hep-ph/xxxxxxx
HRI-04/2006
DESY 06-054
SFB/CPP-06-22

QCD threshold corrections to Higgs decay and to hadroproduction in l^+l^- annihilation

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ABSTRACT

We present threshold enhanced QCD corrections to the bottom quark energy distribution in Higgs boson decay and to hadroproduction in l^+l^- annihilation beyond leading order in the strong coupling constant. This is achieved using the resummed decay distribution obtained using renormalisation group invariance and the mass factorisation theorem that they satisfy and Sudakov resummation of soft gluons.

The Higgs boson (H) which emerges in the electroweak symmetry breaking mechanism [1] is the only particle that is yet to be discovered in the Standard Model(SM). Searches at LEP experiments [2] indicate that the lower bound on its mass is around 114.2 GeV and the upper bound around 260 GeV at 95% confidence level. The hadronic machine Tevatron at Fermi-lab currently running with increased energy and the upcoming Large Hadron Collider (LHC) at CERN have the physics goal of discovering the Higgs boson. At hadronic colliders, among the potential channels through which the Higgs boson can be produced, the vector boson fusion process is one of the promising ones [3]. Here the Higgs boson is produced in a reaction $P_1+P_2 \to V+H$, where P_i are the incoming hadrons, V is the vector boson $W^{\pm}/Z$, followed by the dominant decay mode $H \to b+\overline{b}$. For the heavy Higgs boson searches, the decay to bottom quarks can be accessible in the Two-Higgs Doublet models due to enhanced couplings [4].

A similar process in nature is inclusive hadroproduction in l^+l^- annihilation [5]. Here the incoming leptons annihilate into vector boson such as γ^*/Z which then fragments into hadrons, i.e., $l^+l^- \to \gamma^*, Z \to P+X$, where P is any hadron and X is the remaining final state. The double differential cross section for producing a hadron with energy fraction x of the parton produced in the annihilation is given by

$$\frac{d^2\sigma^P}{dx\, d\cos\theta} = \frac{3}{8}(1+\cos^2\theta)\, \frac{d\sigma_T^P}{dx}(x,q^2) + \frac{3}{4}\sin^2\theta\, \frac{d\sigma_L^P}{dx}(x,q^2) + \frac{3}{4}\cos\theta\, \frac{d\sigma_A^P}{dx}(x,q^2)\,, \qquad (1)$$

where θ is the center of mass angle of the final state hadron. The energy fraction is given by $x = 2p\cdot q/q^2$, where p and q are the momenta of the final state hadron and the intermediate vector boson respectively. The B hadron production in l^+l^- channel is an important process to determine the non-perturbative b quark fragmentation functions [6]. Precise knowledge of them will reduce theory uncertainties in Higgs decays to bottom quarks and also in the top quark mass reconstructions in the top quark decays at Tevatron and LHC.

In the case of the decay process $H \to b+\overline{b}$, we study the energy distribution of one of the bottom quarks. We use the perturbative fragmentation approach [7] which is valid when $m_b \ll m_H$ to factorise the decay distribution into a part which contains the Higgs boson decay to massless partons and a part containing perturbative fragmentation functions denoted by D_I^b that describe the fragmentation of massless partons into massive bottom quarks. In this approach, the large logarithms $\log(m_H^2/m_b^2)$, where m_H is the Higgs boson mass and m_b bottom quark mass, can be resummed using perturbative fragmentation functions. The normalised decay distribution is given by

$$\frac{1}{\Gamma_0}\frac{d\Gamma_b}{dx_b}(x_b,q^2,m_b^2) = \sum_I \int_{x_b}^1 \frac{dz}{z}\, C_I\left(z,q^2,\mu_R^2,\mu_F^2\right) D_I^b\left(\frac{x_b}{z},m_b^2,\mu_F^2\right) \qquad I=b,g, \qquad (2)$$

where Γ_0 is the Born decay distribution, $x_b = 2p_b\cdot q/q^2$ with p_b, q the momenta of b quark and the Higgs boson, with masses m_b and m_H respectively. Here $q^2 = m_H^2$. The renormalisation scale μ_R is due to ultraviolet renormalisation and the factorisation scale μ_F is due to mass factorisation of the collinear singularities. The fixed order result for C_I^b for $I=b$ up to next to leading order (NLO) in strong coupling constant α_s is available in the literature [8]. The soft gluons that result from the outgoing partons lead to large logarithms that can be resummed systematically. The impact of such effects up to the next to leading logarithmic (NLL) level has been studied along with fixed order NLO QCD corrections in [6,8].

The inclusive hadroproduction in l^+l^- annihilation process factorises as

$$\frac{1}{\sigma^{(0)}}\frac{d\sigma_T^P(x,q^2)}{dx} = \sum_I \int_x^1 \frac{dz}{z} C_I(z,q^2,\mu_R^2,\mu_F^2) D_I^P\left(\frac{x}{z},\mu_F^2\right) \qquad I = q,\overline{q},g \tag{3}$$

where $z = 2p_I \cdot q/q^2$ is the partonic energy fraction. Here, p_I is the momentum of the parton that fragments into the final state hadron with momentum p. The non-perturbative function $D_I^P(x/z,\mu_F^2)$ is the fragmentation function that describes the fragmentation of the parton of type I into hadron of type P. The perturbatively calculable coefficient function C_I is known up to NNLO in QCD for $I = q,\overline{q},g$, [9–12]. The NNLL soft gluon resummation to this process is also known in the literature [13–17].

Even though perturbative Quantum Chromodynamics (pQCD) provides a framework to successfully compute various observables involving hadrons at high energies with less theoretical uncertainty for the physics studies, the fixed order QCD predictions often have limitations in applicability due to the presence of various logarithms that become large in some kinematic regions which otherwise can be probed by the experiments. The standard approach to probe these regions is to resum the class of such large logarithms supplemented with fixed order results. This can almost cover the dominant kinematic region of the phase space.

