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Accions

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Abstract

Axion fields provide the most elegant solution to the strong CP problem. In string compactifications it is difficult to obtain an axion whose decay constant is consistent with current cosmological bounds. We examine this question in theories with accidental U(1) symmetries that appear as low energy remnants of discrete symmetries. We refer to the axion-like particles from the spontaneous breakdown of these symmetries as accions. In such systems, the accion decay constant depends on the vacuum configuration and can be lowered to fit the bounds. Furthermore, we find that such accions with consistent decay constant can be embedded in special vacua of $\mathbb{Z}_6$–II orbifold models with realistic features.

1 Introduction

The most elegant and appealing solution to the strong CP-problem is based on the conjecture of an axion field [1, 2]. It requires the existence of an anomalous global Peccei-Quinn symmetry [3] $\mathrm{U}(1)_{\mathrm{PQ}}$ and its spontaneous breakdown at a scale F_a (where F_a denotes the "axion decay constant"). Constraints (mostly) from astrophysics and cosmology require F_a to be in the axion window

$$10^9\,\mathrm{GeV} \leq F_a \leq 10^{12}\,\mathrm{GeV} \tag{1}$$

for the so-called "invisible" axion [4, 5]. The axion field adjusts its vacuum expectation value (VEV) to cancel the θ-parameter of quantum chromodynamics (QCD) to avoid CP-violation due to strong interactions.

In the presence of gravity, the existence of an exact global U(1)-symmetry might be problematic. This holds explicitly in the context of string theory, where potential anomalous U(1)-symmetries appear as gauge (not global) symmetries [6]. Does this exclude the axion solution to the strong CP-problem in the framework of string theory?

In the present work, we focus on a set of realistic string extensions of the minimal supersymmetric standard model (MSSM) in the framework of the $\mathrm{E}_8 \times \mathrm{E}_8$ heterotic string. We address the following two questions:

- how can we possibly obtain a global $\mathrm{U}(1)_{\mathrm{PQ}}$?
- can we disconnect the axion scale F_a from the string scale M_s (which is typically much larger, e.g. $10^{16} - 10^{18}$ GeV)?

The analysis of the models reveals (among the various local U(1)-symmetries) the appearance of a multitude of discrete symmetries of various types [7, 8]. In such a situation we might find accidental U(1)-symmetries [9] if we restrict our attention to superpotentials of a limited degree in the polynomial of the fields. Higher order non-renormalizable terms would break this accidental $\mathrm{U}(1)_{\mathrm{PQ}}$-symmetry back to the exact discrete symmetry. [1] In the standard model of particle physics (SM) such accidental symmetries appear as $\mathrm{U}(1)_{\mathrm{B,L}}$ where B(L) denote baryon(lepton) number, respectively.

Including these accidental symmetries will allow for a multitude of axion candidates in the view of the first question. Apart from the strong CP-problem these fields might be of relevance for hidden sector gauge groups (embedded in the second E_8) or quintessential axions [11, 12].

This new paradigm of string theory, the appearance of accidental global symmetries from exact discrete symmetries, might find interesting applications in the creation of

[1] Similar ideas have been explored in the context of Calabi-Yau compactifications [10].

hierarchical structures as explained in [13]. This might then give us the freedom to lower the axion scale away from the string scale and thus answering our second question.

Within the application to the axion system, pioneering work has been done by Choi, Kim and Kim [14] in a model based on heterotic string constructions via the $\mathbb{Z}_{12}$-orbifold. Unfortunately, however, in this model the axion decay constant of the QCD axion could not be decoupled from the string scale. In the present paper we analyze the models of the so-called heterotic Mini-Landscape [15–19] in view of potential realistic axion candidates. This requires a careful study of a multi-axion system including a multitude of global and discrete symmetries both of anomalous and non-anomalous nature. Our analysis of section 2 reveals the fact that the properties of the system (apart from the structure of the model itself) will strongly depend on the vacuum configuration.

In section 3 we shall then discuss some representative examples of the Mini-Landscape and show their potential suitability for the answers of the questions raised above. This discussion requires extensive computer calculations and we limit ourselves here to the conceptual questions towards the solution of the strong CP-problem via a suitable axion field. The construction of a complete and fully realistic extension of the MSSM will be relegated to future investigations.

We shall see that accidental symmetries that naturally appear in the framework of string constructions are very well suited for satisfactory axion candidates with a decay constant in the axion window. Section 4 will summarize our results, address remaining problems and will give an outlook for future research.

