Reconstructing see-saw models

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Abstract. In this talk we discuss the prospects to reconstruct the high-energy see-saw Lagrangian from low energy experiments in supersymmetric scenarios. We show that the model with three right-handed neutrinos could be reconstructed in theory, but not in practice. Then, we discuss the prospects to reconstruct the model with two right-handed neutrinos, which is the minimal see-saw model able to accommodate neutrino observations. We identify the relevant processes to achieve this goal, and comment on the sensitivity of future experiments to them. We find the prospects much more promising and we emphasize in particular the importance of the observation of rare leptonic decays for the reconstruction of the right-handed neutrino masses.

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INTRODUCTION

Neutrino observations can be well described by adding a dimension five operator to the Standard Model Lagrangian [1]:

$$\mathscr{L}_{eff} = \mathscr{L}_{SM} - \frac{1}{2}\kappa_{ij}(L_i \cdot H)^T (L_j \cdot H) + h.c. \tag{1}$$

so that after the electroweak symmetry breaking a Majorana mass term for the neutrinos is generated:

$$\mathscr{M}_\nu = \kappa_{ij}\langle H^0\rangle^2. \tag{2}$$

One of the most pressing questions in neutrino physics is to determine the origin of this effective operator. Among all the proposals, the type I see-saw mechanism is perhaps the most plausible. It consists on adding to the Standard Model particle content three right-handed neutrino fields, so that the most general Lagrangian compatible with the Standard Model gauge symmetry reads:

$$\mathscr{L}_{eff} = \mathscr{L}_{SM} + \nu_R^{cT}\mathbf{Y}_\nu L \cdot H - \frac{1}{2}\nu_R^{cT}\mathbf{M}_\nu\ \nu_R^c + h.c. \tag{3}$$

If the right-handed neutrino masses are much larger than the scale of electroweak symmetry breaking, the right-handed neutrinos decouple at low energies and the theory can be well described by the effective Lagrangian given by eq.(1), with

$$\kappa_{ij} = \mathbf{Y}_\nu{}^T \mathbf{M}_\nu^{-1}\mathbf{Y}_\nu, \tag{4}$$

which is naturally suppressed by the large mass scale of the right-handed neutrinos, thus providing a natural explanation for the small neutrino masses.

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In this work we would like to discuss the possibility of reconstructing the complete see-saw Lagrangian, eq.(3), using just low energy experiments [2, 3]. There are several motivations to study this problem. Although the effective theory provides a good description of neutrino experiments, we would like to have a deeper understanding of the observed neutrino parameters. In particular, we would like to understand why neutrino masses are so small compared to the rest of the fermion masses, or why there are two large mixing angles, in stark contrast to the quark sector. Clearly, the answers to these questions necessarily lie in the fundamental theory and not in the effective theory.

As a more ambitious goal we would like to unravel the so-called flavour puzzle, namely why fermion masses and mixing angles have the structure they present. The non-trivial flavour structure observed in the leptonic sector indeed provides additional clues to the solution of the flavour puzzle, but again, these clues lie in the fundamental theory rather than in the effective theory. If the fundamental theory responsible for the leptonic flavour physics could be disentangled, one could look for patterns in the eigenvalues of the neutrino Yukawa coupling or in the right-handed neutrino masses, analogous to the intriguing relations observed in the quark sector, $m_u : m_c : m_t \sim \lambda^8 : \lambda^4 : 1$ and $m_d : m_s : m_b \sim \lambda^4 : \lambda^2 : 1$, being λ the Cabibbo angle. In addition, one could look in the fundamental neutrino parameters for similarities with the charged lepton sector or with the quark sector, in the quest for hints of some possible underlying Grand Unified Theory. Needless to say, the ultimate solution to the flavour puzzle would require an understanding of the origin of Yukawa couplings, which is still lacking.

