Neutrino Physics Beyond the Standard Model

Alejandro Ibarra

DESY

IACS

Kolkata, Jan 2007

Outline

- Introduction
- The neutrino conundrum
- Frameworks for neutrino masses
  - Dirac neutrinos
  - Type I see-saw
  - Type II see-saw
- Conclusions

Introduction

- Neutrinos are the most enigmatic particles of the Standard Model.
- They were postulated by Pauli to explain the continuous spectrum of electrons in nuclear decay. He realized that they had to be electrically neutral, very light and very weakly interacting.

Introduction

- Neutrinos are the most enigmatic particles of the Standard Model.
- They were postulated by Pauli to explain the continuous spectrum of electrons in nuclear decay. He realized that they had to be electrically neutral, very light and very weakly interacting.
- If one projects light on a block of glass, it would be necessary a few centimeters to reduce the intensity to one half. The same experiment with neutrinos would require a block of lead ten light-years thick! (six hundred thousand times the distance from the Earth to the Sun)

Introduction

- Neutrinos are the most enigmatic particles of the Standard Model.
- They were postulated by Pauli to explain the continuous spectrum of electrons in nuclear decay. He realized that they had to be electrically neutral, very light and very weakly interacting.
- If one projects light on a block of glass, it would be necessary a few centimeters to reduce the intensity to one half. The same experiment with neutrinos would require a block of lead ten light-years thick! (six hundred thousand times the distance from the Earth to the Sun)
- Neutrinos were discovered in 1956 (the electron neutrino). Later, in 1962 a different type of neutrino was discovered (the muon neutrino), and another different type in 2000 (the tau neutrino). We believe that the list is now complete.

Introduction

- Neutrinos are abundantly produced in the Sun and reach us. Every second, six hundred billion neutrinos go through any of us. Only one will interact with us... in our whole life!
- Neutrinos are also produced in the atmosphere by cosmic rays. There is one atmospheric neutrino going through the tip of my finger every second.
- Neutrinos are ubiquous in the Universe: they are also produced in the early Universe, in supernovas, in the mantle and the nucleus of the Earth, and even inside us!
- But they are so weakly interacting that neutrino experiments require huge detectors and a lot of patience.

Introduction

- Another important feature of neutrinos is that they have tiny masses. Experiments indicated that the mass of the electron neutrino had to be at least one hundred thousand times smaller than the mass of the electron.
- A massless neutrino was also compatible with experiments, so theorists assumed this possibity as a fact. This is one of the ingredients of the Standard Model of Particle Physics.

Introduction

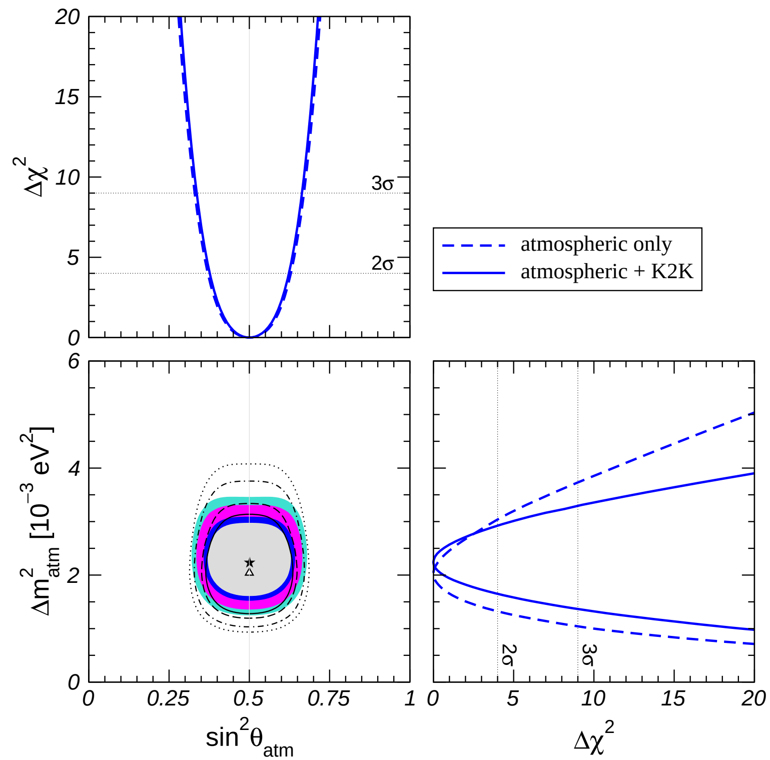
- Another important feature of neutrinos is that they have tiny masses. Experiments indicated that the mass of the electron neutrino had to be at least one hundred thousand times smaller than the mass of the electron.
- A massless neutrino was also compatible with experiments, so theorists assumed this possibity as a fact. This is one of the ingredients of the Standard Model of Particle Physics.
- However, one piece of the jigsaw was not fitting... The number of neutrinos coming from the Sun was around one third smaller than the prediction from Bahcall's solar model. Either we didn't understand the Sun or we didn't understand neutrino properties.

Introduction

- The solar neutrino puzzle could be solved if neutrinos had mass. If this is the case, they would “oscillate” and they would change flavour as they propagate. The Sun produces electron neutrinos, but in their way part of them are converted into muon neutrinos that were not detected.
- Furthermore, interactions of cosmic rays in the atmosphere produce muon neutrinos, that as they propagate are partly converted into tau neutrinos. This deficit of atmospheric neutrinos has also been observed.
- Two independent pieces of evidence that neutrinos have mass and mix in flavour. The Standard Model of Particle Physics is incomplete!!

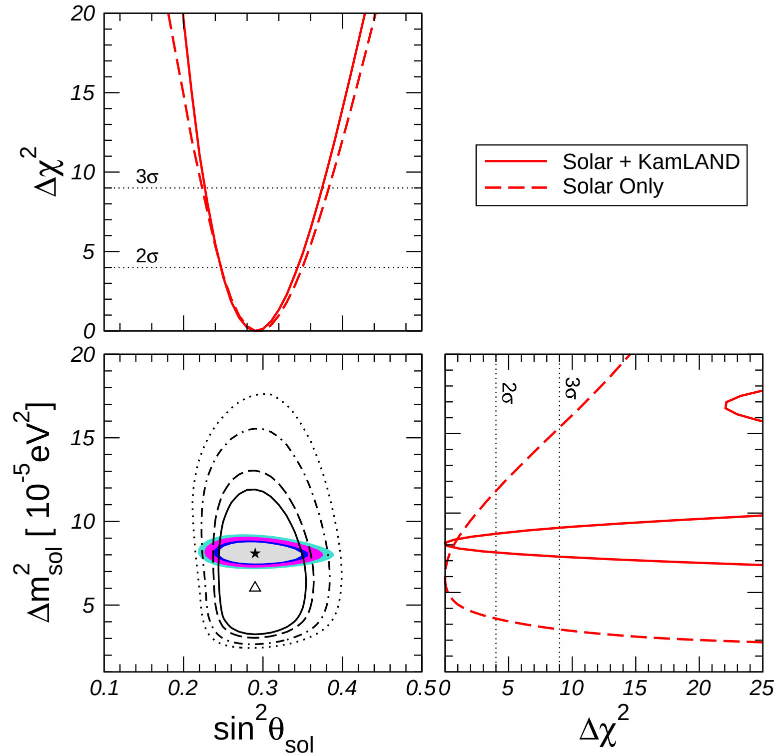
Introduction

There are overwhelming evidences that neutrinos have mass and oscillate. Moreover, data are getting so good that we are entering the era of **neutrino precision data!**



Maltoni *et.al.*

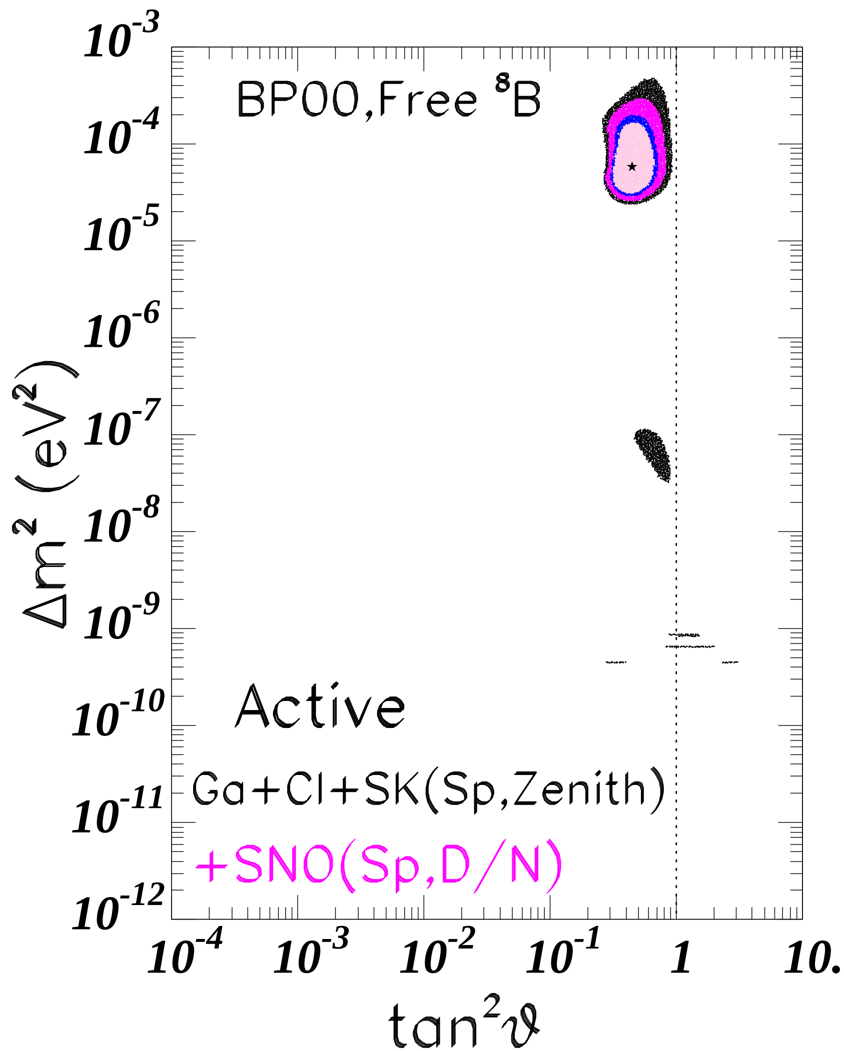
atmospheric and reactor data



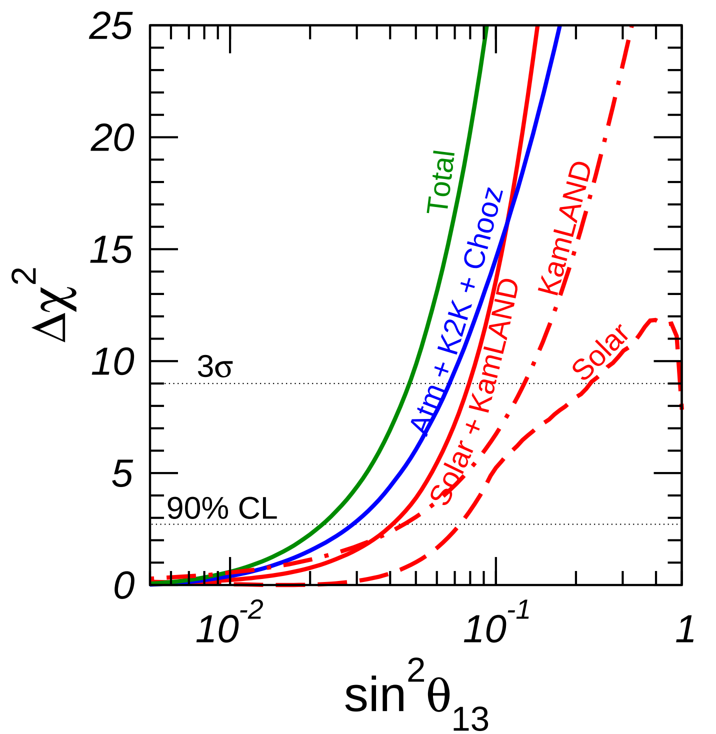
Maltoni *et.al.*

solar and reactor data

Compare with the experimental situation just four years ago!



