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Two-Loop Renormalization in the Standard Model

Part I: Prolegomena*

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In this paper the building blocks for the two-loop renormalization of the Standard Model are introduced with a comprehensive discussion of the special vertices induced in the Lagrangian by a particular diagonalization of the neutral sector and by two alternative treatments of the Higgs tadpoles. Dyson resummed propagators for the gauge bosons are derived, and two-loop Ward-Slavnov-Taylor identities are discussed. In part II, the complete set of counterterms needed for the two-loop renormalization will be derived. In part III, a renormalization scheme will be introduced, connecting the renormalized quantities to an input parameter set of (pseudo-)experimental data, critically discussing renormalization of a gauge theory with unstable particles.

Key words: Feynman diagrams, Multi-loop calculations, Self-energy Diagrams, Vertex diagrams

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Contents

1 Introduction

In a series of papers we developed a strategy for the algebraic-numerical evaluation of two-loop, two-(three-)leg Feynman diagrams appearing in any renormalizable quantum field theory. In [1] the general strategy has been designed and in [2] a complete list of results has been derived for two-loop functions with two external legs, including their infrared divergent on-shell derivatives. Results for one-loop multi-leg diagrams have been shown in [3] and additional material can be found in [4]. Two-loop three-point functions for infrared convergent configurations have been considered in [5], two-loop tensor integrals in [6], two-loop infrared divergent vertices in [7]. As a by-product of our general program we have developed a set of FORTRAN/95 routines [8] for computing everything which is needed, from standard $A_0, \dots, D_0$ functions [9] to two-loop, two-(three-) point functions. This new ensemble of programs, which includes the treatment of complex poles [10], will succeed the corresponding library of `TOPAZO` [11].

The next step in our project has been to introduce all those elements which are necessary for a complete discussion of the two-loop renormalization of the Standard Model (SM). In this paper we introduce basic aspects of renormalization which are needed before the introduction of counterterms. In part II we will present a detailed analysis of the counterterms with special emphasis to the cancellation of ultraviolet poles with non-local residues (the so-called problem of overlapping divergences), while in part III we will deal with finite renormalization deriving renormalization equations, up to two loops, that relate the renormalized parameters of the model to an input parameter set, which always includes the fine structure constant α and the Fermi coupling constant G_F. Renormalization with unstable particles will also be addressed.

Having provided a derivation of the elements which are essential for constructing a renormalization procedure, we will proceed in computing a first set of pseudo-observables, including the running e.m. coupling constant and the complex poles characterizing unstable gauge bosons.

Several authors have already contributed in developing seminal results for the two-loop renormalization of the SM [12]. Here we want to present our own approach, from fundamentals to applications. The whole set of results is completely independent from other sources; furthermore, we wanted to collect in a single place all the formulas and algorithms that can be used for many applications and are never there when you need them.

The code $\mathcal{G}raph\mathcal{S}hot$ [13] synthesizes the algebraic component of the project (for alternative approaches see ref. [14] and references therein) from generation of diagrams, reduction of tensor structures, special kinematical configurations, analytical extraction of ultraviolet/infrared poles [7] and of collinear logarithms and check of Ward-Slavnov-Taylor identities (hereafter WST identities) [15]. The corresponding output is then treated by a FORTRAN/95 code, $\mathcal{L}oop\mathcal{B}ack$ [8], which is able to exploit the multi-scale structure of two-loop diagrams. Future applications will include $H \to \gamma\gamma$ and $H \to gg$, to give an example.

It is worth noticing that there are other solutions to the problems discussed in this paper; noticeably, one can choose to work in the background-field formalism [16]; here we only stress that our solution has been extended up to the two loops and has been implemented in a complete and stand-alone set of procedures for two-loop renormalization.

The outline of the paper is as follows. In Section 2 we discuss the role of tadpoles in a spontaneously broken gauge theory presenting two alternative schemes in Section 2.2 and in Section 2.3. Diagonalization of the neutral sector in the SM is derived in Section 3. WST identities are discussed in Section 4. Dyson resummation is analyzed in Section 5. Bases relevant for renormalization are introduced in Section 6. New sets of Feynman rules, required by our renormalization procedure, are given in the Appendices.

