each S^2 and S^3 a <u>massless scalar field</u> in 4D:

→ 'moduli' : $\# = \mathcal{O}(100)$

S^2 sizes:	S^3 sizes:
volume deformations of the CY_3	shape deformations → how to group X^i into complex z^a...

→ desaster for inflation:
want <u>one</u> controlled scalar field
with: mass < H
<u>but</u> not $\mathcal{O}(100)$ massless!

↓

<u>a)</u>: if we could wean ourselves from 4D $\mathcal{N}=1$ SUSY ... many more 6D spaces than CY_3's, with <u>few</u> or <u>no</u> moduli ...

<u>b)</u>: if we insist on 4D $\mathcal{N}=1$ ...
→ need to stabilize moduli first

↓

look at d.o.f. of 10D effective action from string theory ...

string theory best understood for this purpose:

type IIB: – gravity with $\mathcal{N}=2$ in 10D on closed strings
– matter & gauge fields from open strings ending on D3– or D7-branes, $\mathcal{N}=1$ in 10D

↓

effective action: 10D IIB supergravity

$$S \sim \int d^{10}x\sqrt{-g}\left\{e^{-2\phi}\left[R+(\partial\phi)^2\right]-|F_1|^2-|G_3|^2 - |F_5|^2\right\} + \ldots$$

$$\left\{\begin{aligned} F_p &= dC_{p-1} \\ G_3 &= dC_2-(C_0+ie^{-\phi})\cdot dB_2 \end{aligned}\right.$$

'p-form gauge field strengths'

<u>flux of field strengths F_p can be quantized:</u>

→ compactify on $\mathcal{M}_4 \times C_6$

C_6: compact 6D space

→ if C_6 contains non-trivial p-sphere S^p, $0 < p \leq 6$

$$\Rightarrow \int_{S^p} F_p = N_p \in \mathbb{Z}$$

~ analog to quantized magnetic flux $\int_{S^2} B_2$ for a Dirac monopole

now say, S^p has radius R:

→ then: $\int_{S^p} F_p = N_p \in \mathbb{Z} \Rightarrow F_p \sim \frac{N_p}{R^p}$

and S^p has curvature:

$$^{(p)}R = \frac{p(p-1)}{R^2}$$

thus an action:

$$S \sim \int d^{10}x \sqrt{-g} \left[R - |F_p|^2 \right]$$

$$\supset \int d^4x \int_0^R dr\, r^5 \cdot \left[{}^{(p)}R - |F_p|^2 \right]$$

$$\sim \int d^4x \cdot \underbrace{\left[p(p-1) R^4 - \frac{N_p^2}{R^{2p-6}} \right]}_{\text{scalar potential } V(R)}$$

$$\frac{\partial V}{\partial R} = 0 \Rightarrow \boxed{\langle R \rangle^4 \sim N_p}$$

→ the size modulus R of the p-sphere was <u>stabilized</u> by <u>flux</u> & <u>curvature</u> and is now heavy!

--→ similar arguments (technically more involved) show that:

$\rightsquigarrow$ quantized G_3-flux in IIB stabilizes <u>all</u> the S^3-s of a CY_3 compactification to 4D $\mathcal{N}=1$

$\rightsquigarrow$ <u>"moduli stabilization by flux compactification"</u>

size of the S^2-s of a CY_3?

$\rightsquigarrow$ use non-perturbative effects of the gauge theories on the D7-branes of IIB:

→ D7-brane fills 3+1 and 'wraps' around an "$S^2 \times S^2$"

$$\Rightarrow \left.\frac{1}{g_{YM}^2}\right|_{D7} \sim (\text{size of } S^2)^2$$

→ nonperturbative effects in g_{YM}:

$$\Rightarrow V \supset e^{-\left.\frac{1}{g_{YM}^2}\right|_{D7}}$$

depends on size of S^2

$\Rightarrow$ $\boxed{\text{Can stabilize } \underline{\text{all}}\ CY_3\text{-moduli in IIB.}}$

question: which scalars remain as good inflaton candidates?

~ i) positions of D3-branes on the compact 6D CY_3

~ ii) scalars related to the gauge symmetries of the C_p p-form gauge fields:
→ axions

let's see ...

i) : – D3-brane fills 3+1, is a point in the CY_3. Its 6 coord. x^i on the CY_3 are scalar fields ϕ^i in 4D.

