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Squark pair production at the LHC

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Abstract. We present NNLO cross sections for squark- antisquark production at the LHC. We have calculated new analytic expressions for the scale dependent scaling functions at one and two loop.

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1. SQUARK PAIR PRODUCTION CROSS SECTION AT THE LHC

If Supersymmetry is realised in Nature then it is expected that squark and gluino pairs are produced in large numbers at the LHC. It is possible to probe masses up to the TeV range. Squarks are assumed to be heavier than $\approx 400\,$GeV [1] so these particles are produced near the kinematical production threshold. Therefore one can use the same methods to calculate higher order cross sections as developed for $t\bar{t}$ production [2, 3]. The partonic LO and NLO cross sections are known for long times [4]. Approximate NNLO corrections have been calculated in [5]. The LO partonic cross section and the NLO theshold expansion are known analytically [4]. In this article we present analytical formulae for the scale dependence determining NLO scaling functions and the threshold expansion of the scale dependence determining NNLO scaling functions. For related work, see also Ref. [6]. In Ref. [7], the soft anomalous dimension has been calculated to NNLO accuracy.

The partonic cross section $\hat{s}$ with identified renormalisation and factorisation scale can be expanded as

$$\hat{\sigma}_{ij} = \frac{\alpha_s^2}{m_{\tilde{q}}^2}\left[f_{ij}^{(00)} + 4\pi\alpha_s\left(f_{ij}^{(10)} + f_{ij}^{(11)}L_M\right) + (4\pi\alpha_s)^2\left(f_{ij}^{(20)} + f_{ij}^{(21)}L_M + f_{ij}^{(22)}L_M^2\right)\right],\; L_M = \log(\mu^2/m_{\tilde{q}}^2), \tag{1}$$

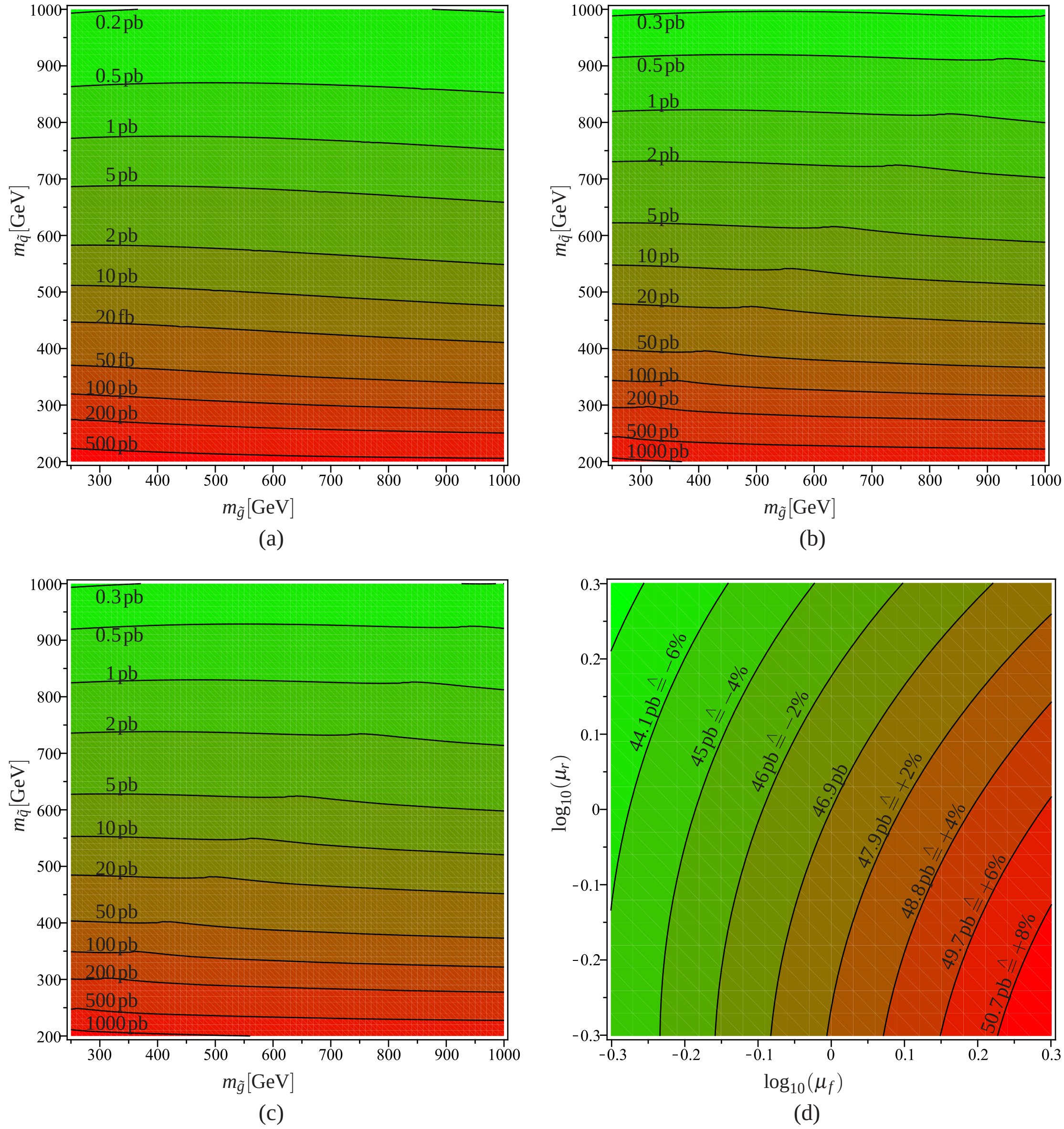
where ij denote the initial states gluon - gluon or quark - antiquark. The full dependence on the renormalisation and factorisation scale is the same as for $t\bar{t}$-production, see Ref. [8]. The hadronic cross section is given as a convolution of the partonic cross section with the parton luminosities L_{ij}:

$$\sigma_{pp\to\tilde{q}\tilde{q}^* X}(s, m_{\tilde{q}}, m_{\tilde{g}}) = \sum_{i,j=q,\bar{q},g}\;\int_{4m_{\tilde{q}}^2}^{s} \mathrm{d}\hat{s}\, L_{ij}\,\hat{\sigma}_{ij}(\hat{s}, m_{\tilde{q}}, m_{\tilde{g}}, \mu). \tag{2}$$

We performed a scan of the LO, NLO, and NNLO squark pair production cross section in the $m_{\tilde{g}}$ - $m_{\tilde{q}}$ - plane using `Prospino` [9] and the formulae presented in [5], see Fig. 1(a) -(c). One clearly sees the strong enhancement of the NLO and NNLO cross section compared to the LO cross section: For squarks and a gluino with mass 200 GeV and 250 GeV, respectively, we have a LO cross section of about 500 pb, but about 1000 pb at NLO and NNLO. At NNLO, the 1000 pb region is even enlarged to higher gluino masses. For a squark mass of 400 GeV and a gluino mass of 500 GeV, we find for the LO, NLO, and NNLO cross section 28.9 pb, 43.1 pb, and 46.9 pb, respectively. The NNLO cross section is 9% larger than the NLO cross section. The contour lines of constant cross section are running nearly parallel to the gluino mass axis: The cross section shows a rather mild dependence on the gluino mass. There is a weak enhancement of the cross section for $m_{\tilde{q}} = m_{\tilde{g}}$ as one can see from the small bump at $m_{\tilde{g}} = m_{\tilde{q}}$. The cross section decreases for more than three orders of magnitude for squark masses from $200 - 1000\,$GeV. This strong mass dependence is well-known from hadronic $t\bar{t}$ pair production.