2 Multi-Axion Systems

The models under consideration have, in addition to some non-anomalous $\mathrm{U}(1)_{\mathrm{gauge}}$ gauge symmetries, (accidental) anomalous U(1) symmetries whose breaking provides many axion candidates. This leads to a complicated vacuum structure that requires a careful investigation. Here we rely on previous work [15–19] where the supersymmetric vacua corresponding to D- and F-flat solutions have been analyzed. The models typically have one[2] anomalous gauge symmetry $\mathrm{U}(1)_{\mathrm{A}}$ [21] with a non-trivial Fayet-Iliopoulos (FI) term which in turn induces non-vanishing VEVs for some of the scalar fields. These break some of the $\mathrm{U}(1)_{\mathrm{gauge}}$ symmetries as well as the $\mathrm{U}(1)_{\mathrm{A}}$ at a scale one or two orders of magnitude below the string scale M_s. We now also have to analyze the fate of the possible accidental (anomalous) U(1) symmetries. To answer the question towards a satisfactory axion within the "allowed window" we have to understand not only the model but also the vacuum configuration, which we discuss now.

[2]More are possible in the blow-up version of the models [20]

2.1 Axions from a Single U(1)

Let us start with the simplest case, a supersymmetric theory with standard model (SM) gauge group and one additional global symmetry, $\mathrm{U}(1)_{\mathrm{PQ}}$, which is assumed to have a mixed $\mathrm{U}(1)_{\mathrm{PQ}}-\mathrm{SU}(3)_C-\mathrm{SU}(3)_C$ anomaly. Consider the matter superfields (φ_i, ψ_i) with $\mathrm{U}(1)_{\mathrm{PQ}}$ charges q^i. If one scalar SM singlet field, say φ_0, with non-vanishing PQ charge attains a VEV v_0, its phase field a becomes an axion and induces the following term in the effective Lagrangian

$$\mathcal{L}_{\mathrm{eff}} \supset \frac{a}{F_a}\frac{g^2}{32\pi^2}\,\mathrm{tr}\left[G^{\mu\nu}\widetilde{G}_{\mu\nu}\right]\,, \tag{2}$$

where g denotes the $\mathrm{SU}(3)_C$ coupling constant, $G_{\mu\nu}$ is the color field strength, and $\widetilde{G}_{\mu\nu}$ its dual. The axion decay constant F_a in this scenario is given by

$$F_a \;=\; \frac{q^0\, v_0}{\mathcal{A}} \tag{3}$$

that does not depend on the normalization of the PQ symmetry. Here $\mathcal{A}$ is the coefficient of the $\mathrm{U}(1)_{\mathrm{PQ}}-\mathrm{SU}(3)_C-\mathrm{SU}(3)_C$ anomaly

$$\mathcal{A} \;=\; \sum_i q^i\,\ell_i \tag{4}$$

with ℓ_i being the $\mathrm{SU}(3)_C$ quadratic index of the fermion ψ_i, such that $\ell_i = 1$ for ψ_i transforming as a triplet. Note that the chiral PQ transformation of the axion, $a \to a + q^0\, v_0\, \alpha$ (with α being the transformation parameter), reproduces as expected the original anomaly,

$$\mathcal{L}_{\mathrm{eff}} \to \mathcal{L}_{\mathrm{eff}} + \alpha\mathcal{A}\,\frac{g^2}{32\pi^2}\,\mathrm{tr}\left[G^{\mu\nu}\widetilde{G}_{\mu\nu}\right]\,. \tag{5}$$

It is straightforward to generalize this result to the case where various scalar fields obtain a VEV. If the fields φ_i attain VEVs v_i, the resulting axion a is given in terms of the phase fields a_i by

$$a \;=\; \frac{1}{M}\left(\sum_i a_i\, q^i\, v_i\right) \;\equiv\; \frac{1}{\sqrt{\sum_i (q^i\, v_i)^2}}\left(\sum_i a_i\, q^i\, v_i\right)\,, \tag{6}$$

where M defines the scale at which $\mathrm{U}(1)_{\mathrm{PQ}}$ is broken. The axion decay constant is then given by

$$F_a \;=\; \frac{M}{\mathcal{A}}\,. \tag{7}$$

Considering the PQ transformation of the phase fields a_i, one recovers eq. (5). We would like to point out that in this case the axion decay constant is dominated by the largest VEV. This behavior might change if the low energy theory contains more than one PQ symmetry, as we shall discuss below.