Bahcall, Gonzalez-Garcia
Peña-Garay
hep-ph/0204314

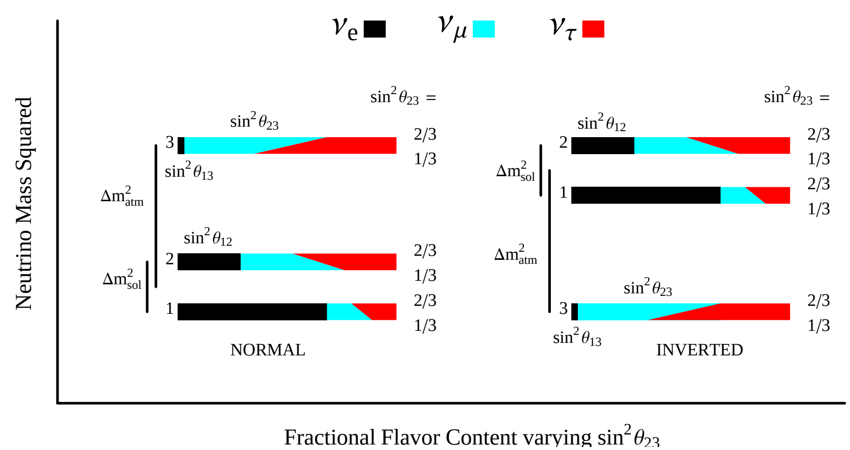


Maltoni *et.al.*

From the theorist's point of view it suffices to know that:

parameter	best fit	2σ range
Δm^2_{sol} $[10^{-5}$ eV$^2]$	7.9	7.3–8.5
Δm^2_{atm} $[10^{-3}$ eV$^2]$	2.6	2.2–3.0
$\sin^2\theta_{sol}$	0.30	0.26–0.36
$\sin^2\theta_{atm}$	0.50	0.38–0.63
$\sin^2\theta_{13}$	0.000	$\leq$ 0.025

Unfortunately, no information yet about the spectrum or CP violation.



Even with this limited information, we can already notice some features:

$\star$ Neutrino masses are tiny, $m_\nu \lesssim \mathcal{O}(0.1\text{eV})$

$\star$ Two large mixing angles ($\theta_{atm} \simeq \pi/4$, $\theta_{sol} \simeq \pi/6$)

One small mixing angle ($\theta_{13} \simeq 0$)

$$U_{lep} \simeq \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

$\star$ The two heaviest neutrinos present a mild mass hierarchy

$$\Delta m^2_{atm} = m_3^2 - m_1^2 \longrightarrow m_3 = \sqrt{\Delta m^2_{atm} - m_1^2}$$

$$\Delta m^2_{sol} = m_2^2 - m_1^2 \longrightarrow m_2 = \sqrt{\Delta m^2_{sol} - m_1^2}$$

Even with this limited information, we can already notice some features:

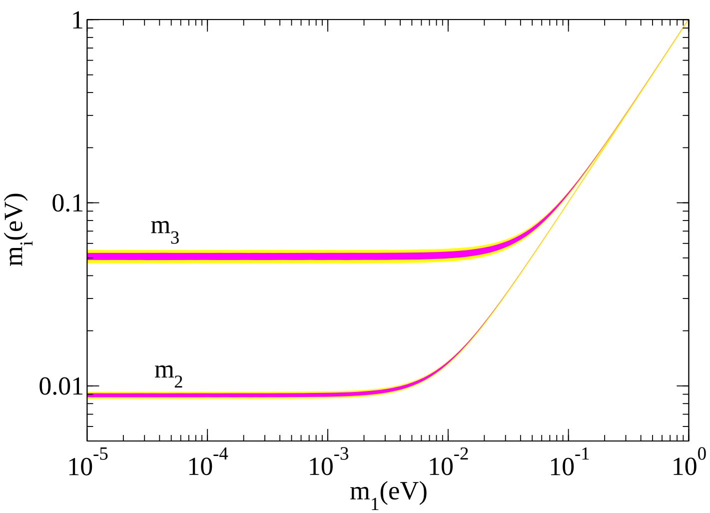
⋆ Neutrino masses are tiny, $m_\nu \lesssim \mathcal{O}(0.1\mathrm{eV})$

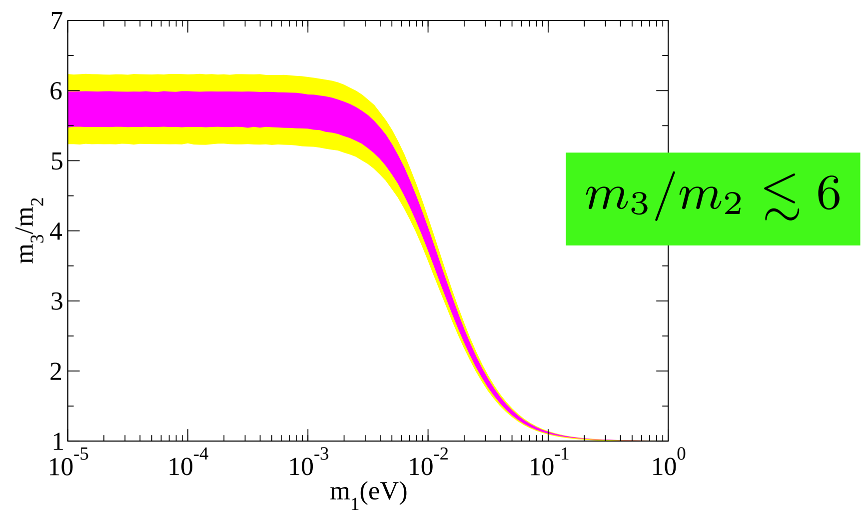
⋆ Two large mixing angles ($\theta_{atm} \simeq \pi/4$, $\theta_{sol} \simeq \pi/6$)

One small mixing angle ($\theta_{13} \simeq 0$)

$$U_{lep} \simeq \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

⋆ The two heaviest neutrinos present a mild mass hierarchy





⋆ Compare with the quark sector:

$$\left.\begin{aligned} m_u &= 1.5 \text{ to } 3.0 \text{ MeV} \\ m_c &= 1.25 \pm 0.09 \text{ GeV} \\ m_t &= 174.2 \pm 3.3 \text{ GeV} \end{aligned}\right\} \quad \begin{aligned} m_t/m_c &\simeq 140 \\ m_c/m_u &\simeq 550 \end{aligned}$$

vs. $m_3/m_2 \lesssim 6$ in ν sector

$$\left.\begin{aligned} m_d &= 3 \text{ to } 7 \text{ MeV} \\ m_s &= 95 \pm 25 \text{ MeV} \\ m_b &= 4.20 \pm 0.07 \text{ GeV} \end{aligned}\right\} \quad \begin{aligned} m_b/m_s &\simeq 44 \\ m_s/m_d &\simeq 19 \end{aligned}$$

$$U_{CKM} \simeq \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.973 & 0.04 \\ 0.008 & 0.04 & 1 \end{pmatrix} \quad \text{vs. } U_{lep} \simeq \begin{pmatrix} 0.82 & 0.56 & 0 \\ -0.41 & 0.56 & -0.71 \\ -0.41 & 0.56 & 0.71 \end{pmatrix}$$

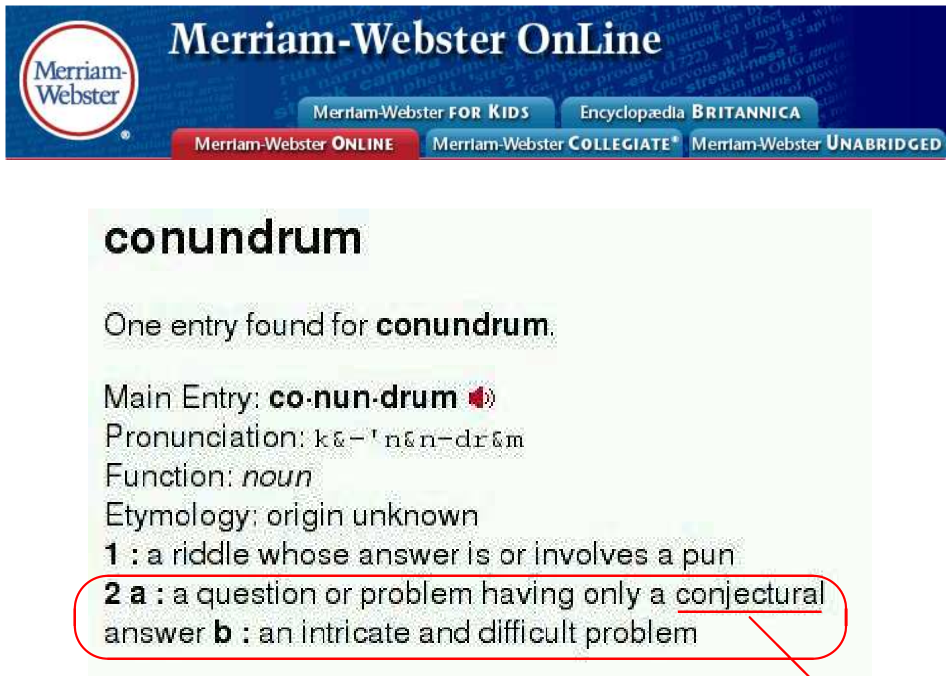
⋆ Compare also with the charged-lepton sector

$$\left.\begin{aligned} m_e &= 0.51 \text{ MeV} \\ m_\mu &= 106 \text{ MeV} \\ m_\tau &= 1.78 \text{ GeV} \end{aligned}\right\} \quad \begin{aligned} m_\tau/m_\mu &\simeq 17 \\ m_\mu/m_e &\simeq 208 \end{aligned}$$

vs. $m_3/m_2 \lesssim 6$ in ν sector

The neutrino sector presents a completely different pattern!!