Finally, it would be interesting to determine the scale of new physics, namely the masses of the right-handed neutrinos. Apart from the intrinsic interest of determining parameters of the complete Lagrangian, determining the scale of new physics could have implications for some models beyond the Standard Model, that are intimately related to some particular mass scale. For example, some Grand Unified Theories predict masses for the right-handed neutrinos close to the gauge unification scale, $M_X \sim 10^{15-16}$GeV. The determination of masses for the right handed neutrinos close to this scale would undoubtedly give strong support to this class of models. Also, the mechanism of leptogenesis to produce the observed baryon asymmetry of the Universe heavily relies on the mass of the lightest right-handed neutrino. The determination of this mass in the range favoured by leptogenesis would also give strong support to this mechanism.

APPROACHES TO DETERMINE SEE-SAW PARAMETERS

The approaches to reconstruct the high-energy see-saw Lagrangian, can be roughly classified into two main classes: top-down approach and bottom-up approach.

The top-down approach consists on selecting a particular model at high energies (a Grand Unified Model, a Froggatt-Nielsen model, a superstring model...), compute the low energy predictions and compare with the experiments. This approach is completely motivated by theory and has the advantage that the analysis is restricted just to scenarios that are very well motivated from the theoretical point of view. Nevertheless, there is still a huge freedom and many different possibilities arise. And what is more frustrating, despite these scenarios are *a priori* the best candidates as fundamental see-saw models, the simplest ideas do not seem to work, indicating perhaps that we are being mislead by our theoretical prejudices.

A radically different approach is the bottom-up approach, that consists on exploiting all the information available at low energies on the leptonic sector, in order to reconstruct the high-energy theory. Contrary to the top-down approach, the bottom-up approach is completely phenomenological and it is impossible to get mislead by aesthetics. However, it is very difficult to realize in practice.

In particular, following this approach in the Standard Model is hopeless. In the basis where the charged lepton Yukawa coupling and the right-handed mass matrix are diagonal, the complete Lagrangian depends on three independent parameters in the right-handed mass matrix (the three right-handed masses), and fifteen independent parameters in the neutrino Yukawa coupling, of which nine are moduli and six are phases. On the other hand, in the effective theory the neutrino mass matrix depends just on nine parameters: three masses, three mixing angles, and three phases. Clearly, part of the information about the complete Lagrangian has been lost in the decoupling process and cannot be recovered just from neutrino experiments, to be precise six real parameters and three phases.

On the other hand, in the Minimal Supersymmetric Standard Model (MSSM) there is additional information about the high-energy see-saw Lagrangian. Namely, radiative corrections on slepton parameters carry information about the combination $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \equiv P$. This information could disentangled from measurements in the slepton sector, provided the mechanism of supersymmetry breaking is specified. Assuming that the slepton mass matrices are proportional to the identity at the high energy scale, quantum effects induced by the right-handed neutrinos would yield at low energies a left-handed slepton mass matrix with a non-trivial structure, whose measurement would provide additional information about the see-saw parameters. To be more specific, the low energy left-handed slepton mass matrices read, in the leading-log approximation [4]:

$$(m^2_{\tilde{\ell},\tilde{\nu}})_{ij} \simeq (\text{diagonal part})_{\tilde{\ell},\tilde{\nu}} - \frac{1}{8\pi^2}(3m_0^2 + A_0^2)(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \log\frac{M_X}{M}\ , \tag{5}$$

where m_0 and A_0 are respectively the soft scalar mass and the soft trilinear term at the cut-off scale, M_X, and M is the mass scale of the right-handed neutrinos. In this formula, “diagonal-part” includes the tree level soft mass matrix, the radiative corrections from gauge and charged lepton Yukawa interactions, and the mass contributions from F- and D-terms (that are different for charged sleptons and sneutrinos). Therefore, the measurement at low energies of rare lepton decays, electric dipole moments and slepton mass splittings would allow the determination of $m^2_{\tilde{\ell},\tilde{\nu}}$, and consequently would provide information about the combination $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \equiv P$.