– D3-branes try to respect SUSY if there → they are BPS-states of the extended SUSY

⇒ positions ϕ^i have no potential for SUSY D3-branes

– D3-branes are charged under C_4 4-form gauge field of IIB ...

analogy
{ EM charge q – coupling:
$\sim q\int A_\mu dx^\mu = q\int A_1$
D3-brane charged under C_4 – coupling:
$\sim \int C_4$

~ there are D3-branes & $\overline{D3}$-antibranes, with opposite charge under C_4:

$\rightarrow$ if D3's are SUSY on a given CY_3

$\Rightarrow$ a $\overline{D3}$ breaks SUSY

~ a pair D3, $\overline{D3}$ separated by distance r in the CY_3:

$\overline{D3}$ breaks SUSY, feels force from C_4 towards the D3 ...

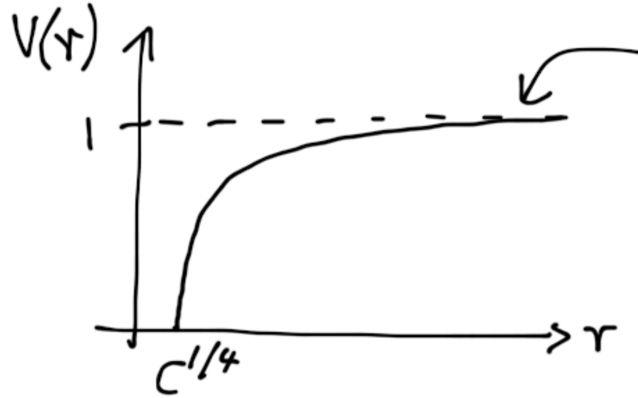
$\Rightarrow$ potential: $V(r) \sim -\dfrac{c}{r^4}$

$\leftrightarrow$ <u>Gauss' law in 6D</u>

and $V(r)$ will be:

- constant > 0 for $r \to \infty$
- vanish for $r \to 0$, as then D3 & $\overline{D3}$ annihilate and SUSY restores ...

$$\Rightarrow V(r) \sim 1 - \frac{c}{r^4}$$



flat: D3-$\overline{D3}$ brane inflation

2 important points:

- D3-brane inflation is small-field

  $\rightsquigarrow$ why?

  a) $r < R_{CY}$: D3-brane moves inside CY_3 ...

  b) relation between M_p^{10D} and M_p in 4D:

$$M_p^2 \sim \left(M_p^{10D}\right)^8 \cdot \text{volume}_{CY}$$

$$\Leftrightarrow \quad \frac{1}{\alpha'^2} \sim \frac{M_p^4}{\text{volume}_{CY}^2} \sim \frac{M_p^4}{R_{CY}^{12}}$$

implies, that all energy forms

scale $\sim \frac{1}{R_{CY}^{12}}$ in 4D if Einstein-Hilbert term is $\sim M_p^2 R$

$\Rightarrow$ kinetic term for r:

$$\frac{1}{\alpha'^2}(\partial r)^2 \sim \frac{(\partial r)^2}{R_{CY}^{12}}$$

$\Rightarrow$ canonically normalized scalar ϕ_r:

$$\phi_r \sim \frac{r}{R_{CY}^6} < \frac{1}{R_{CY}^5} << 1$$

if $R_{CY} >> 1$ in string units.

- the same rescaling introduces a dangerous dim-6 operator into $V(r)$: 55

$$V_{10D}(r) \sim \left(1 - \frac{c}{r^4}\right)$$

$$\rightarrow V_{4D}(r) \sim \frac{V_{10D}(r)}{R_{CY}^{12}}$$

in general, due to backreaction of energy density of ~~SUSY~~ $\overline{D3}$:

$$R_{CY}^{12} = R_{CY}^{12}(r) \sim \left(R_{CY}^{(0)}\right)^{12} - r^2$$

$$\Rightarrow V_{4D}(r) \sim \frac{V_{10D}(r)}{\left(R_{CY}^{(0)}\right)^{12}}\left(1 + \frac{r^2}{\left(R_{CY}^{(0)}\right)^{12}} + \dots\right)$$

$$\sim V_{4D}^{(0)}(\phi_r)\cdot\left(1 + \phi_r^2 + \dots\right)$$

56

the quadratic correction term in the bracket implies, however:

$$\eta = \frac{V''}{V} = \underbrace{\frac{V_{4D}^{(0)\,''}}{V_{4D}^{(0)}}}_{=\eta^{(0)} << 1} + 1$$

<u>'eta problem'</u>

too large for inflation!