In Fig. 1(d) we show the full μ_f - μ_r scale dependence of the NNLO cross section for the example point $m_{\tilde{q}} = 400\,$GeV, $m_{\tilde{g}} = 500\,$GeV. The scale uncertainty is about -8% for $(\mu_f, \mu_r) = (\frac{1}{2}m_{\tilde{q}}, 2m_{\tilde{q}})$ and about $+8\%$ for $(\mu_f, \mu_r) = (2m_{\tilde{q}}, \frac{1}{2}m_{\tilde{q}})$. This is considerably larger than the usual scale uncertainty taken at $\mu_r = \mu_f \equiv \mu$ (in our example $\approx -4\%$ at $\mu = 1/2$ and $+1\%$ at $\mu = 2$). This shows that a full treatment of the scale dependence leads to more reliable estimates of the scale uncertainty.

FIGURE 1. LO (Fig. (a)), NLO (Fig. (b)), and NNLO (Fig. (c)) squark pair production cross sections at the LHC @ 14 TeV. Figure (d) shows the μ_f - μ_r dependence of the NNLO cross section for $m_{\tilde{q}} = 400\,\mathrm{GeV}$ and $m_{\tilde{g}} = 500\,\mathrm{GeV}$. The PDF set is CTEQ6.6.



2. ANALYTICAL FORMULAE

In this section, we present analytical formulae for the $f_{ij}^{(11)}$ scaling functions and the threshold logarithms of the $f_{ij}^{(21)}$ and $f_{ij}^{(22)}$ scaling functions. The $f_{ij}^{(11)}$ scaling functions are determined by the renormalisation group equation:

$$f_{ij}^{(11)} = \frac{1}{8\pi^2}\left(\beta_0 f_{ij}^{(00)} - P_{ij}^{(0)} \otimes f_{ij}^{(00)}\right), \quad ij = gg,\ q\bar{q}. \tag{3}$$

The $P_{ij}^{(0)}$ are the leading order splitting functions, see Ref. [10]. $\otimes$ denotes the standard Mellin convolution. The scale dependent NLO scaling function Eq. (3) depends only on LO functions. Performing the integrations yields as new analytic results the Eq. (4) and (5).

$$f_{gg}^{(11)} = -\frac{1}{384\pi} C_A n_f \rho \Big[\beta\left(10+31\rho\right)L_3 - \rho\left(\rho+16\right)\left(L_3L_2+L_5\right) + \rho\left(\rho-16\right)\left(L_4-\tfrac{1}{2}L_6\right)$$
$$+\left(-20+34\rho+\tfrac{127}{12}\rho^2-16\rho\log(2)+\rho^2\log(2)\right)L_2+\tfrac{1}{90}\beta\left(2606-14763\rho+352\rho^{-1}\right)\Big], \qquad (4)$$

$$f_{qq}^{(11)} = \frac{1}{8\pi^2}\beta_0 f_{qq}^{(00)} - \frac{1}{216\pi}C_F n_f \delta_{ij}\rho\Big[(3\rho-2)L_2+4\beta^3L_3-\tfrac{1}{3}\beta(13-7\rho)\Big]$$
$$-\frac{1}{216\pi}C_F\delta_{ij}\rho\Big\{\rho(1+a)\Big[L_1^2-\tfrac{1}{2}L_1+2L_1L_2+2L_1L_3+L_4-\tfrac{1}{2}L_6-L_7-L_8-\log\left(\tfrac{1+a}{2}\right)L_2$$
$$-4\,\mathrm{Li}_2\left(-\tfrac{2\beta}{1-\beta}\right)+4\,\mathrm{Li}_2\left(-\tfrac{2a\beta}{a(1-\beta)+2}\right)\Big]$$
$$+\tfrac{1}{2}\rho^2a^2\Big[L_9-L_5+L_1L_3\Big]+2(a\rho+2)\Big[\beta L_3-L_2\Big]-\frac{2}{a^2}(1+2a)L_2-\frac{2}{a^2}(1+a)^2L_1+\frac{2}{a}\beta(1-a)\Big\}$$
$$+\frac{1}{72\pi}\rho C_F\Big\{-(a\rho+2)\Big[-2L_9-L_4+L_7+L_8+\tfrac{1}{2}L_6-2L_1L_3+\log\left(\tfrac{1+a}{2}\right)L_2+4\,\mathrm{Li}_2\left(-\tfrac{2\beta}{1-\beta}\right)\Big]$$
$$+\frac{1}{2(a+1)(4a+4+a^2\rho)}\Big[8+16a-a^4\rho-3a^3\rho^2+10a^2\rho+6a^3\rho+4\rho a+8a^2\Big]L_1$$
$$+\frac{2}{(a+1)(4a+4+a^2\rho)}\Big[8a^2+16a+8+4a^2\rho-a^3\rho^2+4a^3\rho\Big]L_2$$
$$-\frac{8\beta}{(4a+4+a^2\rho)}\Big[a^2\rho+2a+2\Big]L_3+\frac{2\beta}{(4a+4+a^2\rho)}\Big[a^2\rho+10a+10\Big]-4L_{10}+a\rho L_2^2\Big\}, \qquad (5)$$

