Resonant diffraction radiation from inclined gratings and bunch length measurements

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Abstract

A simple scheme for the measurement of sub-mm bunch lengths using coherent resonant diffraction radiation (CRDR) from a tilted grating is proposed. The CRDR spectral-angular characteristics have been calculated using an adapted Kirchhoff model, taking into account the pre-wave zone effect. It is shown that the latter leads to a distortion of the CRDR monochromaticity. Choosing the appropriate distance between grating and detector such that the pre-wave zone effect becomes negligible, it is possible to measure the CRDR yield in the sub-THz range by a broadband detector. While changing the grating inclination angle with respect to the beam axis, the CRDR line is shifted and it is possible to obtain information about the bunch length, measuring the signal ratio from two detectors located at fixed observation angles instead of complicated spectral measurements which rely on absolute values of the intensity.

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1. Introduction

There exists considerable interest in studying the properties of coherent Smith–Purcell radiation (CSPR) because of the potential possibility to use such a type of radiation from relativistic electrons for “non-invasive” bunch length diagnostics. In a recent work CSPR was applied for this purpose by measuring the radiated intensity as a function of the observation angle [1]. An array of 11 detectors covering the angular range from $\Theta = 40^\circ$ to 140° above the grating surface was used to detect the coherent threshold in this experiment.

Radiation with wavelengths $\lambda > \sigma$ (σ the bunch length) is emitted coherently, resulting in an emission intensity which scales proportional to N_e^2 (N_e is the number of particles per bunch). For wavelengths $\lambda \leq \sigma$ the radiation intensity scale is proportional to N_e only.

Due to the Smith–Purcell (SP) dispersion relation the radiation wavelength is defined by the grating period and the photon outgoing angle (see Section 2). Therefore, an investigation of the SP radiation yield dependence on the polar angle enables one to observe the intermediate zone where the transition from incoherent to coherent SP radiation takes place, and as a result to extract a bunch length. This approach was already used in the experiments described in [1–3].

In this work, we propose to use an inclined grating with one or two detectors placed at fixed positions (observation angles) instead of a detector array as in [1] or a rotating detector as in [2,3]. In this case, in principle it is possible to obtain the same information. In contrast to CSPR which is emitted from a grating whose surface is oriented parallel to the beam axis ($\Theta_0 = 0$, see Fig. 1), we name the effect coherent resonant diffraction radiation (CRDR) when the grating surface is tilted throughout this paper.

The organization of the paper is as follows. In the next section the theory of resonant diffraction radiation is

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Fig. 1. Generation of resonant diffraction radiation. The grating with period d consists of N identical elements of strip width a and its surface is inclined at an angle Θ_0 with respect to the beam trajectory.

outlined. In the case of long wavelength emission (as it is typically the case for the condition $\lambda > \sigma$) the pre-wave zone effect may influence the radiation characteristics significantly. Therefore, in Section 3 the RDR theory is extended to radiation emission in the pre-wave zone. Based on this model calculations are shown in Section 4, leading to a new scheme for a bunch length diagnostics which is proposed in Section 5. Finally, the paper ends with a short summary and an outlook.

2. Resonant diffraction radiation from an inclined grating in the far-field

According to [4] the Smith–Purcell radiation characteristics from a particle moving along an inclined grating are considered. If the grating is made from conducting strips separated by vacuum gaps the simplest way to treat the radiation characteristics is based on the exact theory of diffraction radiation [5,6]. From this point of view, Smith–Purcell radiation from an inclined grating will be considered in the following as resonant diffraction radiation (RDR).

For the geometry presented in Fig. 1 the intensity is calculated by the formula [4]:

$$\frac{dW_{\text{RDR}}}{\hbar d\omega d\Omega} = \frac{dW_{\text{DR}}}{\hbar d\omega d\Omega} F_{\text{strip}} F_N, \quad (1)$$

where ω is the radiation frequency and $dW_{\text{DR}}/\hbar d\omega d\Omega$ is the intensity of diffraction radiation for a semi-infinite conducting plate. The factor F_{strip} which describes DR from a single strip is calculated as follows:

$$F_{\text{strip}} = 4(\sinh^2 \alpha_a + \sin^2 \phi_a), \quad (2)$$

$$\alpha_a = \frac{a\pi \sin \Theta_0}{\lambda \gamma} \sqrt{1 + \gamma^2 \Theta_x^2}, \quad (3)$$