Note that moduli stabilization fixes $R_{CY}^{(0)}$, not the full $R_{CY}(r)$...

how to fix this?

→ accounted for just <u>one</u> dim-6 term!

→ search for all – and fine-tune them away ...

to do this, note:

IIB on a CY_3 with fluxes G_3 gives 4D $\mathcal{N}=1$ supergravity:

→ scalar potential

$$V(R_{S_i^2}, R_{S_a^3}) = e^K \left(K^{I\bar{J}} D_I W \overline{D_J W} - 3|W|^2 \right)$$

sizes of the S_i^2 & S_a^3 of the CY_3

↳ $I, J = R_{S_i^2}, R_{S_a^3}$

K : <u>Kahlerpotential</u> → kinetic terms

& superpotential W:

$$W = \int_{CY_3} G_3 \wedge \Omega_3(R_{S_a^3}) + \sum_i A_i \, e^{-\frac{1}{g^2_{YM,i}}(R_{S_i^2})}$$

fixes the S_a^3

fixes the $R_{S_i^2}$

→ imagine, some of the A_i are $A_i(r)$ ⟹ further dim-6 contributions to $V_{4D}(r)$, and we can tune ...

warped D3-brane inflation ...

$$W = \int G \wedge \Omega + A(z_i) \cdot e^{-aT}$$

$$K = -3 \ln[T + \bar{T} - k(z_i, \bar{z}_i)]$$

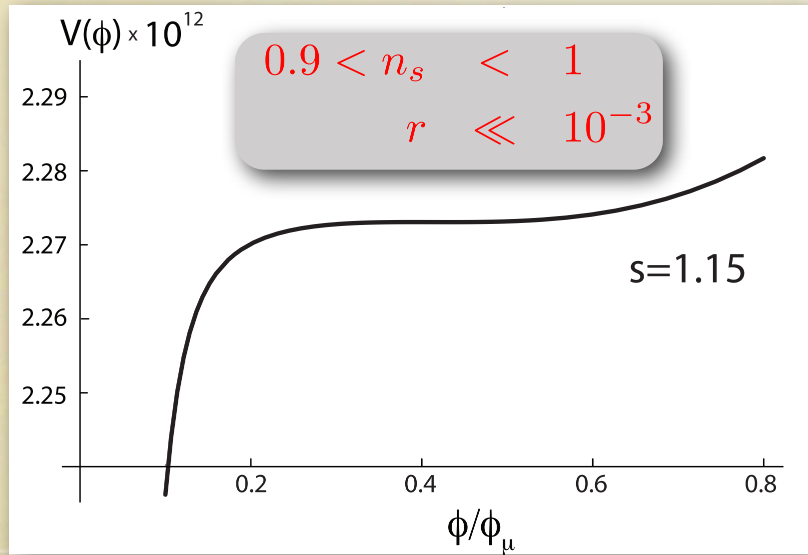
$$k(z_i, \bar{z}_i) \sim \sum_i |z_i|^2 \sim r^2$$