In this paper, we follow the method that we used in Refs. [18,19] to study the soft gluon effects. This generalises our earlier approach to include any infrared safe decay distribution in perturbative QCD. We systematically formulate a framework to resum the dominant soft gluon contributions in z space to the decay distributions, where z is the appropriate scaling variable that enters in the process. We have followed a similar approach described in [18,19] that uses renormalisation group (RG) invariance, mass factorisation and Sudakov resummation of soft gluons as the guiding principles. Using the resummed results in z space, we compute the soft plus virtual part of the dominant partonic decay distributions beyond *NLO*. In Ref. [18] we determined the threshold exponents D_i^I and $B_{i,DIS}$ up to the three loop level for Drell-Yan process, Higgs boson productions and deeply inelastic scattering (DIS) cross sections using our resummed soft as well as soft plus jet distribution functions. Here we extend the similar all order proof which establishes the relation between soft plus jet distribution functions and the threshold resummation exponents and demonstrate the usefulness of this approach to derive higher order threshold enhanced corrections for any infrared safe decay distributions.

We find that the soft plus jet distribution relevant for our present study can be gotten entirely from that of DIS due to the crossing symmetry between them [20–24]. In fact, we find that they are identical. In addition, the Altarelli-Parisi(AP) splitting functions that determine the scale evolution of the fragmentation functions coincide with the AP splitting functions of parton distribution functions in the threshold region. Extensive discussions on this topic can be found in [25–30]. With these two essential ingredients and using the method described in [18, 19] we could successfully reproduce the fixed order NLO soft plus virtual part as well as the NLL resummation exponents [8] for the bottom quark production in Higgs decay and also the fixed order NNLO soft plus virtual part [9–12] and the NNLL resummation exponents [13–17] for the hadron production in l^+l^- annihilation. In addition, we can compute the fixed order NNLO soft plus virtual part of the b quark energy distribution in the Higgs boson decay and also partial but dominant N^iLO results for $i=3,4$. With the same level of accuracy, we can obtain N^iLO with $i=3,4$ soft plus virtual contributions to hadroproduction in l^+l^- annihilation.

In this article, we will mainly concentrate on dominant contributions coming from the threshold effects. This contribution to the energy distribution of the bottom quark in Higgs boson decay and to the hadroproduction in the l^+l^- annihilation process can be obtained by adding the soft part of the decay distributions with the renormalised virtual corrections and performing mass factorisation using appropriate mass factorisation counter terms. We call this infra-red safe combination a "soft plus virtual"(sv) part of the decay distribution. The soft plus virtual part denoted by $\widetilde{\Delta}^{sv}_I(z,q^2,\mu_R^2,\mu_F^2)$ of the perturbative decay distributions C_I after mass factorisation is found to be

$$\widetilde{\Delta}^{sv}_I(z,q^2,\mu_R^2,\mu_F^2) = \mathcal{C}\exp\left(\widetilde{\Psi}^I(z,q^2,\mu_R^2,\mu_F^2,\varepsilon)\right)\Big|_{\varepsilon=0} \tag{4}$$

where $\widetilde{\Psi}^I(z,q^2,\mu_R^2,\mu_F^2,\varepsilon)$ is a finite distribution. For a Higgs boson decaying to bottom quarks, we put $I=b$ and for that decaying to gluon, $I=g$. For the hadroproduction in l^+l^- annihilation, $I=q,\overline{q}$. Here $\widetilde{\Psi}^I(z,q^2,\mu_R^2,\mu_F^2,\varepsilon)$ is computed in $4+\varepsilon$ dimensions, where n denotes the number of space–time dimensions.

$$\begin{aligned}\widetilde{\Psi}^I(z,q^2,\mu_R^2,\mu_F^2,\varepsilon) &= \left(\ln\left(Z^I(\hat{a}_s,\mu_R^2,\mu^2,\varepsilon)\right)^2 + \ln|\hat{F}^I(\hat{a}_s,Q^2,\mu^2,\varepsilon)|^2\right)\delta(1-z)\\ &\quad +2\,\widetilde{\Phi}^{\,I}(\hat{a}_s,q^2,\mu^2,z,\varepsilon) - \mathcal{C}\ln\widetilde{\Gamma}_{II}(\hat{a}_s,\mu^2,\mu_F^2,z,\varepsilon), \qquad I=b,g,q,\overline{q}\end{aligned} \tag{5}$$

The symbol "$\mathcal{C}$" means convolution. For example, $\mathcal{C}$ acting on an exponential of a function $f(z)$ has the following expansion:

$$\mathcal{C}e^{f(z)} = \delta(1-z) + \frac{1}{1!}f(z) + \frac{1}{2!}f(z)\otimes f(z) + \frac{1}{3!}f(z)\otimes f(z)\otimes f(z) + \cdots \tag{6}$$

The function $f(z)$ is a distribution of the kind $\delta(1-z)$ and $\mathcal{D}_i$,

$$\mathcal{D}_i = \left[\frac{\ln^i(1-z)}{(1-z)}\right]_+ \qquad i=0,1,\cdots \tag{7}$$

and the symbol $\otimes$ denotes the Mellin convolution. We drop all the regular functions that result from various convolutions. $\hat{F}^I(\hat{a}_s,Q^2,\mu^2,\varepsilon)$ are the form factors that enter these processes. For the Higgs boson decay to bottom quarks, it is related to the Yukawa type of interaction and to the gluons, it is related to Higgs boson field coupled to the kinetic term of the gauge fields. For the l^+l^- annihilation, the form factor arises from the vector current. In the form factors, $Q^2=-q^2=-m_H^2$ for the Higgs boson decay and $Q^2=-q^2=-q^2_{\gamma^*/Z}$. The functions $\widetilde{\Phi}^{\,I}(\hat{a}_s,q^2,\mu^2,z,\varepsilon)$ are called the soft plus jet distribution functions. The unrenormalised (bare) strong coupling constant $\hat{a}_s$ is defined as $\hat{a}_s = \hat{g}_s^2/(16\pi^2)$ where $\hat{g}_s$ is the strong coupling constant which is dimensionless in $n = 4+\varepsilon$. The scale μ comes from the dimensional regularisation in order to make the bare coupling constant $\hat{g}_s$ dimensionless in n dimensions. The bare coupling constant $\hat{a}_s$ is related to the renormalised one by the following relation:

$$S_\varepsilon \hat{a}_s = Z(\mu_R^2) a_s(\mu_R^2) \left(\frac{\mu^2}{\mu_R^2}\right)^{\frac{\varepsilon}{2}} \tag{8}$$

