

Nonlinear evolution of pomeron and odderon in momentum space

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Abstract

The small x evolution of the QCD pomeron and the QCD odderon is investigated in the mean field limit of the Color Glass Condensate. The resulting system of coupled nonlinear evolution equations is transformed to the momentum space and analyzed at a very small momentum transfer. The main properties of the C -even and C -odd dipole densities in momentum space are obtained analytically. The critical scaling dimension is found for the odderon and the universal asymptotic behavior of the solutions is determined for small and large gluon momenta. We find that the same saturation scale characterizes both the pomeron and the odderon and both densities depend on the momentum only through the geometric scaling variable. The absorptive effects are found to cause a strong suppression of the odderon exchange amplitude for momenta below saturation scale and only a moderate suppression for larger momenta.

1 Introduction

A striking feature of strong scattering amplitudes at high energies is an overwhelming dominance of the exchange with the vacuum quantum numbers – the pomeron. The exchange of the C -odd partner of the pomeron – the odderon – is much more elusive. In fact, despite experimental efforts only some weak indications were found for the odderon contribution in hadronic processes (for a recent review see Ref. [1]). The extraordinary weakness of the odderon exchange is somewhat puzzling. Part of the explanation of this fact is provided in a natural way by general color group arguments, since one needs at least two operators with the gluon quantum numbers to build the pomeron and at least three to build the odderon. Therefore, the high power of α_s entering the odderon exchange amplitude suppresses it with respect to the C -even amplitude. Furthermore, the perturbative QCD pomeron amplitude grows steeply with energy, in contrast to a rather flat dependence of the perturbative odderon on the energy. Indeed, the existing theoretical estimates of the cross sections for various odderon mediated processes [2, 3, 4, 5, 6, 7, 8] predict small cross sections, below the sensitivity of current experiments. The only exception, for which some evidence of the odderon contribution was probably measured, is the elastic pp scattering at non-zero momentum transfer and at ISR energies [9]. Recently, it was realized that in addition to the mechanism described above, an important suppression of the odderon is caused by absorption of the odderon in a dense partonic system [10, 11].

In the high energy limit of perturbative QCD, the dynamics of the two gluon system (the pomeron) is described by the BFKL equation [12], which relies on a systematic resummation of perturbative corrections enhanced by powers of $\log(1/x)$, assuming $\alpha_s \ll 1$, and $\alpha_s \log(1/x) \sim 1$. The evolution of the pomeron amplitude at small x leads to a power like growth of the amplitude, eventually giving rise to sizable unitarity corrections which tame the growth. The perturbative small x evolution equation of the pomeron amplitudes, taking into account non-linear unitarity corrections was derived by Balitsky [13] and Kovchegov [14] for a small and dilute projectile probing a dense and extended target. In the diagrammatic representation, the Balitsky-Kovchegov (BK) equation resums BFKL pomeron fan diagrams in the large N_c limit and it conforms with unitarity constraints at each impact parameter. The BK equation may be also obtained as the mean-field limit of the effective theory of small x gluons in the hadron wavefunction, that is the Color Glass Condensate approach [15].

The perturbative realization of the odderon consists of at least three t -channel gluons in the color singlet state. The small x evolution equation of the odderon (the BKP equation) was derived long time ago [16, 17] but until recent years the properties of the solution have not been known. The spectrum of the QCD odderon Hamiltonian in the space of normalizable functions was found [18] using the holomorphic symmetry of the problem. The odderon intercept appeared to be smaller than unity, meaning that C -odd amplitudes should decrease with energy. In a following analysis, however, it was discovered [19] that the normalizability condition of the holomorphic wave functions may be relaxed, leading to a new odderon solution, having a flat overall dependence on the collision energy.

The new solution has a non-trivial property – two reggeized t -channel gluons occupy the same position in the transverse plane. Both the spectrum and the eigenfunctions of this solution are contained by the standard BFKL set of eigenfunctions with odd conformal spins.

The new odderon solution amplitude has a simple representation in the dipole model framework [10], where it is constructed using a Mueller dipole cascade model [20] with an initial condition with odd spatial parity. The QCD dipole cascade interacts with the target by an exchange of three elementary gluons in the C -odd color singlet state. In the dipole picture, the rescattering effects of the odderon amplitude were obtained [10], in analogy with the Kovchegov derivation of the BK equation [14]. The simultaneous investigation of the odderon and pomeron exchange amplitudes in the Color Glass Condensate lead to a generalization for the abovementioned results in the form of coupled non-linear equations involving both the C -even and C -odd amplitudes [11]. In this letter we shall study in detail the impact of the absorption on the momentum distribution of the gluons in the odderon.

