UNI KASSEL VERSITÄT

Institut für Werkstofftechnik
Metallische Werkstoffe
Prof. Dr.-Ing. B. Scholtes

HELMHOLTZ ZENTRUM BERLIN für Materialien und Energie

APPROACHES TO THE PROBLEM OF THE INVERSE LAPLACE TRANSFORM IN RESIDUAL STRESS ANALYSIS BY COMPARISON OF EXPERIMENTAL RESULTS OF σ(τ) AND σ(z)

Thorsten Manns[1], Ingwer A. Denks[2], Christoph Genzel[2] and Berthold Scholtes[1]

[1] Institute of Materials Engineering, University of Kassel, Mönchebergstr. 3, 34125 Kassel, Germany, e-Mail: manns@uni-kassel.de

[2] Helmholtz Zentrum Berlin, Lise-Meitner-Campus, Glienicker Straße 100, 14109 Berlin, Germany, e-Mail: denks@hmi.de

Introduction

Surface treated 100Cr6 steel samples were analysed concerning their residual stresses. The aim of the project is the development of measurement and evaluation strategies for determining stress gradients by energy and angle dispersive X-ray diffraction. The main problem of conventional Laplace methods is that they only determine real stress values $\sigma(z)$ if no gradient of the residual stresses within the X-ray penetration depth τ occurs. Otherwise the measured Laplace profiles $\sigma(\tau)$ have to be recalculated by inverse Laplace transform to obtain the residual stresses in real space being those from practical relevance for engineering applications.

Material and sample preparation

As sample material the roller bearing steel 100Cr6 in hardened and tempered state was used. The heat treatment yields a fine grained microstructure of martensite, highly dispersed carbides (e.g. Cr-carbides) and residual austenite. From micrographs and phase analyses by X-rays can be concluded that the grain size is in sub-µ-range and the volume fractions of austenite and carbides ($M_{23}C_6$) are 7-9% and 9-10% respectively.

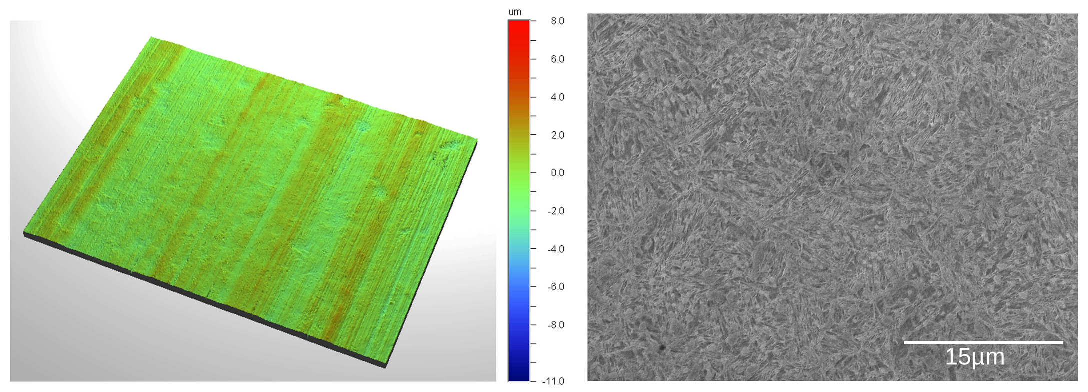

Figure 1:
Surface roughness analysis by optical profiling: Shot peened sample (left), R_a=517nm, R_t=4.90µm.
SEM image (right) showing martensitic microstructure.

Two kinds of surface treatments, each with two different process parameters, were applied to the tempered samples to induce highly varying residual stress distributions $\sigma(z)$. For long range residual stress gradients shot peening processes with low and high Almen intensity were carried out whereas deep grinding with small and large chipping volume was used to induce steep stress gradients in the first few micron from surface. After specimen processing the surface topography was analyzed and is displayed in *Figure 1* for a shot sample.

Experimental setup and results

Strain distributions of the surface treated samples were measured by the energy dispersive method at the EDDI beamline at BESSY up to X-ray energies of approximately 80keV. The used setup allows the analysis of several lattice planes in one measurement and realises penetration depths of more than 100µm. The surface near regions were additionally analysed by monochromatic synchrotron radiation at the HASYLAB beamline G3 at DESY in Hamburg. The applied X-ray energy was equivalent to Cukα and Mokα wavelengths and the samples were measured at the {110} and the {431/510} lattice plane respectively to affirm the energy dispersive results.

Furthermore $\sin^2\psi$ measurements using Crkα radiation were carried out and combined with successive layer removal down to 1µm increments to determine the residual stress state $\sigma(z)$ in real space. Due to the small penetration depth τ_0 of 3.28µm for the {110} and 5.69µm for the {211} lattice plane, the calculated stress values can be taken as references for those calculated from the synchrotron measurements by inverse Laplace transform (ILT).