The scale μ_R is the renormalisation scale at which the renormalised strong coupling constant $a_s(\mu_R^2)$ is defined. The factorisation scale μ_F is due to mass factorisation and the constant $S_\varepsilon = exp\{(\gamma_E - \ln 4\pi)\varepsilon/2\}$ is the spherical factor characteristic of n-dimensional regularisation. The renormalisation constant up to three loop is given by

$$Z(\mu_R^2) = 1 + a_s(\mu_R^2)\frac{2\beta_0}{\varepsilon} + a_s^2(\mu_R^2)\left(\frac{4\beta_0^2}{\varepsilon^2} + \frac{\beta_1}{\varepsilon}\right) + a_s^3(\mu_R^2)\left(\frac{8\beta_0^3}{\varepsilon^3} + \frac{14\beta_0\beta_1}{3\varepsilon^2} + \frac{2\beta_2}{3\varepsilon}\right) \tag{9}$$

The renormalisation constant $Z(\mu_R^2)$ relates the bare coupling constant $\hat{a}_s$ to the renormalised one $a_s(\mu_R^2)$ through Eq. (8). The coefficients β_0,β_1 and β_2 are the coefficients that appear in RG equation for the strong coupling constant up to three loop level [31, 32]

The factors $Z^I(\hat{a}_s, \mu_R^2, \mu^2, \varepsilon)$ are the overall operator renormalisation constants which renormalise the bare form factors $\hat{F}^I(\hat{a}_s, Q^2, \mu^2, \varepsilon)$. For the vector current $Z^q(\hat{a}_s, \mu_R^2, \mu^2, \varepsilon) = 1$, but for the bottom quarks and gluons coupled to the Higgs boson, the corresponding form factors get overall renormalisations [31, 33, 34]. The bare form factors $\hat{F}^I(\hat{a}_s, Q^2, \mu^2, \varepsilon)$ corresponding to the unrenormalised operators satisfy the Sudakov differential equation [35–38]. In dimensional regularisation, the formal solution to the differential equation up to four loop level can be found in [18, 39] in terms of A^I, the standard cusp anomalous dimensions, and the constants $G_i^{\,I}(\varepsilon)$ for both $I = q, b$ and $I = g$ [40] to the required accuracy in ε. The single poles of the form factors contain the combination [18, 19, 39, 41]

$$2\left(B_i^{\,I} - \delta_{I,g}\; i\,\beta_{i-1} - \delta_{I,b}\gamma_{i-1}^b\right) + f_i^{\,I}$$

at order $\hat{a}_s^i$. The terms proportional $-2(\delta_{I,g}\; i\,\beta_{i-1} + \delta_{I,b}\gamma_{i-1}^b)$ come from the large momentum region of the loop integrals that are giving ultraviolet divergences. The poles containing them will go away when the form factors undergo overall operator UV renormalisation through the renormalisation constants $Z^{\,I}$ which satisfy the RG equations

$$\begin{aligned} \mu_R^2\frac{d}{d\mu_R^2}\ln Z^g(\hat{a}_s, \mu_R^2, \mu^2, \varepsilon) &= \sum_{i=1}^{\infty} a_s^i(\mu_R^2)\left(i\,\beta_{i-1}\right) \\ \mu_R^2\frac{d}{d\mu_R^2}\ln Z^b(\hat{a}_s, \mu_R^2, \mu^2, \varepsilon) &= \sum_{i=1}^{\infty} a_s^i(\mu_R^2)\,\gamma_{i-1}^b \end{aligned} \tag{10}$$

where $\varepsilon \to 0$ is set. The constants $i\,\beta_{i-1}$ and γ_{i-1}^b are anomalous dimensions of the renormalised form factors F^g and F^b respectively. After the overall operator renormalisation through $Z^{\,I}$ and coupling constant renormalisation through Z, the remaining poles will be proportional to B_i^I and f_i^I in addition to the standard cusp anomalous dimensions A_i^I. The constants B_i^I and f_i^I are known up to order a_s^3 [40–43].

The collinear singularities resulting from massless partons are removed in $\overline{MS}$ scheme using the mass factorisation kernel $\widetilde{\Gamma}(z,\mu_F^2,\varepsilon)$. The kernel $\widetilde{\Gamma}(z,\mu_F^2,\varepsilon)$ satisfies the following renormalisation group equation:

$$\mu_F^2 \frac{d}{d\mu_F^2}\widetilde{\Gamma}(z,\mu_F^2,\varepsilon) = \frac{1}{2}\widetilde{P}\left(z,\mu_F^2\right)\otimes\widetilde{\Gamma}\left(z,\mu_F^2,\varepsilon\right)\ . \qquad (11)$$

The function $\widetilde{P}(z,\mu_F^2)$ are the well known (matrix-valued) Altarelli-Parisi splitting functions

$$\widetilde{P}(z,\mu_F^2) = \sum_{i=1}^{\infty} a_s^i(\mu_F^2)\widetilde{P}^{(i-1)}(z) \qquad (12)$$

The diagonal terms of the splitting functions $\widetilde{P}^{(i)}(z)$ have the following structure

$$\widetilde{P}_{II}^{(i)}(z) = 2\left[B_{i+1}^I\delta(1-z)+A_{i+1}^I\mathcal{D}_0\right]+\widetilde{P}_{reg,II}^{(i)}(z)\ , \qquad (13)$$

where $\widetilde{P}_{reg,II}^{(i)}$ are regular when $z\to 1$. In the case of the soft plus virtual part of the decay distributions, only the diagonal parts of the kernels contribute. In the $\overline{MS}$ scheme, the kernel contains only poles in ε. The kernel can be expanded in powers of bare coupling $\hat{a}_s$ as

$$\widetilde{\Gamma}(z,\mu_F^2,\varepsilon) = \delta(1-z)+\sum_{i=1}^{\infty}\hat{a}_s^i\left(\frac{\mu_F^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i\widetilde{\Gamma}^{(i)}(z,\varepsilon)\ . \qquad (14)$$

The constants $\widetilde{\Gamma}^{(i)}(z,\varepsilon)$ are expanded in negative powers of ε up to the four loop level can be read from Ref. [18] where similar RG equations were solved.

