Neutrino Dark Energy – Revisiting the Stability Issue

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What is the nature of Dark Energy?

Neutrino Dark Energy (Mass Varying Neutrinos)

[Fardon, Nelson, Weiner '03]

Idea of varying neutrino masses in other contexts

[Kawasaki, Murayama, Yanagida '92, Stephenson et al '97]

- Attractive scalar force between Big Bang relic neutrinos (the analog of the Cosmic Microwave Background (CMB) photons) → **smooth** background, can form a **negative pressure** fluid
 - → acts as a form of Dark Energy → accelerated expansion
 - → neutrino mass m_ν becomes a function of neutrino energy density $\rho_\nu(\mathbf{z})$, which evolves on cosmological time scales (here parametrized in terms of cosmic redshift z)
- **Neutrino mass** not constant, but promoted to a **dynamical** quantity $m_\nu(\mathbf{z})$!

Mass Varying Neutrino (MaVaN) Scenario

The non-SM neutrino interaction mediated by a scalar field

- Introduce a light scalar field ϕ with mass
 $m_{\phi,0} \gg H_0 \sim 10^{-33} \text{eV}$
- Introduce a coupling between neutrinos ν and ϕ
- $\rightarrow$ Consider class of models with

$$\mathcal{L} \supset \mathcal{L}_\phi + \mathcal{L}_{\nu_{\text{kin}}} + \mathcal{L}_{\nu_{\text{mass}}}, \text{ where}$$

$$\mathcal{L}_\phi = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V_\phi(\phi)$$

$$\mathcal{L}_{\nu_{\text{mass}}} = -m_\nu(\phi) \bar{\nu} \nu + h.c.$$

- $\rightarrow$ neutrino mass $m_\nu(\phi)$ is generated from the VEV of ϕ and becomes linked to its dynamics
- $\rightarrow$ neutrinos interact through a new non-SM force

Complex interplay between ν and ϕ

- Neutrino energy density ρ_ν and pressure p_ν are functions of neutrino mass $m_\nu(\phi) \rightarrow \rho_\nu(m_\nu(\phi)), p_\nu(m_\nu(\phi))$
- $\rightarrow$ Neutrinos can stabilize ϕ by contributing to its effective potential $V_{\text{eff}}(\phi) = [\rho_\nu(m_\nu(\phi)) - 3p_\nu(m_\nu(\phi))] + V_\phi(\phi)$
- Evolution of ϕ governed by modified Klein-Gordon equation

$$\ddot{\phi} + 2H\dot{\phi} + a^2 V'_\phi = -a^2 \underbrace{\frac{d \log m_\nu}{d\phi}}_{\text{coupling } \beta} (\rho_\nu - 3p_\nu), \text{ with } (' = d/d\phi)$$

- Extra source term on RHS accounts for energy exchange between ϕ and neutrinos
- As long as neutrinos relativistic, coupling term suppressed ($\rho_\nu - 3p_\nu \sim 0$)

Adiabatic evolution in the non-relativistic regime

- Consider late-time dynamics of MaVaNs in the **non-relativistic limit** $m_\nu \gg T_\nu \rightarrow p_\nu \sim 0$, $\rho_\nu = m_\nu n_\nu$ ($n_\nu \equiv$ neutrino number density)

$$\longrightarrow V_{\text{eff}}(\phi) = \rho_\nu(m_\nu(\phi)) + V_\phi(\phi)$$

- In the limit $H^2 \ll V_{\text{eff}}''(\phi) = m_\phi^2$ adiabatic solution to EOM of ϕ apply (EOM: $\ddot{\phi} + 2H\dot{\phi} + a^2 V_{\text{eff}}'(\phi, z) = 0$, can safely neglect effects of kinetic energy terms)

- $\longrightarrow \phi$ instantaneously tracks the minimum of its effective potential $V_{\text{eff}} \rightarrow$

$$V'_{\text{eff}}(\phi, \mathbf{z}) = V'_\phi(\phi) + \underbrace{\rho'_\nu(m_\nu(\phi), \mathbf{z})}_{m'_\nu(\phi)n_\nu(\mathbf{z})} = 0 \text{ with } (' = \partial/\partial\phi)$$

Crucial effect: $n_\nu(\mathbf{z})$ is diluted by expansion $\rightarrow \phi$ **varies** on cosmological time scales (**slowly**)