2 Formalism

Effects of absorption of the odderon in high energy scattering were considered by Kovchegov, Szymanowski, Wallon [10] in the framework of dipole model. They found a correction term to the odderon evolution equation bilinear in the pomeron and odderon densities. Hatta, Iancu, Itakura and McLerran [11] analyzed C -even and C -odd scattering amplitudes using the formalism of the Color Glass Condensate. The energy evolution of the amplitudes was described by a system of functional equations. In the mean field limit the odderon evolution equation agreed with the result of Ref. [10]. A new non-linear term was found, however, contributing to the pomeron evolution beyond the BK equation, quadratic in the odderon amplitude. Eventually, in the leading logarithmic approximation the small x evolution equations of the C -even amplitude $N(\mathbf{x}, \mathbf{y}; \tau)$ and the C -odd amplitude $O(\mathbf{x}, \mathbf{y}; \tau)$ in the transverse position plane read [11]:

$$\begin{aligned} \frac{\partial N(\mathbf{x}, \mathbf{y}; \tau)}{\partial \tau} &= \frac{\bar{\alpha}_s}{2\pi} \int d^2 z \frac{(\mathbf{x} - \mathbf{y})^2}{(\mathbf{x} - \mathbf{z})^2 (\mathbf{z} - \mathbf{y})^2} [N(\mathbf{x}, \mathbf{z}; \tau) + N(\mathbf{z}, \mathbf{y}; \tau) - N(\mathbf{x}, \mathbf{y}; \tau) \\ &\quad - N(\mathbf{x}, \mathbf{z}; \tau)N(\mathbf{z}, \mathbf{y}; \tau) + O(\mathbf{x}, \mathbf{z}; \tau)O(\mathbf{z}, \mathbf{y}; \tau)], \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{\partial O(\mathbf{x}, \mathbf{y}; \tau)}{\partial \tau} &= \frac{\bar{\alpha}_s}{2\pi} \int d^2 z \frac{(\mathbf{x} - \mathbf{y})^2}{(\mathbf{x} - \mathbf{z})^2 (\mathbf{z} - \mathbf{y})^2} [O(\mathbf{x}, \mathbf{z}; \tau) + O(\mathbf{z}, \mathbf{y}; \tau) - O(\mathbf{x}, \mathbf{y}; \tau) \\ &\quad - O(\mathbf{x}, \mathbf{z}; \tau)N(\mathbf{z}, \mathbf{y}; \tau) - N(\mathbf{x}, \mathbf{z}; \tau)O(\mathbf{z}, \mathbf{y}; \tau)], \end{aligned} \quad (2)$$

with $\tau = \log(1/x)$ and \mathbf{x} , \mathbf{y} and \mathbf{z} representing positions of the end points of color dipoles in the transverse plane. Let us refer to this system as the WHIMIKS equation, after the authors' initials.

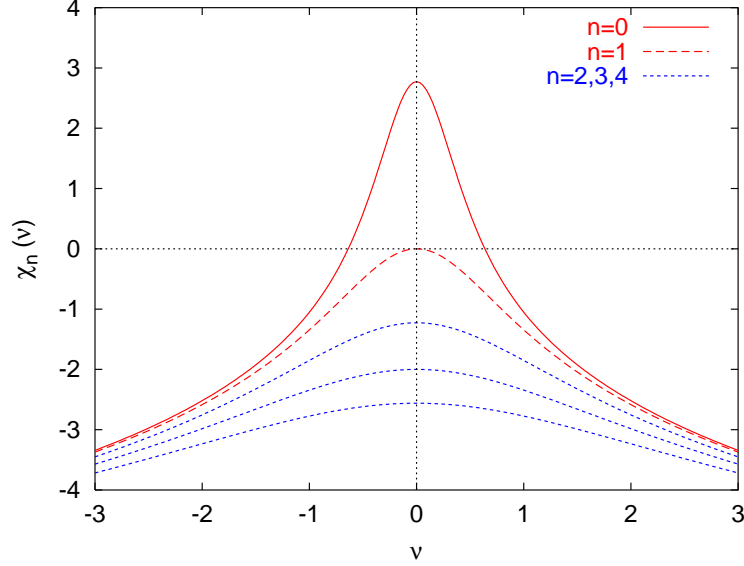


Figure 1: *Eigenvalues of the BFKL kernel $\chi_n(1/2 + i\nu)$ for conformal spins enumerated by n . The values of $\bar{\alpha}_s \chi_n(1/2 + i\nu)$ for $\nu = 0$ with $n = 0$ and $n = 1$ give the intercept of the pomeron and the odderon correspondingly.*