2 Higgs tadpoles

Tadpoles in a spontaneously broken gauge theory have been discussed by many authors (see, for instance [17]). Here we outline those aspects which are peculiar to our approach.

2.1 The basics

Following notation and conventions of ref. [18], the minimal Higgs sector of the SM is provided by the Lagrangian

$$\mathcal{L}_S = -(D_\mu K)^\dagger (D_\mu K) - \mu^2 K^\dagger K - (\lambda/2)(K^\dagger K)^2, \tag{1}$$

where the covariant derivative is given by

$$D_\mu K = \left(\partial_\mu - \frac{i}{2} g B^a_\mu \tau^a - \frac{i}{2} g' B^0_\mu\right) K, \tag{2}$$

$g'/g = -\sin\theta/\cos\theta$, θ is the weak mixing angle, τ^a are the standard Pauli matrices, B^a_μ is a triplet of vector gauge bosons and B^0_μ a singlet. For the theory to be stable we must require $\lambda > 0$. We choose $\mu^2 < 0$ in order to have spontaneous symmetry breaking (SSB). The scalar field in the minimal realization of the SM is

$$K = \frac{1}{\sqrt{2}} \begin{pmatrix} \zeta + i\phi_0 \\ -\phi_2 + i\phi_1 \end{pmatrix}, \tag{3}$$

where ζ and the Higgs-Kibble fields ϕ_0, ϕ_1 and ϕ_2 are real. For $\mu^2 < 0$ we have SSB, $\langle K \rangle_0 \neq 0$. In particular, we choose $\zeta + i\phi_0$ to be the component of K to develop the non-zero vacuum expectation value (VEV), and we set $\langle \phi_0 \rangle_0 = 0$ and $\langle \zeta \rangle_0 \neq 0$. We then introduce the (physical) Higgs fields as $H = \zeta - v$. The parameter v is not a new parameter of the model; its value must be fixed by the requirement that $\langle H \rangle_0 = 0$ (i.e. $\langle K \rangle_0 = (1/\sqrt{2})(v, 0)$), so that the vacuum doesn't absorb/create Higgs particles. To see how this works at the lowest order, consider the part of $\mathcal{L}_S$ containing the Higgs field:

$$-(1/2)(\partial_\mu H)^2 - (\mu^2/2)(H+v)^2 - (\lambda/8)(H+v)^4. \tag{4}$$

These terms generate vertices that imply absorption of H in the vacuum, namely those linear in H,

$$\left[-\mu^2 v - (\lambda/2) v^3\right] H, \tag{5}$$

which correspond to the vertex H ———● . This vertex gives a non-zero value to the diagrams with one H line, and thus a non-zero VEV. We will set it to zero, i.e. $v = (-2\mu^2/\lambda)^{1/2}$ (or $v = 0$, but then, no SSB).

2.2 The parameter β_h

2.2.1 *Definitions and Lagrangian*

More complicated diagrams contribute to $\langle H \rangle_0$ in higher orders of perturbation theory. The parameter v must then be readjusted such that $\langle H \rangle_0 = 0$. First of all, let us introduce the new bare parameters M (the W boson mass), $M_{_H}$ (the mass of the physical Higgs particle) and β_h (the tadpole constant) according to the following definitions:

$$\left\{\begin{array}{lcl} M &=& gv/2 \\ M^2_{_H} &=& \lambda v^2 \\ \beta_h &=& \mu^2 + \frac{\lambda}{2} v^2 \end{array}\right. \quad \Longrightarrow \quad \left\{\begin{array}{lcl} v &=& 2M/g \\ \lambda &=& (gM_{_H}/2M)^2 \\ \mu^2 &=& \beta_h - \frac{1}{2} M^2_{_H} \end{array}\right. \tag{6}$$