Dynamics of the MaVaN Scenario

Neutrino mass varies

- $m_\nu(\phi) = m_\nu(\phi, \mathbf{z})$, $\rightarrow$
 $V_{\text{eff}}(\phi, \mathbf{z}) = V_{\text{eff}}(\mathbf{m}_\nu(\phi), \mathbf{z})$
 $= \underbrace{\rho_\nu(\mathbf{m}_\nu(\phi), \mathbf{z})}_{m_\nu(\phi) n_\nu(\mathbf{z})} + V_\phi(\mathbf{m}_\nu(\phi))$
- $\rightarrow \frac{\partial V_{\text{eff}}(\phi)}{\partial \phi} =$
 $\frac{\partial m_\nu}{\partial \phi} \frac{\partial V_{\text{eff}}(m_\nu)}{\partial m_\nu} \Big|_{m_\nu = m_\nu(\phi)} = 0$
- Neutrino mass variation determined from
 $\frac{\partial V_{\text{eff}}(m_\nu, \mathbf{z})}{\partial m_\nu} = 0 = n_\nu(\mathbf{z}) + \frac{\partial V_\phi(m_\nu)}{\partial m_\nu}$

$\rightarrow$ Combined scalar-neutrino fluid has dynamical Eq. of State $\omega(\mathbf{z}) \equiv \frac{\rho_{\text{DE}}(\mathbf{z})}{\rho_{\text{DE}}(\mathbf{z})}$

$$\omega(\mathbf{z}) + 1 = - \frac{m_\nu(\mathbf{z}) V'_\phi(m_\nu(\mathbf{z}))}{m'_\nu(\mathbf{z}) V_{\text{eff}}(m_\nu(\mathbf{z}))}$$

The Stability Issue in Models of Neutrino Dark Energy

Instabilities? Formation of dense neutrino bound states?

'In the **non-relativistic** neutrino **regime** any realistic MaVaN scenario with $m_\phi^2 \gg H^2 > 0$ is characterized by a **negative sound speed squared** $c_s^2 < 0$ and thus becomes **unstable** to hydrodynamic perturbations...with the likely outcome of the formation of non-linear structures in the neutrino density ('**neutrino nuggets**')

[Afshordi, Kohri, Zaldarriaga '05]

Note: Outcome of neutrino instability is an inherently non-linear process ...but
if 'nuggets' really form, they redshifts similar to cold dark matter with
 $\omega \sim 0 \approx -1 \rightarrow$ no acceleration (Quintessence? Cosmological Constant?)

Reconsider stability issue in framework of linear perturbation theory

Aim: Identification of condition for stabilization of the neutrino density contrast

The Stability Issue in Models of Neutrino Dark Energy

Instabilities

- Neutrino instabilities driven by attractive force mediated by ϕ
- Phenomenon similar to gravitational instabilities of CDM
- Good observational evidence, at early times universe homogeneous and isotropic on all scales
- Apart from small primeval perturbations $\delta\rho_i$ in densities ρ_i of each individual particle i

$$\rho_i(\mathbf{x}, \tau) = \underbrace{\rho_i(\tau)}_{\text{mean background density}} + \underbrace{\delta\rho_i(\mathbf{x}, \tau)}_{\text{small perturbation}}, \quad \underbrace{\delta_i(\mathbf{x}, \tau) \equiv \frac{\delta\rho_i(\mathbf{x}, \tau)}{\rho_i(\tau)}}_{\text{density contrast}}$$

- $\rightarrow$ grew by gravity into observable structure on scales of galaxies and clusters of galaxies
- Small amplitudes $|\delta\rho_i(\mathbf{x}, \tau)| \ll \rho_i(\tau) \leftrightarrow |\delta_i(\mathbf{x}, \tau)| \ll 1 \rightarrow$ growth of fluctuations can be solved from **linear perturbation theory**

The Stability Issue in Models of Neutrino Dark Energy

Gravitational instability in Newtonian theory

- Assume static (non-expanding) universe, consider perfect fluid, density ρ , pressure p , velocity $\mathbf{v}$ (Continuity eq. + Euler eq. + Newtonian gravity)
- Add small perturbations δp , $\delta \rho$, $\delta \mathbf{v}$ and linearise $\rightarrow$ for k^{th} Fourier component

$$\ddot{\delta}_k + \underbrace{(c_s^2 k^2)}_{\text{pressure}} - \underbrace{4\pi G \rho}_{\text{gravity}} \delta_k = 0, \text{ where } \omega = \sqrt{c_s^2 k^2 - 4\pi G \rho}$$