By construction, the pomeron and the odderon exchange amplitudes have definite parities with respect to exchange of the gluon positions, that is:

$$N(\mathbf{y}, \mathbf{x}; \tau) = N(\mathbf{x}, \mathbf{y}; \tau), \quad O(\mathbf{y}, \mathbf{x}; \tau) = -O(\mathbf{x}, \mathbf{y}; \tau). \quad (3)$$

For the sake of simplicity, we restrict our analysis of the nonlinear evolution equations to the translationally invariant case, corresponding to the limit of the target size being much larger than the dipole size. In this limit, the impact parameter dependence of $N(\mathbf{x}, \mathbf{y}; \tau)$ and $O(\mathbf{y}, \mathbf{x}; \tau)$ may be neglected. Thus, we assume

$$N(\mathbf{x}, \mathbf{y}; \tau) = N(\mathbf{y} - \mathbf{x}, \tau), \quad O(\mathbf{x}, \mathbf{y}; \tau) = O(\mathbf{y} - \mathbf{x}, \tau). \quad (4)$$

In the next part, we choose to work in the momentum space. Therefore we define the momentum dependent functions $\Phi(\mathbf{k}, \tau)$ and $\Psi(\mathbf{k}, \tau)$ describing the pomeron and the odderon exchange respectively

$$\Phi(\mathbf{k}, \tau) = \int \frac{d^2 \mathbf{r}}{2\pi r^2} N(\mathbf{r}, \tau) \exp(-i\mathbf{k}\mathbf{r}), \quad (5)$$

$$\Psi(\mathbf{k}, \tau) = \int \frac{d^2 \mathbf{r}}{2\pi r^2} O(\mathbf{r}, \tau) \exp(-i\mathbf{k}\mathbf{r}). \quad (6)$$

In the dipole formalism $\Phi(\mathbf{k}, \tau)$ and $\Psi(\mathbf{k}, \tau)$ represent dipole densities in momentum space. It is straightforward to obtain the form of equations (1) and (2) in this representation,

$$\frac{\partial \Phi(\mathbf{k}, \tau)}{\partial \tau} = \bar{\alpha}_s (K' \otimes \Phi)(\mathbf{k}, \tau) - \bar{\alpha}_s \Phi^2(\mathbf{k}, \tau) + \bar{\alpha}_s \Psi^2(\mathbf{k}, \tau), \quad (7)$$

$$\frac{\partial \Psi(\mathbf{k}, \tau)}{\partial \tau} = \bar{\alpha}_s (K' \otimes \Psi)(\mathbf{k}, \tau) - 2\bar{\alpha}_s \Phi(\mathbf{k}, \tau) \Psi(\mathbf{k}, \tau), \quad (8)$$

with the linear kernel being related to the standard LL BFKL kernel. To be more specific, the action of the kernel K' on the function $f(k^2)$ is given by the BFKL kernel action on the function $k^2 f(k^2)$, for instance

$$(K' \otimes \Phi)(\mathbf{k}, \tau) = \int \frac{d^2 \mathbf{k}'}{(\mathbf{k} - \mathbf{k}')^2} \left[\Phi(\mathbf{k}', \tau) - \frac{k^2 \Phi(\mathbf{k}, \tau)}{\mathbf{k}'^2 + (\mathbf{k} - \mathbf{k}')^2} \right]. \quad (9)$$

In the near-forward scattering limit (the momentum transfer q being much smaller than typical gluon momenta) the eigenfunctions $f_{n,\gamma}(\mathbf{k})$ of the kernel K' take the simple form in the polar coordinates (k, φ) ,

$$f_{n,\gamma}(k, \varphi) = f_{n,\gamma} k^{2\gamma} \cos(n\varphi) \quad (10)$$

and the eigenvalues of the BFKL kernel read

$$\chi_n(\gamma) = 2\psi(1) - \psi(|n|/2 + \gamma) - \psi(|n|/2 + 1 - \gamma). \quad (11)$$

with n and γ being the conformal spin and the scaling dimension respectively. Therefore, the solution to the linear equation

$$\frac{\partial f(k, \varphi; \tau)}{\partial \tau} = \bar{\alpha}_s [K' \otimes f](k, \varphi; \tau) \quad (12)$$

may be expressed as

$$f(k, \varphi; \tau) = \sum_{n=0}^{\infty} \int_{-1/2-i\infty}^{-1/2+i\infty} \frac{d\gamma}{2\pi i} f_{n,\gamma}^{(0)} k^{2\gamma} \cos(n\varphi) e^{\bar{\alpha}_s \chi_n(\gamma+1)\tau} \quad (13)$$