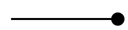
This parameter β_h is the same as β_H of [18] and β_h of [19]. The new set of (bare) parameters is therefore g, g', M, M_H and β_h instead of g, g', μ, λ, v. Remember that β_h (like v) is not an independent parameter. In terms of g, g', M, M_H and β_h, $\mathcal{L}_S^I$ becomes (in ref. [18] some terms have been dropped)

$$\begin{aligned}\mathcal{L}_S^I &= -\mu^2 K^\dagger K - (\lambda/2)(K^\dagger K)^2 = -\beta_h \left[\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac{1}{2}\left(H^2 + \phi_0^2 + 2\phi_+\phi_-\right)\right] \\ &+ \frac{M_H^2 M^2}{2g^2} - \frac{1}{2}M_H^2 H^2 - g\frac{M_H^2}{4M}H\left(H^2 + \phi_0^2 + 2\phi_+\phi_-\right) - g^2\frac{M_H^2}{32M^2}\left(H^2 + \phi_0^2 + 2\phi_+\phi_-\right)^2, \quad (7)\end{aligned}$$

with $\phi_\pm = (\phi_1 \mp i\phi_2)/\sqrt{2}$. Note that $(-\mu^2 K^\dagger K)$ is the only term of $\mathcal{L}_S$ containing β_h (actually, the only term of the whole SM Lagrangian). Let us now set β_h such that the VEV of the Higgs field H remains zero to each order of perturbation theory.

2.2.2 β_h fixing at the lowest order

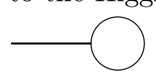
At the lowest order, the only diagram contributing to $\langle H \rangle_0$ is

H  (8)

originated by the term in $\mathcal{L}_S^I$ linear in H, $-(2\beta_h M/g)H$. Therefore, at the lowest order we will simply set $\beta_h = 0$.

2.2.3 β_h fixing up to one loop

Define $\beta_h = \beta_{h_0} + \beta_{h_1} g^2 + \beta_{h_2} g^4 + \cdots$. The lowest-order β_h fixing of the previous section amounts to $\beta_{h_0} = 0$. At the one-loop level, two types of diagrams contribute to the Higgs VEV up to $\mathcal{O}(g)$:

T_0 :  + T_1 :  (9)

where the empty blob in the second term symbolically indicates all the one-loop diagrams containing a scalar field (H, $\phi_\pm$, ϕ_0), a gauge field (Z, $W_\pm$), a Faddeev–Popov ghost field (X_+, X_-, X_Z), or a fermionic field. As an example, consider only the one-loop diagram containing the H field: $T_1^{(H)}$; if this were the only T_1 diagram, in order to have $\langle H \rangle_0 = 0$ it should cancel with T_0, i.e.

$$(2\pi)^4 i \left(-\beta_h^{(H)} \frac{2M}{g}\right) - g\frac{3M_H^2}{4M} i\pi^2 A_0(M_H) = 0, \quad (10)$$

where $i\pi^2 A_0(m) = \mu^{4-n} \int d^n q/(q^2 + m^2 - i\epsilon)$. The solution of this equation is $\beta_{h_0} = 0$ and

$$\beta_{h_1}^{(H)} = \frac{1}{(2\pi)^4 i}\left(\frac{T_1^{(H)}}{2Mg}\right) = -\frac{1}{16\pi^2}\left[\frac{3M_H^2}{8M^2}A_0(M_H)\right]. \quad (11)$$