2.2 Axions from Multiple U(1)s

In a next step we consider a supersymmetric theory with two PQ symmetries, denoted by $U(1)_P \times U(1)_Q$, with respective (non-vanishing) anomalies $\mathcal{A}_P$ and $\mathcal{A}_Q$, analogously defined as in eq. (4). The scalar fields of the theory transform as

$$\varphi_i \to e^{\mathrm{i}\left(\alpha\, q_P^i + \beta\, q_Q^i\right)} \varphi_i \tag{8}$$

under these symmetries. Note that the U(1) resulting from the linear combination

$$q_f^i \;=\; \mathcal{A}_Q\, q_P^i - \mathcal{A}_P\, q_Q^i \tag{9}$$

is always anomaly-free.

Let us suppose that both U(1)s are broken spontaneously. This occurs if, for instance, two fields φ_1 and φ_2 develop VEVs, provided that their charges $(q_P^1,\, q_Q^1)$ and $(q_P^2,\, q_Q^2)$ are linearly independent, i.e.

$$q_P^1\, q_Q^2 \;\neq\; q_Q^1\, q_P^2\,. \tag{10}$$

If the charges do not satisfy eq. (10), only one U(1) would be broken. As before, the spontaneous breaking of these U(1)s induces the term

$$\mathcal{L}_\mathrm{eff} \;=\; \frac{a}{F_a}\frac{g^2}{32\pi^2}\,\mathrm{tr}\left[G^{\mu\nu}\widetilde{G}_{\mu\nu}\right] \;\equiv\; \left(\frac{a_1}{F_{a_1}}+\frac{a_2}{F_{a_2}}\right)\frac{g^2}{32\pi^2}\,\mathrm{tr}\left[G^{\mu\nu}\widetilde{G}_{\mu\nu}\right]\,. \tag{11}$$

The constants F_{a_i} are found by comparing the variation of $\mathcal{L}_\mathrm{eff}$ due to the chiral PQ transformation of the phase fields, $a_i \;\to\; a_i + v_i\,(q_P^i\,\alpha + q_Q^i\,\beta)$, and the one expected from the anomalous nature of $U(1)_P \times U(1)_Q$,

$$\mathcal{L}_\mathrm{eff} \to \mathcal{L}_\mathrm{eff} + (\alpha\mathcal{A}_P + \beta\mathcal{A}_Q)\,\frac{g^2}{32\pi^2}\,\mathrm{tr}\left[G^{\mu\nu}\widetilde{G}_{\mu\nu}\right]\,. \tag{12}$$

The solutions are

$$F_{a_1} \;=\; -\,v_1\,\frac{q_P^1\,q_Q^2 - q_Q^1\,q_P^2}{q_f^2}\,, \qquad\qquad F_{a_2} \;=\; v_2\,\frac{q_P^1\,q_Q^2 - q_Q^1\,q_P^2}{q_f^1}\,. \tag{13}$$

Note that these constants do not depend on the basis chosen for the U(1)s. Then, the axion and its decay constant are respectively given by

$$a \;=\; \frac{-a_1\,q_f^2\,v_2 + a_2\,q_f^1\,v_1}{\sqrt{(q_f^1\,v_1)^2+(q_f^2\,v_2)^2}}\,, \tag{14a}$$

$$F_a \;=\; \left(\left(\frac{1}{F_{a_1}}\right)^2+\left(\frac{1}{F_{a_2}}\right)^2\right)^{-1/2} \;=\; \frac{v_1\,v_2\,(q_P^1\,q_Q^2 - q_Q^1\,q_P^2)}{\sqrt{(q_f^1\,v_1)^2+(q_f^2\,v_2)^2}}\,. \tag{14b}$$

A few comments are in order. First, one can easily verify that, contrary to the previous case, if two U(1)s are broken by two fields attaining VEVs, the axion decay constant given by eq. (14b) is dominated by the smaller VEV: this is the key observation in view of a realization of a decay constant within the allowed "axion window". Secondly, apart from the axion a, clearly the theory must have a Goldstone boson. It is the combination orthogonal to the axion,

$$h \;=\; \frac{a_1\, q_f^1\, v_1 + a_2\, q_f^2\, v_2}{\sqrt{(q_f^1\, v_1)^2 + (q_f^2\, v_2)^2}}\;, \tag{15}$$

associated to the broken anomaly-free symmetry, $\mathrm{U}(1)_f$. If this symmetry is gauged, h is 'eaten' via the Higgs mechanism by the gauge boson of $\mathrm{U}(1)_f$, making it massive. If $\mathrm{U}(1)_f$ is global, h remains as a massless (uncharged) field. As a last remark, from the above discussion we see that in theories with multiple spontaneously broken U(1)s, one could readily identify the axion as the unique combination of the phase fields that is orthogonal to all Goldstone bosons. This observation will help us in the following discussion.