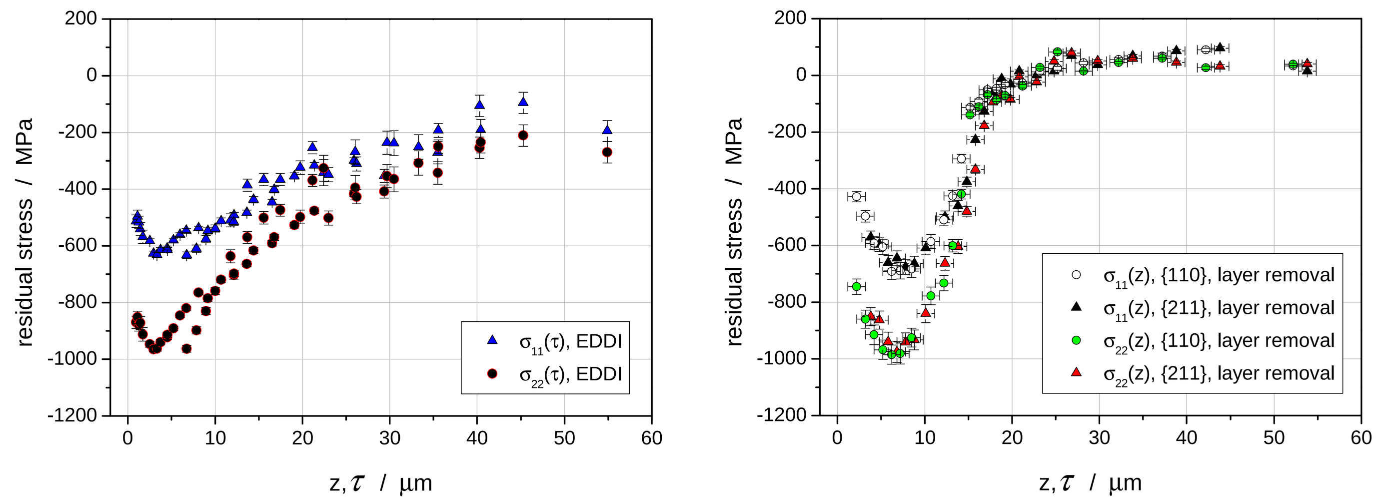

Figure 2: Laplace residual stresses $\sigma(\tau)$ determined by energy dispersive diffraction at BESSY (left). Real space stress distributions $\sigma(z)$ measured by successive layer removal in Kassel (right).

In the diagrams above longitudinal and transverse stress gradients of deep ground samples are presented. Left hand side shows the Laplace values $\sigma(\tau)$ calculated via Universal plot method from the measured strain $\varepsilon(\tau)$, whereas the real space stresses $\sigma(z)$ are plotted on the right. Both diagrams show steep stress gradients and considerably higher amounts in σ_{22} (transverse) direction.

The combination of all measured Laplace values $\sigma_{22}(\tau)$ yields the left diagram in *figure 3* which shows a very good agreement between energy and angle dispersive methods. To compare Laplace stresses with those in real space the combined $\sigma_{22}(\tau)$ profile was transformed via ILT applying different fit functions. The blue curve in the right diagram represents the real space stress $\sigma_{22}(z)$ fitted by an exponentially damped 3rd order polynomial which approximates the residual stress distribution from layer removal very well.