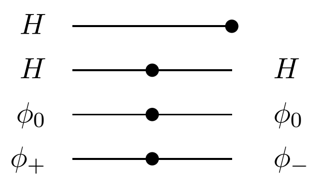
Of course, $\beta_{h_1}^{(H)}$ is just the contribution to β_{h_1} arising from the one-loop tadpole diagram containing the H field; the complete expression for β_{h_1} in the R_ξ gauge is

$$\begin{aligned}\beta_{h_1} &= -\frac{1}{16\pi^2}\Bigg[\frac{3}{2}A_0(M) + \frac{3}{4c_\theta^2}A_0(M_0) + M^2 + \frac{M_0^2}{2c_\theta^2} + \\ &+ \frac{M_H^2}{8M^2}\Big(A_0(\xi_Z M_0) + 2A_0(\xi_W M)\Big) + \frac{3M_H^2}{8M^2}A_0(M_H) - \sum_f \frac{m_f^2}{M^2}A_0(m_f)\Bigg], \quad (12)\end{aligned}$$

where $M_0 = M/c_\theta$ and m_f are the Z and fermion masses, and $c_\theta = \cos\theta$.

2.2.4 β_h vertices in one-loop calculations

Beyond the lowest order, β_h is not zero and the Lagrangian $\mathcal{L}_S^I$ contains the following vertices involving a β_h factor ("β_h vertices", from now on):

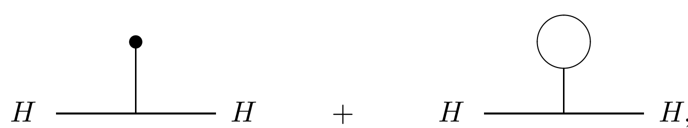


$$(2\pi)^4 i\,(-2M\beta_h/g) \quad (13)$$
$$(2\pi)^4 i\,(-\beta_h) \quad (14)$$
$$(2\pi)^4 i\,(-\beta_h) \quad (15)$$
$$(2\pi)^4 i\,(-\beta_h) \quad (16)$$

(as usual, the combinatorial factors for identical fields are included; see the Appendix D of ref. [19]). Note that only scalar fields appear in the β_h vertices. These β_h vertices must be included in one-loop calculations. Consider, for example, the Higgs self-energy at the one-loop level. The diagrams contributing to this $\mathcal{O}(g^2)$ quantity are

(17)

where the empty blob in the second term represents all the one-loop contributions (two possible topologies). The first diagram, containing a two-leg β_h vertex, shouldn't be forgotten, and plays an important role in the Ward identities (see later). One should also include diagrams containing tadpoles:

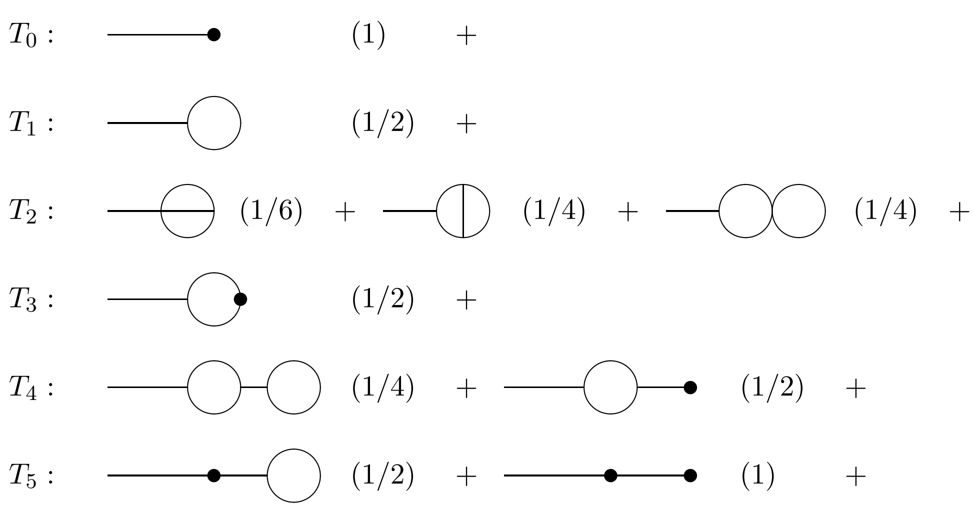


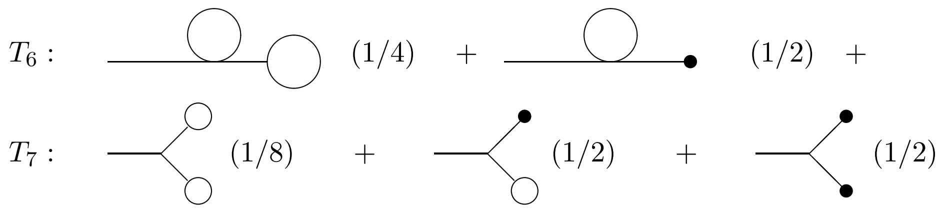
(18)

but these diagrams add up to zero as a consequence of our choice for β_h.