The neutrino conundrum



conundrum

One entry found for **conundrum**.

Main Entry: **co·nun·drum**
Pronunciation: k&-'n&n-dr&m
Function: *noun*
Etymology: origin unknown
1 : a riddle whose answer is or involves a pun
2 a : a question or problem having only a conjectural answer **b :** an intricate and difficult problem

Physics Beyond the SM

ORIGIN OF ν MASSES

- Dirac masses?
- See-saw mechanisms
  - Type I (with SM singlets)
  - Type II (with a $SU(2)$ triplet)
- R-parity violation in SUSY models
- Radiative models (Zee model)
- Large extra dimensions
- Tetrahedral symmetry, A_4
- Permutation symmetry, S_3
- ...
- Anarchy

A model of neutrino masses should address the following questions:

- Why tiny masses?
- Why mild mass hierarchy?
- Why large mixing angles?

And preferably, the model has to be testable.

Dirac masses?

The rest of the known fermions are Dirac particles. Why not neutrinos too? This would require the existence of a right-handed neutrino ν_R.

The leptonic Lagrangian would read:

$$-\mathcal{L}_{lep} = {e_R^c}^T \mathbf{Y_e} L \cdot H^\dagger + {\nu_R^c}^T \mathbf{Y}_\nu L \cdot H + \mathrm{h.c.}$$

$\star$ Why tiny masses? Neutrino masses of $\mathcal{O}$(0.1 eV) require $Y_\nu \sim 10^{-12}$.

Dirac masses?

The rest of the known fermions are Dirac particles. Why not neutrinos too? This would require the existence of a right-handed neutrino ν_R.

The leptonic Lagrangian would read:

$$-\mathcal{L}_{lep} = {e^c_R}^T \mathbf{Y_e} L \cdot H^\dagger + {\nu^c_R}^T \mathbf{Y}_\nu L \cdot H + \mathrm{h.c.}$$

⋆ Why tiny masses? Neutrino masses of $\mathcal{O}$(0.1 eV) require $Y_\nu \sim 10^{-12}$.

Not very natural

Dirac masses?

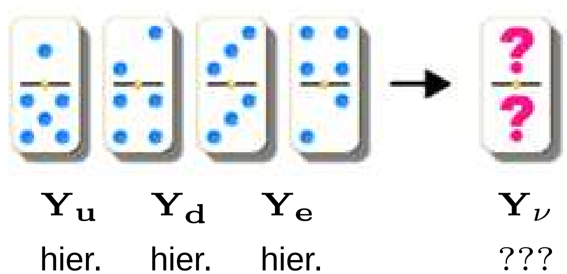
The rest of the known fermions are Dirac particles. Why not neutrinos too? This would require the existence of a right-handed neutrino ν_R.

The leptonic Lagrangian would read:

$$-\mathcal{L}_{lep} = {e_R^c}^T \mathbf{Y_e} L \cdot H^\dagger + {\nu_R^c}^T \mathbf{Y}_\nu L \cdot H + \text{h.c.}$$

⋆ Why tiny masses? Neutrino masses of $\mathcal{O}$(0.1 eV) require $Y_\nu \sim 10^{-12}$. **Not very natural**

⋆ Why mild hierarchy?



Dirac masses?

The rest of the known fermions are Dirac particles. Why not neutrinos too? This would require the existence of a right-handed neutrino ν_R.

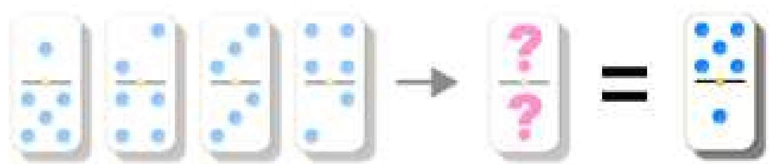
The leptonic Lagrangian would read:

$$-\mathcal{L}_{lep} = {e_R^c}^T \mathbf{Y_e} L \cdot H^\dagger + {\nu_R^c}^T \mathbf{Y}_\nu L \cdot H + \text{h.c.}$$

⋆ Why tiny masses? Neutrino masses of $\mathcal{O}$(0.1 eV) require $Y_\nu \sim 10^{-12}$.

Not very natural

⋆ Why mild hierarchy?



$\mathbf{Y_u}$ hier. $\mathbf{Y_d}$ hier. $\mathbf{Y_e}$ hier. $\mathbf{Y}_\nu$??? $\mathbf{Y}_\nu$ hier. $\longrightarrow \frac{m_3}{m_2} \sim 20 - 150$

but experimentally $\frac{m_3}{m_2} \lesssim 6$

Dirac masses?

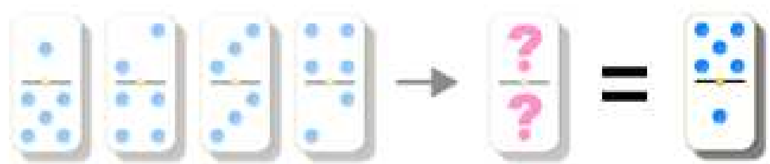
The rest of the known fermions are Dirac particles. Why not neutrinos too? This would require the existence of a right-handed neutrino ν_R.

The leptonic Lagrangian would read:

$$-\mathcal{L}_{lep} = {e^c_R}^T \mathbf{Y_e} L \cdot H^\dagger + {\nu^c_R}^T \mathbf{Y}_\nu L \cdot H + \text{h.c.}$$

⋆ Why tiny masses? Neutrino masses of $\mathcal{O}$(0.1 eV) require $Y_\nu \sim 10^{-12}$. **Not very natural**

⋆ Why mild hierarchy?



$\mathbf{Y_u}$	$\mathbf{Y_d}$	$\mathbf{Y_e}$	$\mathbf{Y}_\nu$	$\mathbf{Y}_\nu \longrightarrow \frac{m_3}{m_2} \sim 20 - 150$
hier.	hier.	hier.	???	hier. but experimentally $\frac{m_3}{m_2} \lesssim 6$

Not very plausible

Dirac masses?

The rest of the known fermions are Dirac particles. Why not neutrinos too? This would require the existence of a right-handed neutrino ν_R.

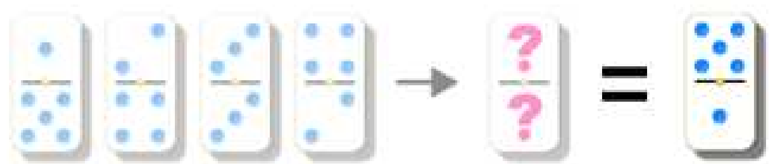
The leptonic Lagrangian would read:

$$-\mathcal{L}_{lep} = {e_R^c}^T \mathbf{Y_e} L \cdot H^\dagger + {\nu_R^c}^T \mathbf{Y}_\nu L \cdot H + \text{h.c.}$$

⋆ Why tiny masses? Neutrino masses of $\mathcal{O}$(0.1 eV) require $Y_\nu \sim 10^{-12}$.

Not very natural

⋆ Why mild hierarchy?



$\mathbf{Y_u}$	$\mathbf{Y_d}$	$\mathbf{Y_e}$	$\mathbf{Y}_\nu$	$\mathbf{Y}_\nu \longrightarrow \frac{m_3}{m_2} \sim 20 - 150$
hier.	hier.	hier.	???	hier. but experimentally $\frac{m_3}{m_2} \lesssim 6$

Not very plausible

⋆ Why large angles? This is possible – no guidance from the other sectors.

Dirac masses?

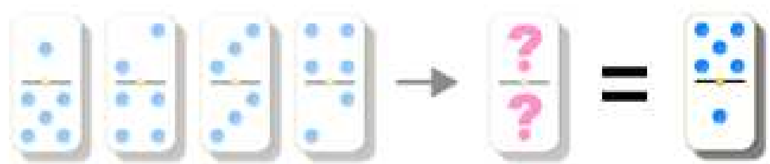
The rest of the known fermions are Dirac particles. Why not neutrinos too? This would require the existence of a right-handed neutrino ν_R.

The leptonic Lagrangian would read:

$$-\mathcal{L}_{lep} = {e_R^c}^T \mathbf{Y_e} L \cdot H^\dagger + {\nu_R^c}^T \mathbf{Y}_\nu L \cdot H + \text{h.c.}$$

⋆ Why tiny masses? Neutrino masses of $\mathcal{O}$(0.1 eV) require $Y_\nu \sim 10^{-12}$. **Not very natural**

⋆ Why mild hierarchy?



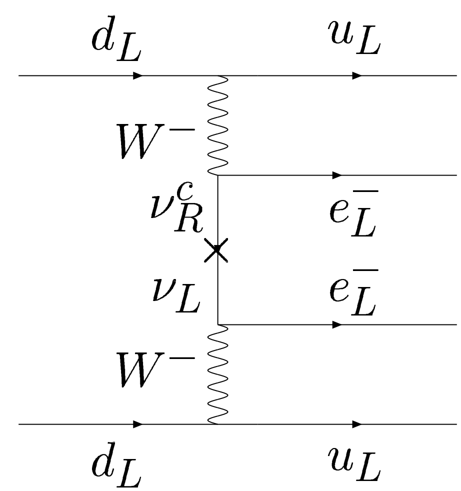
$\mathbf{Y_u}$	$\mathbf{Y_d}$	$\mathbf{Y_e}$	$\mathbf{Y}_\nu$	$\mathbf{Y}_\nu \longrightarrow \frac{m_3}{m_2} \sim 20 - 150$
hier.	hier.	hier.	???	hier. but experimentally $\frac{m_3}{m_2} \lesssim 6$

Not very plausible

⋆ Why large angles? This is possible – no guidance from the other sectors.

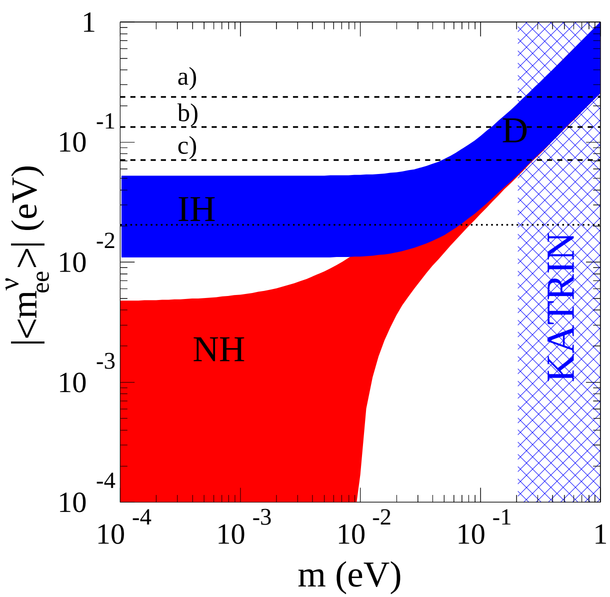
Dirac neutrino masses have ugly features, but **are not excluded!!**

The smoking gun: $0\nu 2\beta$



If neutrinos are Majorana particles, the nuclear process $(A, Z) \to (A, Z + 2) + e^- + e^-$ is allowed.