- Perturbations adiabatic ($c_s^2 = \frac{\dot{p}}{\dot{\rho}}$ adiabatic sound speed squared)
- $\rightarrow$ sign of ω^2 (which depends on c_s^2) determines perturbation evolution
- change of sign of ω^2 at critical value $k_{\text{Jeans}} = \sqrt{4\pi G \rho / c_s^2}$
- for $k < k_{\text{Jeans}}$: $\omega^2 < 0$ (gravity overcomes pressure) $\rightarrow \delta_k \propto e^{\pm|\omega|t}$, **growing solution**
- for $k > k_{\text{Jeans}}$: $\omega^2 > 0 \rightarrow \delta_k \propto e^{\pm i\omega t}$, no growth but acoustic **oscillations**

$\rightarrow$ sound speed squared c_s^2 **governs evolution** of density contrast δ_k

The Stability Issue in Models of Neutrino Dark Energy

Make contact with MaVaN instabilities

- MaVaNs interact through gravity and the force mediated by ϕ (both attractive), $4\pi G \rightarrow 4\pi G_{\text{eff}}(\beta(\phi))$
- Sound speed squared? For a general fluid i (with ' c_g ' general, ' c_s ' adiabatic, ' Γ_i ' intrinsic entropy perturbation)

$$w_i \Gamma_i = (c_{gi}^2 - c_{si}^2) \delta_i, \quad c_g^2 = \frac{\delta p_i}{\delta \rho_i}, \quad c_s^2 = \frac{\dot{p}_i}{\dot{\rho}_i}, \quad \delta_i(\mathbf{x}, \tau) \equiv \frac{\delta \rho_i(\mathbf{x}, \tau)}{\rho_i(\tau)}$$

- Dissipative processes invoke entropy perturbations ($\Gamma_i \neq 0$) [Hu' 98, Bean, Dore '03, ...]
- For MaVaNs? Depends on scales/regimes one considers!
- Relativistic neutrinos:** free-streaming and relativistic pressure support $\rightarrow$ no growth (on all scales)
- Non-relativistic neutrinos:** $p_\nu \sim 0 \rightarrow$ possible growth
- m_ϕ^{-1} sets physical length scales a/k as of which gradient terms become unimportant ($\Gamma_\phi \sim 0$) (for small deviations away from its minimum, ϕ re-adjusts to new minimum on a time scale $m_\phi^{-1} \ll H^{-1}$)

[Afshordi, Kohri, Zalduendo '05, Kaplinghat, Rajaraman '06]

On scales $m_\phi^{-1} < a/k < H^{-1}$ MaVaN **perturbations adiabatic** $\rightarrow \nu - \phi$ system can be treated as unified fluid with $\Gamma_{DE} = 0$ and $c_s^2 = \frac{\dot{p}_{DE}}{\dot{\rho}_{DE}} = \omega - \frac{\dot{\omega}}{3H(1+\omega)}$

Evolution of scalar field perturbations $\delta\phi$

where $\phi(\mathbf{x}, \tau) = \phi(\tau) + \delta\phi(\mathbf{x}, \tau)$

- Perturbed Klein-Gordon equation in the non-relativistic neutrino regime ($\rightarrow$ neglect terms $\propto \rho_\nu, \omega_\nu, c_\nu^2$ and $\dot{\phi}$)

$$\ddot{\delta\phi} + 2H\dot{\delta\phi} + [k^2 + a^2(V''_\phi + \beta'\rho_\nu)]\delta\phi = -a^2\beta\delta_\nu\rho_\nu$$

- Solution of homogenous equation is oscillating with decaying amplitude
- Particular solution given by forcing term on RHS

$$\delta\phi = -\frac{a^2\beta\rho_\nu\delta_\nu}{a^2(V''_\phi + \beta'\rho_\nu) + k^2}$$

[cf. eg. Amendola' 03, Koivisto' 05]

