PROCEEDINGS OF SCIENCE

Heavy-light decay constant at the $1/m$ order of HQET

ALPHA Collaboration

DESY 07-160
SFB/CPP-07-56
CERN-PH-TH/2007-172

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Following the strategy developed by the ALPHA collaboration, we present a method to compute non-perturbatively the decay constant of a heavy-light meson in HQET including the 1/m corrections. We start by a matching between HQET and QCD in a small volume to determine the parameters of the effective theory non-perturbatively. Observables in the effective theory are then evolved to larger volumes. In two steps a large enough volume is reached to determine the physical decay constant. Some preliminary results in the quenched approximation are shown.

The XXV International Symposium on Lattice Field Theory
July 30-4 August 2007
Regensburg, Germany

[*]This work has been supported by the Deutsche Forschungsgemeinschaft (DFG) in SFB Transregio 9 "Computational Particle Physics".

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1. Introduction

A few years ago, a non-perturbative formulation of Heavy Quark Effective Theory (HQET) has been given in [1] - see [2] for a review given at this conference. In particular the problem of power divergences is solved through a finite volume matching. Last year, this has been applied to the quenched computation of the b-quark mass at the $1/m$ order [3]. In the same spirit, we present here a strategy to compute a heavy-light decay constant. We start by writing the Lagrangian at the leading order (i.e. in the static approximation) and add a kinetic and a magnetic piece (we follow the conventions of [3] and set the counterterm δ_m to zero)

$$\mathscr{L}_{\rm HQET} = \overline{\psi}_{\rm h} D_0 \psi_{\rm h} - \omega_{\rm kin} \overline{\psi}_{\rm h} \mathbf{D}^2 \psi_{\rm h} - \omega_{\rm spin} \overline{\psi}_{\rm h} \sigma\cdot\mathbf{B} \psi_{\rm h} \,. \tag{1.1}$$

A precise definition of the operators D_0, $\mathbf{D}^2$ and $\sigma\cdot\mathbf{B}$ can be found in [3]. Here we just note that $\omega_{\rm kin}$ and $\omega_{\rm spin}$ are some bare parameters of the effective theory.

1.1 Schrödinger functional (SF) correlation functions

In QCD, we consider the (renormalized and improved) current to boundary correlators $f_{\rm A}$ and k_V defined - up to improvement factors such as $(1+b_{\rm A} am_{\rm q,b})$ - in the SF by

$$f_{\rm A}(x_0) \;=\; -Z_A Z_\zeta^2 \frac{a^6}{2} \sum_{\mathbf{y},\mathbf{z}} \langle (A_{\rm I})_0(x)\, \overline{\zeta}_{\rm b}(\mathbf{y}) \gamma_5 \zeta_{\rm l}(\mathbf{z})\rangle, \tag{1.2}$$

$$k_{\rm V}(x_0) \;=\; -Z_V Z_\zeta^2 \frac{a^6}{6} \sum_{\mathbf{y},\mathbf{z},k} \langle (V_{\rm I})_k(x)\, \overline{\zeta}_{\rm b}(\mathbf{y}) \gamma_k \zeta_{\rm l}(\mathbf{z})\rangle, \tag{1.3}$$

where the improved currents $A_{\rm I}(x)$ and $V_{\rm I}(x)$ are defined as in [1].
We also consider the boundary to boundary correlators

$$f_1 \;=\; -Z_\zeta^4 \frac{a^{12}}{2L^6} \sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}} \langle \overline{\zeta}'_{\rm l}(\mathbf{u}) \gamma_5 \zeta'_{\rm b}(\mathbf{v})\, \overline{\zeta}_{\rm b}(\mathbf{y}) \gamma_5 \zeta_{\rm l}(\mathbf{z})\rangle, \tag{1.4}$$

$$k_1 \;=\; -Z_\zeta^4 \frac{a^{12}}{6L^6} \sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z},k} \langle \overline{\zeta}'_{\rm l}(\mathbf{u}) \gamma_k \zeta'_{\rm b}(\mathbf{v})\, \overline{\zeta}_{\rm b}(\mathbf{y}) \gamma_k \zeta_{\rm l}(\mathbf{z})\rangle. \tag{1.5}$$