$$\phi_a = \frac{a\pi [\cos(\Theta_y - \Theta_0) - \cos \Theta_0 / \beta]}{\lambda}. \quad (4)$$

with $\beta = v/c$ the normalized electron velocity, γ the Lorentz-factor, λ the wavelength of observation and a the strip width. The projection angle Θ_x is measured from the plane

perpendicular to the grating surface (see Fig. 1). For a grating with period d which consists of N identical elements, the interference factor F_N is written in the following form [4]:

$$F_N = \exp[-(N-1)\alpha_d] \left(\frac{\sin^2(N\phi_d/2) + \sinh^2(N\alpha_d/2)}{\sin^2(\phi_d/2) + \sinh^2(\alpha_d/2)} \right), \quad (5)$$

$$\phi_d = \frac{2\pi d [\cos(\Theta_y - \Theta_0) - \cos \Theta_0 / \beta]}{\lambda}, \quad (6)$$

$$\alpha_d = \left(\frac{2\pi d \sin \Theta_0}{\gamma \lambda} \right) \sqrt{1 + \gamma^2 \Theta_x^2}. \quad (7)$$

It is important to notice that the structure of Eq. (5) is identical to the one of a similar expression for resonant transition radiation from N layers, taking into account the radiation absorption in every layer [7].

First of all, a particular case will be considered which corresponds to the conventional Smith–Purcell geometry ($\Theta_0 = 0$). It is obvious that the decay factor α_d determined by Eq. (7) is equal to zero, therefore Eq. (5) can be rewritten in the well-known form:

$$F_N = \frac{\sin^2(N\phi_d/2)}{\sin^2(\phi_d/2)} \quad (8)$$

If $N \rightarrow \infty$, Eq. (8) transforms into an ordinary δ -function:

$$F_N = 2\pi N \delta(\phi_d - 2n\pi), \quad (9)$$

with n the diffraction order. The presence of the δ -function is an indication of the existence of monochromatic maxima in the RDR spectrum. However, the use of the δ -function for real gratings with limited number of periods is not always justified. Therefore, throughout this article the exact formulas (5) and (8) will be used.

The peak position in the DR spectrum is determined by the phase relation (resonance condition):

$$\phi_d = 2n\pi, \quad (10)$$

resulting in the well-known Smith–Purcell dispersion relation:

$$\lambda_n = \frac{d(\cos \Theta_y - 1/\beta)}{n}. \quad (11)$$

Now the case of the tilted grating is considered. It should be noted that quasi-monochromatic peaks in the DR spectrum can be observed only for small tilt angles Θ_0 of the grating. From Eq. (5) it is possible to obtain criteria for the generation of quasi-monochromatic radiation maxima with a finite full width even for the case $N \rightarrow \infty$ together with a continuous background:

$$\alpha_d N \ll 1 \text{ or } dN \sin \Theta_0 \ll \frac{\gamma \lambda}{2\pi}. \quad (12)$$

Under these conditions an expression for the RDR wavelength can be deduced:

$$\lambda_n = \frac{d[\cos(\Theta_y - \Theta_0) - \cos \Theta_0 / \beta]}{n}, \quad (13)$$

Fig. 2. Diffraction and transmission at a semi-transparent grating in analogy to classical optics.

It is clear that for a parallel grating ($\theta_0 = 0$) Eq. (13) coincides with the Smith–Purcell dispersion relation Eq. (11). Furthermore, it is possible to write the well-known optical relation for electromagnetic wave scattering [8] at an inclined grating, cf. Fig. 2:

$$\text{for a reflected wave: } \lambda_n = d/n(\cos(\theta_0 - \theta_y) - \cos \theta_0) \quad (14a)$$

$$\text{for a transmitted wave: } \lambda_n = d/n(\cos(\theta_0 + \theta_y) - \cos \theta_0) \quad (14b)$$

It is obvious that for the case $\beta \rightarrow 1$ Eqs. (13) and (14a) are identical.