Not observed yet. Lifetime $> 10^{24} - 10^{25}$y



Bahcall, Murayama
Peña-Garay

The rate for $0\nu 2\beta$ depends crucially on the spectrum
If neutrinos are degenerate or inverse hierarchical, $0\nu 2\beta$ could be observed in the next generations of experiments (CUORE, GERDA, Majorana, EXO, MOON, Super-NEMO...)

Type I see-saw

Minkowski
Yanagida
Gell-Mann, Ramond, Slansky
Mohapatra, Senjanović

Add to the Standard model particle content at least two singlets (right-handed neutrinos)

The most general lagrangian compatible with the gauge symmetries is:

$$-\mathcal{L}_{lep} = e_R^{c\,T} \mathbf{Y_e} L \cdot H^\dagger + \nu_R^{c\,T} \mathbf{Y}_\nu L \cdot H - \frac{1}{2} \nu_R^{c\,T} \mathcal{M}\, \nu_R^c + \text{h.c.}$$

$$\Big\downarrow \quad \mathcal{M} \gg \langle H^0 \rangle$$

$$-\mathcal{L}_{eff} = e_R^{c\,T} \mathbf{Y_e} L \cdot H^\dagger - \frac{1}{2} (L \cdot H)^T \mathbf{Y}_\nu{}^T \mathcal{M}^{-1} \mathbf{Y}_\nu (L \cdot H) + \text{h.c.}$$

After the EWSB,

$$\mathcal{M}_\nu = \mathbf{Y}_\nu{}^T \mathcal{M}^{-1} \mathbf{Y}_\nu \, \langle H^0 \rangle^2$$

Naturally small!!

"Most standard extension of the Standard Model"

- Natural, simple and elegant.
- The particle content is left-right symmetric.
- Nicely compatible with GUTs.
- Could account for the observed baryon asymmetry of the Universe. (leptogenesis)

Can the type I see-saw accommodate a mild hierarchy, $m_3/m_2 \lesssim 6$?

$\{Y_\nu, \mathcal{M}\}$ depend on 18 parameters — 12 real, 6 phases

$\{\mathcal{M}_\nu\}$ depends on 9 parameters — 6 real, 3 phases

There is a lot of freedom at high energies

It would not be surprising that the see-saw can accommodate $m_3/m_2 \lesssim 6$

In fact, the see-saw can accommodate anything...

To show that the see-saw can accommodate anything at low energies, we choose to work in the basis where

$$\mathcal{M} = D_{\mathcal{M}} = \text{diag}(M_1, M_2, M_3)$$

Then, it can be checked that

$$\mathbf{Y}_\nu = \frac{1}{\langle H_u^0 \rangle} \sqrt{D_{\mathcal{M}}}\ R\ \sqrt{D_m}\ U_{lep}^\dagger$$

Casas, A.I.

is the most general matrix that satisfi es the see-saw formula:

$$\mathcal{M}_\nu = U_{lep}^* D_m U_{lep}^\dagger = \mathbf{Y}_\nu{}^T \mathcal{M}^{-1} \mathbf{Y}_\nu\ \langle H_u^0 \rangle^2$$

Here, R is an orthogonal matrix (in general, complex)

$$R = \begin{pmatrix} \hat{c}_2 \hat{c}_3 & -\hat{c}_1 \hat{s}_3 - \hat{s}_1 \hat{s}_2 \hat{c}_3 & \hat{s}_1 \hat{s}_3 - \hat{c}_1 \hat{s}_2 \hat{c}_3 \\ \hat{c}_2 \hat{s}_3 & \hat{c}_1 \hat{c}_3 - \hat{s}_1 \hat{s}_2 \hat{s}_3 & -\hat{s}_1 \hat{c}_3 - \hat{c}_1 \hat{s}_2 \hat{s}_3 \\ \hat{s}_2 & \hat{s}_1 \hat{c}_2 & \hat{c}_1 \hat{c}_2 \end{pmatrix}$$

There is an *infinite* set of Yukawa couplings that can accommodate the low energy observations. But there is a price...

The price is that the resulting Yukawa coupling could be "weird".

★ For example, taking $M_1 = 10^9$GeV, $M_2 = 10^{11}$GeV, $M_3 = 10^{13}$GeV, and $R(z_1 = 2i, z_2 = 0, z_3 = 0)$ the matrix

$$\mathbf{Y}_\nu = \begin{pmatrix} 1.9 \times 10^{-4} & -8.6 \times 10^{-5} & 8.6 \times 10^{-5} \\ 0.011 & 0.012 - 0.031\, i & -0.012 - 0.031\, i \\ 0.11\, i & 0.32 + 0.12\, i & 0.32 - 0.12\, i \end{pmatrix}$$

is guaranteed by construction to be compatible with present experiments, using

$$\mathbf{Y}_\nu = \frac{1}{\langle H_u^0 \rangle} \sqrt{D_{\mathcal{M}}}\, R\, \sqrt{D_m}\, U_{lep}^\dagger .$$

Other inputs: $m_3 = 0.05$eV, $m_2 = 0.0083$eV, $\sin^2\theta_{12} = 0.3$, $\sin^2\theta_{23} = 1$ (and $m_1 = m_2/6$, $\theta_{13} = 0$ and no CP violation).

The price is that the resulting Yukawa coupling could be “weird”.

⋆ For example, taking $M_1 = 10^9$GeV, $M_2 = 10^{11}$GeV, $M_3 = 10^{13}$GeV, and $R(z_1 = 2i, z_2 = 0, z_3 = 0)$ the matrix

$$\mathbf{Y}_\nu = \begin{pmatrix} 1.9 \times 10^{-4} & -8.6 \times 10^{-5} & 8.6 \times 10^{-5} \\ 0.011 & 0.012 - 0.031\, i & -0.012 - 0.031\, i \\ 0.11\, i & 0.32 + 0.12\, i & 0.32 - 0.12\, i \end{pmatrix}$$

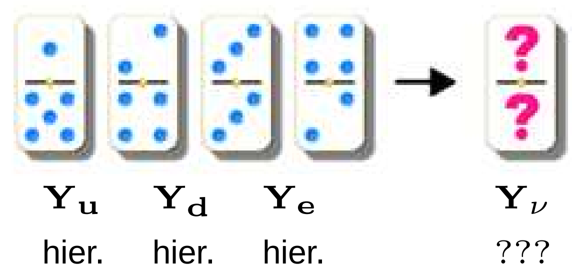
is guaranteed by construction to be compatible with present experiments, using

$$\mathbf{Y}_\nu = \frac{1}{\langle H_u^0 \rangle} \sqrt{D_{\mathcal{M}}}\, R\, \sqrt{D_m}\, U_{lep}^\dagger .$$

Other inputs: $m_3 = 0.05$eV, $m_2 = 0.0083$eV, $\sin^2\theta_{12} = 0.3$, $\sin^2\theta_{23} = 1$ (and $m_1 = m_2/6$, $\theta_{13} = 0$ and no CP violation).

⋆ But the eigenvalues are:

$$\left.\begin{aligned} y_3 &= 0.50 \\ y_2 &= 1.3 \times 10^{-3} \\ y_1 &= 2.2 \times 10^{-4} \end{aligned}\right\} \quad \begin{aligned} y_3/y_2 &= 379 \\ y_2/y_1 &= 6 \end{aligned}$$



Can the type I see-saw accommodate a mild hierarchy, $m_3/m_2 \lesssim 6$, if the neutrino Yukawa eigenvalues have a reasonable hierarchy?

Not so easy... The see-saw mechanism tends to produce large neutrino mass hierarchies.

- "Naive see-saw"

$$m_1 \sim \frac{y_1^2}{M_1} v^2 \qquad m_2 \sim \frac{y_2^2}{M_2} v^2 \qquad m_3 \sim \frac{y_3^2}{M_3} v^2 \qquad \text{Then, } \frac{m_3}{m_2} \sim \frac{y_3^2}{y_2^2} \frac{M_2}{M_3}$$

The Yukawa couplings are hierarchical: $y_1 : y_2 : y_3 = 1 : 20 : 20^2$ or $y_1 : y_2 : y_3 = 1 : 300 : 300^2$

The right-handed neutrino masses, we don't know

hierarchical ν_R	degenerate ν_R	
$\frac{m_3}{m_2} \sim 20 - 300$	$\frac{m_3}{m_2} \sim 400 - 90000$	far from $\frac{m_3}{m_2} \lesssim 6$

Can the type I see-saw accommodate a mild hierarchy, $m_3/m_2 \lesssim 6$, if the neutrino Yukawa eigenvalues have a reasonable hierarchy?

Not so easy... The see-saw mechanism tends to produce large neutrino mass hierarchies.

- "Naive see-saw"

$$m_1 \sim \frac{y_1^2}{M_1} v^2 \qquad m_2 \sim \frac{y_2^2}{M_2} v^2 \qquad m_3 \sim \frac{y_3^2}{M_3} v^2 \qquad \text{Then, } \frac{m_3}{m_2} \sim \frac{y_3^2}{y_2^2} \frac{M_2}{M_3}$$

The Yukawa couplings are hierarchical: $y_1 : y_2 : y_3 = 1 : 20 : 20^2$ or
$y_1 : y_2 : y_3 = 1 : 300 : 300^2$

The right-handed neutrino masses, we don't know

hierarchical ν_R	degenerate ν_R	
$\frac{m_3}{m_2} \sim 20 - 300$	$\frac{m_3}{m_2} \sim 400 - 90000$	far from $\frac{m_3}{m_2} \lesssim 6$

- A more rigurous analysis shows that generically,

$$\frac{m_3}{m_2} \gtrsim \frac{y_3^2}{y_2^2} \frac{M_2}{M_1}$$

hierarchical ν_R: $\frac{m_3}{m_2} \gtrsim \mathcal{O}(10^{3-7})$
degenerate ν_R: $\frac{m_3}{m_2} \gtrsim \mathcal{O}(10^{2-5})$

again far from $\frac{m_3}{m_2} \lesssim 6$

Right-handed mixing at rescue!

