Recent results on unintegrated parton distributions

F. Hautmann^{1,2} and H. Jung³

¹ Oxford University, Department of Theoretical Physics, Oxford OX1 3NP

² LAPTH, F-74941 Annecy-le-Vieux

³ Deutsches Elektronen Synchrotron, D-22603 Hamburg

Abstract

We summarize recent results on i) the use of u-pdf's in the Monte-Carlo simulation of hadronic final states at high-energy colliders, and ii) attempts to characterize u-pdf's with precision in terms of nonlocal operator matrix elements.

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1 Introduction

At forthcoming high-energy colliders a large amount of events are characterized by complex final states with *multiple* hard scales, possibly far apart from each other. QCD methods to address multi-scale hadronic processes involve the use of parton distributions unintegrated in both longitudinal and transverse components of parton's momentum (u-pdf's). Classic examples are provided by Sudakov physics [1, 2] and small-x physics [3, 4]. U-pdf's also govern exclusive processes [5] and spin [6]. They are required, in general, to explore detailed features of hadronic final states [7]. For these reasons, theoretical and phenomenological studies of u-pdf's are being actively pursued.

In the case of small x, u-pdf's can be introduced in a gauge-invariant manner using high-energy factorization [8]. This result was used early on both for Monte-Carlo simulations [9] of $x \rightarrow 0$ parton showers and for numerical resummation programs [10] for $\ln x$ corrections to QCD evolution equations. For structure function's evolution, methods are being developed ([11]-[15] and references therein) to match the $k_\perp$ -dependent, small-x dynamics with perturbative collinear dynamics. For the full simulation of exclu-

sive components of hadronic final states, on the other hand, such matching is more complex (see discussions in [7] and [16]) and is not yet available. This will be critical for turning present event generators based on u-pdf's [17]-[23] into general-purpose Monte-Carlo tools. Importantly, for $x \rightarrow 0$ u-pdf's may provide a useful framework also for discussing the approach to the saturation regime: see e.g. [24, 25] for recent studies.

In the general case, to characterize u-pdf's gauge-invariantly over the whole phase space is more difficult. Open questions on this issue are the subject of much current activity, see e.g. [26]-[40], with wide-ranging applications from semi-inclusive processes to parton-shower algorithms to spin physics.

In this report we concentrate on two topics. In Sec. 2 we consider $x \ll 1$ and discuss recent results from $k_\perp$ Monte-Carlos on the jet structure of small-x final states. In Sec. 3 we describe ongoing progress towards general operator definitions for u-pdf's, focusing on results for the treatment of $x \rightarrow 1$ endpoint divergences.

2 Jet final states from unintegrated parton distributions

There have recently been new calculations of hadronic jet final states from Monte-Carlo generators based on unintegrated parton distributions.

2.1 U-pdf's and shower Monte-Carlo generators

The idea common to the Monte-Carlo event generators based on u-pdf's is to use factor-

ization at fixed $k_{\perp}$ [8] in order to a) generate the hard scattering event, including dependence on the initial transverse momentum, and b) couple this to the evolution of the initial state to simulate the gluon cascade. Event generators of this kind include SMALLX [9], CASCADE [20], LDCMC [22], and the newer tools [17] and [18]. Different generators employ different models for the evolution of the initial state, such as BFKL [41, 42], CCFM [43, 44], LDC [45] evolution. With suitable constraints on the angular ordering of gluon emissions, these can all be set up to give the correct leading $\ln x$ behavior. A number of subleading contributions is however also of importance for realistic simulations of final states. The general approach of these Monte-Carlos and aspects of the differences among them are reviewed in [16]. See also [7] for further references.

Although none of the above generators are nearly as developed as standard Monte-Carlos like PYTHIA [46] or HERWIG [47], they have the potential advantage of a better treatment of high-energy logarithmic effects. Implementing these effects in the shower can be relevant for the simulation of complex final states at LHC energies, see e.g. multi-jet studies in [48], and additional references in [7]. A further potential advantage is that Monte-Carlos with u-pdf's likely provide a more natural framework [49] to simulate the $k_{\perp}$ distribution of the soft underlying event [50] (minijets, soft hadrons) and, possibly, multiple interactions at very high energies.

On the other hand, present Monte-Carlos of this kind do not automatically include contributions from collinear radiation associated to $x \sim 1$. They need to be corrected for this. This is done partially in present implementations, but not yet in a systematic fashion. A recent example of a study in this direction is in [17], based on the procedure of [51, 52].

A related issue concerns the inclusion of quark contributions in the initial state. In present implementations these are either ne-

glected or approximated by lowest-order perturbative evolution [17, 18, 22]. The $k_{\perp}$ -dependent kernel that governs small- x sea-quark evolution to all orders is given in [53]. Universality properties of this kernel are emphasized in [12, 54]. The quark kernel is not yet implemented in current Monte-Carlos.

