Entropy and Equilibrium Property of QCD-Instanton Induced Final State in Deep-Inelastic Scattering^{*}

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Abstract The scaling and additivity properties of Rényi entropy in rapidity space of the instanton final state (IFS) and current jet identified by the r -sorting method from the QCDINS Monte Carlo event sample are studied. Asymptotic scaling of Rényi entropy H_2 is observed for the IFS while the H_2 for current jet tends to saturation with the decreasing of phase space scale. Furthermore, it is found that the additivity of H_2 holds well for IFS in narrow rapidity window at different positions. These results indicate that the IFS produced in the instanton-induced process of deep inelastic scattering has reached local equilibrium.

Key words instanton; quark-gluon fusion; deep-inelastic scattering; entropy; local equilibrium

The basic theory of strong interaction — quantum chromo-dynamics (QCD) is a non-Abelian gauge theory and as such has a complicated vacuum structure. There are degenerate vacua of different topologies, leading to a special effect — tunneling transition between degenerate vacua, referred to as *instanton* [1–4]. Instantons are non-perturbative fluctuations of the gluon field, which are expected to contribute into deep-inelastic scattering (DIS) with a sizable rate [5–7].

Deep-inelastic scattering (DIS) accompanied by instanton-induced hard processes, can not be described by conventional perturbation theory. Therefore, theoretical and experimental studies of the processes induced by instanton are of fundamental significance for a thorough understanding of the non-perturbative sector of QCD [5, 6].

The two experimental groups, H1 and ZEUS, of DESY-HERA in Germany have been searching for instanton induced events in e^+p deep inelastic scatter-

ing experiments, and only upper limits on the cross-section of such processes were set [8, 9]. At present, beside the experimental searching for the instanton induced events, a Monte Carlo study on the physical properties of IFS is also important.

Fig1. The leading QCD-instanton induced process in the DIS regime of e-p scattering.

In figure 1 is shown the leading graph of QCD-instanton induced e-P collision. The incident lepton emits a photon, with 4-momentum q , which in turn transforms into a quark-antiquark pair. One of these quarks with 4-momentum q'' hadronizes to form the

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current jet. The other quark, with 4-momentum q' , fuses with a gluon (with 4-momentum g) from the proton in the presence of an instanton (I). The hadron system produced from the fusion of quark and gluon in the presence of instanton is referred to as *instanton final state*, or IFS in short.

The quark-gluon fusion process with the instanton as background gives rise to a high multiplicity final state. The particles produced from this process, i.e. the IFS, are expected to be isotropically distributed in their center of mass frame^[10].

A problem of high interest is whether the IFS has reached equilibrium. Ref's.^[11, 12] have studied the final-state momentum distribution of IFS using Monte Carlo simulation. An approximate isotropic distribution is found for the IFS in contrast to the high anisotropy of current jet. In the present letter the equilibrium of IFS will be studied by using the Rényi-entropy analysis based on Ma's coincidence probability.

Let us first discuss shortly the concept of *equilibrium*. For a closed macroscopic system, arriving at (thermal) equilibrium, the corresponding ensemble is the micro-canonical one, where the microscopic states of the system are uniformly distributed in the allowed region of phase space of the system (Γ -space). Starting from this ensemble all the thermodynamical properties of the system, including the thermodynamical quantities — entropy, temperature, etc., are derived.

The physical origin of thermal equilibrium of a macroscopic system is the random collisions among the molecules constituting the system. In our case, each parton produced in the instanton-induced quark-gluon-fusion process develops a parton shower, respectively. Hadrons only experience resonance decay, if any. There is no rescattering, and no random thermal motion neither at parton level nor at hadron level. Therefore, no *thermal equilibrium* in the strict sense is possible. However, the distribution of the final states in the allowed region of phase space may be uniform. Such a distribution mimics the micro-canonical ensemble, and the “thermodynamical” properties of the system in consideration can be

derived therefrom. We will refer to such a uniform distribution in phase space also as *equilibrium*. The “equilibrium”, if any, of the IFS and current jet that we will check in the following should be understood in this sense.