Another useful parametrization of the Yukawa couplings is the singular value decomposition:

$$\mathbf{Y}_\nu = V_R D_Y V_L^\dagger$$

Then, the neutrino mass matrix

$$\mathcal{M}_\nu = {\mathbf{Y}_\nu}^T D_{\mathcal{M}}^{-1} \mathbf{Y}_\nu \, \langle H_u^0 \rangle^2 = V_L^* D_Y V_R^T D_{\mathcal{M}}^{-1} V_R D_Y V_L^\dagger \langle H_u^0 \rangle^2$$

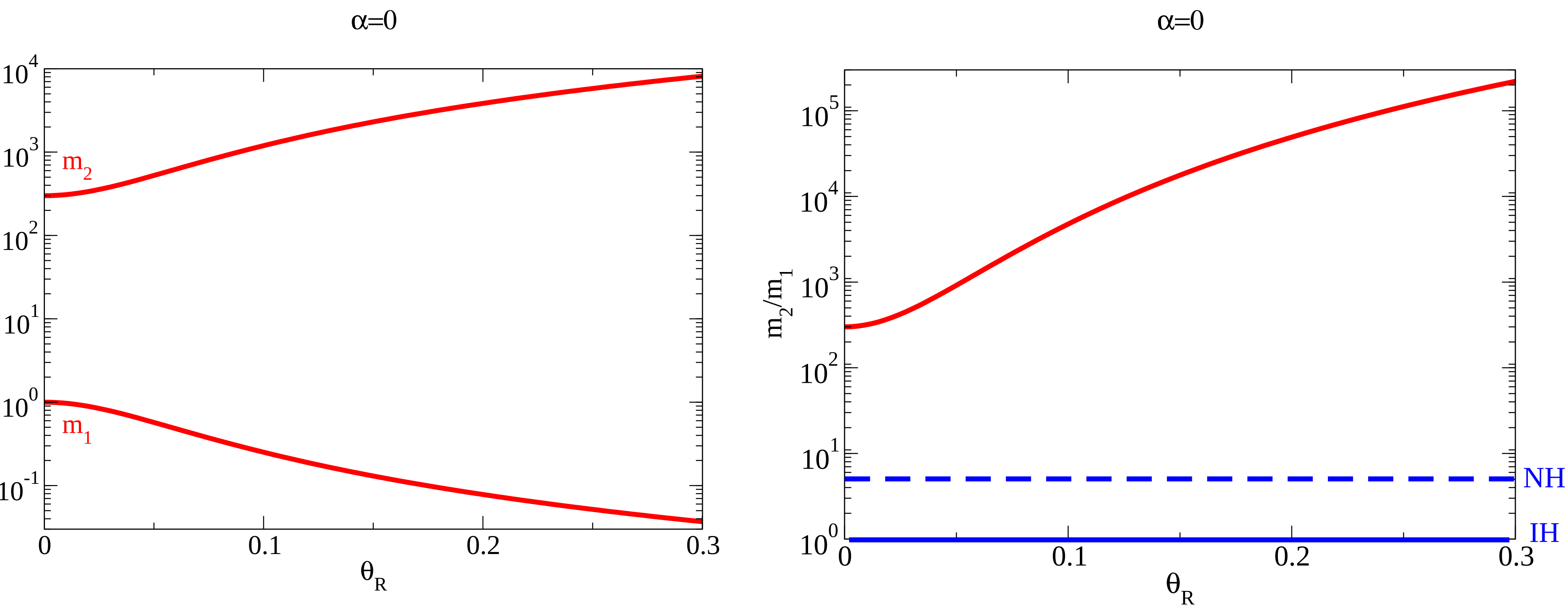
depends on V_L and V_R, but the neutrino mass eigenvalues depend *only* on V_R. For example, for the case with two neutrinos:

$$V_R = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_R & \sin\theta_R \\ -\sin\theta_R & \cos\theta_R \end{pmatrix}$$

Changing the angle θ_R and the phase α, the mass hierarchy between the two neutrinos m_2/m_1 change.

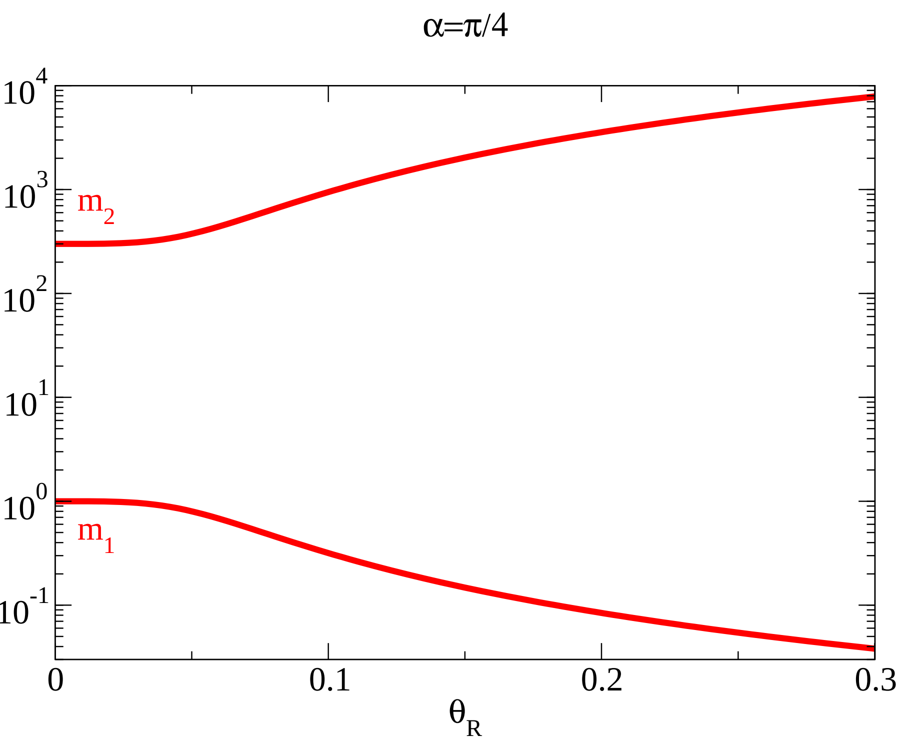
$$y_1 : y_2 = 1 : 300$$
$$M_1 : M_2 = 1 : 300$$

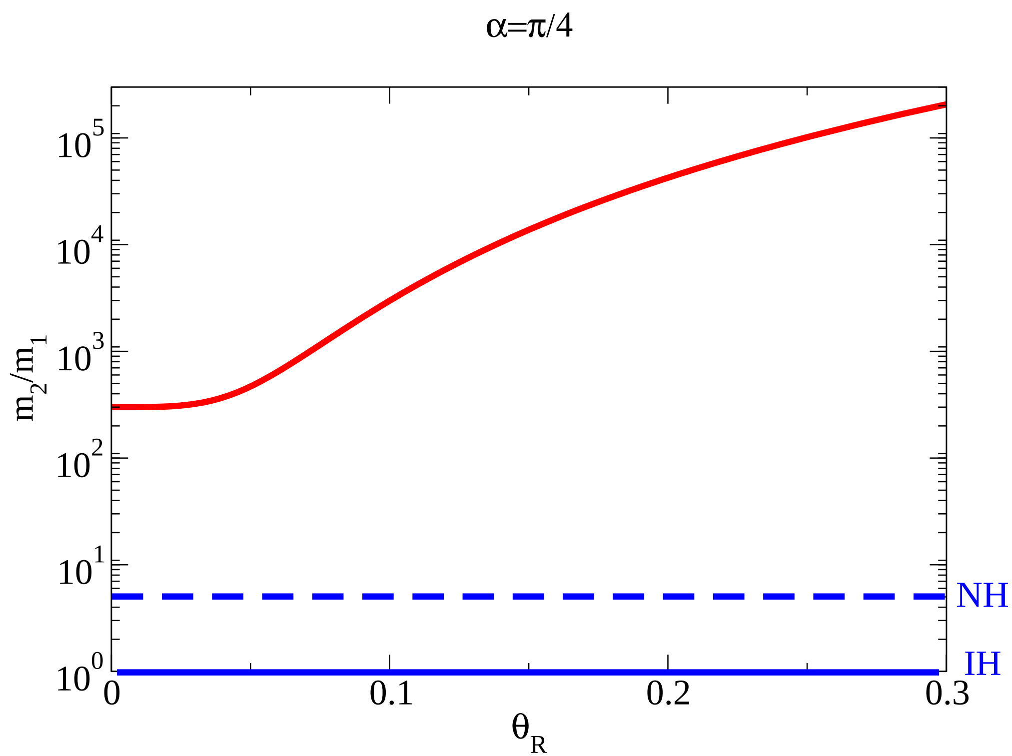
(Inspired by the hierarchy in the up quark sector)



$$y_1 : y_2 = 1 : 300$$
$$M_1 : M_2 = 1 : 300$$

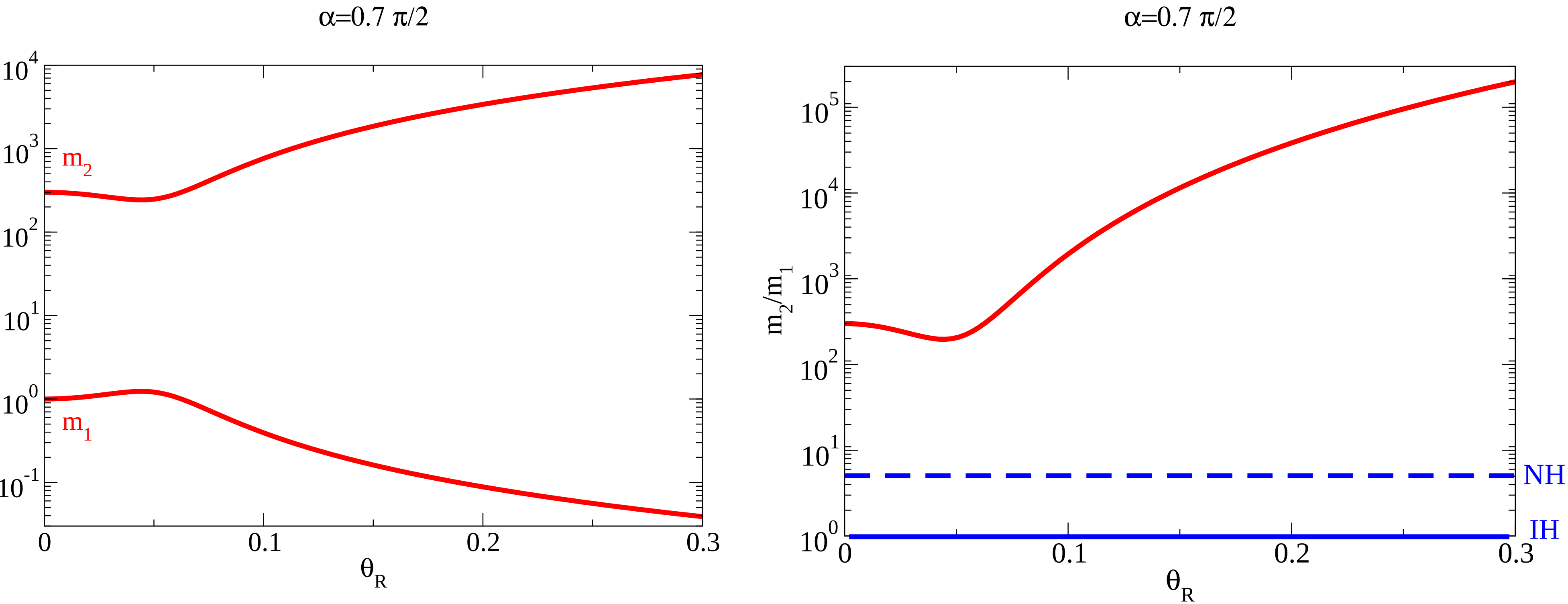
(Inspired by the hierarchy in the up quark sector)