Keeping in mind these limitations, Monte-Carlo event generators with u-pdf's can be used at present within the low- x domain to probe the physical picture of space-like $x \rightarrow 0$ parton showers. A selection of recent results from these event generators is given in the next subsections, going from more inclusive to less inclusive measurements.

2.2 Inclusive cross sections

Available $k_{\perp}$ Monte-Carlos have been used to analyze data for DIS structure functions and for inclusive lepto- and hadro-production of jets, heavy flavors and prompt photons, see [7, 16]. These inclusive analyses serve to test the overall consistency of the physical picture and to perform first determinations of the unintegrated gluon distribution.

A consistent semi-quantitative picture is achieved with sensible results for the evolved unintegrated gluon distribution [16, 19, 51, 55, 56, 57, 58]. However, the gluon at low scales and low x is only poorly constrained by this approach [58, 59]. This is not unexpected, as the summation of higher orders implied by the Monte-Carlo should reduce the sensitivity of predictions at high scales to the form of the input [60].

Examples of results for jet production in ep and $p\bar{p}$ collisions are reported in Figs. 1 and 2. Fig. 1 shows the description of the jet E_T distribution data from H1 [61] obtained with CASCADE using the recent u-pdf fits [62]. Fig. 2 shows the comparison of CDF jet data [63] with the Monte-Carlo [17].

Current uncertainties on the unintegrated gluon are large especially in the lowest x region [16]. Implications of this are studied for heavy-quark structure functions at

Figure 1: Jet E_T distribution at HERA from the $k_\perp$ Monte-Carlo CASCADE and u-pdf fits[62], compared with H1 data[61].

Figure 2: Tevatron jet spectra from[17] compared with CDF data[63].

HERA in [58] and for jet production at RHIC in [57]. Note that first fits of *both* the x - and $k_\perp$ -dependence of the unintegrated gluon are performed in [62].

A potentially important application of these results is to final states containing Higgs bosons at the LHC. The Higgs matrix element coupled to the unintegrated gluon distribution is computed in [64]. First results from Monte-Carlo implementation are given in [19]. Studies of the Higgs transverse-momentum spectrum from NLO + soft-gluon resummation [65, 66] indicate that small- x

terms, although highly subleading from the viewpoint of the soft hierarchy, appear to affect the spectrum at a level comparable to the current theoretical uncertainties estimated as in [67]. Further investigations of these effects [22, 68, 69, 70, 71] are warranted. An additional, special application is to central exclusive production, recently considered for Higgs [72, 73] and scalar χ mesons [74] in the context of u-pdf's.

Figure 3: (top) Azimuth and (bottom) Bjorken- x dependences of di-jet distributions measured by Zeus[78].

We conclude this subsection by observing that recently the small- x definition [8] of u-pdf's has also been used [75, 76] to treat non-leading corrections to jet production at large rapidity separations in hadron-hadron collisions [77], and Monte-Carlo event generators based on the evolution [41, 42] are also becoming available for these processes.

2.3 Multi-jet correlations

More details of the parton $k_\perp$ dynamics can be probed by examining jet correlations in final states containing multiple jets.

Recently the Zeus collaboration has presented measurements of two-jet and three-jet distributions in DIS associated with low x , $10^{-4} < x < 10^{-2}$ [78]. Zeus' jet definition is such that nonglobal logarithms from jet clustering are avoided [79], as are double logarithms from symmetric cuts [80]. Fig. 3 [78] shows two-jet data versus the azimuthal separation $\Delta\phi$ and versus Bjorken x , compared with next-to-leading-order perturbative results [81]. Notice the large variation from the order- α_s^2 to the order- α_s^3 result with decreasing x and decreasing $\Delta\phi$.

Figure 4: Angular jet correlations[60] from CASCADE and HERWIG compared with Zeus data[78]: (top) di-jet cross section; (bottom) three-jet cross section.

Ref. [60] analyzes the multi-jet final states in the framework of u-pdf's. In particular, it studies the contribution to potentially large higher-order corrections arising from a sizeable $k_\perp$ in the initial state when several well-separated hard jets are produced. In this kinematics, effects from the $x \ll 1$ parton shower may be enhanced. These contributions are important at small $\Delta\phi$, when the jets are not close to back-to-back configura-

tions [82]. Fig. 4 shows results for the distribution in the azimuthal separation between the leading jets [60], compared with the measurement [78]. Fig. 5 [60] shows the angular distribution for the third jet, in the cases of small separations and large separations between the leading jets. The $k_\perp$ Monte-Carlo

Figure 5: Cross section in the azimuthal angle between the hardest and the third jet[60], for (top) small and (bottom) large azimuthal separations between the leading jets.