$$L_1=\log\left(\tfrac{(1-\beta)(a(1+\beta)+2)}{(1+\beta)(a(1-\beta)+2)}\right),\qquad L_2=\log\left(\tfrac{1+\beta}{1-\beta}\right),\qquad L_3=\log\left(\tfrac{4\beta^2}{\rho}\right),$$
$$L_4=\mathrm{Li}_2\left(\tfrac{1-\beta}{2}\right)-\mathrm{Li}_2\left(\tfrac{1+\beta}{2}\right),\qquad L_5=\mathrm{Li}_2\left(-\tfrac{2\beta}{1-\beta}\right)-\mathrm{Li}_2\left(\tfrac{2\beta}{1+\beta}\right),$$
$$L_6=\log^2(1+\beta)-\log^2(1-\beta),\qquad L_7=\mathrm{Li}_2\left(\tfrac{a(1-\beta)}{2(1+a)}\right)-\mathrm{Li}_2\left(\tfrac{a(1+\beta)}{2(1+a)}\right),$$
$$L_8=\mathrm{Li}_2\left(\tfrac{-a(1+\beta)}{2}\right)-\mathrm{Li}_2\left(-\tfrac{a(1-\beta)}{2}\right),\qquad L_9=\mathrm{Li}_2\left(-\tfrac{2a\beta}{a(1-\beta)+2}\right)-\mathrm{Li}_2\left(\tfrac{2a\beta}{a(1+\beta)+2}\right),$$
$$L_{10}=\mathrm{Li}_2\left(-\tfrac{2\beta}{1-\beta}\right)+\mathrm{Li}_2\left(\tfrac{2\beta}{1+\beta}\right),\qquad \mathrm{Li}_2(x)=-\int_0^x \mathrm{d}t\frac{\log(1-t)}{t},\qquad a=\frac{m_{\tilde g}^2}{m_{\tilde q}^2}-1\quad. \qquad (6)$$

The leading order scaling functions $f_{gg}^{(00)}$ and $f_{q\bar q}^{(00)}$ can be found in Ref. [5]. The NNLO scale dependent scaling functions follow from the RGE relations

$$f_{ij}^{(21)} = \frac{1}{(16\pi^2)^2}\left(2\beta_1 f_{ij}^{(00)} - f_{kj}^{(00)}\otimes P_{ki}^{(1)} - f_{ik}^{(00)}\otimes P_{kj}^{(1)}\right)+\frac{1}{16\pi^2}\left(3\beta_0 f_{ij}^{(10)} - f_{kj}^{(10)}\otimes P_{ki}^{(0)} - f_{ik}^{(10)}\otimes P_{kj}^{(0)}\right), \qquad (7)$$