2.3 The general case

The above two examples are sufficient to obtain an understanding of the general situation, where we have more than two U(1) symmetries and where the number of SM singlets acquiring VEVs exceeds the number of symmetries that are broken. In order to shed some light on the form of the axion and its decay constant, let us examine the scenario in which $\mathrm{U}(1)_P \times \mathrm{U}(1)_Q$ gets broken when three fields attain VEVs. One combination of the three phase fields a_i will provide the CP axion a, whereas the other two orthogonal combinations will yield a Goldstone boson h that does perceive chiral transformations, and an invariant phase field a'.

To find the axion, we can proceed as before and determine the constants F_{a_i}. A more manageable approach is to identify first the fields a' and h. The axion will be then the combination of a_i orthogonal to them. The result is a generalization of eqs. (14). In particular, assuming a large hierarchy of VEVs, for instance $v_1 \gg v_2 \gg v_3$, the axion decay constant becomes

$$F_a \;\approx\; v_2\, \frac{q_P^1 q_Q^2 - q_Q^1 q_P^2}{q_f^1}\;, \tag{16}$$

that is, F_a is dominated by the second largest VEV (v_2 in our example) and neither by the largest nor by the smallest. The second largest VEV corresponds to the largest scale at which both symmetries, $\mathrm{U}(1)_P$ and $\mathrm{U}(1)_Q$, are already broken. So, it seems natural to expect that, for any number of SM fields attaining VEVs, the axion decay constant will be dominated by the scale at which all PQ symmetries get broken. In the current case, with two PQ symmetries and a large VEV hierarchy, the scale of F_a will be given by the second largest VEV.

We can now state the result for the general case of M U(1) symmetries and $N \geq M$ SM singlet fields that obtain a VEV. By Goldstone's theorem, we have generically M "Goldstone bosons" from the M spontaneously broken symmetries. This means that we have $N - M$ invariant combinations of phase fields, which are orthogonal to these Goldstone bosons. Suppose that one of the Goldstone bosons lies in the anomalous direction and becomes an axion, while the other $M - 1$ ones remain massless, possibly eaten by gauge bosons. From the previous discussion, if the U(1)s are broken at hierarchically different scales, then the axion decay constant F_a is *dominated by the Mth largest VEV*.

2.4 Embedding in the heterotic string theory

In the heterotic string, there are various sources for axions. The spontaneous breakdown of the pseudo-anomalous $U(1)_A$ gauge symmetry together with the Green-Schwarz mechanism produce the so-called model-independent axion, whose decay constant is fixed by the Planck scale (see e.g. [22]), too high to solve the strong CP problem. Model-dependent axions arise from the internal components of the $B-$field. Unfortunately, their decay constants are in general not much lower than in the previous case [22]. Admissible axions, on the other hand, might arise from the breaking of accidental global U(1) symmetries realized as low energy remnants of (stringy) discrete symmetries. In this case, the axion decay constant shall depend on the VEVs of the fields responsible for the breakdown of such global U(1)s.

In general, there is a set of fields with a large VEV dictated by the FI term (one or two orders of magnitude below the string scale). This is the scale where (some of) the gauge U(1) symmetries are broken and where also some of the anomalous U(1)s will be broken. The latter ones should not have a QCD anomaly (they could have e.g. hidden sector anomalies) and/or should be candidates for a quintessential axion. There could in addition be another set of singlet fields that obtain smaller VEVs that break the relevant PQ-symmetry at a scale in the axion window and thus give a hierarchically smaller F_a. The requirements for a satisfactory model are thus clear:

- find a model with an accidental (color)-anomalous $U(1)^*$,
- identify a vacuum configuration where the VEVs driven by the FI term do not break $U(1)^*$,
- search for a vacuum configuration where $U(1)^*$ is broken by a VEV in the axion window (some other gauge U(1)s might be broken here as well),
- check that higher order non-renormalizable terms that break $U(1)^*$ explicitly are sufficiently suppressed to avoid a too "large" axion mass.