Equation of motion of the neutrino density contrast

$$\delta_\nu = \frac{\delta\rho_\nu}{\rho_\nu}$$

- Energy-momentum conservation equations for the coupled neutrinos

$$T_{\gamma;\mu}^\mu = \underbrace{\frac{d \log m_\nu}{d\phi}}_{\text{coupling } \beta} \phi, \gamma T_\alpha^\alpha,$$

where $T_{\mu\gamma}$ is the energy-momentum tensor

- → consider perturbed part in the non-relativistic neutrino regime

(where instabilities can possibly grow)

- → use

$$\delta\phi = -\frac{a^2 \beta \rho_\nu \delta_\nu}{a^2 (V_\phi'' + \beta' \rho_\nu) + k^2}$$

The Stability Issue in Models of Neutrino Dark Energy

Equation of motion of the neutrino density contrast

$$\delta_\nu = \frac{\delta\rho_\nu}{\rho_\nu}$$

In the non-relativistic neutrino regime on length scales $m_\phi^{-1} < a/k < H^{-1}$ with negligible neutrino shear and $p_\nu \sim \omega_\nu \sim 0$, $c_\nu^2 \ll 1$

$$\ddot{\delta} + (c_s^2 k^2 - 4\pi G\rho)\delta = 0$$

Compare: Newtonian theory, static universe, perfect fluid

$$\delta_b \simeq \delta_{\text{CDM}}$$

deep in matter-dom. regime

$$\ddot{\delta}_\nu + H\dot{\delta}_\nu + [c_\nu^2 k^2 - 4\pi a^2 G_{\text{eff}}\rho_\nu]\delta_\nu = 4\pi a^2 G [\rho_{\text{CDM}}\delta_{\text{CDM}} + \rho_b\delta_b]$$

$$G_{\text{eff}} = G \left[1 + \frac{2\beta^2 M_{\text{pl}}^2}{1 + a^2(V_\phi'' + \beta'\rho_\nu)/k^2} \right]$$

$$G[1 + 2\beta^2 M_{\text{pl}}^2] \gtrsim G_{\text{eff}} \gtrsim G$$

The Stability Issue in Models of Neutrino Dark Energy

Equation of motion of the neutrino density contrast δ_ν

In the non-relativistic neutrino regime on length scales $m_\phi^{-1} < a/k < H^{-1}$ with negligible neutrino shear and $p_\nu \sim \omega_\nu \sim 0$, $c_\nu^2 \ll 1$

$$\Omega_i = \frac{8\pi G a^2}{3H^2} \rho_i$$

$$G_{\text{eff}} = G \left[1 + \frac{2\beta^2 M_{\text{pl}}^2}{1 + a^2 (V_\phi'' + \beta' \rho_\nu) / k^2} \right]$$

$$\delta_b \simeq \delta_{\text{CDM}}$$

deep in matter-dominated regime

$$\ddot{\delta}_\nu + \underbrace{H\dot{\delta}_\nu + \left[c_\nu^2 k^2 - \frac{3}{2} H^2 \frac{G_{\text{eff}}}{G} \underbrace{\Omega_\nu}_{\sim 10^{-4} \dots 0.02} \right]}_{\simeq \left(\frac{c_s^2}{(c_s^2 + 1)} k^2 - \frac{3}{2} H^2 \Omega_\nu \right) \delta_\nu} \delta_\nu = \frac{3}{2} H^2 \left[\underbrace{\Omega_{\text{CDM}}}_{\sim 0.22} + \underbrace{\Omega_b}_{\sim 0.04} \right] \delta_{\text{CDM}}$$

The Stability Issue in Models of Neutrino Dark Energy

Equation of motion of the neutrino density contrast δ_ν

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$$\frac{G_{\text{eff}}}{G} \Omega_\nu \ll [\Omega_{\text{CDM}} + \Omega_b]$$

Dynamics of δ_ν governed by CDM $\rightarrow$ moderate growth like ordinary gravitational instabilities ('neutrinos follow CDM') $\rightarrow$ up to the present time
 $\delta_\nu \ll 1$ ($\equiv$ stability) possible

$$\frac{G_{\text{eff}}}{G} \Omega_\nu \gg [\Omega_{\text{CDM}} + \Omega_b]$$

The Stability Issue in Models of Neutrino Dark Energy

Equation of motion of the neutrino density contrast δ_ν

In the non-relativistic neutrino regime on length scales $m_\phi^{-1} < a/k < H^{-1}$ with negligible neutrino shear and $p_\nu \sim \omega_\nu \sim 0$, $c_\nu^2 \ll 1$