where the coefficients $f_{n,\gamma}^{(0)}$ are obtained by a projection of the initial condition $f^{(0)}(k, \varphi) = f(k, \varphi; \tau = 0)$ on the kernel eigenfunctions (10). The rapidity dependence of a solution with a given conformal spin n at large rapidities is driven by the behavior of the corresponding kernel eigenvalue in the vicinity of the point $\gamma = -1/2$ where the saddle point of the integration resides (note the shifted argument of χ_n in (13)). The dependence of $\chi_n(\gamma)$ along the integration contour is illustrated in Fig. 1 for $n = 0, 1, 2, 3$ and $n = 4$, as a function of ν , where $\gamma = 1/2 + i\nu$. For conformal spins $n \geq 2$ the eigenvalue $\chi_n(1/2 + i\nu) < 0$ for all ν (see Fig. 1) thus the components with conformal spins $n \geq 2$ are exponentially suppressed with increasing rapidity. The leading component in the pomeron sector, with $n = 0$, grows exponentially with rapidity, with the famous BFKL intercept $\omega_0 = \chi_0(1/2) = 4 \log 2 \bar{\alpha}_s$. Due to the parity property (3) in the transverse space, the even conformal spins contribute exclusively to the pomeron, and the odd conformal spins to the odderon. The dominant component in the odderon sector at large rapidities corresponds to $n = 1$, with the intercept $\omega_1 = \chi_1(1/2) = 0$ and the main effect of the evolution of the component is a diffusion in the transverse momentum.

Taking into account the rapidity dependence of the components with conformal spins $n \geq 2$ it is justified to retain only the leading conformal spins, that is

$$\Phi(\mathbf{k}, \tau) = \Phi(k, \tau), \quad \Psi(\mathbf{k}, \tau) = \Psi(k, \tau) \cos(\varphi), \quad (14)$$

with φ being the angle between \mathbf{k} and a characteristic direction in the transverse plane, given by the C -odd impact factor or by a small but non-vanishing momentum transfer.

Evolution equations (1) and (2) are consistent with this Ansatz, except of the quadratic odderon term in (1). Thus, we approximate the contribution of this term by its projection on the $n = 0$ (constant) angular component $\Psi^2(k) \cos^2(\varphi) \rightarrow (1/2) \Psi^2(k)$. In this approximation we obtain the following system of equations

$$\frac{\partial \Phi(k, \tau)}{\partial \tau} = \bar{\alpha}_s \int_0^\infty \frac{dk'^2}{k'^2} \left[\frac{k'^2 \Phi(k', \tau) - k^2 \Phi(k, \tau)}{|k'^2 - k^2|} + \frac{k^2 \Phi(k, \tau)}{\sqrt{4k'^2 + k^2}} \right] - \bar{\alpha}_s \Phi^2(k, \tau) + \frac{1}{2} \bar{\alpha}_s \Psi^2(k, \tau), \quad (15)$$

$$\frac{\partial \Psi(k, \tau)}{\partial \tau} = \bar{\alpha}_s \int_0^\infty \frac{dk'^2}{k'^2} \left[\frac{k'^2 (k_{<}/k_{>}) \Psi(k', \tau) - k^2 \Psi(k, \tau)}{|k'^2 - k^2|} + \frac{k^2 \Psi(k, \tau)}{\sqrt{4k'^2 + k^2}} \right] - 2\bar{\alpha}_s \Phi(k, \tau) \Psi(k, \tau), \quad (16)$$

where $k_{<} = \min(k, k')$ and $k_{>} = \max(k, k')$. These equations form the basis of an ongoing numerical analysis [21].

3 Analytical properties of the solution

Let us consider the following Ansatz for the solutions of equations (15,16), in analogy to the similar Ansatz applied in the case of the Balitsky-Kovchegov equation [22, 23, 24]

$$\Phi(k, \tau) = \int \frac{d\gamma}{2\pi i} e^{\gamma t} e^{\omega_P(\gamma)\tau} \phi(\gamma), \quad (17)$$

$$\Psi(k, \tau) = \int \frac{d\gamma}{2\pi i} e^{\gamma t} e^{\omega_O(\gamma)\tau} \psi(\gamma), \quad (18)$$

with $t = \log(k^2/k_0^2)$. and $\omega_P(\gamma)$, $\omega_O(\gamma)$ and $\phi(\gamma)$, $\psi(\gamma)$ being the unknown functions, for which the system is solved. Functions $\omega_R(\gamma)$ with $R = P, O$ determine the rapidity dependence of the component with the scaling dimension γ .