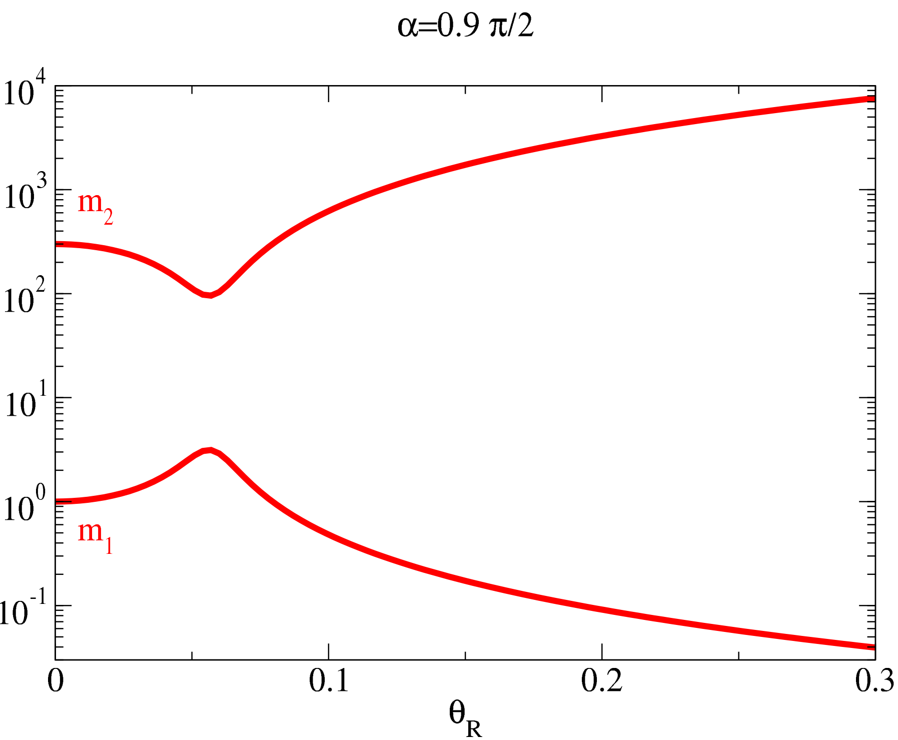
$$y_1 : y_2 = 1 : 300$$
$$M_1 : M_2 = 1 : 300$$

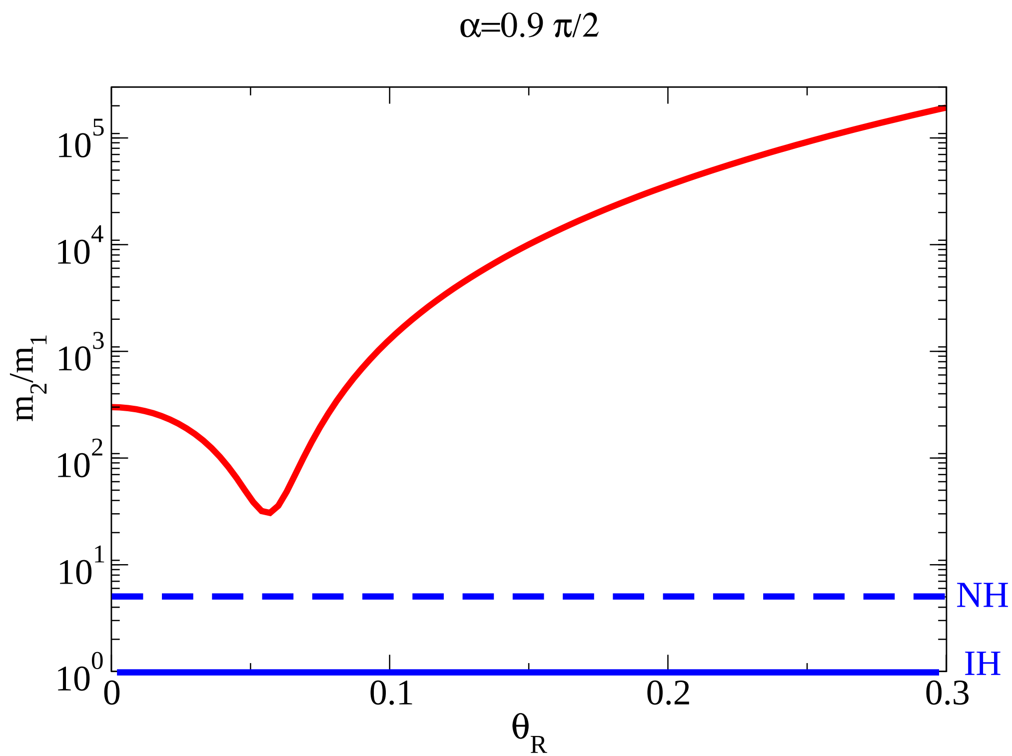
(Inspired by the hierarchy in the up quark sector)



$$y_1 : y_2 = 1 : 300$$
$$M_1 : M_2 = 1 : 300$$

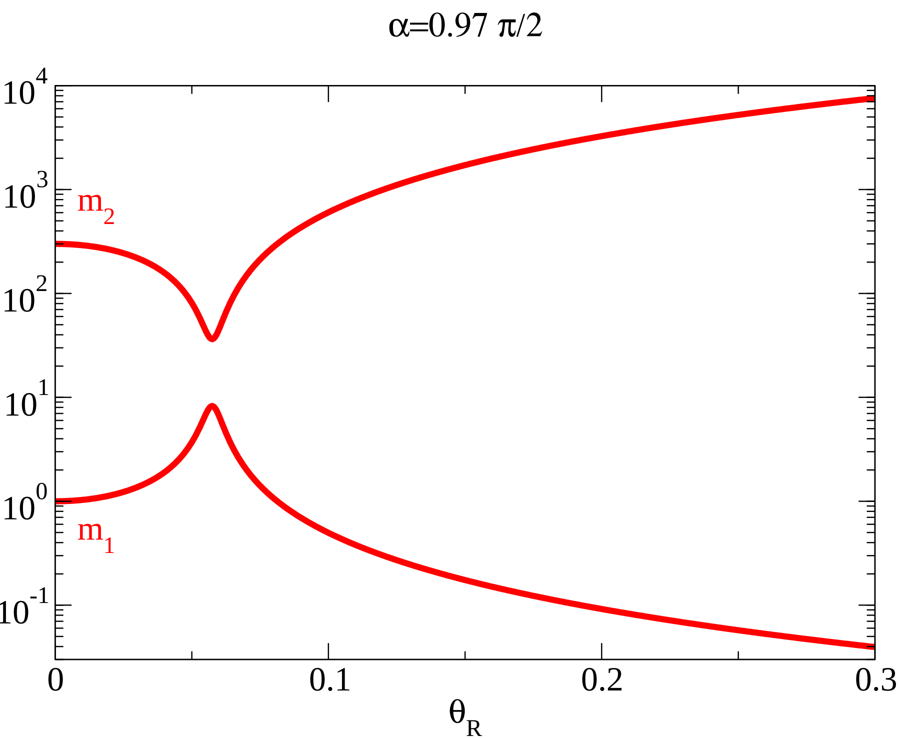
(Inspired by the hierarchy in the up quark sector)