It is natural to expect that the soft plus jet distribution functions have a pole structure in ε similar to that of $\hat{F}^I$ and $\widetilde{\Gamma}_{II}$ so that the decay distributions $\widetilde{\Delta}_I^{sv}$ are finite in the limit $\varepsilon\to 0$. This implies that they satisfy a Sudakov type differential equation that the form factors $\hat{F}^I$ do. Solving the Sudakov differential equation for $\widetilde{\Phi}^{\,I}(\hat{a}_s,q^2,\mu^2,z,\varepsilon)$, we get

$$\begin{aligned}\widetilde{\Phi}^{\,I}(\hat{a}_s,q^2,\mu^2,z,\varepsilon) &= \widetilde{\Phi}^{\,I}(\hat{a}_s,q^2(1-z),\mu^2,\varepsilon)\\ &= \sum_{i=1}^{\infty}\hat{a}_s^i\left(\frac{q^2(1-z)}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i\left(\frac{i\,\varepsilon}{2(1-z)}\right)\hat{\phi}^{\,I,(i)}(\varepsilon)\end{aligned} \qquad (15)$$

where $\hat{\phi}^{\,I,(i)}(\varepsilon)$ coincide with the $\hat{\phi}_{SJ}^{I,(i)}(\varepsilon)$ that appear in the deep-inelastic scattering cross section [19]. This is the result of exact crossing symmetry between the processes under study and the DIS process in the threshold region. The constants $\hat{\phi}_{SJ}^{I,(i)}(\varepsilon)$ for $I=q$ corresponding to DIS are given in [18, 19]. Since the b quark is treated massless, we use the same constants for the Higgs boson decay also.

The threshold corrections that dominate when the partonic scaling variable z approaches its kinematic limit, which is 1, through the distributions $\delta(1-z)$ and $\mathcal{D}_i$ can easily be resummed in Mellin N because the decay distributions appear as convolution of partonic distributions and fragmentation functions. This has been a successful approach due to several important works (see [13–17]). The higher order threshold exponents can be found in [18, 44–46]. We find that the soft plus jet distribution function $\widetilde{\Phi}^I(\hat{a}_s, q^2, \mu^2, z, \varepsilon)$ captures all the features of the N space resummation approach by expressing (15) as

$$\begin{aligned}
\widetilde{\Phi}^{\,I}(\hat{a}_s, q^2, \mu^2, z, \varepsilon) &= \left(\frac{1}{2(1-z)} \left\{ \int_{\mu_R^2}^{q^2(1-z)} \frac{d\lambda^2}{\lambda^2} A_I\left(a_s(\lambda^2)\right) + \overline{G}^{\,I}_{SJ}\left(a_s\left(q^2(1-z)\right), \varepsilon\right) \right\} \right)_+ \\
&\quad + \delta(1-z) \quad \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2}{\mu^2} \right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \, \hat{\phi}^{\,I,(i)}_{SJ}(\varepsilon) \\
&\quad + \left(\frac{1}{2(1-z)} \right)_+ \quad \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{\mu_R^2}{\mu^2} \right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \, \overline{K}^{\,I,(i)}(\varepsilon)
\end{aligned} \tag{16}$$

where

$$\overline{G}^{\,I}_{SJ}\left(a_s\left(q^2(1-z)\right), \varepsilon\right) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2(1-z)}{\mu^2} \right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \overline{G}^{\,I,(i)}_{SJ}(\varepsilon) \,. \tag{17}$$

Here, the constants $\overline{G}^{\,I,(i)}_{SJ}(\varepsilon), \overline{K}^{\,I,(i)}(\varepsilon)$ and $\hat{\phi}^{\,I,(i)}_{SJ}(\varepsilon)$ can be found in [18, 19]. The second line of the equation (16) contains the right poles in ε to cancel those coming from the form factor as well as from the mass factorisation kernel. It also contains the terms that are finite as ε becomes zero through the constants $\overline{\mathcal{G}}^{\,I}_{JS,i}(\varepsilon)$ that appear in $\overline{G}^{\,I,(i)}_{SJ}(\varepsilon)$ (see [19]). Since they are multiplied by $\delta(1-z)$, the fixed order soft plus virtual part of the decay distributions gets contributions from them at higher orders. The third line in the eqn.(16) contains only poles in ε and they all cancel against $\mathcal{D}_0$ parts of the mass factorisation kernel. Hence, adding the eqn.(16) with the renormalised form factor and the mass factorisation kernel and performing the coupling constant renormalisation and then finally taking Mellin moment, we reproduce the exponents in resummation formula(see [13–17]) (after setting $\varepsilon \to 0$). We find that the function $\overline{G}^{\,I}_{SJ}\left(a_s\left(q^2(1-z)\right), \varepsilon\right)$ appearing in the first line of eqn.(16) coincide with the exponent $B^I_{decay}\left(a_s\left(q^2(1-z)\right)\right)$ appearing in the resummation formula

$$\begin{aligned}
B^I_{decay}\left(a_s\left(q^2(1-z)\right)\right) &= \sum_{i=1}^{\infty} a_s^i \left(q^2(1-z)\right) B^I_{decay,i} \\
&= \overline{G}^{\,I}_{SJ}\left(a_s\left(q^2(1-z)\right), \varepsilon\right)\Bigg|_{\varepsilon=0} \,.
\end{aligned} \tag{18}$$

In addition, we get a new exponent that comes from the Mellin moment of $\delta(1-z)$ part and hence N independent. The Mellin space resummation exponents contain the cusp anomalous dimensions A^I and the constants $B^I_{decay,i}$ which we find are identical to those of DIS. These constants are known up to three loop level whose numerical impacts on the physical observables will be important in order to reduce theoretical uncertainties coming from renormalisation and factorisation scales.