From the Mellin transform of (15,16) we get

$$\omega_P(\gamma) \phi(\gamma) e^{\omega_P(\gamma)\tau} = \bar{\alpha}_s \chi_0(\gamma + 1) \phi(\gamma) e^{\omega_P(\gamma)\tau} - \bar{\alpha}_s \int \frac{d\gamma'}{2\pi i} \phi(\gamma - \gamma') \phi(\gamma') e^{[\omega_P(\gamma') + \omega_P(\gamma - \gamma')]\tau}, \quad (19)$$

and

$$\omega_O(\gamma) \psi(\gamma) e^{\omega_O(\gamma)\tau} = \bar{\alpha}_s \chi_1(\gamma + 1) \psi(\gamma) e^{\omega_O(\gamma)\tau} - 2\bar{\alpha}_s \int \frac{d\gamma'}{2\pi i} \phi(\gamma - \gamma') \psi(\gamma') e^{[\omega_O(\gamma') + \omega_P(\gamma - \gamma')]\tau}, \quad (20)$$

where we omitted the subdominant nonlinearity due to $\Psi^2(k, \tau)$ in (19). When the amplitudes are small, the nonlinearities may be neglected, leading to the following form of functions $\omega_{P,O}$ given by the linear (BFKL) part:

$$\omega_P(\gamma) = \bar{\alpha}_s \chi_0(\gamma + 1), \quad \omega_O(\gamma) = \bar{\alpha}_s \chi_1(\gamma + 1). \quad (21)$$

In the absence of the nonlinear rescattering terms, the behavior of equations (15,16) is well known. There, the solution at large rapidities $\tau \gg 1$ is dominated by the saddle point contribution of the inverse Mellin transform, e.g.

$$\Phi(k, \tau) = \int_{-1/2-i\infty}^{-1/2+i\infty} \frac{d\gamma}{2\pi i} \phi_0(\gamma) \exp[\gamma \log(k^2/k_0^2)] \exp[\bar{\alpha}_s \chi_0(\gamma+1)\tau], \quad (22)$$

with $\phi_0(\gamma)$ specified by the initial condition and k_0 being an arbitrary scale. With $\tau \gg \log(k^2/k_0^2)$, the saddle point occurs at $\gamma = -1/2$ and

$$\Phi(k, \tau) \simeq \frac{\phi_0(-1/2)}{\sqrt{2\pi D_0 \tau}} \frac{k_0}{k} \exp(\omega_0 \tau) \exp\left(-\frac{\log^2(k^2/k_0^2)}{2D_0 \tau}\right), \quad (23)$$

with the diffusion coefficient

$$D_0 = 28\zeta(3)\bar{\alpha}_s \simeq 33.66\bar{\alpha}_s. \quad (24)$$

Analogously, for the odderon solution

$$\Psi(k, \tau) \simeq \frac{\psi_0(-1/2)}{\sqrt{2\pi D_1 \tau}} \frac{k_0}{k} \exp\left(-\frac{\log^2(k^2/k_0^2)}{2D_1 \tau}\right), \quad (25)$$

with

$$D_1 = 4\zeta(3)\bar{\alpha}_s \simeq 4.81\bar{\alpha}_s \ll D_0. \quad (26)$$

Note that the odderon solution has no exponential rapidity dependence, as $\omega_1 = \bar{\alpha}_s \chi_1(1/2) = 0$, see (11). Interestingly enough, the diffusion coefficient of the odderon solution is smaller by a significant factor of seven than the diffusion coefficient of the pomeron. Consequently, the saddle point approximation for the odderon (25) is valid for much larger rapidities τ than the asymptotic formula (23) for the pomeron.

Approximate solutions (23) and (25), valid at large τ , are driven by the Gaussian diffusion factor in $\log(k^2/k_0^2)$ distorted by the prefactor $1/k$. Thus, for any starting conditions $\Phi(k, \tau_0)$ and $\Psi(k, \tau_0)$, for sufficiently large τ the diffusion will populate with color dipoles the domain of small momenta, and both $\Phi(k, \tau)$ and $\Psi(k, \tau)$ will become large, switching on non-linear corrections for small k . Note, however, that the much slower diffusion of the hard odderon amplitude makes it safer against the infra-red effects with respect to the pomeron amplitude. The evolution length needed for the odderon to diffuse from the hard initial condition to the infra-red domain is more than 2.5 times longer than it is for the pomeron. It also means that the scattering amplitude of a hard C -odd source should be less affected by absorptive corrections than the C -even amplitude.