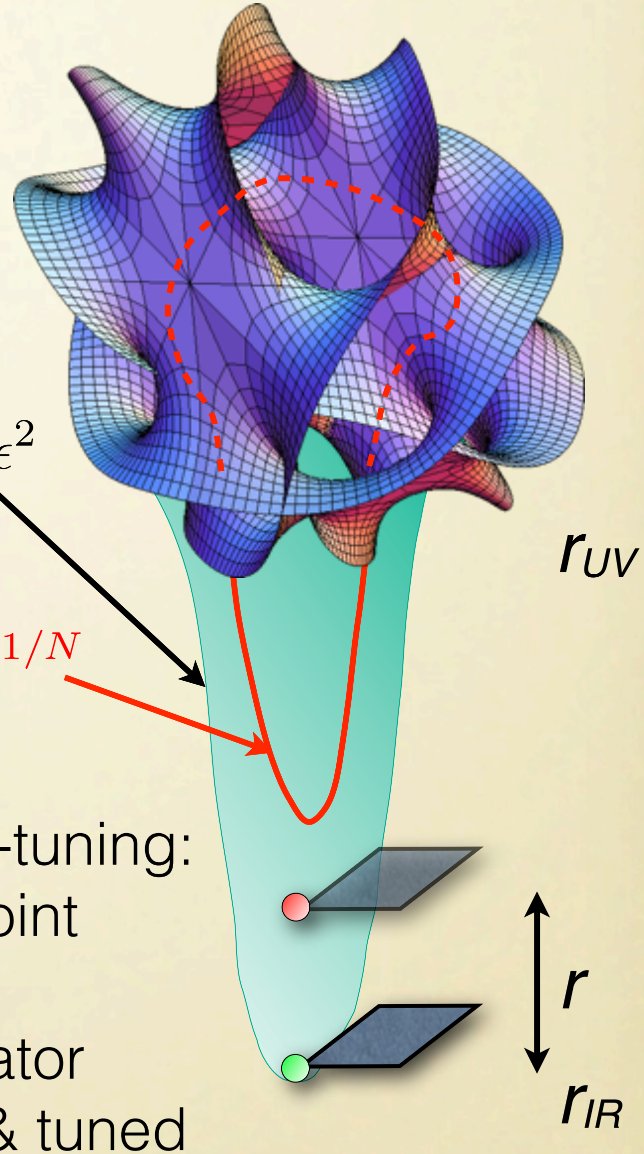
conifold

$$\sum_i z_i^2 = \epsilon^2$$

N embedded D7-branes (fix moduli)

if Kuperstein embedding: $A(z_i) = (z_1 - \mu)^{1/N}$





- explicit fine-tuning: inflection point
- dim-6 operator computed & tuned

[Baumann, Dymarsky, Klebanov, McAllister & Steinhardt '07]

warped D3-brane inflation ...

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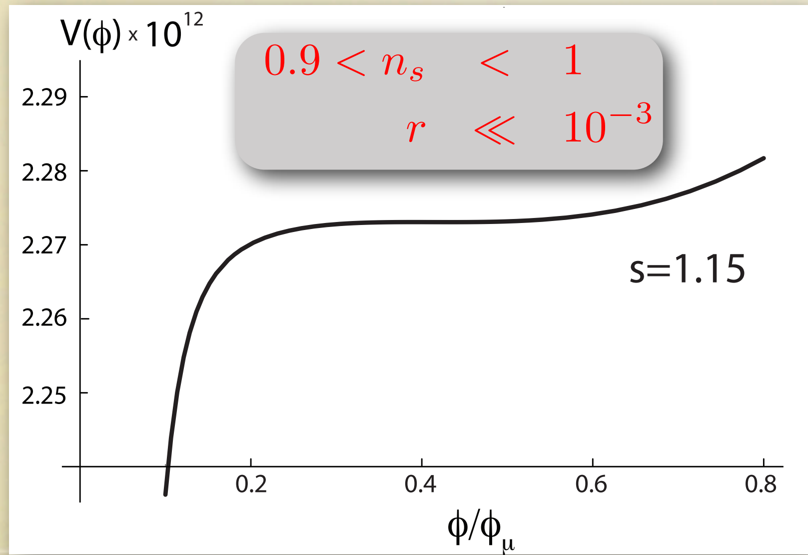

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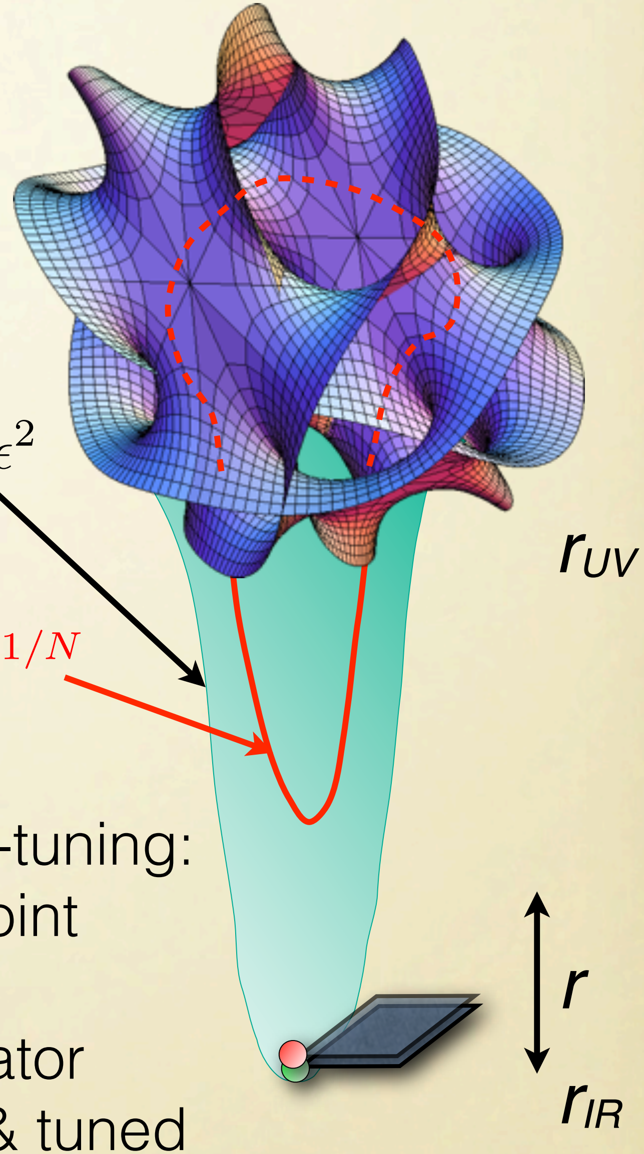