Expanding these correlators at the $1/m$ order of HQET, and using spin-flavor symmetry, one finds [1]

$$\begin{aligned}
f_{\rm A}(x_0) &= Z_{\rm A}^{\rm HQET} Z_{\zeta_{\rm h}} Z_\zeta {\rm e}^{-m_{\rm bare} x_0} \left\{ f_{\rm A}^{\rm stat}(x_0) + c_{\rm A}^{\rm HQET} f_{\delta{\rm A}}^{\rm stat}(x_0) + \omega_{\rm kin} f_{\rm A}^{\rm kin}(x_0) + \omega_{\rm spin} f_{\rm A}^{\rm spin}(x_0) \right\} , \\
k_{\rm V}(x_0) &= -Z_{\rm V}^{\rm HQET} Z_{\zeta_{\rm h}} Z_\zeta {\rm e}^{-m_{\rm bare} x_0} \left\{ f_{\rm A}^{\rm stat}(x_0) + c_{\rm V}^{\rm HQET} f_{\delta{\rm A}}^{\rm stat}(x_0) + \omega_{\rm kin} f_{\rm A}^{\rm kin}(x_0) - \frac{1}{3}\omega_{\rm spin} f_{\rm A}^{\rm spin}(x_0) \right\} , \\
f_1 &= Z_{\zeta_{\rm h}}^2 Z_\zeta^2 {\rm e}^{-m_{\rm bare} T} \left\{ f_1^{\rm stat} + \omega_{\rm kin} f_1^{\rm kin} + \omega_{\rm spin} f_1^{\rm spin} \right\} , \\
k_1 &= Z_{\zeta_{\rm h}}^2 Z_\zeta^2 {\rm e}^{-m_{\rm bare} T} \left\{ f_1^{\rm stat} + \omega_{\rm kin} f_1^{\rm kin} - \frac{1}{3}\omega_{\rm spin} f_1^{\rm spin} \right\} ,
\end{aligned}$$

where $m_{\rm bare}$ is the (linearly divergent) bare quark mass.

[1] The reader can find the definitions of the various correlators $f_{\rm A,1}^{\rm stat}, f_{\rm A,1}^{\rm kin}, f_{\rm A,1}^{\rm spin}, f_{\delta{\rm A}}^{\rm stat}$ in [3].

1.2 Basic observable

We consider a volume L^3 with a time extent $T = L$, and define (in QCD)

$$\Phi_{\rm F}^{\rm QCD}(L) = \ln\left(\frac{-f_{\rm A}(L/2)}{\sqrt{f_1}}\right)$$

In the large volume limit, this observable is related to the decay constant, $F_{\rm B}$, by

$$\Phi_{\rm F}^{\rm QCD}(L) \stackrel{L\gg 1/\Lambda}{\longrightarrow} \ln\left(\frac{1}{2}F_{\rm B}\sqrt{m_{\rm B}L^3}\right). \tag{1.6}$$

In a small volume [2] of space extent $L_1 \simeq 0.4\,\mathrm{fm}$, this observable is matched to its HQET expression

$$\Phi_{\rm F}^{\rm QCD}(L_1) = \Phi_{\rm F}^{\rm HQET}(L_1)\,. \tag{1.7}$$

Using the expansions of the correlators $f_{\rm A}$ and f_1 given previously, one finds for the rhs at the static and at the $1/m$ order

$$\Phi_{\rm F}^{\rm stat}(L) \;=\; \ln Z_{\rm A}^{\rm stat} + \ln\left(\frac{-f_{\rm A}^{\rm stat}(L/2)}{\sqrt{f_1^{\rm stat}}}\right) + {\rm O}(1/m), \tag{1.8}$$

$$\begin{aligned}\Phi_{\rm F}^{\rm stat}(L)+\Phi_{\rm F}^{1/m}(L) \;=&\; \ln Z_{\rm A}^{\rm HQET} + \ln\left(\frac{-f_{\rm A}^{\rm stat}(L/2)}{\sqrt{f_1^{\rm stat}}}\right) + c_{\rm A}^{\rm HQET}\,\frac{f_{\delta\rm A}^{\rm stat}(L/2)}{f_{\rm A}^{\rm stat}(L/2)} \\ &+\omega_{\rm kin}\left(\frac{f_{\rm A}^{\rm kin}(L/2)}{f_{\rm A}^{\rm stat}(L/2)} - \frac{f_1^{\rm kin}}{f_1^{\rm stat}}\right) + \omega_{\rm spin}\left(\frac{f_{\rm A}^{\rm spin}(L/2)}{f_{\rm A}^{\rm stat}(L/2)} - \frac{f_1^{\rm spin}}{f_1^{\rm stat}}\right) + {\rm O}(1/m^2).\end{aligned} \tag{1.9}$$