CASCADE gives a good description of the measurement, and gives large differences to the collinear-based parton shower as implemented in HERWIG, particularly in the region where the azimuthal separations $\Delta\phi$ between the leading jets are small.

The x dependence of the gluon distribution used in the above results for the jet cross sections is shown in Fig. 6 for various scales μ and fixed $k_\perp$.

This analysis indicates that theoretical uncertainties on jet correlations are sizeable at NLO [78] and that the $k_\perp$ Monte-Carlo gives an improved description of multi-jets [60] compared to standard parton showers. This analysis is for DIS. However, despite the

Figure 6: The x dependence of the unintegrated gluon distribution.

much lower energy at HERA, note that owing to the large phase space available for jet production the HERA data may be just as relevant as the Tevatron for understanding the extrapolation to the LHC of initial-state radiation effects [7].

2.4 Forward region

Transverse energy flow and production of jets and particles at forward rapidities are important probes of the initial-state $k_\perp$ [83]. Monte-Carlo results for such observables, however, strongly depend on the details of the model used for evolution of the u-pdf's. This model-dependence is much more pronounced for forward-region observables than for the jet correlations discussed in the previous subsection. A systematic understanding of u-pdf's evolution [84] will be relevant for more precise interpretations of forward measurements. See [40] for recent work on evolution equations, including discussion of target fragmentation. We refer the reader to [7, 16], and references therein, for discussion of current forward-region results.

3 Towards precise characterizations of u-pdf's

The possibility to turn the event generators discussed in Sec. 2 into general tools to describe hadronic final states over the

whole phase space depends on theoretical progress about unintegrated parton distributions. This will involve precise characterizations of the distributions, factorization and evolution equations. In what follows we discuss recent progress in the understanding of operator matrix elements for u-pdf's and their lightcone limits.

3.1 Gauge invariant matrix elements

The relevance of consistent operator definitions for parton $k_\perp$ distributions was emphasized long ago, see e.g. [5, 85]. Ensuring gauge invariance at $k_\perp \neq 0$, however, is nontrivial. To this end the approach commonly used is to generalize the coordinate-space matrix elements that serve to identify ordinary pdf's to the case of field operators at non-lightcone distances. E.g., for quarks one has [6, 86, 87]

$$\tilde{f}(y) = \langle P | \bar{\psi}(y) V_y^\dagger(n) \gamma^+ V_0(n) \psi(0) | P \rangle \quad (1)$$

Here ψ are the quark fields evaluated at distance $y = (0, y^-, y_\perp)$, $y_\perp$ is in general nonzero, and V are eikonal-line operators in direction n required to make the matrix element gauge-invariant. The unintegrated quark distribution is obtained from the double Fourier transform in y^- and $y_\perp$ of $\tilde{f}$. An extra gauge link at infinity [86] is to be taken into account in the case of physical gauge.

There are subtleties, however, to using Eq. (1) beyond tree level. As realized early on [85], parton distributions at fixed $k_\perp$ are no longer protected by the KLN mechanism [88] against uncanceled lightcone divergences near the $x = 1$ endpoint [87, 89]. It is only after supplying the above matrix element with a regularization prescription that the distribution is well-defined.

Importantly, similar to what observed in [90] for the case of the Sudakov form factor, the choice of a particular regularization method for the lightcone divergences also affects the distributions integrated over $k_\perp$ and

the ultraviolet subtractions. A further set of questions being studied [27, 51, 91, 92] are in fact associated with the coefficient functions that govern the expansion of unintegrated distributions in terms of ordinary ones, and the relation of the integral of u-pdf's with ordinary distributions.

The above issues can be analyzed by explicit calculation at one loop [27, 93]. Expansion in powers of y^2 of the result for the coordinate-space matrix element at this order (Fig. 7) yields

$$\begin{aligned} \tilde{f}_1(y) = & \frac{\alpha_s C_F}{\pi} p^+ \int_0^1 dv \frac{v}{1-v} \\ & \times \left\{ [e^{ip \cdot y v} - e^{ip \cdot y}] \Gamma(2 - \frac{d}{2}) \left(\frac{4\pi\mu^2}{\rho^2} \right)^{2-d/2} \right. \\ & + e^{ip \cdot y v} \pi^{2-d/2} \Gamma(\frac{d}{2} - 2) (-y^2 \mu^2)^{2-d/2} \\ & \left. + \dots \right\} , \end{aligned} \quad (2)$$

where μ is the dimensional-regularization scale and ρ is an infrared mass regulator. The lightcone singularity $v \rightarrow 1$, corresponding to the exclusive boundary $x = 1$, cancels for ordinary pdf's (first term in the right hand side of Eq. (2)) but it is present, even at $d \neq 4$ and finite ρ , in subsequent terms. This implies that, using the matrix element (1), the $1/(1-x)$ factors from real emission probabilities do not combine with virtual corrections to give $1/(1-x)_+$ distributions, but leave uncanceled divergences at fixed $k_\perp$.