[Baumann, Dymarsky, Klebanov, McAllister & Steinhardt '07]

Can determine complete catalog of relevant higher-dim. contributions to $V_{4D}(r)$ from string theory $\rightarrow$ showcase for necessity & sufficiency of string theory as a UV completion of small-field inflation!

warped D3-brane inflation ...

- can determine dim ≥ 6 operators for a whole class of setups:
  - a conifold-throat has a CFT dual (a la AdS-CFT)
  - CFT computes *all* operators of dim. $\Delta = 4 + \delta_{LM}$ from CY at r_{UV}
  - define $x = r / r_{UV}$ & 3-brane angular position Ψ:

$$V(x,\Psi) = V_0 + V_{\mathcal{C}}(x) + V_{\mathcal{R}}(x) + V_{\text{bulk}}(x,\Psi)$$

$$V_{\mathcal{C}}(x) \sim \left(1 - \frac{C}{x^4}\right) \qquad V_{\text{bulk}}(x) \sim V_{\mathcal{C}}(x) \sum_{L,M} c_{LM} x^{\delta(L)} f_{LM}(\Psi)$$

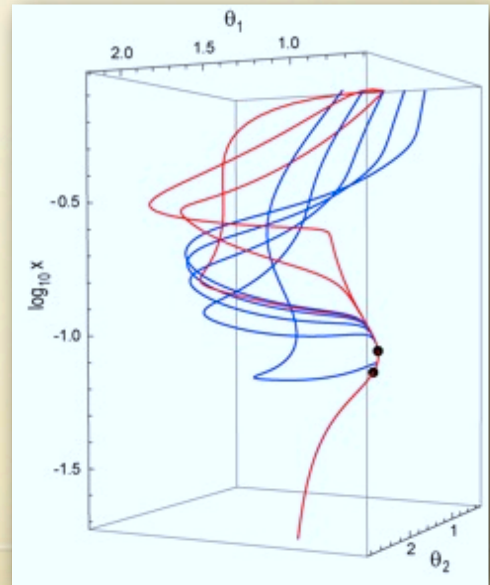
$$V_{\mathcal{R}}(x) \sim V_{\mathcal{C}}(x) \cdot x^2 \qquad \delta = 1,\, 3/2,\, 2,\, \sqrt{28} - 3,\, 5/2,\, \ldots$$

[Baumann, Dymarsky, Kachru, Klebanov & McAllister '09/'10]

- universality of D3-brane inflation:
  can sample c_{LM}'s & initial conditions
  - Markov Chain Monte Carlo;

  still often get an effective
  single-field inflection point dynamics

[Agarwal, Bean, McAllister & Xu '11]



A fundamental property of stringy inflaton candidates:

field range $\Delta\phi < M_P$

$\rightsquigarrow$ have seen this for brane position

$\rightsquigarrow$ also true for case ii) candidates:

$\longrightarrow$ gauge symmetries of p-form gauge fields C_p correspond to scalars with shift symmetries & periodicity:

$\rightarrow$ p-form axions: $c_p = \int_{\Sigma^p} C_p$

- they enjoy shift symmetries from the C_p - gauge symmetries...

