DESY 05-003, SFB/CPP-06-06 hep-ph/xxyymmm

$O(\alpha_s^3)$ contributions to $F_L^{Q\overline{Q}}(x, Q^2)$ for large virtualities

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The $O(\alpha_s^3)$ contributions to the heavy flavor Wilson coefficients for the structure function $F_L(x, Q^2)$ are calculated in the region $Q^2 \gg m^2$ using the renormalization group method.

1. INTRODUCTION

The heavy flavor contributions to unpolarized deep-inelastic structure functions are large in the region of small values of x, see e.g. [1]. At present they are known to $O(\alpha_s^2)$ [2], while the anomalous dimensions and Wilson coefficients for the light parton contributions were calculated to $O(\alpha_s^3)$ [3,4]. Since the scaling violations of the light parton and heavy flavor terms are different, the knowledge of both contributions at the same order of precision is highly desirable to perform QCD analyses. While a complete calculation of the $O(\alpha_s^3)$ heavy flavor Wilson coefficients is still a technical problem, the calculation of these quantities in the asymptotic regime $Q^2 \gg m^2$ seems feasable. Calculations of this kind were carried out at the $O(\alpha_s^2)$ level before [5,6].

In the region of smaller values of x the structure function $F_L(x, Q^2)$ can be used to put stringent constraints on the gluon distribution [7]. It is therefore importnat to calculate this quantity as precisely as possible. In this paper we give a brief outline of the calculation of the heavy flavor Wilson coefficients for the longitudinal structure function $F_L(x, Q^2)$ in the asymptotic region. Details are presented in [8].

2. THE METHOD

The twist–2 contributions to structure functions in deeply inelastic scattering due to massless partons are described as convolutions between the (massless) parton densities and the Wilson coefficients due to mass factorization. Since the scale evolution of parton densities is free of quark mass effects, the heavy flavor contributions to the structure functions result form the associated Wilson coefficients only. In the limit $Q^2 \gg m^2$ the non–power contributions to these coefficient functions obey the factorization relation

$$H_{I,l}^{\mathrm{S,NS},g}\left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right) = A_{k,l}^{\mathrm{S,NS},g}\left(\frac{m^2}{\mu^2}\right) \otimes C_{I,k}^{\mathrm{S,NS},g}\left(\frac{Q^2}{\mu^2}\right), \qquad (1)$$

with μ the factorization scale, $A_{k,l}$ partonic operator matrix elements and $C_{I,k}$ the respective light-parton Wilson coefficients. The operator-matrix elements are process-independent quantities. Eq. (1) allows to calculate all logarithmic contributions and the constant term in the on-mass-shell scheme. For the $O(\alpha_s^3)$ asymptotic heavy flavor Wilson coefficients for $F_2(x, Q^2)$ the 3-loop operator matrix elements are required. In case of $F_L(x, Q^2)$ the $O(\alpha_s^3)$ contributions depend on the 2-loop operator matrix elements and the massless Wilson coefficients at $O(\alpha_s^3)$ [4] only, since at leading order the Wilson coefficient is scale-independent.

3. HEAVY FLAVOR WILSON COEFFICIENT FOR $F_L(x, Q^2)$

To $O(\alpha_s^3)$ three heavy flavor Wilson coefficients contribute: $H_{L,g}^{\mathrm{S}}, H_{L,q}^{\mathrm{PS}}, H_{L,q}^{\mathrm{NS}}$. In the asymptotic

*This work was supported in part by DFG Sonderforschungsbereich Transregio 9, Computergestützte Theoretische Physik.

region $Q^2 \gg m^2$ they are given in terms of Mellin convolutions between the light-parton Wilson coefficients and the corresponding operator matrix elements $A_{i,j}^{(k)}$:

$$H_{L,g}^{\rm S}\left(\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\right) = a_s \widehat{C}_{L,g}^{(1)}\left(\frac{Q^2}{\mu^2}\right) + a_s^2\left[A_{Q,g}^{(1)}\left(\frac{\mu^2}{m^2}\right)\otimes C_{L,q}^{(1)}\left(\frac{Q^2}{\mu^2}\right) + \widehat{C}_{L,g}^{(2)}\left(\frac{Q^2}{\mu^2}\right)\right] + a_s^3\left[A_{Q,g}^{(2)}\left(\frac{\mu^2}{m^2}\right)\otimes C_{L,q}^{(1)}\left(\frac{Q^2}{\mu^2}\right) + A_{Q,g}^{(1)}\left(\frac{\mu^2}{m^2}\right) \otimes C_{L,q}^{(2)}\left(\frac{Q^2}{\mu^2}\right) + \widehat{C}_{L,g}^{(3)}\left(\frac{Q^2}{\mu^2}\right)\right]$$