It is remarkable that the matrix P encodes precisely the necessary information to reconstruct the high-energy see-saw parameters [2]. This can be easily understood from the following procedure. Using the singular value decomposition $\mathbf{Y}_\nu = V_R D_Y V_L^\dagger$, with V_R and V_L unitary matrices and $D_Y = \text{diag}(y_1, y_2, y_3)$ the diagonal matrix of the Yukawa eigenvalues, the matrix P reads:

$$P \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu = V_L^\dagger D_Y^2 V_L. \tag{6}$$

Then, from the diagonalization of P, the matrices V_L and D_Y could be straightforwardly determined. On the other hand, from $\mathscr{M}_\nu = \mathbf{Y}_\nu^T D_{\mathbf{M}_\nu}^{-1} \mathbf{Y}_\nu \langle H_u^0 \rangle^2$ and the singular value decomposition of $\mathbf{Y}_\nu$, it follows that

$$D_Y^{-1} V_L^* \mathscr{M}_\nu V_L^\dagger D_Y^{-1} = V_R^* D_{\mathbf{M}_\nu}^{-1} V_R^\dagger, \tag{7}$$

where the left hand side of this equation is known ($\mathscr{M}_\nu$ is one of our inputs, and V_L and D_Y were obtained from eq. (6)). Therefore, V_R and $D_{\mathbf{M}_\nu}$ can also be determined. This simple procedure shows that starting from the low energy observables $\mathscr{M}_\nu$ and P it is possible to determine *uniquely* the matrices $D_{\mathbf{M}_\nu}$ and $\mathbf{Y}_\nu = V_R D_Y V_L^\dagger$.

In general, although the reconstruction of the high energy see-saw parameters from low energy observables is possible in theory, in practice it could be very difficult, if not impossible. The reconstruction procedure requires the measurement of all the parameters in $\mathscr{M}_\nu$ and P, which is not feasible at least with the present and proposed experiments. In particular, the measurement of the Majorana phases in the neutrino mass matrix, the mass splitting between the lightest sleptons, and the electric dipole moments of the muon and the tau do not seem possible even with the next round of experiments.

In the view of the limitations of the previous approaches, one might try to pursue a more humble approach, taking elements from both the top-down and the bottom-up approaches. This hybrid approach would consist on a bottom-up approach with some well-motivated hypotheses about the high-energy theory. For example, we could assume that the neutrino Yukawa coupling is symmetric, or that the neutrino Yukawa eigenvalues are in the relation $m_u : m_c : m_t$, inspired in $SO(10)$ Grand Unified Theories. Another possibility is to assume that the Yukawa couplings present texture zeros, inspired by the success of the Gatto-Sartori-Tonin relation in the quark sector. Finally we could assume that the Standard Model is extended with just two right-handed neutrinos instead of three, the so-called two right-handed neutrino (2RHN) model. In the rest of the talk we will concentrate in this last hypothesis: the two right-handed neutrino model.

THE TWO RIGHT-HANDED NEUTRINO MODEL

The phenomenological motivation to consider this model is that oscillation experiments indicate that two new mass scales have to be introduced, in order to account for the solar and atmospheric mass splittings. These two mass scales could be associated to the masses of two right-handed neutrinos, therefore a see-saw model with just two right-handed neutrinos can already accommodate all the observations, being the third one in principle not necessary.

Another motivation to consider the 2RHN model is that it corresponds to interesting limits of the complete three right-handed neutrino (3RHN) model, that is undoubtedly more plausible from the theoretical point of view. Let us discuss first the situations where the 3RHN model can be well approximated by the 2RHN model from the point of view of the neutrino mass matrix, and then the situations where it can be approximated also from the point of view of the radiative corrections, parametrized by the matrix P [3]. In the basis where the right-handed neutrino mass matrix is diagonal, the low energy effective neutrino mass matrix reads:

$$(\mathcal{M}_\nu)_{ij} = \frac{y_{1i}y_{1j}}{M_1} + \frac{y_{2i}y_{2j}}{M_2} + \frac{y_{3i}y_{3j}}{M_3}, \tag{8}$$

where $y_{ij} = (\mathbf{Y}_\nu)_{ij}$. Two right-handed neutrinos dominate the see-saw when

$$\begin{aligned} \frac{y_{1i}y_{1j}}{M_1} &\ll \frac{y_{2i}y_{2j}}{M_2}, \frac{y_{3i}y_{3j}}{M_3} \quad \text{or} \\ \frac{y_{2i}y_{2j}}{M_2} &\ll \frac{y_{1i}y_{1j}}{M_1}, \frac{y_{3i}y_{3j}}{M_3} \quad \text{or} \\ \frac{y_{3i}y_{3j}}{M_3} &\ll \frac{y_{1i}y_{1j}}{M_1}, \frac{y_{2i}y_{2j}}{M_2} \quad \text{for all } i,j = 1,2,3. \end{aligned} \tag{9}$$