  $\Rightarrow$ no potential in perturbation theory

- string theory is invariant under:

$$c_p \to c_p + 2\pi \quad \text{'periodicity'}$$

and again: $\mathcal{L}^{4D}_{kin.} \sim \frac{(\partial c_p)^2}{R_{CY}^{12}}$

$$\Rightarrow \phi_{c_p} \sim \frac{c_p}{R_{CY}^6} < M_P$$

- one exception:

$\rightarrow$ Dp-branes couple to C_p!

action of a Dp-brane:

$$S_{Dp} \sim \frac{1}{\alpha'^{\frac{p+1}{2}}} \int d^{p+1}\xi \sqrt{-\det(g_{\mu\nu}+B_{\mu\nu})}$$

'DBI-action'

simple example:

put B_2 on a S^2 in CY_3

$\Rightarrow$ axion $b = \int_{S^2} B_2$

there are D5-branes, which see B_2 as above:

$$\Rightarrow S_{D5} \sim \frac{1}{\alpha'^3} \int_{M^4 \times S^2} d^6x \sqrt{-g_{4D}(g_{S^2}+B_2)}$$

$$\sim \frac{1}{\alpha'^2} \int d^4x \sqrt{-g_{4D}} \sqrt{\mathrm{vol}_{S^2}^2 + b^2}$$

$\Rightarrow$ scalar potential

$$V(b) \sim \sqrt{\mathrm{vol}_{S^2}^2 + b^2} \sim b$$

at large b !

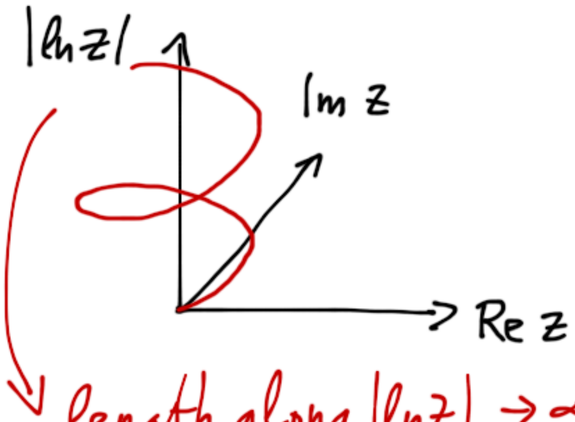
$\rightarrow$ potential is not periodic in b, although everything else is:

"V(b) has monodromy in b."

$\Rightarrow$ gives parametrically unbounded field range:

- no periodicity
- no relation to limiting size of the CY_3

$\rightsquigarrow$ analogy: $|\ln z|$, $z \in \mathbb{C}, |z|=1$



length along $|\ln z| \to \infty$, z periodic

$\Rightarrow$ axion monodromy inflation with 5-branes gives large-field inflation in string theory with:

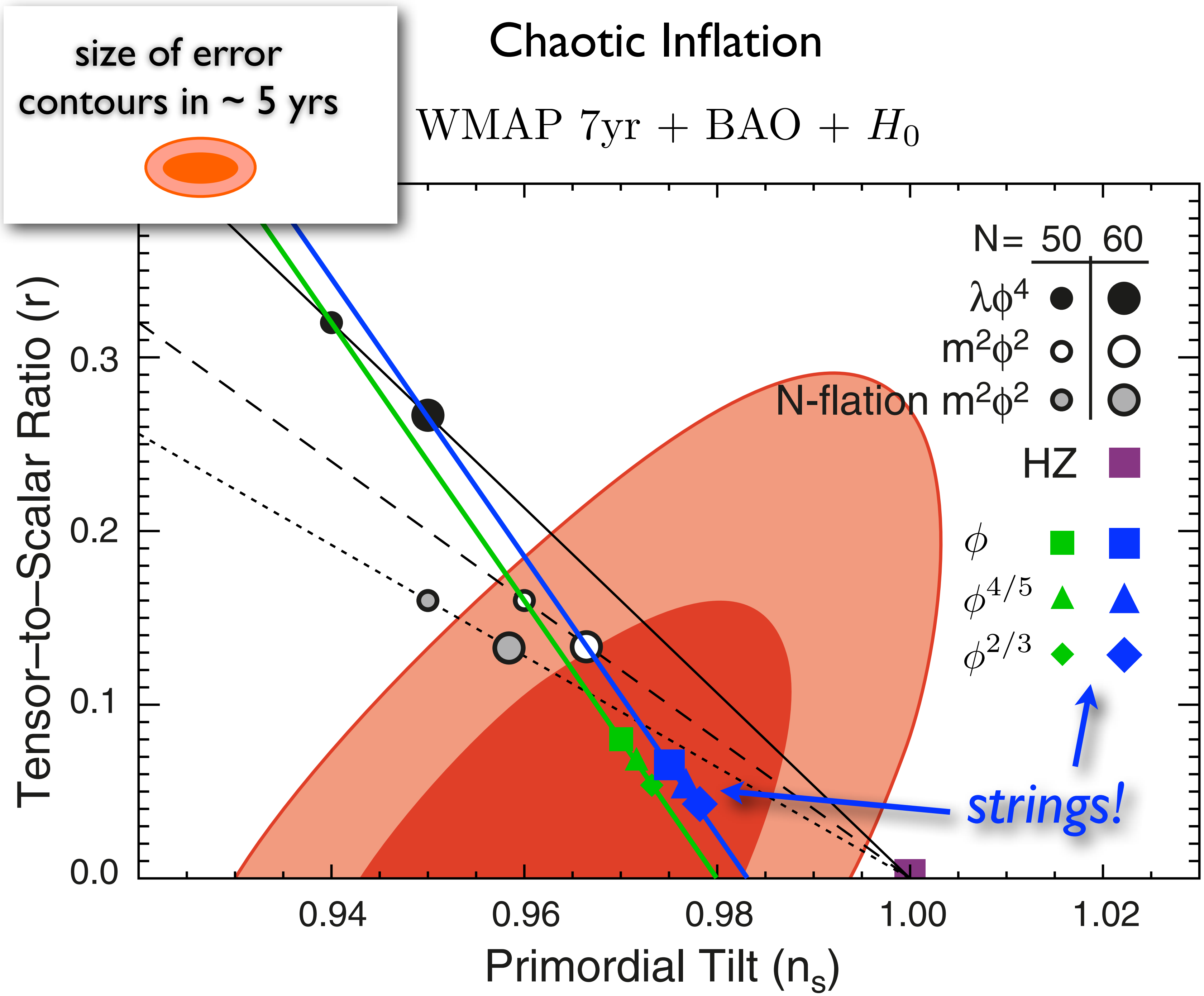
$$\boxed{V(\phi) \sim \phi \quad \text{at } \phi \gg M_p}$$

