Collective Flavor transitions of supernova neutrinos

- Setup: Equations of Motion
- Synchronized and bipolar oscillations
- Relevance for Core Collapse Supernovae
- Decoherence of oscillations between different angular and energy modes
- Three-flavor effects
- Open Questions and Conclusions

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Motivation from Observed Neutrino Mixing

Three-flavor oscillations characterized by six parameters, two squared mass differences, three mixing angles, and one CP violating phase δ . For current experimental situation see, e.g., [Schwetz, Tórtola, Valle, arXiv:0808.2016](#):

$$\begin{aligned}\Delta m_{\text{solar}}^2 &= m_2^2 - m_1^2 = 7.65_{-0.20}^{+0.23} \times 10^{-5} \text{ eV}^2; \quad \sin^2 \theta_{12} = 0.304_{-0.016}^{+0.022} \\ \Delta m_{\text{atm}}^2 &= |m_3^2 - m_2^2| = 2.40_{-0.11}^{+0.12} \times 10^{-3} \text{ eV}^2; \quad \sin^2 \theta_{23} = 0.50_{-0.06}^{+0.07} \\ \sin^2 \theta_{13} &\leq 0.04 \quad (2\sigma)\end{aligned}$$

Since most “action” in collective effects occur for inverted hierarchy, and solar hierarchy is known to be normal, in a two-flavor description,

Δm_{atm}^2 and θ_{13} is most relevant.

The frequencies of vacuum oscillations, matter effects, and self-interactions can be parametrized by

$$\omega = \frac{|\Delta m^2|}{2E}; \quad \lambda_r = \sqrt{2} G_F n_e(r); \quad \mu_r = \sqrt{2} G_F n_{\bar{\nu}_e} \langle 1 - \cos \theta \rangle, \quad ,$$

with E the neutrino energy, $n_e(r)$ the density of electrons minus positrons, $n_{\bar{\nu}_e}$ the electron-anti-neutrino density, and θ the angle between modes.

Equations of Motion

Defining the neutrino density matrices

$$(\rho_p)_{ij} = \langle a_i^+ a_j \rangle_p; \quad (\bar{\rho}_p)_{ij} = \langle \bar{a}_i^+ \bar{a}_j \rangle_p$$

the oscillation frequencies in the mass basis $\Omega_p = U \text{diag}(m_1^2, m_2^2, m_3^2) U^\dagger / (2p)$ with $p = |\mathbf{p}|$, U the vacuum mixing matrix, $|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$, and the charged lepton densities, $L = \text{diag}(n_e, n_\mu, n_\tau)$ and an analogous $\bar{L}$ for the anti-lepton densities, all in the weak interaction basis, the general equations of motion are

$$\begin{aligned} \partial_t \rho_p &= -i \left[\Omega_p + \sqrt{2} G_F \left(L - \bar{L} + \int \frac{d^3 \mathbf{q}}{(2\pi)^3} (\rho_q - \bar{\rho}_q) (1 - \cos \theta_{pq}) \right), \rho_p \right] \\ \partial_t \bar{\rho}_p &= +i \left[\Omega_p - \sqrt{2} G_F \left(L - \bar{L} + \int \frac{d^3 \mathbf{q}}{(2\pi)^3} (\rho_q - \bar{\rho}_q) (1 - \cos \theta_{pq}) \right), \bar{\rho}_p \right] \end{aligned}$$

vacuum

matter

Density term(isotropic)
self-maintained coherence

flux term
self-induced decoherence

self-interactions

Considering only a single two-flavor mode $\mathbf{p}$ one can expand $\rho = (f + \mathbf{P} \cdot \boldsymbol{\sigma})/2$, analogously for $\bar{\rho}$, $L - \bar{L} = (n_0 + n_e \mathbf{L} \cdot \boldsymbol{\sigma})/2$, and $\Omega = (\Omega_0 + |\Delta m^2|/(2p) \mathbf{B} \cdot \boldsymbol{\sigma})/2$. With the vacuum mixing angle θ , $\mathbf{B} = (\sin 2\theta, 0, -\cos 2\theta)$, one arrives at

$$\begin{aligned}\partial_t \mathbf{P} &= \left[+\omega \mathbf{B} + \lambda \mathbf{L} + \mu (\mathbf{P} - \bar{\mathbf{P}}) \right] \times \mathbf{P} \\ \partial_t \bar{\mathbf{P}} &= \left[-\omega \mathbf{B} + \lambda \mathbf{L} + \mu (\mathbf{P} - \bar{\mathbf{P}}) \right] \times \bar{\mathbf{P}}\end{aligned}$$