The DR intensity for an inclined semi-infinite conducting target (inclination angle $\theta_0 \gg \gamma^{-1}$) and for a particle moving with the shortest distance to the target $h_1 = h - \frac{Nd \sin \theta_0}{2}$ (h is the impact-parameter taken from the middle of the grating, see Fig. 1) is calculated as follows [9]:

$$\begin{aligned} \frac{d^2 W_{\text{DR}}}{\hbar d\omega d\Omega} &= \frac{\alpha}{4\pi^2} \exp \left[-(\omega/\omega_c) \sqrt{1 + \gamma^2 \theta_x^2} \right] \\ &\quad \{ \gamma^{-2}(1 + \cos \theta_0) \times [1 - \cos(\theta_y - \theta_0)] + \\ &\quad + 2\theta_x^2 [1 - \cos \theta_0 \times \cos(\theta_y - \theta_0)] \} \\ &\quad [(\gamma^{-2} + \theta_x^2) \sin(\theta_y/2) \times \sin^2(\theta_0 - \theta_y/2)]^{-1}, \end{aligned} \quad (15)$$

with $\omega_c = \gamma c/2h_1$ the DR characteristic frequency and α is the fine structure constant. The Eq. (15) is valid for $\theta_x \ll 1, \theta_y \gg \gamma^{-1}$. The characteristics of coherent RDR can be calculated in full analogy to the one of coherent Smith–Purcell radiation under the assumption of a Gaussian distributed longitudinal bunch profile [10,11] as

$$\frac{d^2 W_{\text{CRDR}}}{\hbar d\omega d\Omega} \approx N_e \exp \left\{ -\frac{2\pi^2 \sigma^2}{\lambda^2} \right\} \frac{d^2 W_{\text{RDR}}}{\hbar d\omega d\Omega}, \quad (16)$$

with N_e the number of particles per bunch and σ the one-sigma bunch length. Eq. (16) is valid if the transverse beam size (perpendicular to the grating) is much smaller than the impact-parameter.

3. Resonant diffraction radiation in the pre-wave zone

In the work [12] the authors applied a vector Kirchhoff method for the problem of resonant diffraction radiation (Smith–Purcell radiation) in the far-field. In the recent work [13] the similar technique was used to demonstrate the presence of pre-wave zone effects for Smith–Purcell radiation as well (as it is a familiar effect for transition radiation). It was shown that the well-known monochromaticity law $\Delta\lambda/\lambda \sim 1/N$ is valid only for the far-field and the wave zone criterion for SPR can be written as [13,14]

$$r_0 \gg \frac{(Nd)^2}{\lambda_n} \sin^2 \theta, \quad (17)$$

where r_0 is a distance from “source” to an observation point. Although this criterion was derived for the case of parallel orientation ($\theta_0 = 0$) we can apply it for the resonant diffraction radiation, assuming small inclination angles ($\theta_0 \ll \pi/2$).

Thus, the far-field distance depends on the grating length Nd quadratically, and the SPR characteristics in the pre-wave zone do not correspond to the widely-studied radiation properties [15,16]. Hence, if SPR shall be applied for bunch length diagnostics it is important to take into account the finite distance between grating and detector, because pre-wave zone effects may play a significant role for distances which do not satisfy the criterion Eq. (17) and thus resulting in a distortion of the emission spectrum.

In the following, a model for resonant diffraction radiation will be deduced which takes into account possible pre-wave zone effects. Based on an analog of the vector Kirchhoff integral in standard diffraction theory (Eq. (10.87) in [8]):

$$\mathbf{E}^R(r_0, \omega) = \frac{1}{4\pi} \oint_S \mathbf{E}^R(r, \omega) (\mathbf{n} \nabla g) dS, \quad (18)$$

with $\mathbf{n}, g$ the unit vector normal for the grating surface and the semi-space Green’s function (that is why the second term in Jackson’s expression has vanished), it is possible to deduce an analytical expression for the radiation field $\mathbf{E}^R(r_0, \omega)$ at the observation point r_0 . Applying the boundary conditions for the tangential components of the total field which is the sum of the particle self field $\mathbf{E}^0(r, \omega)$ and the radiation field $\mathbf{E}^R(r, \omega) = \mathbf{E}(r, \omega) - \mathbf{E}^0(r, \omega)$, Eq. (18) can be rewritten:

$$E_{x,z}^R(r_0, \omega) = -\frac{1}{4\pi} \int E_{x,z}^0(r, \omega) (\mathbf{n} \nabla g) dS. \quad (19)$$