Using the z space resummed expression given in Eq. (4) and the known exponents, we present here the results for $\widetilde{\Delta}_I^{sv,(i)}$ for the bottom quark energy distribution in Higgs boson decay beyond leading order in QCD. We present the complete soft plus virtual contribution to order N^iLO for $i=1,2$ and a partial N^3LO result, i.e., without $\delta(1-z)$ part. In addition, we extend this approach to N^4LO order where we can obtain the *partial* soft plus virtual contribution coming from all $\mathcal{D}_j$ except $j=0,1$ for the N^4LO coefficient $\widetilde{\Delta}_b^{sv,(4)}$. Here also we cannot determine the $\delta(1-z)$ part. We repeat this for the hadroproduction in l^+l^- annihilation up to N^4LO order. These fixed order results are expected to reduce the renormalisation and factorisation scale uncertainties. Expanding $\widetilde{\Delta}_I^{sv}(z,q^2,\mu_R^2,\mu_F^2)$ in powers of $a_s(\mu_R^2)$ as

$$\widetilde{\Delta}_I^{sv}(z,q^2,\mu_R^2,\mu_F^2) = \sum_{i=0}^{\infty} a_s^i(\mu_R^2)\, \widetilde{\Delta}_I^{sv,(i)}(z,q^2,\mu_R^2,\mu_F^2) \tag{19}$$

and choosing $\mu_R^2=\mu_F^2=q^2$ for simplicity, we find

$$\widetilde{\Delta}_b^{sv,(0)} = \delta(1-z) \tag{20}$$

$$\widetilde{\Delta}_b^{sv,(1)} = \delta(1-z)\,\Big[C_F\,\Big(3+8\,\zeta_2\Big)\Big] + \mathcal{D}_0\,\Big[C_F\,\Big(-3\Big)\Big] + \mathcal{D}_1\,\Big[C_F\,\Big(4\Big)\Big] \tag{21}$$

$$\begin{aligned}\widetilde{\Delta}_b^{sv,(2)} = {} & \delta(1-z)\,\Big[n_f\,C_F\,\Big(-91/12-14/3\,\zeta_2+4/3\,\zeta_3\Big)+C_F\,C_A\,\Big(1691/24+95/3\,\zeta_2\\ & -49/5\,\zeta_2^2+32/3\,\zeta_3\Big)+C_F^2\,\Big(109/8+31\,\zeta_2+30\,\zeta_2^2-78\,\zeta_3\Big)\Big]\\ & +\mathcal{D}_0\,\Big[n_f\,C_F\,\Big(247/27-8/3\,\zeta_2\Big)+C_F\,C_A\,\Big(-3155/54+44/3\,\zeta_2+40\,\zeta_3\Big)\\ & +C_F^2\,\Big(-21/2-8\,\zeta_3\Big)\Big]+\mathcal{D}_1\,\Big[n_f\,C_F\,\Big(-58/9\Big)+C_F\,C_A\,\Big(367/9-8\,\zeta_2\Big)\\ & +C_F^2\,\Big(21+16\,\zeta_2\Big)\Big]+\mathcal{D}_2\,\Big[n_f\,C_F\,\Big(4/3\Big)+C_F\,C_A\,\Big(-22/3\Big)+C_F^2\,\Big(-18\Big)\Big]\\ & +\mathcal{D}_3\,\Big[C_F^2\,\Big(8\Big)\Big]\end{aligned} \tag{22}$$

$$\begin{aligned}\widetilde{\Delta}_b^{sv,(3)} = {} & \mathcal{D}_0\,\Big[n_f\,C_F\,C_A\,\Big(160906/729-9920/81\,\zeta_2+208/15\,\zeta_2^2-776/9\,\zeta_3\Big)\\ & +n_f\,C_F^2\,\Big(16423/108-722/27\,\zeta_2-16\,\zeta_2^2-60\,\zeta_3\Big)+n_f^2\,C_F\,\Big(-8714/729\\ & +232/27\,\zeta_2-32/27\,\zeta_3\Big)+C_F\,C_A^2\,\Big(-599375/729-176/3\,\zeta_2\,\zeta_3+32126/81\,\zeta_2\\ & -652/15\,\zeta_2^2+21032/27\,\zeta_3-232\,\zeta_5\Big)+C_F^2\,C_A\,\Big(-31151/72+80\,\zeta_2\,\zeta_3\\ & +3365/27\,\zeta_2+39\,\zeta_2^2+1988/9\,\zeta_3-120\,\zeta_5\Big)+C_F^3\,\Big(-479/8-64\,\zeta_2\,\zeta_3\\ & -45\,\zeta_2+6\,\zeta_2^2+178\,\zeta_3+432\,\zeta_5\Big)\Big]+\mathcal{D}_1\,\Big[n_f\,C_F\,C_A\,\Big(-15062/81+512/9\,\zeta_2\end{aligned}$$