For a single mode define

$$\begin{aligned}\mathbf{Q} &= \mathbf{P} + \bar{\mathbf{P}} - \frac{\omega}{\mu} \mathbf{B} \approx \mathbf{P} + \bar{\mathbf{P}}; \quad \mathbf{q} = \mathbf{Q}/Q \\ \mathbf{D} &= \mathbf{P} - \bar{\mathbf{P}}\end{aligned}$$

In the absence of matter, $\lambda=0$, or, approximately for small vacuum mixing angle, the equations of motion are

$$\dot{\mathbf{Q}} = \mu \mathbf{D} \times \mathbf{Q}; \quad \dot{\mathbf{D}} = \omega \mathbf{B} \times \mathbf{Q}$$

This implies

$$|\mathbf{Q}| = \text{const} ; \sigma \equiv \mathbf{D} \cdot \mathbf{Q} = \text{const}$$

$$\mathbf{D} = \frac{\mathbf{q} \times \dot{\mathbf{q}}}{\mu} + \sigma \mathbf{q}$$

These are the equations for a spinning top of spin σ , angular momentum $\mathbf{q} \times \dot{\mathbf{q}} / \mu$ and moment of inertia $I = m\ell^2 = \mu^{-1}$. The potential energy is

$$E = E_{\text{pot}} + E_{\text{kin}} = \omega (\mathbf{B} \cdot \mathbf{Q} + Q) + \frac{\mu}{2} \mathbf{D}^2 = \omega (\mathbf{B} \cdot \mathbf{Q} + Q) + \frac{1}{2\mu} \dot{\mathbf{q}}^2 + \frac{\mu}{2} \sigma^2$$

potential

energy

kinetic

The general initial conditions are $\mathbf{P}=(0,0,1+\epsilon)$, $\bar{\mathbf{P}}=(0,0,1)$, where the asymmetry parameter in terms of the fluxes of different neutrino flavors is

$$\epsilon = \frac{F(\nu_e) - F(\nu_x)}{F(\bar{\nu}_e) - F(\bar{\nu}_x)} - 1 = \frac{F(\nu_e) - F(\bar{\nu}_e)}{F(\bar{\nu}_e) - F(\bar{\nu}_x)}.$$

The spinning top equations then exhibit the following three regimes:

Synchronized oscillations

Bipolar oscillations

Vacuum oscillations

Schematic example for the inverted hierarchy

Bipolar regime for inverted hierarchy

In the limit of small vacuum mixing angle $\tilde{\theta} \equiv \pi/2 - \theta$ and for constant μ and λ the time scale for bipolar conversion is

$$\tau_{\text{bipolar}} \approx -\kappa^{-1} \ln \left[\tilde{\theta} \frac{\kappa}{(\kappa^2 + \lambda^2)^{1/2}} \right].$$

The main effect of the matter term is thus to decrease the effective mixing angle to $\tilde{\theta} \kappa / (\kappa^2 + \lambda^2)^{1/2}$ and thus delay the onset of bipolar oscillations.

For varying neutrino density and in the limit of small vacuum mixing angle, angular momentum and adiabatic energy conservation in the bipolar regime gives

$$P(\nu_e \rightarrow \nu_e) = \frac{1}{2} (1 + P_z) \propto \mu^{1/2}$$

for the electron neutrino survival probability.

For smaller mixing angle it takes longer to "tip over"
 $\Rightarrow$ bipolar transition less adiabatic
 $\Rightarrow$ Larger nutation amplitude

Matter decreases effective mixing angle:
 Later onset of bipolar transition
 and larger nutation amplitude

Bipolar conversion in inverted hierarchy is essentially pair conversion
 $\nu_e \bar{\nu}_e \rightarrow \nu_x \bar{\nu}_x$ from the second highest state m_1 to the lowest state m_3 ,
 separated by the atmospheric splitting Δm_{atm}^2 :

Experimentally still allowed hierarchies

ν_μ', ν_τ' are the mass eigenstates in the $\bar{\nu}$ - $\bar{\nu}$ sector

Normal atmospheric hierarchy:
System already in lowest state
 $\Rightarrow$ not much happens

Inverted atmospheric hierarchy:
System in second-highest state
 $\Rightarrow$ pendulum will fall

Note: In inverted hierarchy, bipolar oscillations occur for arbitrarily small Θ_{13} .