A comment regarding the latter requirement is in order. Higher order terms in the superpotential break explicitly the accidental $U(1)^*$ symmetry. This can pull the axion off from the CP-conserving vacuum and generate an additional contribution to the axion mass m_*^2 [23, 24]. m_*^2 is constrained by the bounds on the electric dipole moment of the neutron [25, 26]:

$$m_*^2 \lesssim 10^{-11}\ m_{QCD}^2 \approx 10^{-15}\ \frac{\text{GeV}^4}{F_a^2}\,, \tag{17}$$

where $m_{QCD}^2 \approx 10^{-4}\ \text{GeV}^4/F_a^2$ is the axion mass induced by QCD instantons. Not every $U(1)^*$–violating superpotential term, however, contributes to the axion mass. The magnitude of the axion mass, induced by the explicit breaking of the $U(1)^*$ symmetry, does not necessarily depend on the order of the lowest term in the superpotential that breaks this symmetry. It rather depends on the order of the (lowest) $U(1)^*$–violating term that contributes to the axion mass directly. Note e.g. that if a superpotential term breaks $U(1)^*$ explicitly, but contains two or more fields that do not develop VEVs, it does not contribute to the axion mass. It follows then that verifying whether eq. (17) is satisfied depends strongly on the specifics of the vacuum configuration and must be analyzed case by case.

We shall explore some candidate models in the next section.

3 QCD Axions in Orbifold Models

Let us turn now to the question of whether we can find suitable QCD axions in heterotic orbifold models. Recent progress in this framework has revealed that models resembling many features of the MSSM can be constructed [27–31]. In particular, we analyze a subset of the models found in [15] and identify those with the following properties: a) their superpotentials $\mathcal{W}$ must be symmetric under an accidental PQ symmetry, and b) they should admit vacua such that the PQ symmetry gets spontaneously broken at a phenomenologically admissible scale (see eq. (1)).

Identifying PQ symmetries of the superpotential $\mathcal{W}$ of a model requires to verify whether all superpotential terms of a given order preserve some U(1) symmetry other than the gauged ones. To find these accidental symmetries, it is convenient to use the method of [14], which we briefly summarize in the following. The superpotential of a model can be written as

$$\mathcal{W} = \sum_i^{\#\,\text{terms}} \mathcal{W}_i \qquad \text{with} \qquad \mathcal{W}_i = \prod_j^{\#\,\text{fields}} \Phi_j^{\lambda_{ij}}\,, \tag{18}$$

where all coefficients are set to one and λ_{ij} are non-negative integers constrained by the order n of the superpotential according to $\sum_j \lambda_{ij} \leq n$, for all i. Each element of the kernel (or null space) of the matrix $\left(\lambda^T\lambda\right)$ defines the charges of the fields under an Abelian symmetry of $\mathcal{W}$. Subtracting those symmetries contained in the gauge group, one is left with all accidental symmetries denoted by $\mathrm{U}(1)_{\mathrm{acc}}$.

Not every accidental symmetry of the theory can play the role of a $\mathrm{U}(1)_{\mathrm{PQ}}$. It is crucial that such symmetries also produce mixed $\mathrm{U}(1)_{\mathrm{acc}} - \mathrm{SU}(3) - \mathrm{SU}(3)$ anomalies, that is,

$$\mathcal{A} = \sum_j \ell_j \, q^j_{\mathrm{acc}} \neq 0\,, \tag{19}$$

where q^j_{acc} denote the accidental charges of the fields φ_j and ℓ_j refer, as before, to the quadratic indices of the SU(3) representations.

Using the method outlined above, we search for the accidental symmetries in a subset of realistic $\mathbb{Z}_6$–II orbifold models found previously in the so-called Mini-Landscape [15, 17, 18]. We use the notation introduced there. We analyze 55 orbifold models based on the Mini-Landscape shift $V^{\mathrm{SO}(10),1}$ exhibiting the following properties:

- SM gauge group,
- non-anomalous hypercharge $\mathrm{U}(1)_Y \subset \mathrm{SU}(5)$,
- spectrum = 3 generations + vector-like exotics,
- heavy top, and
- supersymmetric vacua.