$$+16\,\zeta_3\Big) + n_f\, C_F^2\, \Big(- 1343/9 + 64/3\,\zeta_2 + 112/3\,\zeta_3\Big) + n_f^2\, C_F\, \Big(940/81 - 32/9\,\zeta_2\Big)$$
$$+C_F\, C_A^2\, \Big(50689/81 - 680/3\,\zeta_2 + 176/5\,\zeta_2^2 - 264\,\zeta_3\Big) + C_F^2\, C_A\, \Big(13783/18$$
$$-352/3\,\zeta_2 - 196/5\,\zeta_2^2 - 592/3\,\zeta_3\Big) + C_F^3\, \Big(181/2 + 4\,\zeta_2 - 104/5\,\zeta_2^2 - 360\,\zeta_3\Big)\Big]$$
$$+\mathcal{D}_2\, \Big[n_f\, C_F\, C_A\, \Big(1552/27 - 16/3\,\zeta_2\Big) + n_f\, C_F^2\, \Big(827/9 - 64/3\,\zeta_2\Big)$$
$$+n_f^2\, C_F\, \Big(- 116/27\Big) + C_F\, C_A^2\, \Big(- 4649/27 + 88/3\,\zeta_2\Big) + C_F^2\, C_A\, \Big(- 10009/18$$
$$+460/3\,\zeta_2 + 240\,\zeta_3\Big) + C_F^3\, \Big(- 153/2 + 72\,\zeta_2 + 16\,\zeta_3\Big)\Big]$$
$$+\mathcal{D}_3\, \Big[n_f\, C_F\, C_A\, \Big(- 176/27\Big) + n_f\, C_F^2\, \Big(- 280/9\Big) + n_f^2\, C_F\, \Big(16/27\Big)$$
$$+C_F\, C_A^2\, \Big(484/27\Big) + C_F^2\, C_A\, \Big(1732/9 - 32\,\zeta_2\Big) + C_F^3\, \Big(60\Big)\Big] + \mathcal{D}_4\, \Big[n_f\, C_F^2\, \Big(40/9\Big)$$
$$+C_F^2\, C_A\, \Big(- 220/9\Big) + C_F^3\, \Big(- 30\Big)\Big] + \mathcal{D}_5\, \Big[C_F^3\, \Big(8\Big)\Big] \qquad (23)$$
$$\widetilde{\Delta}_b^{sv,(4)} = \mathcal{D}_2\, \Big[n_f\, C_F\, C_A^2\, \Big(17189/9 - 5096/9\,\zeta_2 + 176/5\,\zeta_2^2 - 352\,\zeta_3\Big)$$
$$+n_f\, C_F^2\, C_A\, \Big(964334/243 - 57524/27\,\zeta_2 + 2332/15\,\zeta_2^2 - 11032/9\,\zeta_3\Big)$$
$$+n_f\, C_F^3\, \Big(12299/9 - 2336/3\,\zeta_2 - 728/5\,\zeta_2^2 - 936\,\zeta_3\Big) + n_f^2\, C_F\, C_A\, \Big(- 7403/27$$
$$+688/9\,\zeta_2 + 16\,\zeta_3\Big) + n_f^2\, C_F^2\, \Big(- 71776/243 + 4088/27\,\zeta_2 + 304/9\,\zeta_3\Big)$$
$$+n_f^3\, C_F\, \Big(940/81 - 32/9\,\zeta_2\Big) + C_F\, C_A^3\, \Big(- 649589/162 + 4012/3\,\zeta_2 - 968/5\,\zeta_2^2$$
$$+1452\,\zeta_3\Big) + C_F^2\, C_A^2\, \Big(- 6034493/486 - 832\,\zeta_2\,\zeta_3 + 63764/9\,\zeta_2 - 2450/3\,\zeta_2^2$$
$$+25336/3\,\zeta_3 - 1392\,\zeta_5\Big) + C_F^3\, C_A\, \Big(- 12529/3 - 768\,\zeta_2\,\zeta_3 + 13718/3\,\zeta_2$$
$$+1094/5\,\zeta_2^2 + 13300/3\,\zeta_3 - 720\,\zeta_5\Big) + C_F^4\, \Big(- 420 - 448\,\zeta_2\,\zeta_3$$
$$+198\,\zeta_2 + 324\,\zeta_2^2 + 1584\,\zeta_3 + 4128\,\zeta_5\Big)\Big]$$
$$+\mathcal{D}_3\, \Big[n_f\, C_F\, C_A^2\, \Big(- 9502/27 + 352/9\,\zeta_2\Big) + n_f\, C_F^2\, C_A\, \Big(- 358142/243$$
$$+11984/27\,\zeta_2 + 1216/9\,\zeta_3\Big) + n_f\, C_F^3\, \Big(- 5660/9 + 2608/9\,\zeta_2 + 1888/9\,\zeta_3\Big)$$
$$+n_f^2\, C_F\, C_A\, \Big(1540/27 - 32/9\,\zeta_2\Big) + n_f^2\, C_F^2\, \Big(25966/243 - 736/27\,\zeta_2\Big)$$
$$+n_f^3\, C_F\, \Big(- 232/81\Big) + C_F\, C_A^3\, \Big(55627/81 - 968/9\,\zeta_2\Big) + C_F^2\, C_A^2\, \Big(2259107/486$$
$$-47104/27\,\zeta_2 + 864/5\,\zeta_2^2 - 13024/9\,\zeta_3\Big) + C_F^3\, C_A\, \Big(26720/9 - 16192/9\,\zeta_2$$