1.3 Evolution to larger volumes, in the static approximation

In order to clarify the discussion, we first explain the strategy in the static approximation, the generalization to the $1/m$ order will be done in the next section. We start by the matching of HQET to QCD in the volume L_1, at the static order : $\Phi_{\rm F}^{\rm QCD}(L_1) = \Phi_{\rm F}^{\rm stat}(L_1)$. The evolution to a volume $L_\infty = L_3 = 2L_2 = 4L_1$ is then done, within the effective theory, in the following way:

$$\Phi_{\rm F}(L_\infty) = [\Phi_{\rm F}^{\rm stat}(L_\infty) - \Phi_{\rm F}^{\rm stat}(L_2)] + [\Phi_{\rm F}^{\rm stat}(L_2) - \Phi_{\rm F}^{\rm stat}(L_1)] + \Phi_{\rm F}^{\rm QCD}(L_1)\,. \tag{1.10}$$

We note that $Z_{\rm A}^{\rm stat}$ cancels in the differences $[\Phi_{\rm F}^{\rm stat}(2L) - \Phi_{\rm F}^{\rm stat}(L)]$. Using the renormalized SF coupling $\bar{g}^2(L)$ [4], we define the static step scaling function (ssf)

$$\sigma_{\rm F}^{\rm stat}(u) \;=\; \left[\Phi_{\rm F}^{\rm stat}(2L) - \Phi_{\rm F}^{\rm stat}(L)\right]_{\bar{g}^2(L)=u} = \lim_{a/L\to 0}\Sigma_{\rm F}^{\rm stat}(u,a/L) \tag{1.11}$$

$$\Sigma_{\rm F}^{\rm stat}(u,a/L) \;=\; \left[\zeta^{\rm stat}(2L) - \zeta^{\rm stat}(L)\right]_{\bar{g}^2(L)=u} \;\text{ where } \zeta^{\rm stat}(L) = \ln\left(\frac{-f_{\rm A}^{\rm stat}(L/2)}{\sqrt{f_1^{\rm stat}}}\right)\,. \tag{1.12}$$

We can now rewrite the rhs of eq (1.10) as the sum of three continuum terms

$$\Phi_{\rm F}(L_\infty) = \sigma_{\rm F}^{\rm stat}(u_2) + \sigma_{\rm F}^{\rm stat}(u_1) + \Phi_{\rm F}^{\rm QCD}(L_1)\,,\ \text{where } u_{\rm k} = \bar{g}^2(L_{\rm k})\,. \tag{1.13}$$

Before discussing the $1/m$ corrections we close this section by a few remarks :

[2] The matching is done in a small volume, in order to be able to simulate a b-quark with the discretization errors under control.

- The different terms in the eq (1.12) have to be computed at the same value of the lattice spacing (in order to insure that Σ_F has a well defined continuum limit). This is due to divergences proportional to the logarithm of the lattice spacing that one has to cancel.
- In eq. (1.13), the entire quark mass dependence comes from $\Phi_{\mathrm{F}}^{\mathrm{QCD}}(L_1)$.
- At this order, since there is only one matching constant to eliminate ($Z_{\mathrm{A}}^{\mathrm{stat}}$), it is sufficient to match one observable (Φ_{F}).