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$\frac{G_{\text{eff}}}{G} \Omega_\nu \ll [\Omega_{\text{CDM}} + \Omega_b]$	$\frac{G_{\text{eff}}}{G} \Omega_\nu \gg [\Omega_{\text{CDM}} + \Omega_b]$
<p>Dynamics of δ_ν governed by CDM $\rightarrow$ moderate growth like ordinary gravitational instabilities ('neutrinos follow CDM') $\rightarrow$ up to the present time</p> <p>$\delta_\nu \ll 1$ ($\equiv$ stability) possible</p>	<p>Dynamics of δ_ν governed by the strong coupling $\rightarrow$ depending on coupling function $\beta(\phi(z))$ (faster than) exponential growth $\rightarrow \delta_\nu \gg 1$ ($\equiv$ instability)</p>

The Stability Issue in Models of Neutrino Dark Energy

Any realistic MaVaN scenario $c_s^2 < 0$?

- Require $c_s^2 = \frac{\dot{\rho}_{\text{DE}}}{\rho_{\text{DE}}} = \omega - \frac{\dot{\omega}}{3H(1+\omega)} \geq 0$ for $m_\nu(z) \gg T_\nu(z)$ (take into account finite temperature effects)

$$\rightarrow \sum_{i=1}^3 \frac{\partial m_{\nu_i}(z)}{\partial z} \left(1 - \frac{5\alpha T_{\nu,0}^2(1+z)^2}{3m_{\nu_i}^2(z)} \right) + \sum_{i=1}^3 \frac{25\alpha T_{\nu,0}^2(1+z)}{3m_{\nu_i}(z)} \geq 0, \text{ with } \alpha \equiv \frac{\int_0^\infty \frac{dy y^4}{e^y + 1}}{2 \int_0^\infty \frac{dy y^2}{e^y + 1}}$$

[Takahashi, Tanimoto '06]

- Assume degenerate mass spectrum with $m_{\nu_i}(0) \sim m_\nu(0) = 0.312$ eV, $i = 1, 2, 3$
 $\rightarrow$ determine maximally allowed neutrino mass variation

Requirement of $c_s^2 \geq 0$ strongly restricts the allowed mass variation at late times

The Stability Issue in Models of Neutrino Dark Energy

A concrete model

Consider model proposed in the context of 'Chameleon cosmologies'

[Khoury, Weltman '03, Brax, van de Bruck, Davis, Khoury, Weltman '04, ...]

- **Recall:** evolution of ϕ determined by $V'_{\text{eff}}(\phi) = 0 = V'_\phi(\phi) + \rho'_\nu(m_\nu(\phi))$

- Exponential potential

$$V_\phi(\phi) = M^4 e^{\frac{M^n}{\phi^n}}$$

- Exponential dependence of m_ν on ϕ

$$m_\nu(\phi) = \tilde{m} e^{\beta\phi}, \text{ where } \overbrace{\beta}^{\text{coupling}} = \frac{d \log m_\nu}{d\phi} = \text{const.}$$

- Typically, $\beta\phi \ll 1 \rightarrow m_\nu$ very weakly depends on changes in the neutrino energy density $\rightarrow m_\nu$ **hardly evolves with time**
- $\rightarrow$ attractive force between neutrinos essentially time independent

The Stability Issue in Models of Neutrino Dark Energy

A stable model

- Normalization?
 - For $k = 0.11 h \text{ Mpc}^{-1}$
$$\Delta^2(k) = \frac{k^3 P(k)}{2\pi^2} \propto \delta_{\text{CDM}}^2 \ll 1$$
→ **linear**
- [Percival et al.'06]
- Since $\tilde{\delta}_\nu^2 < \tilde{\delta}_{\text{CDM}}^2 \rightarrow$ neutrino density contrast linear, $\delta_\nu^2 \ll 1 =$ **no 'neutrino nuggets'!**
- → Adiabatic model of Neutrino Dark Energy **stable** also in the highly non-relativistic regime → **viable dark energy candidate**

$$k = 0.11 h \text{ Mpc}^{-1}, \beta = 1/M_{\text{pl}}, m_{\nu_i}(z=0) = 0.312 \text{ eV}$$