Here the integration is performed only along the grating surface with $\mathbf{r}$ the vector of integration, since the field $\mathbf{E}^R$ satisfies radiation conditions for the infinity. It should be noted that the basic formula of the following analysis (19) leads to the same results as for transition radiation according to the recently proposed approach in [17]. The expression for the particle field in the laboratory reference frame where the z' -axis is directed along the velocity direction is given for example in [8] (see Eq. (13.80) where the

numerical factor outside the brackets differs due to the symmetric Fourier transform):

$$\mathbf{E}^{0r}(\mathbf{r}', \omega) = \frac{e\omega}{\pi v^2 \gamma} \left(\frac{\rho'}{\rho'} K_1 \left[\frac{\omega \rho'}{v \gamma} \right] - \frac{i}{\gamma} \frac{\mathbf{v}}{v} K_0 \left[\frac{\omega \rho'}{v \gamma} \right] \right) e^{i\frac{\omega \mathbf{r}'}{v}},$$

$$\rho' = (x', y'), \quad (20)$$

$\mathbf{r}' = (\rho', z')$ is the radius vector and the functions K_0, K_1 represent modified Bessel functions of the second kind. As next step, the transformation is performed into the reference frame where the z -axis is directed along the wave vector, so the field has only two transverse components ($E_{z,k}^R = 0$). After simple transformations it is possible to derive the following expression:

$$E_{x,k}^R = \frac{E_x^R}{\sqrt{1 - (\sin \Theta \sin \Phi)^2}},$$

$$E_{y,k}^R = -E_z^R \frac{\sqrt{1 - (\sin \Theta \sin \Phi)^2}}{\sin \Theta \cos \Phi} + E_x^R \frac{\cos \Theta \tan \Phi}{\sqrt{1 - (\sin \Theta \sin \Phi)^2}}, \quad (21)$$

with Θ, Φ the ordinary polar and azimuthal angles which are connected to the “projection” angles by the following formulas: $\sin \Theta_x = \sin \Theta \sin \Phi, \sin(\Theta_y - \Theta_0) = \sin \Theta \cos \Phi / \sqrt{1 - (\sin \Theta \sin \Phi)^2}$. Here it was taken into account that the angle Θ_y is measured from the particle velocity direction, whereas the angles Θ, Φ do not depend on the grating inclination (see Fig. 1).

Finally, the particle field components (20) at the right-hand side of Eq. (19) have to be expressed in the grating reference frame. It is also performed using simple coordinate transformations. If the particle moves at an angle Θ_0 with respect to the grating surface, from Eqs. (19)–(21) it is straightforward to derive the following expressions:

$$E_{x,k}^R = -\frac{2ie}{\gamma \lambda^2 c} \frac{1}{r_0} \frac{\sin \Theta \cos \Phi}{\sqrt{1 - (\sin \Theta \sin \Phi)^2}} \int \frac{x}{\rho'} K_1 \left[\frac{2\pi}{\gamma \lambda} \rho' \right] e^{i\varphi} dx dz,$$

$$E_{y,k}^R = -\frac{2ie}{\gamma \lambda^2 c} \frac{1}{r_0} \sqrt{1 - (\sin \Theta \sin \Phi)^2} \times \int \left[\frac{1}{\rho'} K_1 \left[\frac{2\pi}{\gamma \lambda} \rho' \right] \left(x \frac{\sin \Theta \cos \Theta \sin \Phi}{1 - (\sin \Theta \sin \Phi)^2} + \sin \Theta_0 (h \cos \Theta_0 - 2z \sin \Theta_0) \right) + \frac{i}{\gamma} K_0 \left[\frac{2\pi}{\gamma \lambda} \rho' \right] \cos \Theta_0 \right] e^{i\varphi} dx dz. \quad (22)$$

Here, the phase shift can be expressed as $\varphi = k(|r - r_0| + \beta^{-1} (z \cos 2\Theta_0 \cos^{-1} \Theta_0 + h \sin \Theta_0))$ and $\rho' = \sqrt{x^2 + (h \cos \Theta_0 - 2z \sin \Theta_0)^2}$. The integrals in Eq. (22) represent the sum of the integrals along the surface of each strip of the grating. For reasons of simplicity these integrals will be evaluated numerically. These results will not be

compared to the ones of Section 2 for the far-field because the interest is only in the monochromaticity distortion due to the pre-wave zone effect. An article devoted to the detailed comparison of the method developed here with previously known results is under preparation. Using Eq. (22) it is possible to calculate the spectral-angular radiation density for a specific observation geometry (i.e. for a distance $\gamma \lambda \ll r_0 < (Nd)^2 \sin^2 \Theta \cos^2 \Theta_0 / \lambda_n$ and angles Θ, Φ) as follows:

$$\frac{d^2 W}{\hbar d\omega d\Omega} = cr_0^2 |\mathbf{E}_k^R|^2 = cr_0^2 (|E_{x,k}^R|^2 + |E_{y,k}^R|^2). \quad (23)$$

It should be noted that at large distances to the observer, the spherical wave front is almost flat within a small region. Thus, the angular distribution obtained for the pre-wave zone can be considered as the coordinate distribution onto the surface of a small flat detector (so called “dotted detector” according to [18]).