1.4 Including $1/m$ corrections

At this order, there are three more matching parameters in Φ_{F} compared to the static case [3]. Therefore, to determine them, we introduce three other observables defined in a volume of space extent L and we give their expressions at the $1/m$ order

$$\Phi_1(L) \equiv \frac{1}{4}(R_1^{\mathrm{P}} + 3R_1^{\mathrm{V}}) - R_1^{\mathrm{stat}} = \omega_{\mathrm{kin}} R_1^{\mathrm{kin}} \,, \tag{1.14}$$

$$\Phi_2(L) \equiv \frac{3}{4}\ln\left(\frac{f_1}{k_1}\right) = \omega_{\mathrm{spin}} \frac{f_1^{\mathrm{spin}}}{f_1^{\mathrm{stat}}} \qquad \text{with } T = L/2 \,, \tag{1.15}$$

$$\Phi_3(L) \equiv R_{\mathrm{A}}(L/2) - R_{\mathrm{A}}^{\mathrm{stat}}(L/2) = c_{\mathrm{A}}^{\mathrm{HQET}} R_{\delta\mathrm{A}}(L/2) + \omega_{\mathrm{kin}} R_{\mathrm{A}}^{\mathrm{kin}}(L/2) + \omega_{\mathrm{spin}} R_{\mathrm{A}}^{\mathrm{spin}}(L/2) \,, \tag{1.16}$$

where the definitions of the ratios R can be found in [3] [4]. Together with Φ_{F}, given at this order by (1.9), we then have a set of four observables. In these observables, we have chosen to subtract the static part (when existing) from the QCD one, as we did in the mentioned reference. This is perfectly legitimate because they both have a continuum limit, and this simplifies the equations.
Like in the static case, the matching is imposed in the volume L_1. This allows us to replace in (1.9) the parameters ω_{kin}, ω_{spin} and $c_{\mathrm{A}}^{\mathrm{HQET}}$ by a combination of QCD and HQET quantities.
The evolution to the volume L_2 is given by

$$\Phi_{\mathrm{F}}(L_2) = \Phi_{\mathrm{F}}^{\mathrm{HQET}}(L_2) - \Phi_{\mathrm{F}}^{\mathrm{HQET}}(L_1) + \Phi_{\mathrm{F}}^{\mathrm{QCD}}(L_1) \tag{1.17}$$

$$= \left[\Phi_{\mathrm{F}}^{\mathrm{stat}}(L_2) - \Phi_{\mathrm{F}}^{\mathrm{stat}}(L_1)\right] + \left[\Phi_{\mathrm{F}}^{1/m}(L_2) - \Phi_{\mathrm{F}}^{1/m}(L_1)\right] + \Phi_{\mathrm{F}}^{\mathrm{QCD}}(L_1) \,. \tag{1.18}$$

The ssf for the static term has already been given in the previous part, and for the $1/m$ part we write

$$\Phi_{\mathrm{F}}^{1/m}(2L) - \Phi_{\mathrm{F}}^{1/m}(L) = \sum_{\mathrm{i}} \sigma_{\mathrm{i}}(\bar{g}^2(L))\Phi_{\mathrm{i}}(L) \,. \tag{1.19}$$

The expressions for the ssf can be found from the last equation by using (1.9) together with (1.14), (1.15), (1.16) in the volume L_1. The explicit definitions are given in the appendix.
In the step $L_2 \to L_\infty$, we need $\Phi_{\mathrm{i}}(L_2, M)$, and we are then lead to define the ssf for the Φ_{i} :

$$\Phi_{\mathrm{i}}(L_2) = \sum_j \sigma_{\mathrm{ij}}(\bar{g}^2(L))\Phi_{\mathrm{j}}(L_1) \,. \tag{1.20}$$

We can write down the final equation for Φ_{F}

$$\Phi_{\mathrm{F}}(L_\infty) = \sigma_{\mathrm{F}}^{\mathrm{stat}}(u_2) + \sigma_{\mathrm{F}}^{\mathrm{stat}}(u_1) + \sum_{\mathrm{ij}} \sigma_{\mathrm{i}}(u_2)\sigma_{\mathrm{ij}}(u_1)\Phi_{\mathrm{j}}(L_1) + \sum_{\mathrm{i}} \sigma_{\mathrm{i}}(u_1)\Phi_{\mathrm{i}}(L_1) + \Phi_{\mathrm{F}}^{\mathrm{QCD}}(L_1) \tag{1.21}$$

[3] Also $Z_{\mathrm{A}}^{\mathrm{HQET}}$ is different than $Z_{\mathrm{A}}^{\mathrm{stat}}$, but as in the static case, it drops out in the differences

[4] We remind the reader that the quantities defined with an subscript 1 are "boundary to boundary" observables. Because the noise over signal ratio grows exponentially with the time, we impose $T = L/2$ for all these observables.