$$+248/5\,\zeta_2^2-11392/9\,\zeta_3\Big)+C_F^4\,\Big(533/2-376\,\zeta_2-1488/5\,\zeta_2^2-1072\,\zeta_3\Big)\Big]$$
$$+\mathcal{D}_4\,\Big[n_f\,C_F\,C_A^2\,\Big(242/9\Big)+n_f\,C_F^2\,C_A\,\Big(8120/27-80/3\,\zeta_2\Big)+n_f\,C_F^3\,\Big(6070/27$$
$$-560/9\,\zeta_2\Big)+n_f^2\,C_F\,C_A\,\Big(-44/9\Big)+n_f^2\,C_F^2\,\Big(-640/27\Big)+n_f^3\,C_F\,\Big(8/27\Big)$$
$$+C_F\,C_A^3\,\Big(-1331/27\Big)+C_F^2\,C_A^2\,\Big(-24040/27+440/3\,\zeta_2\Big)+C_F^3\,C_A\,\Big(-35755/27$$
$$+4160/9\,\zeta_2+400\,\zeta_3\Big)+C_F^4\,\Big(-150+240\,\zeta_2+400/3\,\zeta_3\Big)\Big]$$
$$+\mathcal{D}_5\,\Big[n_f\,C_F^2\,C_A\,\Big(-704/27\Big)+n_f\,C_F^3\,\Big(-164/3\Big)+n_f^2\,C_F^2\,\Big(64/27\Big)$$
$$+C_F^2\,C_A^2\,\Big(1936/27\Big)+C_F^3\,C_A\,\Big(998/3-48\,\zeta_2\Big)+C_F^4\,\Big(78-32\,\zeta_2\Big)\Big]$$
$$+\mathcal{D}_6\,\Big[n_f\,C_F^3\,\Big(56/9\Big)+C_F^3\,C_A\,\Big(-308/9\Big)+C_F^4\,\Big(-28\Big)\Big]+\mathcal{D}_7\,\Big[C_F^4\,\Big(16/3\Big)\Big]\quad(24)$$

Similarly for hadroproduction in l^+l^- annihilation, we find

$$\widetilde{\Delta}_b^{sv,(0)}-\widetilde{\Delta}_q^{sv,(0)} = 0 \quad (25)$$
$$\widetilde{\Delta}_b^{sv,(1)}-\widetilde{\Delta}_q^{sv,(1)} = \delta(1-z)\Big[C_F\Big(12\Big)\Big] \quad (26)$$
$$\widetilde{\Delta}_b^{sv,(2)}-\widetilde{\Delta}_q^{sv,(2)} = \delta(1-z)\Big[n_fC_F\Big(-365/18+8\,\zeta_2\Big)+C_FC_A\Big(5269/36-40\,\zeta_2-36\,\zeta_3\Big)$$
$$+C_F^2\Big(-111/4+70\,\zeta_2\Big)\Big]+\mathcal{D}_0\Big[C_F^2\Big(-36\Big)\Big]+\mathcal{D}_1\Big[C_F^2\Big(48\Big)\Big] \quad (27)$$
$$\widetilde{\Delta}_b^{sv,(3)}-\widetilde{\Delta}_q^{sv,(3)} = \mathcal{D}_0\Big[n_fC_F^2\Big(3071/18-56\,\zeta_2\Big)+C_F^2C_A\Big(-41047/36+296\,\zeta_2+588\,\zeta_3\Big)$$
$$+C_F^3\Big(261/4+78\,\zeta_2-96\,\zeta_3\Big)\Big]+\mathcal{D}_1\Big[n_fC_F^2\Big(-1426/9+32\,\zeta_2\Big)$$
$$+C_F^2C_A\Big(9673/9-256\,\zeta_2-144\,\zeta_3\Big)+C_F^3\Big(-3+88\,\zeta_2\Big)\Big]$$
$$+\mathcal{D}_2\Big[n_fC_F^2\Big(16\Big)+C_F^2C_A\Big(-88\Big)+C_F^3\Big(-216\Big)\Big]+\mathcal{D}_3\Big[C_F^3\Big(96\Big)\Big] \quad (28)$$
$$\widetilde{\Delta}_b^{sv,(4)}-\widetilde{\Delta}_q^{sv,(4)} = \mathcal{D}_2\Big[n_fC_F^2C_A\Big(27908/27-176\,\zeta_2-48\,\zeta_3\Big)+n_fC_F^3\Big(4148/3-1304/3\,\zeta_2\Big)$$
$$+n_f^2C_F^2\Big(-2122/27+32/3\,\zeta_2\Big)+C_F^2C_A^2\Big(-169535/54+1936/3\,\zeta_2$$
$$+264\,\zeta_3\Big)+C_F^3C_A\Big(-26519/3+8252/3\,\zeta_2+3528\,\zeta_3\Big)+C_F^4\Big(459/2$$
$$+1332\,\zeta_2+192\,\zeta_3\Big)\Big]+\mathcal{D}_3\Big[n_fC_F^2C_A\Big(-704/9\Big)+n_fC_F^3\Big(-4820/9$$
$$+64\,\zeta_2\Big)+n_f^2C_F^2\Big(64/9\Big)+C_F^2C_A^2\Big(1936/9\Big)+C_F^3C_A\Big(31322/9$$

$$-704\,\zeta_2-288\,\zeta_3\Big)+C_F^4\Big(210-208\,\zeta_2\Big)\Big]+\mathcal{D}_4\Big[n_fC_F^3\Big(160/3\Big)$$
$$+C_F^3C_A\Big(-880/3\Big)+C_F^4\Big(-360\Big)\Big]+\mathcal{D}_5\Big[C_F^4\Big(96\Big)\,,\Big] \qquad (29)$$

where the colour factors for the $SU(N)$ gauge group are $C_A=N, C_F=(N^2-1)/(2N)$ and n_f is the number of active flavours. The above expressions are easily transformed to Mellin space. The functions $\mathcal{D}_k$ are represented by polynomials of single harmonic sums only, cf. [47]. The soft resummation terms supplement the corresponding representations for the 2–loop Wilson coefficients of the production cross sections in Mellin space derived in [48].

To summarise, we have systematically studied the soft plus virtual contributions to the bottom quark energy distribution in Higgs boson decay and the hadroproductions in l^+l^- annihilation processes using the formalism of threshold resummation to infrared safe decay distributions. This was achieved using renormalisation group invariance and Sudakov resummation of soft gluons and the factorisation property of these decay distributions. We have also shown how these resummed distributions are related to resummation exponents that appear in Mellin space. Using this approach we have computed the soft plus virtual decay distributions at $NNLO$ and partial results at N^3LO and N^4LO in perturbative QCD for the bottom quark energy distribution in Higgs decay and hadroproductions in l^+l^- annihilation.

Acknowledgments: VR would like to thank Prakash Mathews for his constant encouragements and discussions. This work was supported in part by DFG Sonderforschungsbereich Transregio 9, Computergestützte Theoretische Physik.