The most interesting cases are the first and the third. The first one corresponds to the case in which the Yukawa couplings for the first generation of right handed neutrinos are tiny, $y_{1i} \ll y_{2i}, y_{3i}$, for $i = 1,2,3$. If this is the case, the radiative corrections are also dominated by the same two right-handed neutrinos, the two heaviest ones. Therefore, in this case the 3RHN model can be well approximated by a 2RHN model, both from the point of view of neutrino masses as of radiative corrections. Since the two relevant right-handed neutrinos are the two heaviest ones, the corresponding Yukawa couplings could be large, and the radiative corrections could be sizable.

The third case corresponds to the situation where the mass of the heaviest right-handed neutrino is much larger than the mass of the other two, $M_3 \gg M_1, M_2$. However, in general the heaviest right-handed neutrino will produce sizable contributions to the radiative corrections. If this is the case, the 3RHN model could be reduced to a 2RHN model only from the point of view of neutrino masses, but not from the point of view of the radiative corrections. Nevertheless, there are some circumstances in which the heaviest right-handed neutrino indeed does not contribute to the radiative corrections and does not leave any imprint in P, so that this matrix is only determined by the Yukawa couplings of the two lightest generations of right-handed neutrinos. If this occurs, the 3RHN model would also be well approximated by a 2RHN model from the point of view of the radiative corrections. This situation arises for example when the mass of the heaviest right-handed neutrino is very close to the Planck mass, although this possibility seems a bit contrived.

A more plausible situation arises in models with gauge mediated supersymmetry breaking. So far, we have implicitly assumed that the boundary conditions for the soft breaking terms are set at the Planck scale. However, if the mass of the messenger particles involved in the supersymmetry breaking mechanism is smaller than M_3 but larger than M_2, then the heaviest right-handed neutrino would decouple at an energy larger than the energy at which supersymmetry breaking is communicated to the observable sector. Consequently, it would not participate in the radiative corrections of the parameters of the Lagrangian. If this is the case, only the two lightest right-handed neutrinos would contribute to the radiative corrections and to the neutrino mass generation, and therefore the 3RHN model could be well approximated by a 2RHN model.

PARAMETRIZATIONS OF THE 2RHN MODEL

For the purposes of this study the most interesting feature of the 2RHN model is that it depends on many less parameters than the complete 3RHN model, while is still capable to accommodate neutrino observations. In the basis where the right-handed mass matrix is diagonal, the model is described by two right-handed neutrino masses and a 2×3 neutrino Yukawa coupling, that depends on six independent real parameters and there phases. In consequence, the complete Lagrangian depends on eleven parameters, of which eight are moduli and three are phases. On the other hand at low energies, in principle experiments could measure a total of eighteen parameters, of which twelve are moduli and six are phases. We find then that in the 2RHN model many predictions arise, both in the neutrino mass matrix and in the radiative effects [3].

To be specific, the effective neutrino mass matrix in the 2RHN model is rank 2, and therefore the lightest neutrino mass automatically vanishes and only two possible spectra may arise:

- Normal hierarchy: $m_1 = 0$, $m_2 = \sqrt{\Delta m^2_{sol}}$, $m_3 = \sqrt{\Delta m^2_{atm}}$
- Inverted hierarchy: $m_1 = \sqrt{\Delta m^2_{atm} - \Delta m^2_{sol}}$, $m_2 = \sqrt{\Delta m^2_{atm}}$, $m_3 = 0$

The only Majorana phase corresponds to the phase difference between the two non-vanishing mass eigenvalues. Therefore, the number of unmeasurable parameters in the 2RHN model is reduced to three moduli and one phase.