2. Results in the quenched approximation

We used basically the same data as in [3], in which the reader can find the details of the simulation. The simulations are done with non-perturbatively $\mathrm{O}(a)$-improved Wilson fermions. The light quark mass is fixed to the strange one. Concerning HQET, we used the HYP actions [5], which help to have a reasonable statistical precision. We show the continuum extrapolations in fig. 1.

The extrapolations are done linearly in $(a/L)^2$ for QCD as well as for the static part, but in a/L

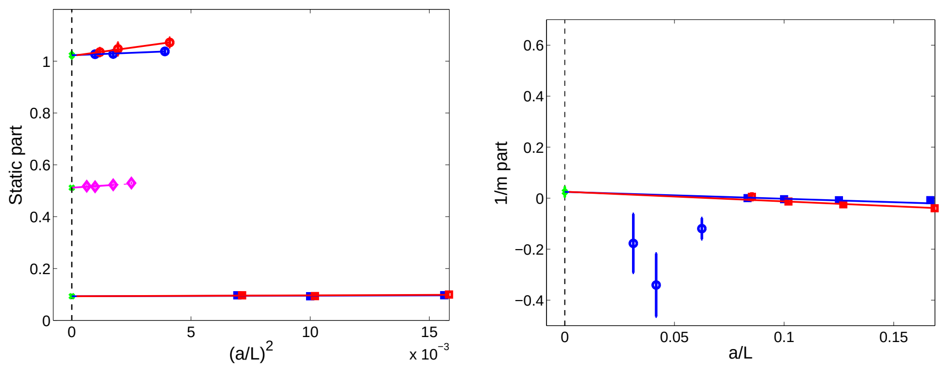

Figure 1: Continuum extrapolations of the various terms appearing in eq. (1.21). The QCD contribution $\Phi_{\mathrm{F}}^{\mathrm{QCD}}$ (diamonds) and the static part are shown on the left (the circles represent the large volume contribution $\sigma_{\mathrm{F}}^{\mathrm{stat}}(u_2)$, and the squares the small volume part $\sigma_{\mathrm{F}}^{\mathrm{stat}}(u_1)$). The $1/m$ correction is plotted on the right, the circles represent the large volume terms $\sum_{\mathrm{ij}} \sigma_{\mathrm{i}}(u_2)\sigma_{\mathrm{ij}}(u_1)\Phi_{\mathrm{j}}(L_1)$, and the squares the small volume part $\sum_{\mathrm{i}} \sigma_{\mathrm{i}}(u_1)\Phi_{\mathrm{i}}(L_1)$. The color blue stands for HYP2, and red for HYP1 [5].

for the $1/m$ term, because of the absence of $\mathrm{O}(a)$ improvement. The ordinate scale is the same in order to compare the relative size of the different contributions. Concerning the precision, one can see that the total error is largely dominated by the one of the $1/m$ part in the large volume. Since for this part the results are not yet completely satisfactory, we refrain from performing a continuum extrapolation. We will use the result at the finest lattice spacing only ($\beta \sim 6.45$, $L/a = 32$).

Our preliminary results for F_{B_s} are shown in table 1. In the first column we give the results in the static approximation, while in the other columns, we have included the $1/m$ corrections. We observe that in the static approximation, depending on the matching condition represented here by θ [5], the result can change by 7%. This variation disappears when the $1/m$ terms are included. Note that differences of $F_{\mathrm{B_s}}^{\mathrm{stat}+1/\mathrm{m}}$, table 1, have much smaller error than their individual values, for example

$$F_{\mathrm{B_s}}^{\mathrm{stat}+1/\mathrm{m}}(\theta_0 = 0, \theta_1 = 1, \theta_2 = 0) - F_{\mathrm{B_s}}^{\mathrm{stat}+1/\mathrm{m}}(\theta_0 = 1, \theta_1 = 0, \theta_2 = 0.5) = 4 \pm 2\ \mathrm{MeV}. \tag{2.1}$$

The other information is that the $1/m$ term contributes (with a minus sign) up to $\sim 15\%$ to the final result. One can see that adding the $1/m$ terms increases the size of the statistical errors, as expected from the previous plots. This is due to the fact that the signal for the $1/m$ part in large

[5]The quark fields are periodic in space up to a phase θ.

volume is more difficult to extract than in the static case, and also because of the absence of O(a)-improvement at this order. We also note that our result is compatible with a recent computation done with a different method, but which also goes beyond the leading order of HQET [6].