$$\Rightarrow \boxed{\phi_{60} \simeq 11\, M_p,\ n_s \simeq 0.975,\ r \simeq 0.08}$$

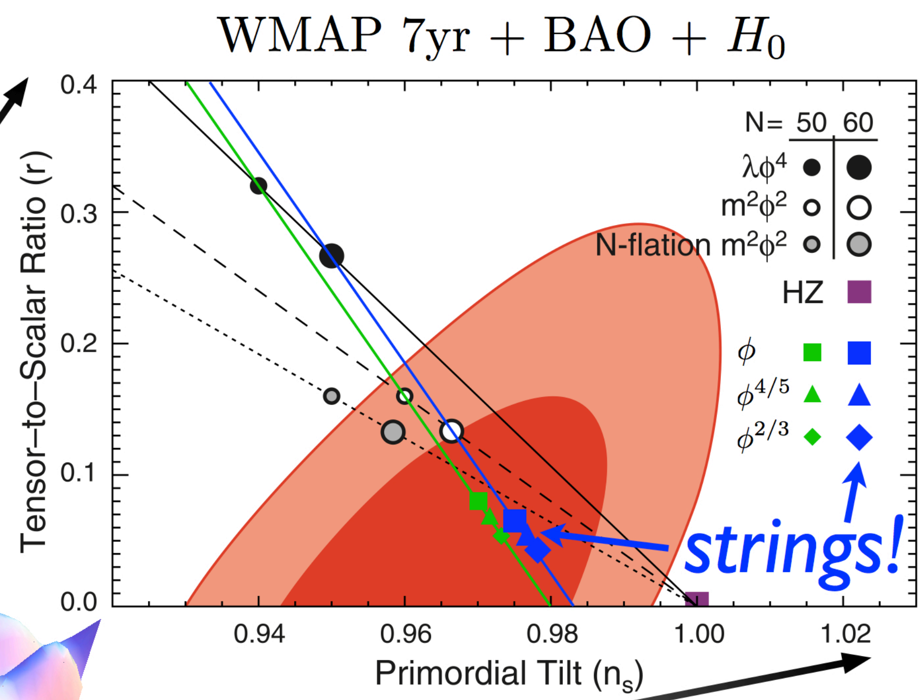
(controlled setups by: McAllister, Silverstein & AW in 2008)