$$H_{L,q}^{\rm PS}\left(\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\right) = a_s^2 \widehat{C}_{L,q}^{\rm PS,(2)}\left(\frac{Q^2}{\mu^2}\right) + a_s^3\left[A_{Qq}^{\rm PS,(2)}\left(\frac{\mu^2}{m^2}\right) \otimes C_{L,q}^{(1)}\left(\frac{Q^2}{\mu^2}\right) + \widehat{C}_{L,q}^{\rm PS,(3)}\left(\frac{Q^2}{\mu^2}\right)\right]$$

$$H_{L,q}^{\rm NS}\left(\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\right) = a_s^2 \widehat{C}_{L,q}^{\rm NS,(2)}\left(\frac{Q^2}{\mu^2}\right) + a_s^3\left[A_{qq,Q}^{\rm NS,(2)}\left(\frac{\mu^2}{m^2}\right)\otimes C_{L,q}^{(1)}\left(\frac{Q^2}{\mu^2}\right) + \widehat{C}_{L,q}^{\rm NS,(3)}\left(\frac{Q^2}{\mu^2}\right)\right] ,$$

with $\hat{f} = f(N_F+1) - f(N_F)$. The operator matrix elements have the form

$$A_{i,j}^{(k)}\left(\frac{m^2}{\mu^2}\right) = \sum_{l=1}^{k} \hat{a}_{i,j}^{l,(k)} \ln^l\left(\frac{m^2}{\mu^2}\right) + a_{i,j}^{(k)} .$$

They were calculated in [5] up to $k = 2$ in z-space. The operator matrix elements take their most simple structure in Mellin space. As an example, the coefficient $a_{Qg}^{(2)}(N)$ reads

$$\begin{aligned}
a_{Qg}^{(2)}(N) = 4C_FT_R\Biggl\{&\frac{N^2+N+2}{N(N+1)(N+2)}\Biggl[-\frac{1}{3}S_1^3(N-1)+\frac{4}{3}S_3(N-1)-S_1(N-1)\\
&\times S_2(N-1)-2\zeta_2S_1(N-1)\Biggr]\\
&+\frac{2}{N(N+1)}S_1^2(N-1)\\
&+\frac{N^4+16N^3+15N^2-8N-4}{N^2(N+1)^2(N+2)}S_2(N-1)\\
&+\frac{3N^4+2N^3+3N^2-4N-4}{2N^2(N+1)^2(N+2)}\zeta_2\\
&+\frac{N^4-N^3-16N^2+2N+4}{N^2(N+1)^2(N+2)}S_1(N-1)\\
&+\frac{P_2(N)}{2N^4(N+1)^4(N+2)}\Biggr\}\\
+4C_AT_R\Biggl\{&\frac{N^2+N+2}{N(N+1)(N+2)}\Biggl[4\mathbf{M}\left[\frac{\mathrm{Li}_2(x)}{1+x}\right](N)\\
&+\frac{1}{3}S_1^3(N)+3S_2(N)S_1(N)+\frac{8}{3}S_3(N)\\
&+\beta''(N+1)-4\beta'(N+1)S_1(N)-4\beta(N+1)\zeta_2\\
&+\zeta_3\Biggr]-\frac{N^3+8N^2+11N+2}{N(N+1)^2(N+2)^2}S_1^2(N)\\
&-2\frac{N^4-2N^3+5N^2+2N+2}{(N-1)N^2(N+1)^2(N+2)}\zeta_2\\
&-\frac{P_3(N)}{(N-1)N^2(N+1)^2(N+2)^2}S_2(N)\\
&-\frac{P_4(N)}{N(N+1)^3(N+2)^3}S_1(N)\\
&-4\frac{\left(N^2-N-4\right)}{(N+1)^2(N+2)^2}\beta'(N+1)\\
&+\frac{P_5(N)}{(N-1)N^4(N+1)^4(N+2)^4}\Biggr\} ,
\end{aligned}$$