	$F_{\mathrm{B_s}}^{\mathrm{stat}}$	$F_{\mathrm{B_s}}^{\mathrm{stat+1/m}}$		
θ_0		$\theta_1 = 0$ $\theta_2 = 0.5$	$\theta_1 = 0.5$ $\theta_2 = 1$	$\theta_1 = 1$ $\theta_2 = 0$
0	224 ± 3	185 ± 21	186 ± 22	189 ± 22
0.5	220 ± 3	185 ± 21	187 ± 22	189 ± 22
1	209 ± 3	184 ± 21	185 ± 21	188 ± 22

Table 1: Results for F_{B_s} in MeV with and without the $1/m$ corrections, for different values of the θ angles.

3. Conclusion

We have shown how to perform a non-perturbative computation of a heavy-light decay constant at $1/m$ order of HQET, and we have given preliminary numerical results in the quenched approximation. The inclusion of the dynamical quarks is on the way [7, 8]. Applying this method for $N_f > 0$ should allow for precise computations of the heavy-light decay constant, with a good control on the systematic errors. Note in particular that eq. (2.1) is a good sign of the absence of significant $1/m^2$ corrections. On the numerical side, the cancellations of the divergences require sufficient statistical precision, and we hope that the all-to-all propagator, like proposed in [9, 10] will be of great help there.

Acknowledments
We thank NIC for allocating computer time on the APE computers to this project and the APE group at Zeuthen for its support.

4. Appendix: The step scaling functions

In order to have more compact notations, we replace Φ_{F} by Φ_4, such that the ssf σ_{i} introduced in eq. (1.19) are now represented by $\sigma_{4\mathrm{i}}$. We can rewrite eq. (1.19) together with eq.(1.20) as

$$\Phi_{\mathrm{i}}(L_{\mathrm{k}+1}, M) = \sum_{j=1}^{4} \sigma_{\mathrm{ij}}(u_{\mathrm{k}})\Phi_{\mathrm{j}}(L_{\mathrm{k}}, M) + \delta_{4\mathrm{i}}\sigma_{\zeta}(u_{\mathrm{k}})\,. \tag{4.1}$$

The ssf are then given by a four by four matrix

$$\left[\sigma_{\mathrm{ij}}\right] = \begin{pmatrix} \sigma_{11} & 0 & 0 & 0 \\ 0 & \sigma_{22} & 0 & 0 \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & 0 \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & 1 \end{pmatrix}$$

To give their explicit expressions, we define

$$\Psi^{\rm kin}(L) = \left(\frac{f_{\rm A}^{\rm kin}(T/2)}{f_{\rm A}^{\rm stat}(T/2)} - \frac{f_1^{\rm kin}}{f_1^{\rm stat}} \right) , \quad \Psi^{\rm spin}(L) = \left(\frac{f_{\rm A}^{\rm spin}(T/2)}{f_{\rm A}^{\rm stat}(T/2)} - \frac{f_1^{\rm spin}}{f_1^{\rm stat}} \right) \quad \text{and} \quad \rho_{\delta_{\rm A}}(L) = \frac{f_{\delta {\rm A}}^{\rm stat}(T/2)}{f_{\rm A}^{\rm stat}(T/2)} .$$