To derive the predictions on the radiative effects it is convenient to express the Yukawa coupling in the following form [5]

$$\mathbf{Y}_\nu = D_{\sqrt{\mathbf{M}_\nu}} R D_{\sqrt{m}} U^\dagger / \langle H_u^0 \rangle, \tag{10}$$

where $D_{\sqrt{m}} = \mathrm{diag}(\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3})$ is the diagonal matrix of the square roots of the light neutrino mass eigenvalues (which has $m_1 = 0$ for the normal hierarchy and $m_3 = 0$ for the inverted hierarchy), $D_{\sqrt{\mathbf{M}_\nu}} = \mathrm{diag}(\sqrt{M_1}, \sqrt{M_2})$ is the diagonal matrix of the square roots of the right handed neutrino masses, U is the leptonic mixing matrix, and R is an orthogonal matrix that in the 2RHN model with normal hierarchy has the following structure (in what follows we will concentrate on the case with normal hierarchy, since the analysis for the case with inverted hierarchy is completely analogous):

$$R = \begin{pmatrix} 0 & \cos z & \xi \sin z \\ 0 & -\sin z & \xi \cos z \end{pmatrix}, \tag{11}$$

with z a complex parameter and $\xi = \pm 1$ a discrete parameter that accounts for a discrete indeterminacy in R. In consequence, the elements of the neutrino Yukawa matrix read:

$$\begin{aligned} (\mathbf{Y}_\nu)_{1\alpha} &= \sqrt{M_1}(\sqrt{m_2}\cos z\, U^*_{\alpha 2} + \xi \sqrt{m_3}\sin z\, U^*_{\alpha 3})/\langle H_u^0 \rangle, \\ (\mathbf{Y}_\nu)_{2\alpha} &= \sqrt{M_2}(-\sqrt{m_2}\sin z\, U^*_{\alpha 2} + \xi \sqrt{m_3}\cos z\, U^*_{\alpha 3})/\langle H_u^0 \rangle, \end{aligned} \tag{12}$$

The three moduli and the phase that are not determined by low energy experiments are identified in this parametrization with the two right-handed masses M_1 and M_2, and the complex parameter z.

Notice that we have included all the low energy phases in the definition of the matrix U, *i.e.* we have written the leptonic mixing matrix in the form $U = V\ \mathrm{diag}(1, e^{-i\phi/2}, 1)$, where ϕ is the Majorana phase and V has the form of the CKM matrix:

$$V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \qquad (13)$$

so that the neutrino mass matrix is $\mathscr{M}_\nu = U^* \mathrm{diag}(m_1, m_2, m_3) U^\dagger$. It is straightforward to check that the Yukawa coupling eq.(10) indeed satisfies the see-saw formula $\mathscr{M}_\nu = \mathbf{Y}_\nu^T \mathrm{diag}(M_1^{-1}, M_2^{-1}) \mathbf{Y}_\nu \langle H_u^0 \rangle^2$.

The Yukawa coupling affects the renormalization group equation of the slepton parameters through the combination $P = \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, that depends in general on six moduli and three phases. Since the Yukawa coupling depends in the 2RHN model on only three unknown moduli and one phase, so does P, and consequently it is possible to obtain predictions on the moduli of three P-matrix elements and the phases of two P-matrix elements. Namely, from eq.(10) one obtains that:

$$U^\dagger P U = U^\dagger \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu U = D_{\sqrt{m}} R^\dagger D_{\mathbf{M}_\nu} R D_{\sqrt{m}} / \langle H_u^0 \rangle^2. \qquad (14)$$

In the case with normal hierarchy $m_1 = 0$, from where it follows that $(U^\dagger P U)_{1i} = 0$, for $i = 1, 2, 3$, leading to three relations among the elements in P. For instance, one could derive the diagonal elements in P in terms of the off-diagonal elements:

$$\begin{aligned} P_{11} &= -\frac{P_{12}^* U_{21}^* + P_{13}^* U_{31}^*}{U_{11}^*}, \\ P_{22} &= -\frac{P_{12} U_{11}^* + P_{23}^* U_{31}^*}{U_{21}^*}, \\ P_{33} &= -\frac{P_{13} U_{11}^* + P_{23} U_{21}^*}{U_{31}^*}. \end{aligned} \qquad (15)$$

The observation of these correlations would be non-trivial tests of the 2RHN model.