4. Calculation of RDR properties

Due to the change of the effective grating period, in the case of an inclined grating it is possible to observe a shift of the CRDR line. Based on the model derived in the previous section, the line shift and its broadening can be estimated for the situation when the detector is disposed in the pre-wave zone, i.e. it is possible now to overcome the restriction to the so-called far-field zone case where the source of radiation is considered as point-like. Measurements in the pre-wave zone are often the case for real observation geometries, especially for ultra-relativistic beams and mm wavelength regions of radiation. In other words, the finite distance between detector and grating as well as the finite grating length can be taken into account with the model developed in the previous paragraph.

As first step, the evolution of the RDR spectral lines is investigated with decreasing distance between grating and detector. Calculations have been carried out for two grating inclination angles $\Theta_0 = 0^\circ, 5^\circ$ and for a single electron with an energy of 100 MeV (i.e. $\gamma = 200$) which pass the grating at the distance (or impact-parameter, measured from the grating's midpoint) of $h = 2$ mm. The grating is assumed to consist of $N = 30$ identical strips with period length $d = 1.2$ mm and strip width $a = d/2$. The detector is located at the fixed observation angle $\Theta = 90^\circ$ (as detector no. 1 in Fig. 3) and the distance between the grating and detector is varied. The results are shown in Figs. 4–6. Due to the relative small number of grating periods ($N = 30$) there are some “satellite” peaks, and part of them will be enhanced by the bunch form-factor which is also shown in Fig. 4. It is clear that for the case $\lambda \geq \sigma$ the radiation intensity increases by about N_e times while the short wavelength part of the spectrum is strongly suppressed.

From Figs. 4–6, it is concluded that for the geometry assumed in the calculation, at the distance of 0.5 m and larger the spectral line distortion due to the pre-wave zone effect is negligible. In other words, the FWHM of the

Fig. 3. Experimental setup proposed for the investigation of resonant diffraction radiation and the application as bunch length monitor. The grating is tilted in its center by Θ_0 versus the beam axis. Two detectors located at fixed observation angles measure the radiation yield as a function of Θ_0 .

Fig. 4. Spectral line shape of RDR for two different inclination angles $\Theta_0 = 0^\circ, 5^\circ$ and a distance of $r_0 = 1$ m between grating and detector. The additional parameters for the calculation are as follows: $\gamma = 200$, $d = 1.2$ mm, $N = 30$, $h = 2$ mm, $\Theta = 90^\circ$, $\Phi = 0$.

Fig. 5. Spectral line shape of RDR for the same parameters as in Fig. 4 but for a distance of $r_0 = 0.5$ m between grating and detector.

Fig. 6. Spectral line shape of RDR for the same parameters as in Figs. 4, 5 but for a distance of $r_0 = 0.25$ m between grating and detector. The distortion of the monochromaticity is clearly visible due to the pre-wave zone influence.

spectral line differs from the far-field law $\sim 1/N$ not more than 10% [4]. An evolution of the radiation line width with changing the distance r_0 for two values of the tilt angle Θ_0 is presented in Fig. 7.

As next step, calculations have been carried out with the same grating parameters as before but for various grating inclination angles Θ_0 with the detector located at a fixed distance $r_0 = 1$ m. Instead of a single electron now a Gaussian shaped longitudinal electron bunch is assumed with width $\sigma = 0.44$ mm and $N_e = 10^9$ particles. These electron beam parameters correspond to the ones of the injector linac of the Swiss Light Source SLS [19].

With increasing inclination angle Θ_0 the peak wavelength measured with both detectors indicated in Fig. 3 is shifted, as can be seen in Figs. 8 and 9. This is due to the change of the effective grating period, and the line is shifted to the “hard” part of the spectrum for the first detector (Fig. 8) and in the opposite direction for the second one (Fig. 9). For an inclination angle of $\Theta_0 = 5^\circ$ the peak wavelength changes by about 10%, and the intensity in both detectors varies due to the additional contribution from the bunch form-factor, cf. Fig. 4.