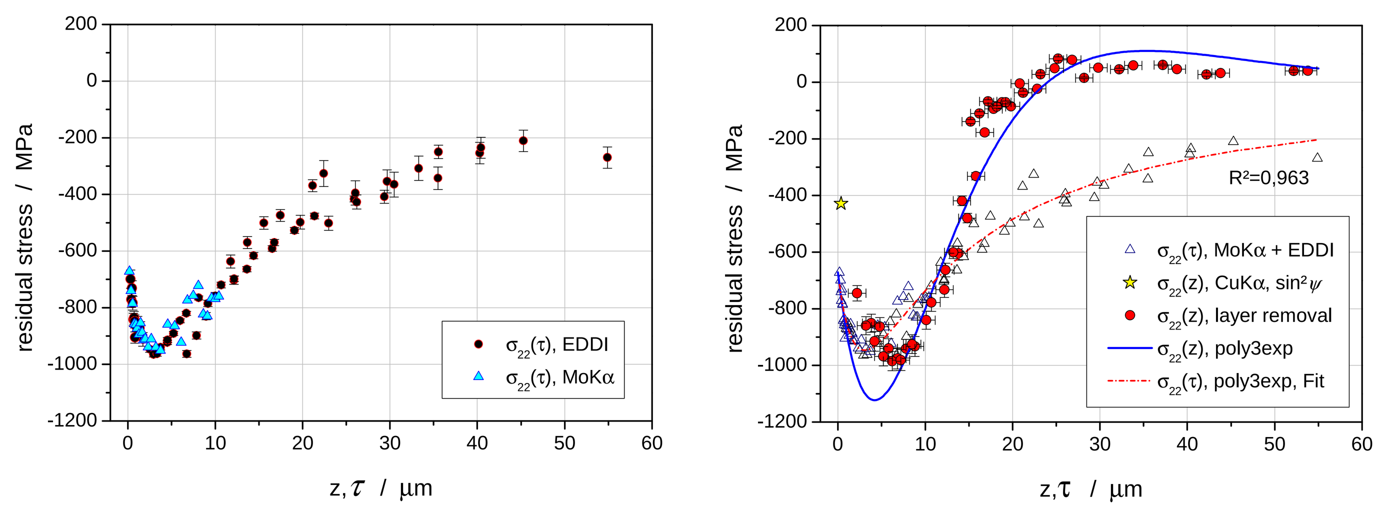

Figure 3: Laplace stress values $\sigma_{22}(\tau)$ determined by energy and angle dispersive diffraction at BESSY and DESY respectively (left).
Comparison of real space stresses $\sigma_{22}(z)$ measured by successive layer removal and $\sigma_{22}(z)$ profiles calculated from $\sigma_{22}(\tau)$ applying of inverse Laplace transform (right).

Trail functions

Real space:

$$\sigma(z) = \sum_0^n a_n \cdot z^n \cdot \exp(b \cdot z)$$

Laplace space:

$$\sigma(\tau) = \sum_0^n \frac{a_n}{\left(\frac{1}{\tau} - b\right)^{n+1}} \cdot \frac{1}{\tau}$$

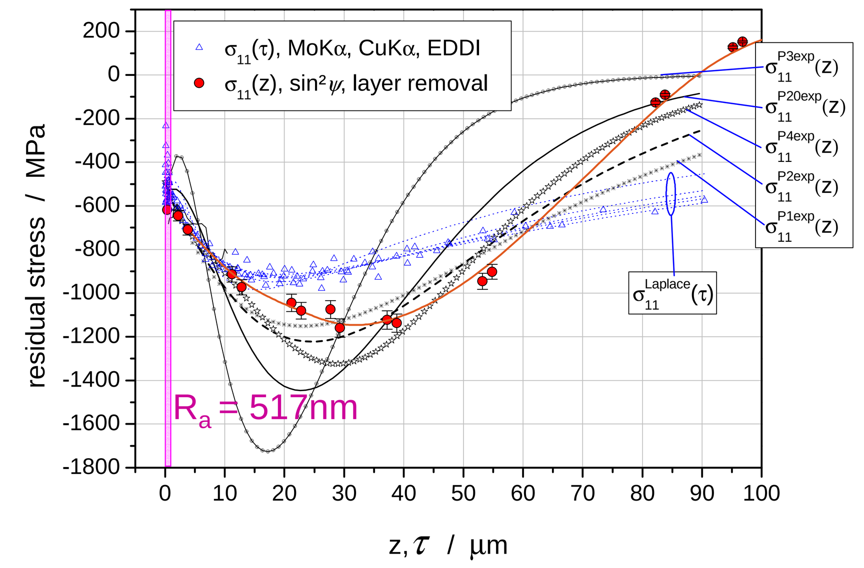

Figure 4: Applied exponential damped trail functions where n describes the order of polynomial (left)
Real space stress values $\sigma_{11}(z)$ determined by layer removal (red dots) versus calculated ones by ILT. Influence of various Laplace trail functions $\sigma_{11}(\tau)$ on the transformed stress profiles.

The diagram in *Figure 4* illustrates the evaluated stress profiles of a shot peened specimen. Five different trail functions were fitted to the experimentally obtained Laplace profile and afterwards transformed in real space. The resulting stress distributions are strongly dependent on the applied fit function. It can be seen that even if in Laplace space a sufficient approximation of the measured data exists, the transformed residual stress may tend to oscillate. This behaviour typically increases with higher order polynomials. Another reason can be assigned to scattering strain values determined very close to the surface which are probably affected by the sample topography. Due to the high density of measurement points here the transformed curves might be falsified by these values and should be discarded up to the roughness R_a.

Conclusions

- Both Laplace and real space methods yield accurate residual stress depth profiles in their respective spaces.
- The Laplace space method is non-destructive and the results are obtained with an expenditure of time that is short compared to the real space method.
- Direct access to the real space stress profiles $\sigma(z)$ is only possible by means of real space methods. However layer removal leads to a (partial) destruction of the sample.
- It was shown that the inverse Laplace transform (ILT) of the discrete Laplace stress profiles leads to reasonable results for $\sigma(z)$ in case of experimental data being of excellent quality.
- Further work is necessary to improve the stability of the ILT solutions and to find appropriate ways that ensure a quantitative assessment of the goodness of the calculated $\sigma(z)$ profiles.

Acknowledgements

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