The relations for the phases arise from the hermicity of P, since the diagonal elements in P have to be real. Taking as the independent phase the argument of P_{12}, one can derive from eq.(15) the arguments of the remaining elements:

$$\begin{aligned} e^{i\arg P_{13}} &= \frac{-i\,\mathrm{Im}(P_{12}U_{21}U_{11}^*) \pm \sqrt{|P_{13}|^2 |U_{11}|^2 |U_{31}|^2 - [\mathrm{Im}(P_{12}U_{21}U_{11}^*)]^2}}{|P_{13}| U_{31} U_{11}^*}, \\ e^{i\arg P_{23}} &= \frac{i\,\mathrm{Im}(P_{12}U_{21}U_{11}^*) \pm \sqrt{|P_{23}|^2 |U_{21}|^2 |U_{31}|^2 - [\mathrm{Im}(P_{12}U_{21}U_{11}^*)]^2}}{|P_{23}| U_{31} U_{21}^*}, \end{aligned} \qquad (16)$$

where the $\pm$ sign has to be chosen so that the eigenvalues of P are positive. It is important to remark that the hermicity of P is not guaranteed for any value of P_{12}, $|P_{13}|$, $|P_{23}|$; only some particular ranges for the parameters are allowed, corresponding to the values for which the arguments of the square roots in eq.(16) are positive.

We conclude then that the P-matrix parameters P_{12}, $|P_{13}|$ and $|P_{23}|$ can be regarded as independent and can be used in an alternative parametrization of the 2RHN model. Together with the five moduli and the two phases of the neutrino mass matrix, sum up to the eight moduli and the three phases necessary to reconstruct the high-energy Lagrangian of the 2RHN model. Interestingly enough, most of the low energy parameters required to reconstruct the 2RHN model have good prospects to be measured or at least to be further constrained by future experiments. Concerning the parameters of the neutrino mass matrix, two mixing angles and the three masses have already been determined, under the assumption of normal hierarchy. On the other hand θ_{13} is already constrained to be small, $\sin^2\theta_{13} < 0.046$ at 3σ [1], and will be further constrained by future experiments, the phase δ could be measured with superbeams, and the only parameter that will remain poorly determined is the Majorana phase. Finally, the parameters from the matrix P could be determined or constrained by measurements of rare lepton decays and the electric dipole moment of the electron, for which there are good perspectives in the next few years.

THE RECONSTRUCTION PROCEDURE

In Section 2 we have discussed that the complete Lagrangian can be written in terms of the five moduli and two phases of the neutrino mass matrix, and the three independent moduli and two phases of the matrix P, that is involved in the radiative corrections of the slepton parameters. In this section we will derive *exact* formulas for the high energy parameters in terms of these low energy parameters.

To this end, we will use the parametrization of the Yukawa couplings in eq.(10), so that all our ignorance of the high energy theory is encoded in the right-handed neutrino masses, M_1 and M_2, and the complex parameter in the matrix R, z. Let us define the hermitian matrix $Q \equiv U^\dagger P U$, that depends *exclusively* on parameters that in principle could be measured in low energy experiments. The first row and column vanish and yield the relations among the P-matrix elements already presented in eq.(15). On the other hand, the remaining elements Q_{22}, Q_{23}, Q_{33}, can be written in terms of the high-energy parameters M_1, M_2 and z. Therefore, one can invert the equations to derive exact expressions for the high-energy parameters in terms of the low energy parameters in Q. These expressions are [3] :