Fig. 7. Dependence of the spectral line width as a function of distance between the grating and detector.

Fig. 8. Spectral line shape of CRDR with grating inclination angle Θ_0 as parameter, as seen by detector D1.

Fig. 9. Spectral line shape of CRDR with grating inclination angle Θ_0 as parameter, as seen by detector D2.

Assuming the grating oriented parallel to the beam axis ($\Theta_0 = 0^\circ$) and the previous mentioned parameters, the CRDR yield for the line with mean wavelength of $\lambda = 1.2$ mm is calculated to be

$$\frac{dW_{\text{CRDR}}}{d\Omega} \sim 3 \cdot 10^{13} \frac{\text{eV}}{\text{sr} \times \text{bunch}}.$$

For a detector with entrance aperture $S_{\text{det}} = 1 \text{ cm}^2$ which is placed at a distance $r_0 = 100 \text{ cm}$ from the grating the detected intensity level of CRDR is

$$\Delta W_{\text{CRDR}} \sim 0.5 \text{ nJ/bunch}.$$

The power level is estimated as $P_{\text{CRDR}} \sim \Delta W_{\text{CRDR}}/\tau_B$. Here, τ_B indicates the bunch duration which is assumed to be $\tau_B \sim 2.36\sigma/c \sim 3.5 \text{ ps}$. With these values the power may achieve a level of $P_{\text{CRDR}} \sim 140 \text{ W}$. Such a high power level even allows the use of broadband room-temperature detectors.

In Fig. 10, the dependency of the measured intensity ΔW_{CRDR} on the grating tilt angle Θ_0 is shown for both detectors. As can be seen, the intensity in the first detector D1 decreases with increasing tilt angle. This can be understood by the fact that a counter-clockwise tilt of the grating as indicated in Fig. 3 leads to a line shift to smaller wavelengths (c.f. Fig. 8) and as result to decreasing of CRDR yield due to reduced contribution from the bunch form factor as it is shown in Fig. 4. From this it is obvious that the signal from detector D2 shows opposite behavior in accordance with Eqs. (14a), (14b).

5. Application for bunch length diagnostics

The spectral decomposition of the emitted radiation at the grating enables the use of CRDR as a tool for bunch length diagnostics similar to conventional Smith–Purcell radiation. As explanation the basic relation of coherent radiation diagnostics connecting the radiation spectrum of a bunch of electrons $dW_{\text{CRDR}}/\hbar d\omega$ to that of a single electron $dW_{\text{RDR}}/\hbar d\omega$ is recalled:

$$\frac{dW_{\text{CRDR}}}{\hbar d\omega} = \frac{dW_{\text{RDR}}}{\hbar d\omega} (N_e + (N_e - 1)) |F(\omega)|^2 \quad (24)$$

Fig. 10. CRDR yield as function of the inclination angle Θ_0 for detector D1 (left) and D2 (right), assuming both detectors with aperture $S = 1 \text{ cm}^2$ and located at a distance $r_0 = 100 \text{ cm}$ from the grating.

Here, N_e is the number of particles per bunch and the bunch form factor $F(\omega)$ is determined by the Fourier transform of the longitudinal bunch distribution

$$F(\omega) = \int_{-\infty}^{\infty} dz S(z) e^{i\omega z} \quad (25)$$

From a measurement of the spectral radiation content it is therefore possible to reconstruct the bunch shape according to Eq. (25) by an inverse Fourier transform and additionally to extract the bunch length. For more details see for example [20] and the references therein. The main difference in coherent radiation diagnostics techniques is therefore the way how the spectral contents are resolved.

In the case of CSPR based diagnostics used in [1] a detector array of 11 individual detectors above the grating was used to measure the radiation yield at distinct wavelengths, exploiting the relation between observation angle and wavelength according to the Smith–Purcell dispersion relation Eq. (11). In the case of CRDR diagnostics the fundamental relation is given by Eq. (13) which connects the grating tilt angle Θ_0 and λ , i.e. the radiation yield can be measured with only one detector located at a fixed observation angle ($\Theta = +90^\circ$ in this case) while the grating orientation is varied.