$$f_{ij}^{(22)} = \frac{1}{(16\pi^2)^2}\Bigg(f_{kl}^{(00)}\otimes P_{ki}^{(0)}\otimes P_{lj}^{(0)}+\frac{1}{2}f_{in}^{(00)}\otimes P_{nl}^{(0)}\otimes P_{lj}^{(0)}+\frac{1}{2}f_{nj}^{(00)}\otimes P_{nk}^{(0)}\otimes P_{ki}^{(0)}+3\beta_0^2 f_{ij}^{(00)}$$
$$-\frac{5}{2}\beta_0 f_{ik}^{(00)}\otimes P_{kj}^{(0)}-\frac{5}{2}\beta_0 f_{kj}^{(00)}\otimes P_{ki}^{(0)}\Bigg) \qquad (8)$$

i,j,k,l,n are parton indices with implied summation over repeated indices. The threshold expansion is derived by computing the Mellin transformation of each of the involved factors and inverting the products back to ρ space. The scaling function $f_{gq}^{(ij)}$ is very small near threshold so we did not include them. The constants $a_1^{qq,gg}$ can be found in Ref. [5]. The coefficients of the QCD β - function are given as $\beta_0 = 11-(2/3)n_f$ and $\beta_1 = 102-(38/3)n_f$. We used the threshold expansion to fit the numerical determined values of the NNLO scaling functions.

$$f_{gg}^{(21)} = \frac{f_{gg}^{(00)}}{(16\pi^2)^2}\Big[-4608\log^3(\beta) + \Big(-18432\log(2) + \tfrac{109920}{7} - 64n_f\Big)\log^2(\beta)$$
$$+\Big(6766.94811 - 66178.09806a_1^{gg} - 192\log(2)n_f + \tfrac{4048}{21}n_f - \tfrac{176}{7}\tfrac{\pi^2}{\beta}\Big)\log(\beta)$$
$$-3572.87371 + 35472.75010a_1^{gg} + 55.41606408n_f - 919.1402509a_1^{gg}n_f$$
$$+\tfrac{56.86772061}{\beta} - \tfrac{3.446528522n_f}{\beta}\Big] \tag{9}$$

$$f_{gg}^{(22)} = \frac{f_{gg}^{(00)}}{(16\pi^2)^2}\Big[1152\log^2(\beta) + \Big(16n_f - 2568 + 2304\log(2)\Big)\log(\beta)$$
$$2568 + 1152\log^2(2) - 2568\log(2) - 144\pi^2 + 16n_f\log(2) - 16n_f\Big] \tag{10}$$

$$f_{q_i\bar{q}_j}^{(21)} = \frac{f_{q_i\bar{q}_j}^{(00)}}{(16\pi^2)^2}\Big[-\tfrac{8192}{9}\log^3(\beta) + \Big(-\tfrac{32768}{9}\log(2) + \tfrac{34688}{9} - \tfrac{256}{3}n_f\Big)\log^2(\beta)$$
$$+\Big(1150.2835 - 2412.743158a_1^{qq} + \tfrac{5392}{27}n_f - 256\log(2)n_f - \tfrac{448}{9}\tfrac{\pi^2}{\beta}\Big)\log(\beta)$$
$$-1374.416616 + 3567.790429a_1^{qq} + 70.72319322n_f - 226.1946711a_1^{qq}n_f$$
$$+\tfrac{9235}{\beta} - \tfrac{46.05815389n_f}{\beta}\Big] \tag{11}$$

$$f_{q_i\bar{q}_j}^{(22)} = \frac{f_{q_i\bar{q}_j}^{(00)}}{(16\pi^2)^2}\Big[\tfrac{2048}{9}\log^2(\beta) + \Big(-\tfrac{7840}{9} + \tfrac{320}{9}n_f + \tfrac{4096}{9}\log(2)\Big)\log(\beta)$$
$$+\tfrac{9415}{9} + \tfrac{2048}{9}\log^2(2) - \tfrac{7840}{9}\log(2) - \tfrac{256}{9}\pi^2 - \tfrac{596}{9}n_f + \tfrac{320}{9}\log(2)n_f + \tfrac{4}{3}n_f^2\Big] \tag{12}$$

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