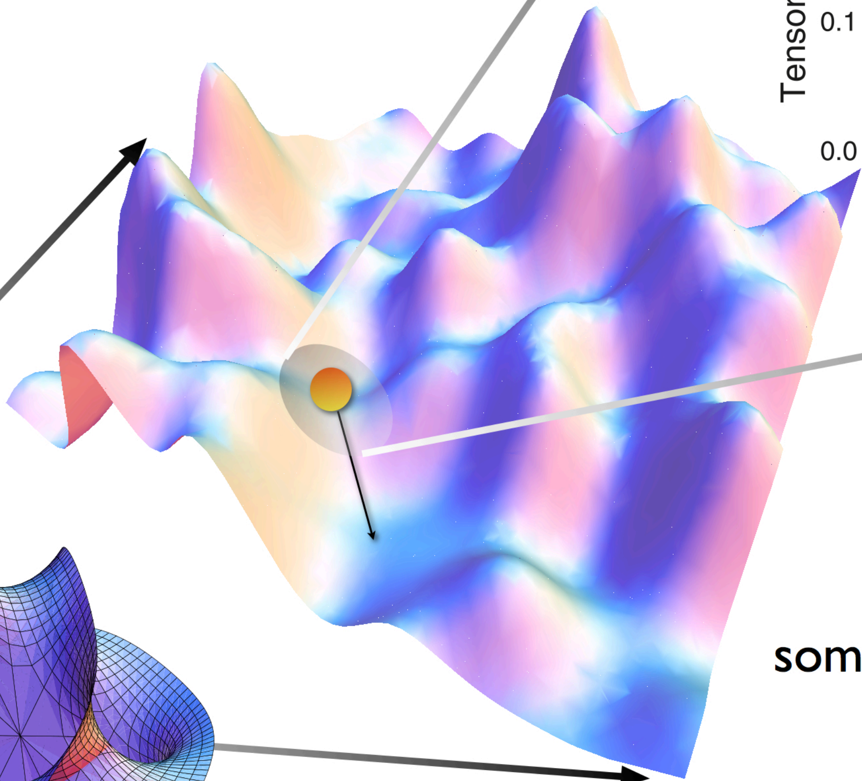
crucial: string theory generated the shift symmetry from SUSY & stringy gauge symmetry $\longrightarrow$

concrete UV completion of
large-field model class!

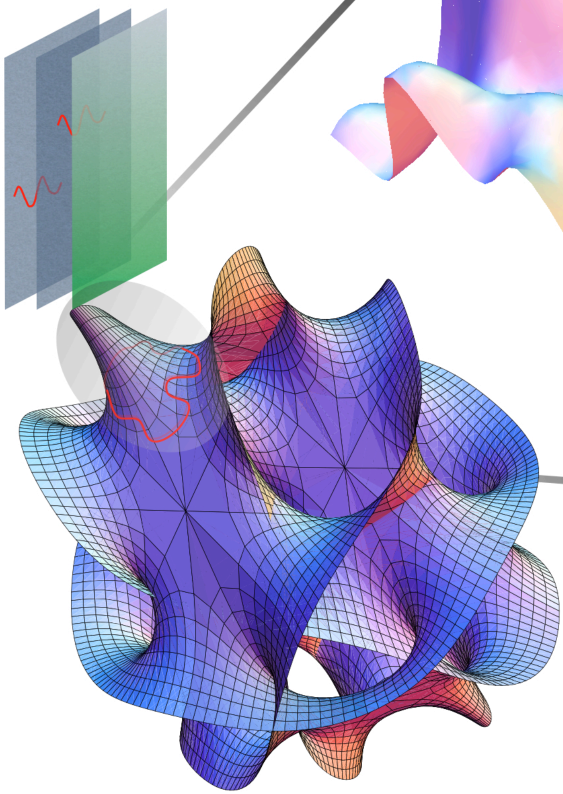

size of error
contours in ~ 5 yrs
Chaotic Inflation
WMAP 7yr + BAO + H_0
N= 50 60
$\lambda\phi^4$
$m^2\phi^2$
N-flation $m^2\phi^2$
HZ
ϕ
$\phi^{4/5}$
$\phi^{2/3}$
strings!
Tensor–to–Scalar Ratio (r)
0.3
0.2
0.1
0.0
0.94
0.96
0.98
1.00
1.02
Primordial Tilt (n_s)

cosmological data:
spectral tilt of density fluctuations (n_S) vs
relative power of gravity waves (r)





the string theory *landscape*:
many isolated *vacua*,
some mountain slopes drive *inflation*



string theory's 6 compact dimensions:
strings , branes & fluxes