This can be used as experimental test by observing a supernova with mega-ton detectors that are mostly sensitive to electron-antineutrinos:

Following Dasgupta, Dighe, Mirizzi,, arXiv:0802.1481, see afternoon talk
relate fluxes $F(\bar{\nu}_e)$ at detector to fluxes $F_{\nu}(\bar{\nu}_e)$ at neutrino-sphere:

No swapping of $\bar{\nu}_e$ and $\bar{\nu}_{\tau}'$ if either normal hierarchy or inverted hierarchy with $\sin^2 \theta_{13} \geq 10^{-2}$ (such that bipolar transition followed by adiabatic MSW):

$$F(\bar{\nu}_e) \approx \cos^2 \theta_{13} F_{\nu}(\bar{\nu}_e) + \sin^2 \theta_{13} F_{\nu}(\bar{\nu}_{\mu}')$$

Swapping of $\bar{\nu}_e$ and $\bar{\nu}_{\tau}'$ if inverted hierarchy with $\sin^2 \theta_{13} \leq 10^{-2}$ (such that bipolar transition followed by non-adiabatic MSW) with $F_{\nu}(\bar{\nu}_{\tau}') = F_{\nu}(\bar{\nu}_{\mu}')$:

$$F(\bar{\nu}_e) \approx \cos^2 \theta_{13} F_{\nu}(\bar{\nu}_{\tau}') + \sin^2 \theta_{13} F_{\nu}(\bar{\nu}_{\mu}') = F_{\nu}(\bar{\nu}_{\tau}')$$

For Earth matter effects $\cos^2 \theta_{13} \rightarrow P(\bar{\nu}_{\mu}' \rightarrow \bar{\nu}_e)$

Placing a detector behind and in front of the Earth then results in the flux difference

$$\Delta F(\bar{\nu}_e) = \left[P(\bar{\nu}_\mu' \rightarrow \bar{\nu}_e) - \cos^2 \theta_{12} \right] \left[F_\odot(\bar{\nu}_e) - F_\odot(\bar{\nu}_\mu') \right]$$

for scenario 1 and

$$\Delta F(\bar{\nu}_e) = \cdot$$

for scenario 2.

This is relatively insensitive to source physics and may be distinguishable experimentally:

Signature for inverted hierarchy and $\sin^2 \Theta_{13} < 10^{-5}$

The Role of Energy and Angular Modes

The momentum modes $\mathbf{p}$ essentially consist of energy and angular modes. For axisymmetry one has the radius r as integration variable

$$\mathbf{p} \rightarrow (E \approx |\mathbf{p}|, u = \sin^2 \theta_R); \quad \rho_{\mathbf{p}} \rightarrow \rho_{E,u}$$

where θ_R is the emission angle at the neutrino sphere, $r=R$.

Energy bins $\rightarrow$ spectral splits

angular bins $\rightarrow$ possible decoherence

We define the flux matrices

$$J_{E,u,r} = \frac{E^2 \rho_{E,u,r}}{4\pi (4\pi)}$$

With the radial velocity

$$v_{u,r} = v_{p,r} = \cos \theta_r = \sqrt{1 - u(R/r)}$$

one has the integral flux and number density matrices

$$J_r = \frac{r^\gamma}{R^\gamma} \int \frac{d^\gamma \mathbf{p}}{(\gamma \pi)^\gamma} \rho_p v_{p,r} = \int_\gamma du \int_\gamma^\infty dE J_{E,u,r},$$

$$N_r = \int_\gamma du \int_\gamma^\infty dE \frac{J_{E,u,r}}{v_{u,r}}$$

For an average matter velocity v_e one then has the general equations of motion:

$$i \partial_r J_{E,u,r} = \left[\frac{\Omega_E + \lambda_r L(\gamma - v_{u,r} v_e)}{v_{u,r}} + \sqrt{\gamma} G_F \frac{R^\gamma}{r^\gamma} \left(\frac{N_r - \bar{N}_r}{v_{u,r}} - (J_r - \bar{J}_r) \right), J_{E,u,r} \right];$$

analog for antineutrinos with $\Omega_E \rightarrow -\Omega_E$.