In the saturation regime, the saddle point evaluation of the integrand over γ' in the r.h.s of (19) leads to the following condition [22, 23, 24]

$$\omega_P(\gamma) = 2\omega_P(\gamma/2), \quad (27)$$

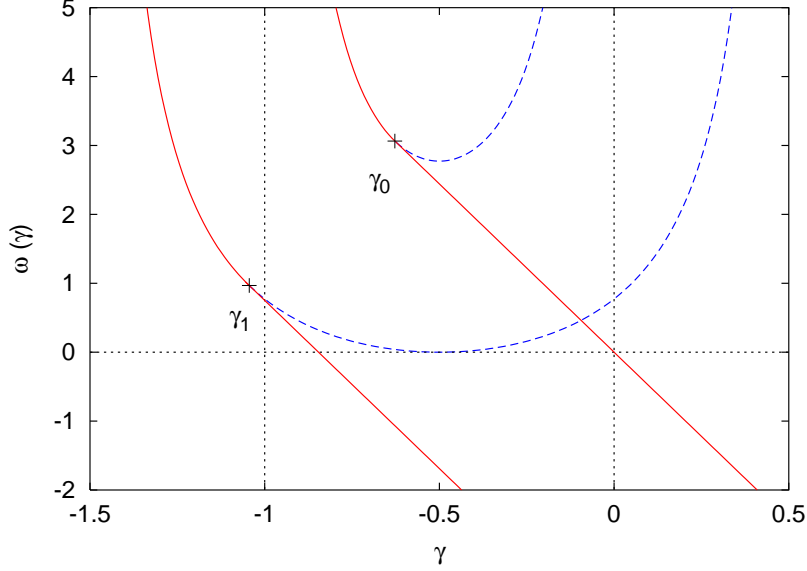


Figure 2: *Matching of $\omega(\gamma)$ in the linear and saturated regimes for the pomeron (upper set) and the odderon (lower set) dipole densities as functions of the real part of scaling dimension γ . The critical scaling dimensions are indicated for the pomeron (γ_0) and for the odderon (γ_1).*

with the solution

$$\omega_P(\gamma) = C\gamma. \quad (28)$$

Recall that such dependence of $\omega_P(\gamma)$ implies the geometric scaling property [22, 23, 25, 24],

$$\Phi(k, \tau) = \int \frac{d\gamma}{2\pi i} \phi(\gamma) \exp(\gamma t + C\gamma\tau) = \Phi(k^2 \exp(C\tau)). \quad (29)$$

At sufficiently large rapidity τ , the integral over γ' in the r.h.s of (20) is also dominated by a contribution from the saddle point $\gamma' = \gamma_s(\gamma)$, such that

$$\frac{\partial}{\partial \gamma'} [\omega_O(\gamma') + \omega_P(\gamma - \gamma')] \Big|_{\gamma'=\gamma_s(\gamma)} = 0. \quad (30)$$

Then we obtain

$$\omega_O(\gamma) = \omega_O(\gamma_s(\gamma)) + \omega_P(\gamma - \gamma_s(\gamma)), \quad (31)$$

thus, using (28)

$$\omega_O(\gamma) - \omega_O(\gamma_s(\gamma)) = C(\gamma - \gamma_s(\gamma)). \quad (32)$$

The equation is fulfilled for any $\gamma_s(\gamma)$ if $\omega_O(\gamma)$ is a linear function,

$$\omega_O(\gamma) = A + C\gamma. \quad (33)$$

After applying the inverse Mellin transform (18), one finds that in the saturation regime

$$\Psi(k, \tau) = \int \frac{d\gamma}{2\pi i} e^{\gamma t} e^{(A+C\gamma)\tau} \psi(\gamma) = \Psi(k^2 \exp(C\tau)) e^{A\tau}, \quad (34)$$

so $\Psi(k, \tau)$ depends on the transverse momentum only through the scaling variable $\xi = k^2 \exp(C\tau)$ of the BK solution, as $\omega_P(\gamma)$ and $\omega_O(\gamma)$ in the saturation region are characterized by the same coefficient C . In contrast to the pomeron case, however, it follows from (34) that the odderon dipole density Ψ depends on rapidity not only through ξ but also by an overall decreasing (as it will be shown that $A < 0$) exponential suppression factor.

Following the Refs. [23, 24] we impose the condition that of the smooth transition (with the first derivative) of $\omega_P(\gamma)$ between the linear and saturation regimes given by (21) and (28) respectively. Then, the transition point γ_0 is determined by

$$(\gamma_0 + 1) \chi_0(\gamma_0 + 1) = \chi'_0(\gamma_0 + 1), \quad (35)$$

thus $\gamma_0 \simeq -0.6275$ and $C = \bar{\alpha}_s \chi'_0(1 + \gamma_0) = -4.8834 \bar{\alpha}_s$. The analogous condition imposed on $\omega_O(\gamma)$ defines the transition point γ_1 between the linear and saturated regime in the odderon sector. From (28) and (33) we deduce that

$$\chi'_1(\gamma_1 + 1) = \chi'_0(\gamma_0 + 1). \quad (36)$$

and $\gamma_1 \simeq -1.0441$. Now, it is straightforward to determine the coefficient A in (33):

$$A = \bar{\alpha}_s [\chi_1(\gamma_1 + 1) - (\gamma_1 + 1) \chi'_1(\gamma_1 + 1)]. \quad (37)$$