From experimental point of view, such a diagnostics scheme has the advantage to avoid a complicated and expensive setup because only one detector (for example D1, see Fig. 3) is required. Furthermore, a measurement with several detectors has the disadvantage that the sensitivity of each individual detector must be known with high accuracy because the reconstruction of the bunch shape is based on the determination of absolute intensities, which is a complicated task at longer wavelengths. Finally, the CSPR based measurement with several detectors has the drawback that only a limited number of wavelengths are accessible. The number of wavelengths is given by the number of detectors which is limited by the space above the grating and the angular extension of each detector element. Therefore, a precise reconstruction of the bunch shape by inverse Fourier transform according to Eq. (25) is not possible with a CSPR based monitor as described in [1]. In contrast to that the number of wavelengths for the CRDR diagnostics is limited by the number of grating inclination angles Θ_0 which is in principle determined by the accuracy of the rotary measuring stage used for the alignment of the grating surface versus the beam axis. This number is typically much larger and allows in principle a full bunch shape reconstruction according to Eq. (25). Nevertheless, the CSPR based bunch shape measurement has the advantage of single-shot capability which is not possible for the proposed CRDR diagnostics scheme.

In order to increase the accuracy a second detector D2 can be located at a different observation angle, c.f. Fig. 3 ($\Theta = -90^\circ$ in this case). In this diagnostics scheme the information of both individual detectors is available with only one scan of the grating inclination angle. Furthermore, under abandonment of the need for bunch recon-

Fig. 11. Dependence of the signal ratio from detector D1 and D2 on the inclination angle for different bunch lengths σ .

struction and restricting to the case of bunch length measurement the information of both detectors can be combined by taking the intensity ratio R of the signals from detectors D1 and D2. The advantage of such a diagnostics scheme is that one has not to rely on absolute values of the radiation yield, avoiding the need to know the individual sensitivity of each detector to a high level of accuracy.

Fig. 11 presents the signal ratio R for different bunch lengths as function of the grating inclination angle Θ_0 , assuming a Gaussian shaped longitudinal bunch distribution. As can be seen the functional dependency of R on Θ_0 strongly depends on the bunch length, i.e. such a type of measurement can easily be applied as basic scheme for a simple bunch length diagnostics monitor.

6. Summary and outlook

The basic properties of RDR from an inclined grating were presented. It was shown that the pre-wave zone effect affects the radiation emission and leads to a strong monochromatic distortion. For possible applications a measurement should take place in the far-field zone providing the known dependency for the monochromaticity $\sim 1/N$, therefore a proper choice of the distance between grating and detector is essential.

Furthermore, it was demonstrated that a variation of the grating inclination angle Θ_0 leads to a wavelength shift of the emitted radiation at a fixed observation point, i.e. the wavelength can easily be tuned by changing the grating orientation with respect to the beam axis. As an example an inclination angle of $\sim 15^\circ$ provides a shift in the CRDR wavelength by about 30%. This shift can be used to measure in the vicinity of the coherent threshold and to investigate the transition from incoherent to coherent radiation.

The intensity of CRDR is sufficiently high for the parameter set under investigation here so that conventional room-temperature and broadband THz detectors can be used. For the example above with an inclination angle of $\sim 15^\circ$ the CRDR yield will be changed by about 40% in contrast to the yield from a parallel oriented grating. These

radiation properties are well suited for the application as an accurate and non-invasive bunch shape diagnostics. While a single-shot bunch profile measurement is not possible with the principle of scanning the grating inclination angle, the high number of available wavelengths in the scanning process offers the possibility for a bunch reconstruction from the inverse Fourier transform of the bunch form factor.

Moreover, if the longitudinal bunch shape is known and the determination of a bunch length is sufficient, for typical bunch profiles as for example a Gaussian one Eq. (25) can be carried out analytically and a measurement at a single wavelength would be sufficient for the extraction of a bunch length. In this case, this monitor would have even a single-shot capability for bunch length determination.

If the detection scheme is extended to the use of two identical detectors located at fixed observation angles, the information of both of them can be combined by taking the signal ratio. This has the advantage that such a diagnostics scheme does not rely on absolute values of the radiation yield, avoiding the need to know the individual sensitivity of each detector to a high level of accuracy.

As a conclusion, one may state that the use of CRDR diagnostics offers the possibility for an easy and compact bunch length monitor with single-shot capability and with the advantage of not relying on absolute intensities. Therefore, in the nearest future a proof-of-principle experiment is planned in order to verify the radiation properties and to investigate the sensitivity limits of such a kind of monitor.

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