To check the equilibrium of IFS and jet we will make use of the Rényi-entropy analysis based on Ma's coincidence probability^[13–15]. According to Ma^[16] an event is characterized by a set of integer number $s \equiv \{m_i\}$, $i = 1, \dots, M$, where a certain phase space region Δ is partitioned equally into M bins with the size of each bin $\delta = \frac{\Delta}{M}$, m_i is the number of particles in the i th bin. The number of coincidence of k configurations is defined as

$$N_k = \sum_s n_s(n_s - 1) \cdots (n_s - k + 1), \quad (1)$$

where n_s is the appearing time of the s -configuration in the whole event sample.

The coincidence probability of k configurations is given by

$$C_k = \frac{N_k}{N(N-1) \cdots (N-k+1)}, \quad (2)$$

where N is the total number of events in the sample, and the Rényi entropies are defined as^[17]

$$H_k \equiv -\frac{\ln C_k}{k-1}. \quad (3)$$

The Rényi entropies are closely related to the Shannon entropy (S), which is formally equal to the limit of H_k as $k \rightarrow 1$ and can be obtained through extrapolation^[14, 15]. It has been shown that the extrapolation method for determining S is in general not unique and will produce additional uncertainty^[14, 15], while the Rényi entropies are of interest by themselves and can provide information on the equilibrium of the system, cf. Eq's. (5), (6) below. So our study will be concentrated on the behavior of Rényi entropies, in particular on H_2 .

For a system close to equilibrium the Rényi entropies obey the scaling property:

$$H_k(M) = H_k(M_0) + d \ln \frac{M}{M_0} \quad (4)$$

for sufficiently fine-grained subdivision of phase space^[13, 14], where d is a constant related to the di-

mension of space, M is bin number with the size of each bin $\delta = \Delta/M$, M_0 is another bin number with the size of each bin $\delta_0 = \Delta/M_0$. Substituting $M = \frac{\Delta}{\delta}$, we can rewrite Eq. (4) as

$$H_k(M) = A - d \ln \delta \quad \text{for } M \rightarrow \infty, \quad (5)$$

where $A = H_k(M_0) + d \ln \delta_0$ is a constant.

Furthermore, the additivity property^[15]:

$$H_k^{(R)}(2M) = H_k^{(R_1)}(M) + H_k^{(R_2)}(M), \quad (6)$$

should hold for two non-overlapping and independent phase space regions R_1 and R_2 with R the union. Note that the equality of the two sides of Eq. (6) is examined at the same scale $\delta = \frac{\Delta}{M} = \frac{2\Delta}{2M}$.

The above-mentioned method has been applied to hadron-hadron (h-h) collision data from both PYTHIA-JETSET Monte Carlo generator^[18] and NA22 experiment^[19]. It turns out that in both cases the Rényi entropy H_2 versus $-\ln \delta y$ tends to saturate instead of approaching a straight line as expected by the scaling law Eq. (5) and the additivity Eq. (6) does not hold, showing that there is no equilibrium in the final state system of hadron-hadron collisions.

In the present letter we apply the method to the hadron system produced in the quark-gluon fusion process $q' + g \xrightarrow{(1)} X$ in the background of instanton. Our study is based on the Monte Carlo code QCDINS^[20, 21], which is a package for instanton induced events embedded in the HERWIG^[22, 23] event generator. The default parameters of the QCDINS 2.0 version are used in our study, i.e. $x' > 0.35$, $Q'^2 > 113 \text{ GeV}^2$ and the number of quark flavors is set to be $n_f = 3$.