The matching procedure is illustrated in Fig. 2. The above results for $\omega_{P,O}(\gamma)$ in the saturated regime may be summarized in the following way,

$$\omega_P(\gamma) = \bar{\alpha}_s \chi'_0(\gamma_0) \gamma \simeq -4.8834 \bar{\alpha}_s \gamma, \quad (38)$$

$$\omega_O(\gamma) = \bar{\alpha}_s [\chi_1(\gamma_1 + 1) + (\gamma - \gamma_1) \chi'_1(\gamma_1 + 1)] \simeq \bar{\alpha}_s [-4.8834 \gamma - 4.1311]. \quad (39)$$

It was shown [14] that at low k the solution to the Balitsky-Kovchegov equation is dominated by $\gamma \simeq 0$, close to the pole of $\chi_0(\gamma+1)$, giving rise to the asymptotic behavior $\Phi(k, \tau) \simeq \log(Q_s(\tau)/k)$, with $Q_s(\tau)$ being the saturation scale. Analogously, the C -odd dipole density $\Psi(k, \tau)$ in the low k region is dominated by the pole of $\chi_1(\gamma+1)$ at $\gamma = 1/2$. This corresponds to the behavior $\Psi(k, \tau) \sim k/Q_s(\tau)$ at $k \rightarrow 0$. Indeed, for $\Phi(k, \tau) \sim \log k$ and $\Psi(k, \tau) \sim k^\alpha$ the nonlinear term $\Phi(k, \tau) \Psi(k, \tau) \sim k^\alpha \log k$ has a double pole in the Mellin space $\sim 1/(\gamma - \alpha)^2$. In equation (8) this double pole can be matched only by a contribution from $(K' \otimes \Psi)(k, \tau)$, which in the Mellin space has the pole structure of $\chi_1(\gamma+1) \psi(\gamma)$, see also (20). Thus, the position of the pole in $\psi(\gamma)$ must coincide with the anti-collinear pole of $\chi_1(\gamma+1)$ at $\gamma = 1/2$.

At the large momentum asymptotics $k \gg Q_s(\tau)$, the nonlinearity in the evolution equation is not relevant, as both $\Phi(k, \tau)$ and $\Psi(k, \tau)$ exhibit a power like decrease at large k . Therefore the large k tails of $\Psi(k, \tau)$ and $\Phi(k, \tau)$ should be driven by the largest scaling dimensions γ for which the linear

term dominates the evolution in equations (19) and (20). It follows that the large k behavior of $\Phi(k, \tau)$ and $\Psi(k, \tau)$ is characterized by the scaling dimensions corresponding to the matching points γ_0 and γ_1 respectively: $\Phi(k, \tau) \sim k^{2\gamma_0}$ and $\Psi(k, \tau) \sim k^{2\gamma_1}$. Note, however, that the asymptotical dependencies at a given value of k (small or large) are reached only after the evolution length τ is sufficiently long for the diffusion to occur from the initial condition to the region of momenta close to k .

Finally, let us determine the rapidity dependencies of the dipole densities at a fixed momentum k and a large τ . It follows from the previous considerations, that for $k < Q_s(\tau)$, $\Phi(k, \tau) \sim \log(\tau) + \text{const}$ and the C -odd density is strongly suppressed, $\Psi(k, \tau) \sim \exp((C/2 + A)\tau) \simeq \exp(-6.55\bar{\alpha}_s\tau)$. At large k , $\Phi(k, \tau)$ grows as $\exp(C\gamma_0\tau) \sim \exp(3.05\bar{\alpha}_s\tau)$ and $\Psi(k, \tau)$ decreases with rapidity rather mildly, $\Psi(k, \tau) \sim \exp((C\gamma_1 + A)\tau) \simeq \exp(-0.97\bar{\alpha}_s\tau)$.

In the forthcoming study [21] the above results will be compared to the results of numerical investigations.

4 Summary and remarks

Let us recapitulate the approximate results:

1. The scale parameter (saturation scale) $Q_s(\tau)$ of the solutions to the nonlinear, coupled evolution equations for the pomeron and the odderon exchange depends exponentially on the rapidity $Q_s(\tau) \simeq Q_0 \exp(|C|\tau/2)$, with the coefficient C that coincides with its analogue in the solution of the Balitsky-Kovchegov equation.
2. The pomeron solution exhibits approximate geometric scaling $\Phi(k, \tau) \simeq \Phi(\xi)$ with $\xi = k^2 \exp(C\tau)$, and the shape of the odderon solution depends on the same scaling variable ξ but the overall normalization decreases with rapidity: $\Psi(k, \tau) \simeq \Psi(\xi) \exp(A\tau)$.
3. In the saturation region $k \ll Q_s(\tau)$ one gets the following leading behavior of the solutions:

$$\Phi(k, \tau) \simeq \log(Q_s(\tau)/k), \quad \Psi(k, \tau) \sim k/Q_s(\tau) \exp(A\tau),$$

note that the normalizing prefactor of $\Psi(k, \tau)$ is not uniquely determined in our approach.