The last property deserves a comment. The pseudo-anomalous symmetry $\mathrm{U}(1)_{\mathrm{A}}$ (generically present in orbifold models) induces the FI term [32]

$$D_{\mathrm{A}} \;\approx\; \sum_j q^j_{\mathrm{A}} \, |\langle\varphi_j\rangle|^2 + \frac{M^2_{\mathrm{Pl}}}{192\pi^2} \sum_j q^j_{\mathrm{A}}\,. \tag{20}$$

Therefore, in supersymmetric vacua a set of fields φ_j have to acquire non-vanishing VEVs, such that $D_{\mathrm{A}} = 0$. In general, solutions of the $D = 0$ conditions can be expressed by gauge invariant monomials [33],

$$I \;=\; \varphi_1^{m_1} \varphi_2^{m_2} \ldots\,, \tag{21}$$

where m_i are positive integers. In the case of an anomalous U(1), such a monomial must be invariant only with respect to the non-anomalous gauge symmetries in order to cancel the FI term, eq. (20). Since in our conventions $\sum_j q^j_{\mathrm{A}} > 0$, a suitable vacuum configuration exists if there is a set of SM singlets, $s_i \subset \varphi_j$, that build a monomial with a total negative anomalous charge, $\sum_j m_j \, q^j_{\mathrm{A}} < 0$ [30]. Therefore, the condition $D_{\mathrm{A}} = 0$ sets the VEVs of the singlets s_i and, thereby, the scale of the breakdown of $\mathrm{U}(1)_{\mathrm{A}}$. In heterotic orbifolds, this scale can be as high as $\sim 10^{17}$ GeV. All models studied here have vacua satisfying $D = 0$ and preserving the SM gauge group unbroken (including

Order of $\mathcal{W}$	# Models with some $\mathrm{U}(1)_{\mathrm{acc}}$	# Models with some $\mathrm{U}(1)_{\mathrm{PQ}}$
3	55	55
4	55	54
5	55	15
6	34	0

Table 1: Statistics of frequency of accidental PQ symmetries in a subset of promising $\mathbb{Z}_6$–II heterotic orbifold models.

hypercharge). [3] This contradicts recent statements about model building in heterotic string orbifolds [34].

For each model, we choose a monomial suitable to cancel the FI term that involves only a few SM singlets. The VEVs of the fields in such a monomial is set close to the string scale by eq. (20). Further VEVs for other SM singlets which we will consider later are assumed to be significantly smaller. The global U(1)s in which we are interested are those of the effective superpotential at finite orders obtained after the cancelation of the FI term. We find that the studied orbifold models have generically one or more accidental symmetries. As presented in table 1, although most of the models have approximate global symmetries up to high orders, only at orders 3 and 4 most of them exhibit $\mathrm{U}(1)_{\mathrm{acc}}-\mathrm{SU}(3)-\mathrm{SU}(3)$ anomalies. In fact, at order 6 in $\mathcal{W}$, no model of the limited sample analyzed in this study has symmetries which can serve as a $\mathrm{U}(1)_{\mathrm{PQ}}$.

The explicit breaking of all accidental PQ symmetries at order 6 in these models could spoil the CP-conserving vacuum. However, from the discussion of sec. 2.4, a solution to the strong CP problem is still possible provided that the PQ-violating couplings contributing to the axion mass appear at higher orders. In addition, we have seen that requiring a hierarchy of VEVs lets us set the axion decay constant at an intermediate scale. Such a hierarchy could arise in this type of models from other symmetries of the low energy theory [13]. In the following we examine a model with only one $\mathrm{U}(1)_{\mathrm{PQ}}$.

3.1 A $\mathbb{Z}_6$–II Model with a QCD Axion

A $\mathbb{Z}_6$–II orbifold model which can lead to a suitable QCD axion is defined by the following gauge shift and Wilson lines:

$$\begin{aligned} V^{\mathrm{SO}(10),1} &= \left(\frac{1}{3},-\frac{1}{2},-\frac{1}{2},0,0,0,0,0\right)\left(\frac{1}{2},-\frac{1}{6},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right), \\ W_3 &= \left(-\frac{1}{2},-\frac{1}{2},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6}\right)\left(1,0,-\frac{1}{3},-\frac{2}{3},\frac{2}{3},-\frac{5}{3},-\frac{2}{3},\frac{1}{3}\right), \end{aligned}$$

[3] As we only know the form of the superpotential without coefficients, we have to assume that $F=0$ can be fulfilled and stabilizes the remaining D-flat directions. For generic coefficients of order 1, this has been checked in some examples.