References

[1] A. Djouadi, arXiv:hep-ph/0503172 and references therein.

[2] [ALEPH Collaboration], arXiv:hep-ex/0509008.

[3] C. Buttar *et al.*, arXiv:hep-ph/0604120.

[4] A. Djouadi, arXiv:hep-ph/0503173.

[5] G. Altarelli, R. K. Ellis, G. Martinelli and S. Y. Pi, Nucl. Phys. B **160**, 301 (1979).

[6] G. Corcella and V. Drollinger, Nucl. Phys. B **730**, 82 (2005) [arXiv:hep-ph/0508013].

[7] B. Mele and P. Nason, Nucl. Phys. B **361**, 626 (1991).

[8] G. Corcella, Nucl. Phys. B **705**, 363 (2005) [Erratum-ibid. B **713**, 609 (2005)] [arXiv:hep-ph/0409161].

[9] P. J. Rijken and W. L. van Neerven, Phys. Lett. B **386**, 422 (1996) [arXiv:hep-ph/9604436]. Nucl. Phys. **B488**, 233 (19yy).

[10] P. J. Rijken and W. L. van Neerven, Nucl. Phys. B **487**, 233 (1997) [arXiv:hep-ph/9609377].

[11] P. J. Rijken and W. L. van Neerven, Phys. Lett. B **392**, 207 (1997) [arXiv:hep-ph/9609379].

[12] A. Mitov, S. Moch and A. Vogt, arXiv:hep-ph/0604053.

[13] M. Cacciari and S. Catani, Nucl. Phys. B **617**, 253 (2001) [arXiv:hep-ph/0107138].

[14] G. Sterman, Nucl. Phys. B **281**, 310 (1987).

[15] S. Catani and L. Trentadue, Nucl. Phys. B **327**, 323 (1989).

[16] J. Kodaira and L. Trentadue, Phys. Lett. B **112** (1982) 66.

[17] A. Vogt, Phys. Lett. B **497**, 228 (2001) [arXiv:hep-ph/0010146].

[18] V. Ravindran, to appear in Nuclear Physics **B**,[arXiv:hep-ph/0512249].

[19] V. Ravindran, arXiv:hep-ph/0603041.

[20] S. D. Drell, D. J. Levy and T. M. Yan, Phys. Rev. D **1**, 1617 (1970).

[21] A. Suri, Phys. Rev. D **4**, 570 (1971).

[22] R. Gatto and G. Preparata, Nucl. Phys. B **47**, 313 (1972).

[23] H. D. Dahmen and F. Steiner, Phys. Lett. B **43**, 217 (1973).

[24] W. Furmanski and R. Petronzio, Z. Phys. C **11**, 293 (1982).

[25] M. Stratmann and W. Vogelsang, Nucl. Phys. B **496**, 41 (1997) [arXiv:hep-ph/9612250].

[26] E. G. Floratos, C. Kounnas and R. Lacaze, Nucl. Phys. B **192**, 417 (1981).

[27] J. Blümlein, V. Ravindran and W. L. van Neerven, Acta Phys. Polon. B **29**, 2581 (1998) [arXiv:hep-ph/9806355].

[28] J. Blümlein, V. Ravindran and W. L. van Neerven, Nucl. Phys. B **586**, 349 (2000) [arXiv:hep-ph/0004172].

[29] Y. L. Dokshitzer, G. Marchesini and G. P. Salam, arXiv:hep-ph/0511302.

[30] J. Blümlein and V. Ravindran, Nucl. Phys. **B** in print, [arXiv:hep-ph/0604019].

[31] T. van Ritbergen, J. A. M. Vermaseren and S.A. Larin, Phys. Lett. B **400**, 379 (1997) [arXiv:hep-ph/9701390];

[32] M. Czakon, Nucl. Phys. B **710**, 485 (2005) [arXiv:hep-ph/0411261].

[33] K. G. Chetyrkin, B. A. Kniehl and M. Steinhauser, Nucl. Phys. B **510**, 61 (1998) [arXiv:hep-ph/9708255].

[34] J. Gracey, Phys. Lett. **B488** (2000) 175.

[35] V. V. Sudakov, Sov. Phys. JETP **3**, 65 (1956) [Zh. Eksp. Teor. Fiz. **30**, 87 (1956)].

[36] A. H. Mueller, Phys. Rev. D **20**, 2037 (1979).

[37] J. C. Collins, Phys. Rev. D **22**, 1478 (1980).

[38] A. Sen, Phys. Rev. D **24**, 3281 (1981).

[39] S. Moch, J. A. M. Vermaseren and A. Vogt, JHEP **0508**, 049 (2005) [arXiv:hep-ph/0507039].

[40] S. Moch, J. A. M. Vermaseren and A. Vogt, Phys. Lett. B **625**, 245 (2005) [arXiv:hep-ph/0508055].

[41] V. Ravindran, J. Smith and W. L. van Neerven, Nucl. Phys. B **704**, 332 (2005) [arXiv:hep-ph/0408315].

[42] S. Moch, J. A. M. Vermaseren and A. Vogt, Nucl. Phys. B **688**, 101 (2004) [arXiv:hep-ph/0403192].

[43] A. Vogt, S. Moch and J. A. M. Vermaseren, Nucl. Phys. B **691**, 129 (2004) [arXiv:hep-ph/0404111].

[44] S. Moch, J. A. M. Vermaseren and A. Vogt, Nucl. Phys. B **726**, 317 (2005) [arXiv:hep-ph/0506288].

[45] E. Laenen and L. Magnea, arXiv:hep-ph/0508284.

[46] A. Idilbi, X. d. Ji, J. P. Ma and F. Yuan, arXiv:hep-ph/0509294.

[47] J. Blümlein and S. Kurth, Phys. Rev. D **60** (1999) 014018.

[48] J. Blümlein and V. Ravindran, Nucl. Phys. B **716** (2005) 128 [hep-ph/0501178].