Figure 7: One-loop contributions to lightcone divergences in the matrix element (1).

Different methods have been employed in the literature to provide u-pdf's with consistent regularization for the endpoint region.

We consider these methods briefly in the next subsection.

3.2 Cut-off vs. subtractive method

A possible approach to treat the endpoint is to implement a cut-off by taking the eikonal line n to be non-lightlike, as in the early works [85] and [94]. The cut-off in x at fixed $k_\perp$ is of order $1 - x \geq k_\perp / \sqrt{4\eta}$, where $\eta = (p \cdot n)^2 / n^2$ and p is the incoming momentum. Evolution equations in the cut-off parameter η are investigated in [36, 85, 95, 96]. A thorough analysis of factorization at leading order has recently been given in [26] based on this approach.

A potential drawback of this approach is that cut-off regularization is not very well-suited for applications beyond the leading order. Also, as the two lightcone limits $y^2 \rightarrow 0$ and $n^2 \rightarrow 0$ do not commute, the integral over $k_\perp$ of the distribution has a finite dependence on the regularization parameter η ,

$$\int dk_\perp f(x, k_\perp, \mu, \eta) = F(x, \mu, \eta) \neq \text{ordinary pdf} , \quad (3)$$

which makes the relation with the standard operator product expansion not so transparent.

An alternative route is based on the subtractive method [90, 97]. In this case the direction n is kept to be lightlike but the divergences are canceled by multiplicative, gauge-invariant counterterms given by vacuum expectation values of eikonal operators. The counterterms contain in general both lightlike and non-lightlike eikonals. For this reason studies of u-pdf's in this framework introduce auxiliary eikonals in direction $u = (u^+, u^-, 0_\perp)$ with u^+ and u^- nonzero [27, 87]. The counterterms have compact all-order expressions in coordinate space. At one loop they provide the regularization of the $x \rightarrow 1$ endpoint through a $k_\perp \neq 0$ extension of the plus-distribution

regularization, whose specific form is determined in [27].

The subtractive method is more systematic than the cut-off, and likely more suitable for using unintegrated parton distributions at subleading-log level. In particular, the specific choice of the counterterms [27] is such that the dependence on the non-lightlike eikonal u cancels in the matrix element at $y_\perp = 0$, corresponding to the ordinary pdf. The subtractive method may be helpful for global NLO analyses incorporating Sudakov resummation [98, 99], and construction of parton-shower algorithms beyond leading order [91, 100]. See [101, 102, 103] for studies of subtractive methods similar to those of [90, 97] in relation to effective-theory techniques.

3.3 Further issues in higher order

Further issues arise in higher order. Non-universality of $k_\perp$ -dependent parton distributions [28] has recently been studied [29]–[32] in the hadroproduction of back-to-back high- p_t particles. Potential factorization-breaking effects from soft gluons coupling initial and final states are shown in Fig. 8 [30]. These would enter at a high order of perturbation theory (N^3 LO correction to dihadron production). Although the role of these corrections is yet to be fully established, it is interesting that Coulomb/radiative mixing terms also appear to break color coherence [104] in high-order contributions to cross sections for dijets with a gap in rapidity. A satisfactory understanding of factorization should likely require a full clarification of these issues.

4 Conclusion

Monte-Carlo event generators are being developed based on gauge-invariant definition of u-pdf's at small x (Sec. 2.1). Compared to standard parton shower approaches, these Monte-Carlos have the advantage of taking into account QCD initial-state radiation pro-

Figure 8: Soft gluon exchange with spectator partons.

cesses that depend on the large- $k_\perp$ tail of partonic distributions and matrix elements.

Such processes are likely to be relevant to the simulation of complex final states characterized by multiple hard scales. An example is the DIS multi-jet correlations discussed in Sec. 2.3. But we expect analogous considerations to apply for multi-jet final states in hadron-hadron collisions at LHC energies.

$k_\perp$ Monte-Carlos are currently being tuned and validated using HERA and Tevatron data, as discussed in Secs. 2.2 and 2.3. Determinations of the unintegrated gluon density are performed.

It will be important to extend the notion of u-pdf's beyond $x \ll 1$ (Sec. 3) to turn the above event generators into general-purpose tools.

This involves a challenging theoretical program, raising new questions on operator matrix elements (Sec. 3.1), factorization and lightcone divergences (Sec. 3.2), and possibly new Coulombic effects (Sec. 3.3).

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