#	Representation	Label	#	(Anti-)Repr.	Label	#	Representation	Label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})_{1/6}$	q_i				4	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_0$	m_i
9	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{-1/2}$	ℓ_i	6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{1/2}$	$\bar{\ell}_i$	2	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{2})_0$	n_i
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_1$	$\bar{e}_i$				65	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_0$	s_i^0
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{-2/3}$	$\bar{u}_i$				12	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_0$	h_i
8	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{1/3}$	$\bar{d}_i$	5	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{-1/3}$	d_i	10	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})_0$	w_i
12	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{1/2}$	s_i^+	12	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{-1/2}$	s_i^-	10	$(\mathbf{1},\mathbf{1};\overline{\mathbf{4}},\mathbf{1})_0$	$\bar{w}_i$
2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{1/2}$	x_i^+	2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{-1/2}$	x_i^-			

Table 2: Massless spectrum. The quantum numbers are shown with respect to $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(4) \times \mathrm{SU}(2)$, the hypercharge is given by the subscript.

$$W_2 \;=\; \left(\frac{3}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4},-\frac{1}{4},-\frac{1}{4},-\frac{1}{4}\right)\left(\frac{5}{2},-\frac{3}{2},-2,-\frac{5}{2},-\frac{5}{2},-2,-2,2\right). \tag{22}$$

The unbroken gauge group after compactification is

$$\mathcal{G} \;=\; \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times [\mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{U}(1)_\mathrm{A} \times \mathrm{U}(1)^7]\,, \tag{23}$$

where $\mathrm{U}(1)_Y$ denotes the standard SU(5) hypercharge and $\mathrm{U}(1)_\mathrm{A}$, the anomalous U(1). The massless matter spectrum is given in table 2; it contains three MSSM generations plus vector-like exotics with respect to $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. All non-Abelian and Abelian charges of the massless particles are provided at our web page [35].

The selected vacuum has to preserve $\mathcal{N}=1$ SUSY. With this purpose, we require that the fields contained in the gauge invariant monomial

$$I_\mathrm{FI} = \left(s^0_{19}\right)^2 s^0_{49} \left(s^0_{52}\right)^3 s^0_{57} \left(s^0_{59}\right)^2 \tag{24}$$

(with total $\mathrm{U}(1)_\mathrm{A}$ charge $q_\mathrm{A}(I_\mathrm{FI}) = -25/6$) acquire VEVs such that the FI term is canceled. As a consequence, the gauge symmetry is further broken to

$$\mathcal{G}' \;=\; \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times [\mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{U}(1)^3]\,. \tag{25}$$

After symmetry breaking, the effective superpotential $\mathcal{W}$ of this model contains 870 couplings of order up to five and is invariant with respect to 19 accidental U(1)s, in addition to the gauge symmetry group $\mathcal{G}'$. Only one accidental U(1), that we denote $\mathrm{U}(1)_\mathrm{PQ}$, presents mixed PQ-$\mathrm{SU}(3)_C$ anomalies:

$$\mathcal{A} = \sum_i \ell_i q^i_\mathrm{PQ} = 1 \neq 0\,. \tag{26}$$

If the fields entering the gauge invariant monomial

$$I \;=\; s^0_1\, s^0_{36}\, s^0_{37} \tag{27}$$

Label	q_A	q_Y	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{PQ}
s_1^0	$\frac{5}{3}$	0	-1	-3	0	0	-10	0	0	-2
s_{19}^0	$-\frac{11}{18}$	0	$\frac{1}{2}$	$\frac{11}{6}$	$-\frac{5}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{2}{3}$	0
s_{36}^0	$\frac{22}{9}$	0	0	4	0	0	2	0	0	0
s_{37}^0	$\frac{13}{9}$	0	1	-1	0	0	8	0	0	1
s_{49}^0	$\frac{7}{9}$	0	$-\frac{1}{2}$	$\frac{4}{3}$	$\frac{5}{6}$	0	12	$-\frac{2}{3}$	$\frac{4}{3}$	0
s_{52}^0	$-\frac{22}{9}$	0	0	-4	0	0	-2	0	0	0
s_{57}^0	$-\frac{4}{9}$	0	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$	0	-14	$-\frac{2}{3}$	$\frac{4}{3}$	0
s_{59}^0	$-\frac{13}{9}$	0	$-\frac{1}{2}$	$\frac{10}{3}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{9}{2}$	$\frac{1}{3}$	$-\frac{2}{3}$	0