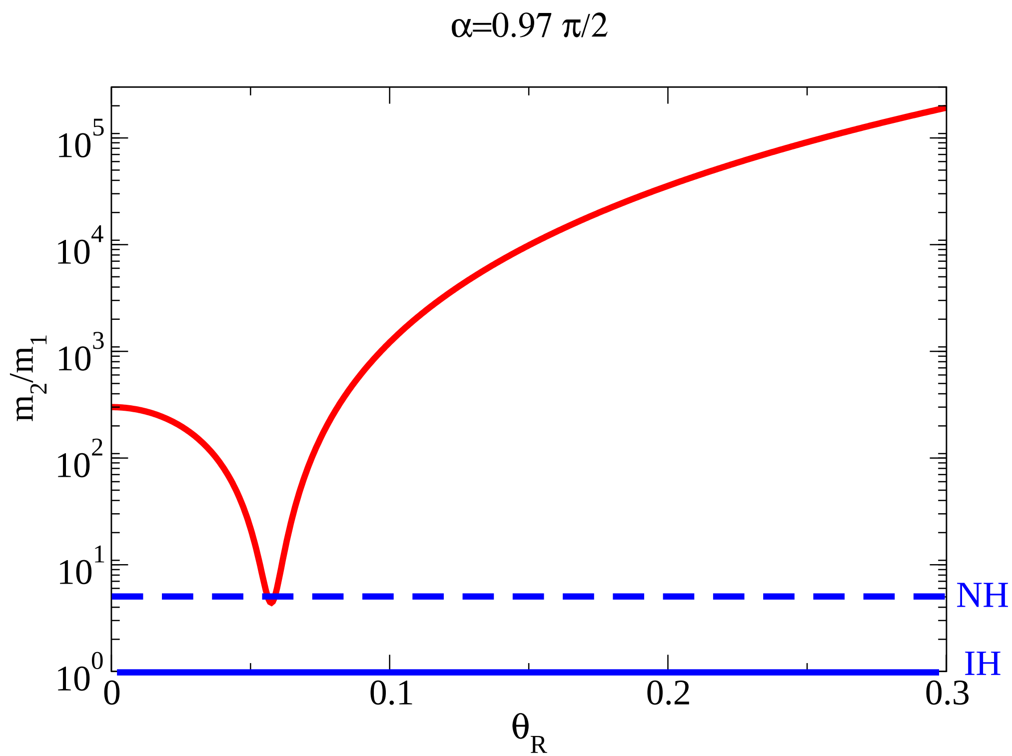




$$y_1 : y_2 = 1 : 300$$
$$M_1 : M_2 = 1 : 300$$

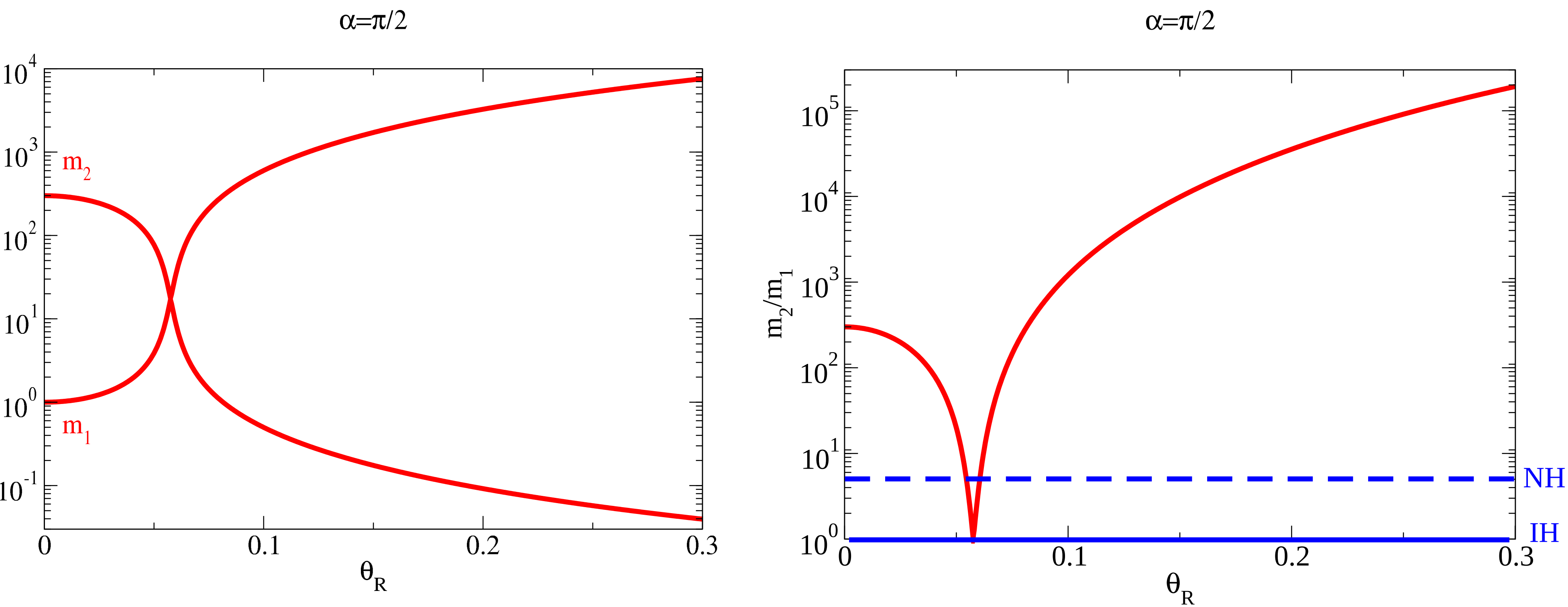
(Inspired by the hierarchy in the up quark sector)





$$y_1 : y_2 = 1 : 300$$
$$M_1 : M_2 = 1 : 300$$

(Inspired by the hierarchy in the up quark sector)



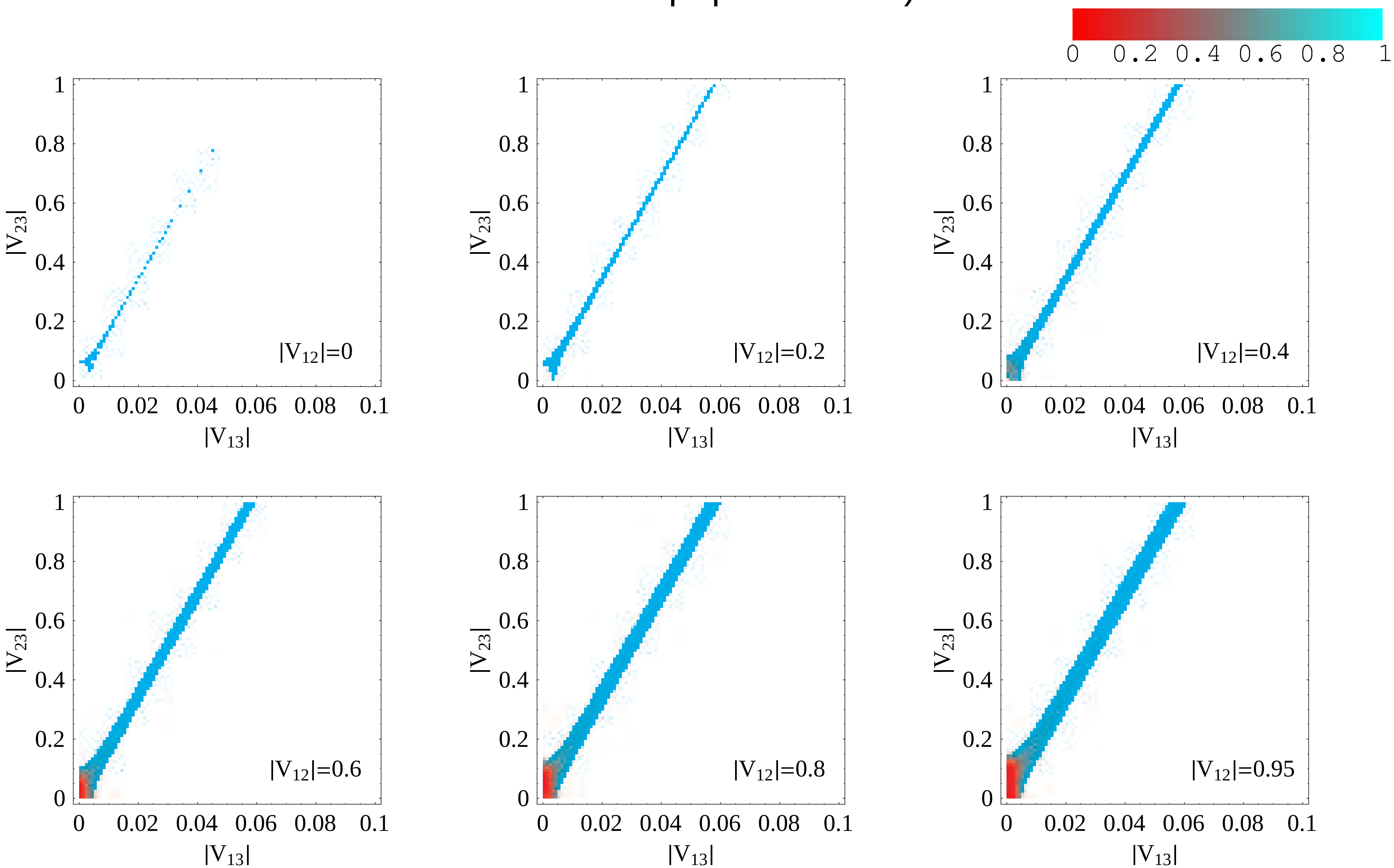
The see-saw mechanism (with two neutrinos) *can* accommodate the mild hierarchy and this essentially fi xes the parameters in the right-handed sector!

$$\cos^2\theta_R \simeq \frac{M_2 y_2^2 - M_1 y_1^2}{(M_1+M_2)(y_2^2-y_1^2)}, \quad \alpha \simeq \pi/2$$

Three generation case

$$y_1 : y_2 : y_3 = 1 : 300 : 300^2$$
$$M_1 : M_2 : M_3 = 1 : 300 : 300^2$$

(Inspired by the hierarchy in the up quark sector)



The see-saw mechanism (with three neutrinos) *can* accommodate the mild hierarchy and this constrains severely the parameters in the right-handed sector!

⋆ What about the mixing angles?

The observed mixing matrix can always be accommodated, even if the neutrino Yukawa eigenvalues are hierarchical.

Unfortunately, here we cannot get any guidance from the quark sector: the CKM matrix has small mixing angles, but the matrices that diagonalize the Yukawa couplings could have large or small mixing angles.

Is the see-saw mechanism testable?

⋆ The physics responsible for ν masses is not directly accessible to experiments.

1 TeV Linear Collider $\sim$ 100 MW
10^{12}GeV Linear Collider $\sim 10^{11}$ MW

Is the see-saw mechanism testable?

$\star$ The physics responsible for ν masses is not directly accessible to experiments.

1 TeV Linear Collider $\sim$ 100 MW
10^{12}GeV Linear Collider $\sim 10^{11}$ MW
annual power consumption in the world $\sim 10^{7}$ MW

Is the see-saw mechanism testable?

⋆ The physics responsible for ν masses is not directly accessible to experiments.

1 TeV Linear Collider $\sim$ 100 MW
10^{12}GeV Linear Collider $\sim 10^{11}$ MW
annual power consumption in the world $\sim 10^{7}$ MW

⋆ Indirect tests.

- In the Standard Model, hopeless.
  Part of the information is lost in the decoupling process and there is no way of recovering it.

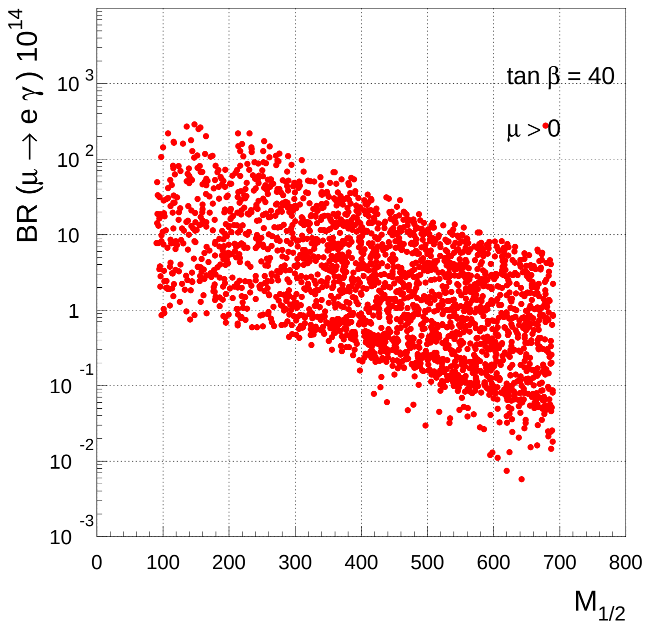
- In the Minimal Supersymmetric Standard Model, there could be indirect tests

The observation of processes involving supersymmetric particles could provide indications of neutrino Yukawa couplings

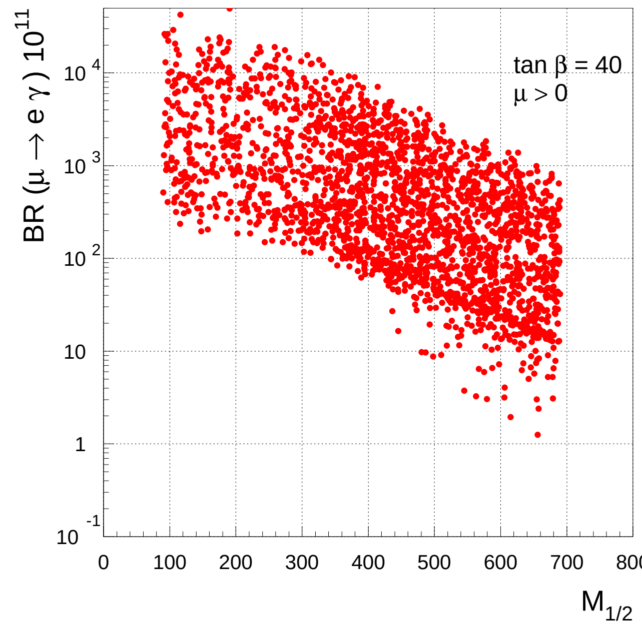
⋆ Rare lepton decays

process	present bound	future sensitivity
$BR(\mu \to e\gamma)$	$< 1.2 \times 10^{-11}$ (MEGA)	10^{-14} (MEG)
$BR(\tau \to \mu\gamma)$	$< 3.1 \times 10^{-7}$ (Belle) $< 6.8 \times 10^{-8}$ (BaBar)	$10^{-8} - 10^{-9}$ (SuperB-fact)
$BR(\tau \to e\gamma)$	$< 3.9 \times 10^{-7}$ (Belle)	$10^{-8} - 10^{-9}$(SuperB-fact)
$R(\mu^- \mathrm{Ti} \to e^- \mathrm{Ti})$	$< 6.1 \times 10^{-13}$ (SINDRUM II)	10^{-18} (PRISM/PRIME) 10^{-19} (CERN ν-fact)

⋆ Rare lepton decays



$V_L = V_{CKM}$



$V_L = V_{MNS}$

⋆ Slepton mass splittings

At future colliders, it could be possible to determine the mass splitting between selectron and stau, so disentangling the effect of the radiative corrections from the tau and the heaviest RH neutrino (but what about the other mass splitting?)