where

$$\begin{aligned}
P_2(N) &= 12N^8+54N^7+136N^6+218N^5\\
&\quad +221N^4+110N^3-3N^2-24N-4\\
P_3(N) &= 7N^5+21N^4+13N^3+21N^2+18N\\
&\quad +16\\
P_4(N) &= N^6+8N^5+23N^4+54N^3+94N^2\\
&\quad +72N+8\\
P_5(N) &= 2N^{12}+20N^{11}+86N^{10}+192N^9\\
&\quad +199N^8-N^7-297N^6-495N^5\\
&\quad -514N^4-488N^3-416N^2-176N\\
&\quad -32\\
\beta(N) &= \frac{1}{2}\left[\psi\left(\frac{N+1}{2}\right)-\psi\left(\frac{N}{2}\right)\right] .
\end{aligned}$$

Similar, even simpler, expressions are obtained for the other matrix elements [8]. The non-logarithmic contributions to $H_{L,g(q)}^{\rm S,PS,NS}$ contain the respective light-parton Wilson coefficients of given order and a term, which consists out of convolutions of operator matrix elements, splitting functions and lower order Wilson coefficients.

Since

$$\mathbf{M}\left[\frac{\mathrm{Li}_2(x)}{1+x}\right](N) - \zeta_2\beta(N) = (-1)^N\left[S_{-2,1}(N)(N-1) + \frac{5}{8}\zeta_3\right] \quad (2)$$

also the heavy flavor Wilson coefficients to $O(\alpha_s^3)$ belong to the class which can be represented by harmonic sums *without* an index $k = -1$, as observed earlier for all known 2–loop Wilson coefficients [9] and the 3–loop anomalous dimensions [10,11]. Due to this the general class of harmonic sums [12] is drastically reduced [10,13] even before referring to structural relations.

4. SMALL x LIMIT

In the small-x limit the asymptotic heavy flavor Wilson coefficient for $F_L^S(x, Q^2)$ obey

$$H_L^S(z) \propto a_s^2\,\frac{d_1^{(1)}}{z} + \sum_{k=2}^{\infty} a_s^{k+1} \times \left[d_k^{(1)}\frac{\ln^{k-1}(z)}{z} + d_k^{(2)}\frac{\ln^{k-2}(z)}{z} + \ldots\right]. \quad (3)$$

for $Q^2 = \mu^2$. The lowest order result $d_{1,i}^{(1)} = -32C_iT_R/9,\ i = A, F$ [14] agrees with the corresponding limit of the complete calculation [5]. At $O(\alpha_s^3)$ the leading terms are due to the massless contributions [4] in the $\overline{\rm MS}$ scheme, while the small x heavy flavor contributions emerge in $d_k^{(2)}$ for the first time[2]

$$d_{2,i}^{(2)} = -32C_iC_FT_R \times\left[\frac{1}{3}\ln^2\left(\frac{Q^2}{m^2}\right) - \frac{10}{9}\ln\left(\frac{Q^2}{m^2}\right) + \frac{28}{27}\right] + \tilde{d}_{2,i}^{(2)} - \delta_{iA}\frac{256}{27}C_FT_R^2(2N_f+1)\ln\left(\frac{Q^2}{m^2}\right) \quad (4)$$

[2]We corrected typographical errors contained in Nucl. Phys. (Proc. Suppl.) **157** (2006) 2.

Here, $\tilde{d}_{2,i}^{(2)}$ are contributions due to the massless Wilson coefficient. While the genuine heavy flavor terms in the sub-leading order scale with the color factor, the $\tilde{d}_{2,i}^{(2)}$-terms do not.

5. CONCLUSIONS

In the region $Q^2 \gg m^2$ the 3–loop heavy flavor Wilson coefficients for the structure function $F_L(x, Q^2)$ can be calculated using the renormalization group method. They depend on the massless $\overline{\rm MS}$ scheme Wilson coefficients up to 3–loop order and massive operator matrix elements and anomalous dimensions up to 2–loop order. The complexity of the structure of the Wilson coefficients beyond the $\overline{\rm MS}$ light–parton 3–loop contributions is that of the 2–loop anomalous dimensions. Their representation in Mellin space shows that indices $k = -1$ do not occur in the respective harmonic sums, as observed for a large variety of other cases before. The pure heavy flavor terms at $O(\alpha_s^3)$ contribute at one order less than the leading logarithmic order in the small x region.

Acknowledgment. I would like to thank the organizers of the conference for arranging a very interesting meeting in a splendid atmosphere.

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