$$\begin{aligned}
M_1 &= \frac{1}{2}\left[\sqrt{\left(\frac{Q_{33}}{m_3}+\frac{Q_{22}}{m_2}\right)^2+\frac{(Q_{23}-Q_{23}^*)^2}{m_2 m_3}}-\sqrt{\left(\frac{Q_{33}}{m_3}-\frac{Q_{22}}{m_2}\right)^2+\frac{(Q_{23}+Q_{23}^*)^2}{m_2 m_3}}\right]\langle H_u^0\rangle^2,\\
M_2 &= \frac{1}{2}\left[\sqrt{\left(\frac{Q_{33}}{m_3}+\frac{Q_{22}}{m_2}\right)^2+\frac{(Q_{23}-Q_{23}^*)^2}{m_2 m_3}}+\sqrt{\left(\frac{Q_{33}}{m_3}-\frac{Q_{22}}{m_2}\right)^2+\frac{(Q_{23}+Q_{23}^*)^2}{m_2 m_3}}\right]\langle H_u^0\rangle^2,\\
\cos 2z &= \left(\frac{Q_{33}^2}{m_3^2}-\frac{Q_{22}^2}{m_2^2}+\frac{(Q_{23}+Q_{23}^*)(Q_{23}-Q_{23}^*)}{m_2 m_3}\right)\frac{\langle H_u^0\rangle^4}{M_2^2-M_1^2}. \qquad (17)
\end{aligned}$$

To complete the reconstruction procedure, the Yukawa coupling would be derived from these parameters using eq.(10) and where the discrete parameter ξ in eq.(11) is deter-

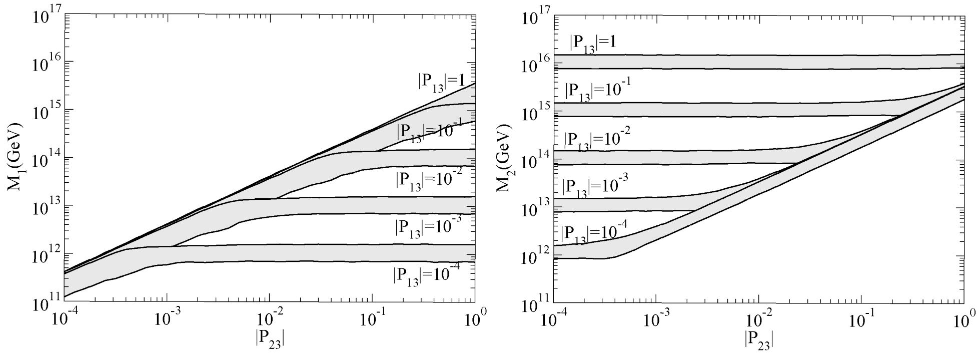

FIGURE 1. Reconstructed right-handed neutrino masses for different values of $|P_{13}|$ and $|P_{23}|$, in the limit $|P_{12}| \ll |P_{13}|, |P_{23}|$. Here, we have taken $\theta_{23} \simeq \pi/4$ and $\theta_{12} \simeq \pi/6$, and we have sampled over different values for ϕ and δ for $\theta_{13} = 0.1$. The shaded areas represent the regions at 2σ from the main value.

mined by:

$$\xi = \frac{\sqrt{m_2 m_3}}{Q_{23}\langle H_u^0\rangle^2}(M_1 \sin z \cos z^* - M_2 \cos z \sin z^*). \tag{18}$$

We would like to illustrate now the reconstruction procedure for a particular choice of the matrix P, namely when $|P_{12}| \ll |P_{13}|, |P_{23}|$. Physically, this limit would correspond to a situation where $BR(\mu \to e\gamma) \ll BR(\tau \to e\gamma), BR(\tau \to \mu\gamma)$. Note that in this limit the only independent phase in P is irrelevant, therefore, the only independent parameters in P are $|P_{13}|$ and $|P_{23}|$, that are related to the rate for $\tau \to e\gamma$ and $\tau \to \mu\gamma$, respectively. On the other hand, in the neutrino mass matrix we know all the parameters except θ_{13}, that is constrained by present experiments to be small, and the phases δ and ϕ. We can then reconstruct the high energy parameters in terms of the low energy data, including an indeterminacy stemming from our ignorance on the phases and the angle θ_{13}. The result is shown in Fig.1, where we show the regions at 2σ from the main value for the reconstructed right-handed masses as a function of $|P_{23}|$, for different values of $|P_{13}|$. It is apparent from this figure that in this limit, $|P_{12}| \ll |P_{13}|, |P_{23}|$, the observation of rare lepton decays could allow the reconstruction of the right-handed masses up to a factor of three.