In total 493400 instanton-induced DIS events are generated with the energies of electron and proton being equal to 27.5 and 820 GeV, respectively. The IFS and current jet are identified using the r -sorting method proposed in Ref.^[11]. This method is to define a distance in the θ - ϕ -plane in the hadronic center-of-mass frame, i.e.

$$r(\theta, \phi) = \sqrt{\frac{(\frac{\theta - \theta_0}{\pi})^2 + (\frac{\phi - \phi_0}{\pi})^2}{2}}, \quad (7)$$

where $(\theta_0, \phi_0 = 0)$ is the position of the current quark. This variable measures how far every final state particle is from the jet axis. Choosing an appropriate value for r_0 the particles with $r(\theta, \phi) < r_0$ are attributed to jet and those with $r(\theta, \phi) > r_0$ to IFS. First discard the proton remnant by an 1D cut. Renumber the left n particles by their r values, let $r_1 < r_2 < \dots < r_n$. Choose the value of r_0 in between r_k and r_{k+1} ($r_k < r_0 < r_{k+1}$), i.e. take particles from 1 to k as current jet, from $k+1$ to n as IFS. Accumulating the energies from particle 1 to particle k gives $E_k = \sum_{i=1}^k \varepsilon_i$; simultaneously accumulating energies from $k+1$ to n gives $E'_k = \sum_{i=k+1}^n \varepsilon_i$. The energy reconstruction errors for jet and instanton are $\Delta E_{\text{jet}} = \frac{E_k - E_C}{E_C} \times 100\%$ and $\Delta E_I = \frac{E'_k - E_I}{E_I} \times 100\%$, respectively, where E_C is the energy of current parton, E_I is that of the quark and gluon included in IFS at partonic level. Then choose the value of k to optimize the energy reconstruction, i.e. make $\Delta E = 0.4 \times |\Delta E_{\text{jet}}| + 0.6 \times |\Delta E_I|$ the minimum. The particles in the IFS and those in the current jet are then transformed to their own rest frames, respectively, for entropy analysis. The following referred rapidity y are always in their rest frame. In Fig. 2 are shown the reconstruction error ΔE_{jet} for jet energy and ΔE_I for instanton energy. It can be seen that the reconstruction is good and the resulting IFS and jet are reliable.

Fig2. The distributions of the reconstruction error ΔE_{jet} for jet energy (a) and ΔE_I for instanton energy (b) in r -sorting method.

Fig3. The Rényi entropy H_2 versus $-\ln \delta y$ in central rapidity regions $|y| < 2$ (circles) and $|y| < 0.5$ (triangles) for (a) IFS and (b) current jet. The straight lines in (a) are linear fit to the high $-\ln \delta y$ part of H_2 as function of $-\ln \delta y$. The slopes of the two lines are 6.820 ± 0.013 (upper line) and 6.304 ± 0.005 (lower line), respectively.

In Fig's. 3 (a) and (b) are shown the Rényi entropy H_2 versus $-\ln \delta y$ in central rapidity regions $|y| < 2$ (circles) and $|y| < 0.5$ (triangles) for IFS and current jet, respectively. A striking feature is that, in case of IFS the behavior of H_2 versus $-\ln \delta y$ differs qualitatively from those for DIS-current jet and h-h collision final states. It bends upwards and tends to a straight line for large $-\ln \delta y$, cf. Fig. 3(a), in contrast to the downward bending and saturation tendency for current jet (Fig 3(b)) and h-h collision final-state systems [18, 19]. This is a strong indication showing that unlike the violation of scaling law in cases of DIS-current jet and h-h collision final-states the scaling law Eq. (5) is satisfied for IFS for sufficiently fine-grained subdivision of phase space.

Fig4. The sum of Rényi entropy H_2 from IFS versus $-\ln \delta y$ for two regions R_1 and R_2 and

that directly calculated from the whole region $R = R_1 + R_2$. The legend shows the rapidity regions corresponding to each set of points.

Let us now check the additivity property Eq.(6) for the instanton final state IFS. In Fig. 4 are shown the Rényi entropies H_2 of IFS obtained from taking the sum of the results calculated from two regions R_1 and R_2 , in comparison with those directly calculated from the union $R = R_1 + R_2$. The legend in the figure shows the rapidity regions corresponding to each set of points. For example, the solid circle in Fig. 4(a) is the sum of the H_2 's from the regions $-2 < y < 0$ (R_1) and $0 < y < 2$ (R_2), while the open circle is the H_2 from the union $R = R_1 + R_2$ ($-2 < y < 2$). It can be seen from Fig. 4(a) that when the widths of regions R_1 and R_2 are narrow (~ 0.5) the additivity holds excellently, while for wider widths (~ 2) the additivity holds only approximately, especially for large M . In Fig. 4(b) the additivity is checked for R_1 and R_2 of the same width ($=1$) but located at different places. It can be seen that when R_1 and R_2 are adjacent the additivity holds well, while when they are separated by a gap the additivity holds only approximately.