The Stability Issue in Models of Neutrino Dark Energy

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Another model

Proposed by Fardon, Nelson, Weiner '05

- Logarithmic scalar potential (V_0 fixed by requirement of $\Omega_{\text{DE}} \sim 0.7$)

$$V_\phi(\phi) = V_0 \log(1 + \kappa\phi), \text{ with } V_0, \kappa = \text{const.}$$

- Mass dependence on ϕ as preferred in the MaVaN literature [Fardon, Nelson, Weiner '05, '06, Afshordi, Kohri, Zaldarriaga '05, Spitzer '06...]

$$m_\nu(\phi) = \frac{\tilde{m}^2}{\phi}, \text{ where } \overbrace{\beta}^{\text{coupling}} = \frac{d \log m_\nu}{d\phi} = -\frac{1}{\phi} \neq \text{const.}$$

- dependence $m_\nu(\phi)$ naturally arises from integrating out a heavier sterile state, whose mass varies linearly with the value of ϕ ('MaVaN seesaw')
- m_ν strongly depends on changes in the scalar field VEV
- since ϕ decreases, $|\beta|$ increases with time $\rightarrow$ attractive force between neutrinos increases with time

The Stability Issue in Models of Neutrino Dark Energy

An unstable model

- Rapid evolution of $m_\nu(z)$
- $m_\nu(z) \ll T_\nu(z)$ (non-relativistic regime): **pressure** support **deminishes** $\rightarrow c_s^2$ driven to **negative** values

Recall:

$$\ddot{\delta}_\nu + H\dot{\delta}_\nu + [c_\nu^2 k^2 - \frac{3}{2}H^2 \frac{G_{\text{eff}}}{G}\Omega_\nu]\delta_\nu = \frac{3}{2}H^2 [\Omega_{\text{CDM}} + \Omega_b]\delta_{\text{CDM}}$$

- As soon as coupling is large enough to compensate for small neutrino mass (and thus Ω_ν) $\rightarrow$
- $\delta_\nu \gg 1 \rightarrow$ model **unstable** before today $\rightarrow$ **excluded** as DE candidate

$$k = 0.11 h \text{Mpc}^{-1}, \beta \neq \text{const.}, m_{\nu_i}(z=0) = 0.312 \text{ eV}$$

The Stability Issue in Models of Neutrino Dark Energy

An unstable model

- Rapid evolution of $m_\nu(z)$
- $m_\nu(z) \ll T_\nu(z)$ (non-relativistic regime): **pressure support diminishes** $\rightarrow c_s^2$ driven to **negative** values

Recall:

$$\ddot{\delta}_\nu + H\dot{\delta}_\nu + [c_\nu^2 k^2 - \frac{3}{2}H^2 \frac{G_{\text{eff}}}{G} \Omega_\nu] \delta_\nu = \frac{3}{2}H^2 [\Omega_{\text{CDM}} + \Omega_b] \delta_{\text{CDM}}$$

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$$k = 0.11 h \text{Mpc}^{-1}, \beta \neq \text{const.}, m_{\nu_i}(z=0) = 0.312 \text{ eV}$$

Summary

- Reconsideration of the **stability issue** in models of adiabatic neutrino dark energy
- Other cosmic components (CDM and baryons) can have stabilizing effect on MaVaN perturbations
- If $\frac{G_{\text{eff}}}{G} \Omega_\nu \ll [\Omega_{\text{CDM}} + \Omega_b] \rightarrow$ **moderate growth** of perturbations as in general relativity $\rightarrow \delta_\nu \ll 1$ ($\equiv$ **stability**) **possible**
- If **strong coupling** compensates for relative smallness of $\Omega_\nu \rightarrow \delta_\nu \gg 1$ ($\equiv$ **instability**) in the non-relativistic regime
- Viable model of neutrino dark energy found with $c_s^2 > 0 \rightarrow$ allowed **mass variation** strongly **restricted** at late times
- Note: non-adiabatic models of neutrino dark energy with $m_\phi \sim H$ are stable
- Note: 'Hybrid' models involving two light scalar fields can be stable until the present time even in the presence of unstable neutrino component

[Brookfield, van de Bruck, Mota, Tocchini-Valentini '06, Afshordi, Kohri, Zalzarriaga '05]

[Fardon, Nelson, Weiner '06, Spitzer '06]