Then, one finds:

$$\begin{aligned}
\Sigma_{11}(u) &= \left[R_1^{\rm kin}(2L)/R_1^{\rm kin}(L)\right]_{\bar{g}^2(L)=u} , \qquad \Sigma_{22}(u) = \left[\rho_1^{\rm spin}(2L)/\rho_1^{\rm spin}(L)\right]_{\bar{g}^2(L)=u} , \\
\Sigma_{31}(u) &= \left[\frac{1}{R_1^{\rm kin}(L)} \left(R_{\rm A}^{\rm kin}(2L) - \frac{R_{\rm A}^{\rm kin}(L)\, R_{\delta_{\rm A}}(2L)}{R_{\delta_{\rm A}}(L)} \right) \right]_{\bar{g}^2(L)=u} , \\
\Sigma_{32}(u) &= \left[\frac{1}{\rho_1^{\rm spin}(L)} \left(R_{\rm A}^{\rm spin}(2L) - \frac{R_{\rm A}^{\rm spin}(L) R_{\delta_{\rm A}}(2L)}{R_{\delta_{\rm A}}(L)} \right) \right]_{\bar{g}^2(L)=u} , \qquad \Sigma_{33}(u) = \left[\frac{R_{\delta_{\rm A}}(2L)}{R_{\delta_{\rm A}}(L)} \right]_{\bar{g}^2(L)=u} , \\
\Sigma_{41}(u) &= \left[\frac{1}{R_1^{\rm kin}(L)} \left(\Psi^{\rm kin}(2L) - \Psi^{\rm kin}(L) - \frac{\rho_{\delta_{\rm A}}(2L) - \rho_{\delta_{\rm A}}(L)}{R_{\delta_{\rm A}}(L)}\, R_{\rm A}^{\rm kin}(L) \right) \right]_{\bar{g}^2(L)=u} , \\
\Sigma_{42}(u) &= \left[\frac{1}{\rho_1^{\rm spin}(L)} \left(\Psi^{\rm spin}(2L) - \Psi^{\rm spin}(L) - \frac{\rho_{\delta_{\rm A}}(2L) - \rho_{\delta_{\rm A}}(L)}{R_{\delta_{\rm A}}(L)}\, R_{\rm A}^{\rm spin}(L) \right) \right]_{\bar{g}^2(L)=u} , \\
\Sigma_{43}(u) &= \left[\frac{\rho_{\delta_{\rm A}}(2L) - \rho_{\delta_{\rm A}}(L)}{R_{\delta_{\rm A}}(L)} \right]_{\bar{g}^2(L)=u} .
\end{aligned}$$

References

[1] **ALPHA** Collaboration, J. Heitger and R. Sommer, *Non-perturbative heavy quark effective theory*, *JHEP* **02** (2004) 022 [`hep-lat/0310035`].

[2] M. Della Morte, *Standard model parameters and heavy quarks on the lattice*, *PoS* **LAT2007** 008.

[3] M. Della Morte, N. Garron, M. Papinutto, and R. Sommer, *Heavy quark effective theory computation of the mass of the bottom quark*, *JHEP* **01** (2007) 007 [`hep-ph/0609294`].

[4] **ALPHA** Collaboration, S. Capitani, M. Lüscher, R. Sommer, and H. Wittig, *Non-perturbative quark mass renormalization in quenched lattice QCD*, *Nucl. Phys.* **B544** (1999) 669 [`hep-lat/9810063`].

[5] M. Della Morte, A. Shindler, and R. Sommer, *On lattice actions for static quarks*, *JHEP* **08** (2005) 051 [`hep-lat/0506008`].

[6] D. Guazzini, R. Sommer, and N. Tantalo, *$m_{\rm b}$ and $f_{\rm b_s}$ from a combination of HQET and QCD*, *PoS* **LAT2006** (2006) 084 [`hep-lat/0609065`].

[7] M. Della Morte, P. Fritzsch, J. Heitger, H. Meyer, H. Simma, and R. Sommer, *Towards a non-perturbative matching of HQET and QCD with dynamical light quarks*, *PoS* **LAT2007** 246.

[8] M. Della Morte, P. Fritzsch, B. Leder, H. Meyer, H. Simma, R. Sommer, S. Takeda, O. Witzel, and U. Wolff, *Preparing for $N_f = 2$ simulations at small lattice spacings*, *PoS* **LAT2007** 255.

[9] J. Foley *et. al.*, *Practical all-to-all propagators for lattice QCD*, *Comput. Phys. Commun.* **172** (2005) 145 [`hep-lat/0505023`].

[10] M. Lüscher, *Local coherence and deflation of the low quark modes in lattice QCD*, *JHEP* **07** (2007) 081 [`hep-lat/07062298`].