As θ_{13} becomes smaller the indeterminacy in the reconstruction process is reduced, and some analytical formulas can be derived for the reconstructed parameters in different limits:

- $|P_{12}| \ll |P_{13}| \ll |P_{23}|$ (or $BR(\mu \to e\gamma) \ll BR(\tau \to e\gamma) \ll BR(\tau \to \mu\gamma)$)

$$\begin{aligned} M_1 &\simeq 2\sqrt{\frac{2}{3}}\frac{|P_{13}|}{m_2}\langle H_u^0\rangle^2, \\ M_2 &\simeq \frac{2|P_{23}|}{m_3}\langle H_u^0\rangle^2, \end{aligned}$$

$$\mathbf{Y}_\nu \simeq \sqrt{|P_{23}|}\begin{pmatrix} \sqrt{\frac{|P_{13}|}{\sqrt{6}|P_{23}|}}e^{i\phi/2} & \sqrt{\sqrt{\frac{3}{8}}\frac{|P_{13}|}{|P_{23}|}}e^{i\phi/2} & -\sqrt{\sqrt{\frac{3}{8}}\frac{|P_{13}|}{|P_{23}|}}e^{i\phi/2} \\ -\frac{|P_{13}|}{2|P_{23}|} & 1 & 1 \end{pmatrix}. \quad (19)$$

Note that in this limit the lightest right-handed mass is essentially determined by the rate for the process $\tau \to e\gamma$, while the heaviest one, by the process $\tau \to \mu\gamma$.

- $|P_{12}| \ll ||P_{23}| \ll |P_{13}|$ (or $BR(\mu \to e\gamma) \ll BR(\tau \to \mu\gamma) \ll BR(\tau \to e\gamma)$)

$$M_1 \simeq \frac{8|P_{23}|\langle H_u^0\rangle^2}{\sqrt{9m_2^2+16m_3^2+24m_2m_3\cos\phi}},$$

$$M_2 \simeq \frac{|P_{13}|\sqrt{9m_2^2+16m_3^2+24m_2m_3\cos\phi}}{\sqrt{6}m_2m_3}\langle H_u^0\rangle^2,$$

$$\mathbf{Y}_\nu \simeq \sqrt{\sqrt{6}\frac{\Delta}{\Delta^*}|P_{13}|}\begin{pmatrix} \sqrt{\frac{\sqrt{6}|P_{23}|}{|P_{13}|}}\frac{m_2e^{i\phi}}{|\Delta|^2} & \sqrt{\frac{|P_{23}|}{\sqrt{6}|P_{13}|}}\frac{\Delta^*}{\Delta} & \sqrt{\frac{|P_{23}|}{\sqrt{6}|P_{13}|}}\frac{-3m_2e^{i\phi}+4m_3}{|\Delta|^2} \\ -\frac{e^{i\phi/2}}{\sqrt{6}} & \frac{\sqrt{6}m_2e^{-i\phi/2}}{\Delta^2}\frac{|P_{23}|}{|P_{13}|} & e^{i\phi/2} \end{pmatrix} \quad (20)$$

where $\Delta = \sqrt{3m_2e^{-i\phi}+4m_3}$. Contrary to the previous limit, the lightest right-handed mass is essentially determined by the process $\tau \to \mu\gamma$, and the heaviest by $\tau \to e\gamma$.

Finally, in the limit in which all the parameters are real, the expressions are further simplified, by substituting $\phi = 0$ or π in these formulas, to account for the cases where light neutrinos have the same or opposite CP parities.

CONCLUSIONS

In this work we have studied the possibility of reconstructing the high energy see-saw Lagrangian in terms of low energy observables. We have discussed that the reconstruction of the complete see-saw model with three right-handed neutrinos could be possible in theory, but probably not in practice. However, the reconstruction of the model with two right-handed neutrinos could indeed be possible. We have also stressed the fact that the model with two right-handed neutrinos is not just a toy model for neutrino masses, but it also corresponds to interesting and physically plausible limits of the three right-handed neutrino model.

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