The asymptotic scaling and additivity of IFS are special and impressive. These properties have never been observed in other systems, *e.g.* hadronic jets and h-h collision final states, studied up to now. They strongly indicate that the system produced in quark-gluon fusion in the background of instanton, *i.e.* IFS, has reached local equilibrium.

Our Monte Carlo study is based on the QCDINS code, where the 4-momenta of the $n = 2n_f - 1 + n_g$ produced partons in the instanton rest frame (for convenience, in the following we will refer to the partons produced in the instanton-induced quark-gluon fusion process as IQGF) are uniformly generated in energy-weighted phase space [20], according to the leading-order matrix element [24] with different energy weights for gluons and quarks. Every event of IQGF is a realization of the instanton-induced fusion process and can be regarded as a microscopic state of the system. The fact that the 4-momenta of the partons in IQGF are uniformly generated in energy-weighted phase space means that the micro-

scopic states of IQGF are uniformly distributed in phase space, mimicking the micro-canonical ensemble of a macroscopic system in thermal equilibrium. In this sense the local equilibrium in parton level is built in the model due to theoretical consideration on the property of instanton.

What we have checked above is the equilibrium of the IFS, which is the hadron system after the hadronization of IQGF. It is unclear whether and how the equilibrium property of IQGF is preserved in the hadronization process. Our work shows that the local equilibrium is present in the IFS and can be represented by the scaling and additivity of Rényi entropy.

Since the IQGF and/or IFS are in local equilibrium we can derive the “thermodynamical” quantities for them according to the usual thermodynamic formulae. However, the meaning of the obtained values of these quantities are different from those of a macroscopic system. For example, the temperature of a macroscopic system measures the average kinetic energy of molecules, but the “temperature” in our case do not have this kind of meaning. This explains why the hadron system IFS can have temperature much higher than the critical temperature ($\sim 160\text{--}170\text{ MeV}$) of deconfining phase transition, as shown in Ref. [11].

In conclusion, we have studied the scaling and additivity properties of Rényi entropy H_2 for IFS and current jet in the instanton-induced process of deep inelastic scattering produced from the QCDINS Monte Carlo generator identified by the r -sorting method.

The Rényi entropy H_2 for IFS is found to obey the scaling law, Eq. (5), for sufficiently fine-grained phase space region and the additivity property, Eq. (6), holds in narrow phase space window at different positions, especially in adjacent intervals. These results provide strong indication that the IFS has reached local equilibrium.

In order to get a better understanding about the equilibrium of IFS, further investigation along this line is suggested, *e.g.* to study the entropy properties in 3 dimensional momentum space, transforming the variables to the corresponding cumulant ones [13], or to use other characteristics of equilibrium, *e.g.* $|K_l(E, n)|^{2/l} \ll K_2(E, n), l = 3, 4, \dots$, [25–27] which guarantees the smallness of the higher-order energy correlations $K_l(E, n) = \langle \prod_{k=1}^l (\epsilon_k - \langle \epsilon \rangle) \rangle$ with ϵ_k being the energy of the k th particle.

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深度非弹性散射中瞬子末态的熵分析及热平衡性质*

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摘要 在QCDINS 产生的瞬子参与的ep深度非弹性散射事件中用r 排序法得到的瞬子末态和喷注样本, 研究了Renyi 熵在快度空间中的标度性和相加性。在瞬子末态中观察到了Renyi 熵随着相空间尺度的减小展现出来的渐进标度性, 而喷注展现出饱和趋势。在瞬子末态中还观察到在不同快度位置的窄窗口中Renyi 熵的相加性是很好地成立的。这些现象表明在瞬子参与的深度非弹性散射子中的瞬子末态达到了局域热平衡。

关键词 瞬子 夸克胶子融和 深度非弹性散射 熵分析 热平衡性质

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