Integrating and writing $J_r - \bar{J}_r = [F(\nu) - F(\bar{\nu}) + \mathbf{D} \cdot \boldsymbol{\sigma}] / \gamma$ in the limit of small effective mixing angle one gets

$$\partial_r \mathbf{D} = \mathbf{e}_z \times \mathbf{M}$$

with $\mathbf{M}$ an (irrelevant) integral of $J_{E,u,r}$.

Thus $[J_r - \bar{J}_r]_{\gamma\gamma} - [J_r - \bar{J}_r]_{\gamma\gamma} = F(\nu_e) - F(\bar{\nu}_e) - [F(\nu_x) - F(\bar{\nu}_x)] = \text{const.}$

Since total lepton number is conserved, electron and x-lepton number are conserved separately.

Beyond the Bipolar Range

Energy Modes and Spectral Splits

The spectral split is governed by lepton number conservation for both flavors separately:

Anti-neutrinos swap completely; to compensate, neutrinos can only swap partially because

$$\left| F(\nu_x) - F(\nu_e) \right| > \left| F(\bar{\nu}_x) - F(\bar{\nu}_e) \right|$$

Angular Modes and Decoherence

Decoherence can occur either from **the flux term in the self-interactions (self-induced decoherence)** or **kinematically in the matter terms**:

$$i \partial_r J_{E,u,r} = \left[\frac{\Omega_E + \lambda_r L(1 - v_{u,r} v_e)}{v_{u,r}} + \sqrt{\gamma} G_F \frac{R^\gamma}{r^\gamma} \left(\frac{N_r + \bar{N}_r}{v_{u,r}} - (J_r - \bar{J}_r) \right), J_{E,u,r} \right]$$

The matter term can be transformed away up to the u -dependence of $v_{u,r}$

Duan, Fuller, Qian, PRD 74 (2006) 123004; Hannestad, Raffelt, Sigl, Wong, PRD 74 (2006) 105010

This u -dependence can cause decoherence of the angular modes.

See afternoon talk by Esteban-Pretel

Self-induced decoherence from self-interaction flux terms

Final location of 500 anti-neutrino polarization vectors for inverted hierarchy

$\Lambda=0$, no matter

When ε is small, self-induced decoherence takes place even in the normal hierarchy.

The critical asymmetry ε below which decoherence occurs depends on Θ , ω , and μ :

Practically, in type II supernovae, ε is likely large enough to avoid decoherence.

Angular Decoherence from matter effects

The matter-induced multi-angle effect becomes important when

$$n_{e^-} - n_{e^+} \gg n_{\bar{\nu}_e}$$

Three Flavor Oscillations and the μ - τ Potential

The energy shift between muon and tau neutrinos is described by

$$\Delta V_{\mu\tau} = \sqrt{2} G_F Y_\tau^{\text{eff}} n_B$$

$$Y_\tau^{\text{eff}} = \frac{\sqrt{2} G_F m_\tau}{(\sqrt{2} \pi)^2} \left[\ln \left(\frac{m_W}{m_\tau} \right) - 1 + \frac{Y_n}{\sqrt{2}} \right] = 2.7 \times 10^{-6}$$

This would lead to a MSW resonance at density $\rho = 3 \times 10^7 \text{ g cm}^{-3}$ corresponding to $\lambda \sim 10^4 \text{ km}^{-1}$. Oscillations are then sensitive to Θ_{23} and these effects can be relevant in the accretion phase.

However, at such high densities the matter decoherence effects are probably strong enough to make this effect of little relevance.

General Case: Three Flavors and μ - τ Potential

General Case: Three Flavors and μ - τ Potential

Energy modes only

Open Questions and Conclusions

Neutrino self-coupling can play a major role in the oscillations of neutrinos in type II supernovae

The one-mode approximation is meanwhile well understood and can be thought of as a spinning top

Asymmetry between neutrinos and anti-neutrinos leads to spectral splits in the context of energy modes and, if sufficiently large, prevents self-induced decoherence of angular modes

Matter effects can also lead to decoherence if charged lepton density is larger than neutrino density. This probably happens before $\mu\tau$ mixing can occur.

There are still **unresolved issues** in collective neutrino oscillations., including

- Absence of axisymmetry, non-symmetric case
- Damping due to matter inhomogeneities
- ...