Table 3: U(1) charges of the relevant non-Abelian singlets of a $\mathbb{Z}_6$–II orbifold model with a PQ symmetry.

acquire VEVs, $\mathrm{U}(1)_{\mathrm{PQ}}$ is broken spontaneously and the axion

$$a = \frac{1}{M}\left(q^1_{\mathrm{PQ}} v_1 a_1 + q^{36}_{\mathrm{PQ}} v_{36} a_{36} + q^{37}_{\mathrm{PQ}} v_{37} a_{37}\right) \tag{28}$$

appears in the theory. Here a_i, q^i_{PQ} and v_i denote respectively the phase components, PQ charges and VEVs of the singlets of eq. (27). The $\mathrm{U}(1)_{\mathrm{PQ}}$ breakdown scale M is read off from eq. (6).

Along the D-flat direction described by eq. (27), the VEVs of the singlets are equal. Thus, the axion decay constant is given by

$$F_a \;=\; \frac{M}{\mathcal{A}} \;=\; \sqrt{\left(q^1_{\mathrm{PQ}} v_1\right)^2 + \left(q^{36}_{\mathrm{PQ}} v_{36}\right)^2 + \left(q^{37}_{\mathrm{PQ}} v_{37}\right)^2} = \sqrt{5}\, v_1\,, \tag{29}$$

where we have considered the singlet charges displayed in table 3. Provided that the VEVs v_i are well below the scale of the FI term, say at $\sim 10^{12}$ GeV, the inverse axion coupling can agree with the current experimental bounds, eq. (1). Note that we have disregarded the axion-like components of the fields that acquire VEVs at the larger scale (see eq. (24)) as their contributions are subdominant.

There are two issues that still have to be addressed. First, since the $\mathrm{U}(1)_{\mathrm{PQ}}$ is broken at order 6, we have to verify whether there appear large contributions to the axion mass. In the current model, the first contribution to the axion mass from the explicit breaking of $\mathrm{U}(1)_{\mathrm{PQ}}$ arises at order 16. We have estimated the contribution to the axion mass to be $m_*^2 \sim 10^{-18}\ \mathrm{GeV}^4/F_a^2$, consistent with the current experimental bound, eq. (17). Second, in the vacuum configuration chosen here, most of the exotics remain massless. This problem is due to the small number of fields that are chosen to develop large VEVs (see eq. (24)). However, as shown in [15–19], there exist supersymmetric vacua where all exotics acquire masses because (almost) all the SM singlets attain VEVs. In the current model, we find a monomial I_{FI} containing 32 fields that leaves a $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry unbroken and provides masses for many more exotics. Whether there exist configurations where all exotics are decoupled and simultaneously a $U(1)_{PQ}$ is retained is a question beyond the scope of the current letter and will be addressed elsewhere. Note that this is related to the magnitude of m_*^2, as integrating out heavy exotics could induce further contributions to the axion mass.

4 Conclusions

Discrete symmetries (as found abundantly in models of the heterotic brane world) are suited to solve the strong CP problem in a satisfactory way. They might give rise to accidental global $U(1)_{PQ}$ symmetries as "low energy" remnants. Accions – the axion like particles of the spontaneous breakdown of these symmetries – could have a decay constant F_a in the allowed axion window, eq. (1).

The key observation for this latter fact has been explained in section 2, formula (14). In a system with an anomalous and a non-anomalous U(1) symmetry broken by two fields with nontrivial VEVs, it is the smallest of these VEVs that corresponds to the accion decay constant F_a. This allows a decoupling of F_a from the large VEVs required to cancel the FI terms (which appear to be only one or two orders of magnitude below the string scale). The actual value of F_a depends strongly on the vacuum configuration of the system and requires further model building. In this framework of models with accidental U(1) symmetries, such a hierarchy of the scales could be potentially created along the lines explained in ref. [13].

The implementation of the accion-system in the framework of realistic MSSM models from the heterotic string is still in its infancy. The identification of suitable candidates for accidental U(1)'s requires an enormous amount of computer work. In section 3 we have examined a few candidate models from our previous MSSM search that can serve as an existence proof for the accion solution to the strong CP-problem. A few things remain to be done: the decoupling of exotics at higher order in the superpotential and a detailed discussion of the accion mass arising from the explicit breakdown of the accidental U(1) symmetry. Such an analysis is beyond the scope of the present paper. It would require substantially higher computer power and/or new algorithms to efficiently attack the problem. Work along this direction is under way.

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