4. In the region of linear evolution $k \gg Q_s(\tau)$, one obtains

$$\Phi(k, \tau) \sim [k/Q_s(\tau)]^{2\gamma_0}, \quad \Psi(k, \tau) \sim [k/Q_s(\tau)]^{2\gamma_1} \exp(A\tau).$$

5. The approximate numerical values of the parameters read: $C \simeq -4.88\bar{\alpha}_s$, $A \simeq -4.11\bar{\alpha}_s$, $\gamma_0 \simeq -0.63$ and $\gamma_1 \simeq -1.04$.

Clearly, the accuracy of the applied approximations is limited, so the obtained characteristics of the solution to the system of equations (7) and (8) may differ from the exact results. For instance, in the case of the BK equation, the value of the exponent $C \simeq -4.2\bar{\alpha}_s$ was determined in a numerical analysis, smaller than the value $C \simeq -4.88\bar{\alpha}_s$ found in the approximate scheme [24]. The accuracy level of the present analysis is expected to be similar. Moreover, the studied evolution equations are given in the leading $\log(1/x)$ approximation and thus the energy dependence of the saturation scale is by far too steep. Consequently the absorptive effects are exaggerated.

Let us also point out the importance of the squared odderon term in the evolution equation (7). This term may change qualitatively the behavior of the solution for both the pomeron and the odderon in the case when the initial condition contains only the C -odd part, that is $\Phi(k, \tau = 0) = 0$. Then, in the absence of $\Psi^2(k, \tau)$, the pomeron solution would vanish for all rapidities, $\Phi(k, \tau) = 0$, and the system (7,8) would reduce to the linear odderon evolution equation. The quadratic linear term, however, acts as a source for the pomeron amplitude, driving it away from zero. Actually, due to positive rapidity dependence of $\Phi(k, \tau)$ and the negative one of $\Phi(k, \tau)$, at large rapidities the pomeron amplitude will dominate anyway, generating the saturation scale and leading to the generic behavior of the both amplitudes, which was summarized above.

The results of our analysis may be compared to the approximate asymptotic solution to the WHIMIKS equation obtained in Ref. [11],

$$O(\mathbf{x}, \mathbf{y}) \simeq \exp \left[-\frac{|C|}{2} \bar{\alpha}_s^2 (\tau - \tau_0)^2 \right], \quad \text{for } |\mathbf{x} - \mathbf{y}| \gg 1/Q_s(\tau). \quad (40)$$

We confirm the conclusion of Ref. [11] that the saturation scale generated in the pomeron sector drives the absorptive effects in the odderon sector, imposing its rapidity dependence on the C -odd scattering amplitude. It is clear, however, that the current analysis in the momentum space provides novel information about the universal features of the solution to the WHIMIKS equation, like the universal scaling behavior for $k \ll Q_s(\tau)$ and $k \gg Q_s(\tau)$ and the overall rapidity dependence. Besides that, it should be useful to have the direct insight into the momentum dependence of the odderon exchange.

It should be mentioned that a similar approach to the one proposed in this letter may be applied to study the evolution of components with higher conformal spins of the BK equation or of the WHIMIKS equation. We anticipate that the nonlinear effects will cause strong damping of the higher conformal spin components, in analogy to the odderon $n = 1$ component. Finally, it would be worthwhile to continue the analysis of WHIMIKS equation keeping the non-zero momentum transfer and investigate the phenomenological consequences of the absorption for some classical odderon mediated processes, like, for instance, the η_c photoproduction off a proton [2, 7].

5 Conclusions

The system of coupled nonlinear small x evolution equations for the pomeron and the odderon (WHIMIKS equation) was analyzed with approximate analytical methods in momentum space. The system generates the saturation scale, growing exponentially with rapidity, with the same exponent as emerges from the BK equation. With respect to the BK case, the change was not significant for properties of the C -even dipole density at large rapidity. The C -odd dipole density was found to depend on the momentum only through the geometric scaling variable of the BK equation, but in addition an overall dependence on rapidity was found in the form of a decreasing exponential prefactor. Low and large momentum asymptotics of the dipole densities was determined. The main conclusion from this paper is that the effects of rescattering lead to a strong damping of the odderon for momenta below the saturation scale and a rather moderate suppression at large momenta. Therefore, the absorptive effects reduce substantially the odderon contribution to semihard processes with a scale smaller than the saturation scale, which may explain the observed weakness of the odderon exchange amplitude.

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