⋆ Slepton mass splittings

At future colliders, it could be possible to determine the mass splitting between selectron and stau, so disentangling the effect of the radiative corrections from the tau and the heaviest RH neutrino (but what about the other mass splitting?)

⋆ Electric Dipole Moments

EDM	present bound	future sensitivity
d_e	$< 1.6 \times 10^{-27}$ e cm (Berkeley group)	10^{-29} e cm (Yale group, 3 years) 10^{-31} e cm (Yale group, 5 years) 10^{-35} e cm (LANL group)
d_μ	$< 7 \times 10^{-19}$ e cm	10^{-24} e cm (BNL) 10^{-26} e cm (ν-fact)
d_τ	$-2.2 < \mathrm{Re}(d_\tau) < 4.5(\times 10^{-17})$ e cm	—

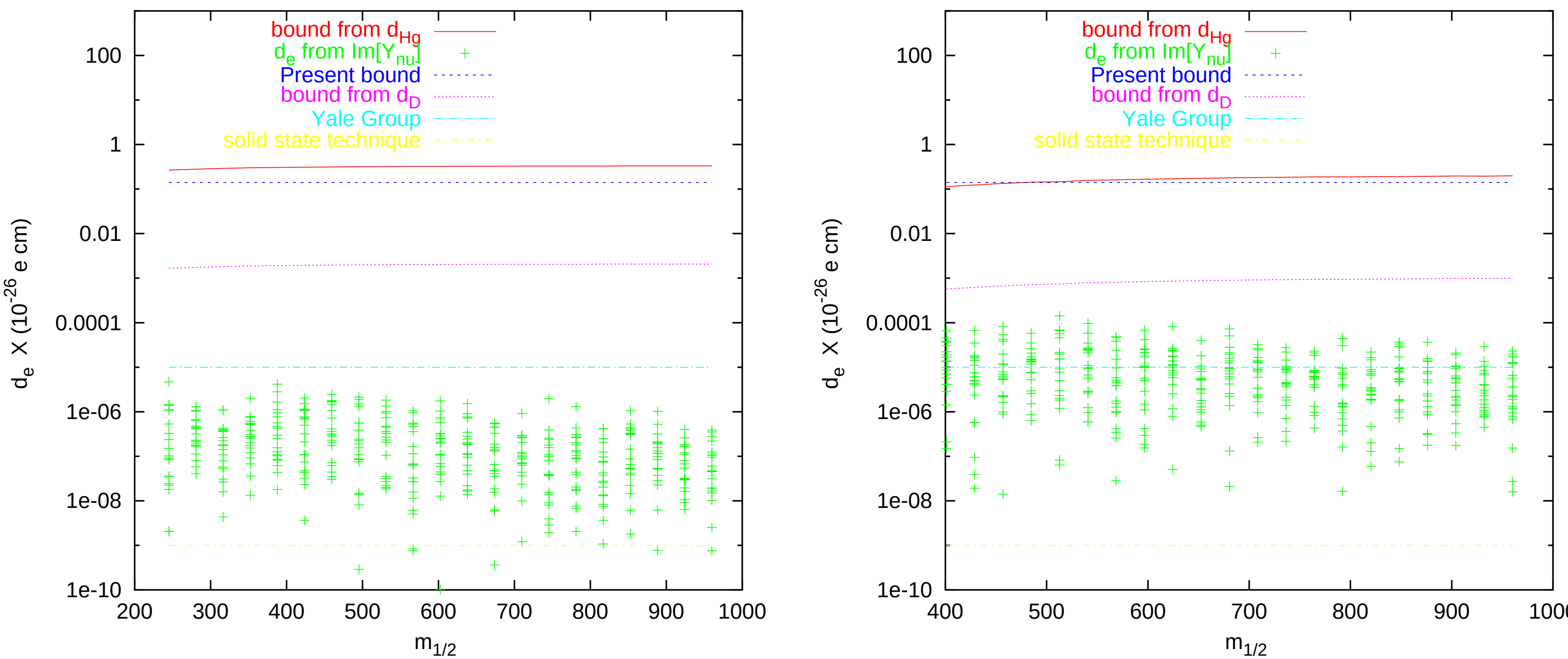
⋆ Slepton mass splittings

At future colliders, it could be possible to determine the mass splitting between selectron and stau, so disentangling the effect of the radiative corrections from the tau and the heaviest RH neutrino (but what about the other mass splitting?)

⋆ Electric Dipole Moments

$$\tan\beta = 20, \ \mathrm{sign}(\mu) = +$$

Demir, Farzan



$A_0 = 0$ $\qquad$ $A_0 = 1000$ GeV

Leptogenesis

- In the Universe there is more matter than antimatter, to be precise

$$\eta_B = (n_B - n_{\bar{B}})/s \simeq (0.3 - 0.9) \times 10^{-10}$$

- The decay of the right-handed neutrinos in the early Universe can explain this number, through the mechanism of leptogenesis (Fukugita, Yanagida)
- The leptogenesis mechanism relies on the physics of right-handed neutrinos, that is not directly accessible to experiments. However, the observed mass splittings in oscillations experiments constrain the leptogenesis mechanism:
  - $M_1 \gtrsim 10^9$GeV $\rightarrow t \lesssim 10^{-15} s$ (Davidson, A.I)
  - $m_3 \lesssim 0.1$eV (Buchmüller, di Bari, Plümacher)

• The type I see-saw mechanism can accommodate naturally the small neutrino masses, the mild mass hierarchy and the observed pattern of mixing angles. As a bonus, it can also explained the observed baryon asymmetry of the Universe.

• Also, if SUSY exists in nature, new phenomena due to the see-saw could be observed, like rare decays and electric dipole moments.

• But, is it type I see-saw or something else? Unfortunately, no smoking gun for the type I see-saw has been found. More work in this direction is needed.

Type II see-saw mechanism

Add to the Standard model particle content one Higgs triplet:

$$T = \begin{pmatrix} T^0 & -\frac{1}{\sqrt{2}}T^+ \\ -\frac{1}{\sqrt{2}}T^+ & -T^{++} \end{pmatrix}$$

T couples to the lepton doublets

$$-\mathcal{L}_{lep} \supset \mathbf{Y^T_{ij}} L_i T L_j = \mathbf{Y^T_{ij}}[\nu_L T^0 \nu_L - \sqrt{2}\nu_L T^+ e_L - e_L T^{++} e_L]$$

If T^0 acquires a vev, neutrinos become massive.

⋆ Small neutrino masses?

The most general Higgs potential with one doublet and one triplet (without imposing any lepton number conservation):

$$V = m_H^2 H^\dagger H + \frac{1}{2}\lambda_1 (H^\dagger H)^2 + M_T^2 T^\dagger T + \frac{1}{2}\lambda_2 (T^\dagger T)^2 + \lambda_3 (H^\dagger H)(T^\dagger T) + \mu_{\not{L}} H^\dagger T H^\dagger$$

When $M_T \gg m_H$, the minimum of the potential lies at:

$$\langle H^0 \rangle^2 \simeq \frac{-m_H^2}{\lambda_1 - 2\mu_{\not{L}}^2/M_T^2} \qquad \langle T^0 \rangle \simeq \frac{-\mu_{\not{L}}\langle H^0\rangle^2}{M_T^2}$$

Another seesaw!

⋆ Mild mass hierarchy?

This Yukawa coupling is not really analogous to Y_u, Y_d, Y_e. The naturalness of the mild mass hierarchy remains unanswered. (But a mild hierarchy can be accommodated)

⋆ Large mixing angles?

Again, no guidance from Y_u, Y_d, Y_e. (But large mixing angles can be accommodated)

⋆ Is it testable?

Without SUSY, hopeless. With SUSY, promising.

$$\frac{BR(\tau\to\mu\gamma)}{BR(\mu\to e\gamma)} \approx \left(\frac{m_3}{m_2}\right)^4 \left(\frac{\sin 2\theta_{atm}}{\sin 2\theta_{sol}\,\cos\theta_{atm}}\right)^2 \frac{BR(\tau\to\mu\nu_\tau\bar{\nu}_\mu)}{BR(\mu\to e\nu_\mu\bar{\nu}_e)} \approx 10^3$$

$$\frac{BR(\tau\to e\gamma)}{BR(\mu\to e\gamma)} \approx \tan^2\theta_{atm}\,\frac{BR(\tau\to e\nu_\tau\bar{\nu}_e)}{BR(\mu\to e\nu_\mu\bar{\nu}_e)} \approx 0.1$$

The observation of these correlations would point to the type II see-saw

Conclusions

- Many new proposals to explain the recent neutrino oscillation experiments. These proposals have to address the following questions:
  - Why neutrino masses are tiny
  - Why the mass hierarchy is mild
  - Why there are two large mixing angles

  And if possible, they should be testable in future experiments.

Conclusions

- Many new proposals to explain the recent neutrino oscillation experiments. These proposals have to address the following questions:
  - Why neutrino masses are tiny
  - Why the mass hierarchy is mild
  - Why there are two large mixing angles

  And if possible, they should be testable in future experiments.
- Some possibilities:

	tiny masses	mild hierarchy	large angles	testability
Dirac	unnatural	unnatural	?, but possible	$\nu 0\beta\beta$
type I see-saw	natural	natural	?, but possible	potentially testable (SUSY)
type II see-saw	natural	?, but possible	?, but possible	ratios of BRs (SUSY)

Conclusions

- Many new proposals to explain the recent neutrino oscillation experiments. These proposals have to address the following questions:
  - Why neutrino masses are tiny
  - Why the mass hierarchy is mild
  - Why there are two large mixing angles

  And if possible, they should be testable in future experiments.
- Some possibilities:

	tiny masses	mild hierarchy	large angles	testability
Dirac	unnatural	unnatural	?, but possible	$\nu 0\beta\beta$
type I see-saw	natural	natural	?, but possible	potentially testable (SUSY)
type II see-saw	natural	?, but possible	?, but possible	ratios of BRs (SUSY)

More work needed!