2.2.5 β_h fixing up to two loops

Up to terms of $\mathcal{O}(g^3)$, $\langle H\rangle_0$ gets contributions from the following diagrams:





The coefficients in parentheses indicate the combinatorial factors of each diagram when all fields are identical. Owing to our previous choice for β_{h_0} and β_{h_1}, all the reducible diagrams add up to zero: $T_4 = T_5 = T_6 = T_7 = 0$. The equation

$$\sum_{i=0}^{3} T_i = 0 \tag{19}$$

provides then β_{h_2}:

$$\beta_{h_2} = \frac{1}{(2\pi)^4 i} \left(\frac{T_2 + T_3}{2Mg^3} \right). \tag{20}$$

2.2.6 β_h vertices in two-loop calculations

The two-leg β_h vertices in Eqs. (14,15,16) should be included in all the appropriate diagrams at the two-loop level, while all graphs (up to two loops) containing tadpoles will add up to zero as a consequence of our choice for β_{h_0}, β_{h_1} and β_{h_2}. Note that two-leg β_h vertices will also appear in the $\mathcal{O}(g^4)$ self-energies of fields which do not belong to the Higgs sector; for example, in diagrams like these:

which are representative of the only two irreducible $\mathcal{O}(g^4)$ Z self-energy topologies containing β_h vertices (excluding tadpoles, of course).

2.3 The β_t Scheme

2.3.1 Definitions and Lagrangian

Tadpoles do not depend on any particular scale other than their internal mass, and cancel in any renormalized self-energy. However, they play an essential role in proving the gauge invariance of all the building blocks of the theory. In order to exploit this option, we will now consider a slightly different strategy to set the Higgs VEV to zero. Instead of using Eqs. (6), the "β_h scheme", we will define the new bare parameters M' (the W boson mass), M'_H (the mass of the physical Higgs particle) and β_t (the tadpole constant) according to the following "β_t scheme":

$$\left\{ \begin{array}{lcl} M'(1+\beta_t) & = & gv/2 \\ (M'_H)^2 & = & \lambda\,(2M'/g)^2 \\ 0 & = & \mu^2 + \frac{\lambda}{2}\,(2M'/g)^2 \end{array} \right. \Longrightarrow \left\{ \begin{array}{lcl} v & = & 2M'(1+\beta_t)/g \\ \lambda & = & \left(gM'_H/2M'\right)^2 \\ \mu^2 & = & -\frac{1}{2}(M'_H)^2 \end{array} \right. \tag{21}$$

The new set of bare parameters is therefore g, g', M', M'_H and β_t instead of g, g' and μ, λ, v or g, g' M, M_H and β_h. Remember that β_t (like v and β_h) is not an independent parameter. Note that, contrary to β_h, the parameter β_t appears in the Higgs doublet K via $\zeta = H + v$, with $v = 2M'(1+\beta_t)/g$. As a consequence, all three terms of the Lagrangian $\mathcal{L}_S$ in Eq.(1) depend on this parameter. In particular, the interaction part of $\mathcal{L}_S$ becomes

$$\mathcal{L}_S^I = -\mu^2 K^\dagger K - (\lambda/2)(K^\dagger K)^2 \tag{22}$$

$$\begin{aligned} &= (1+\beta_t)^2\Big(1-\beta_t\,(2+\beta_t)\Big)\frac{M_H'^2 M'^2}{2g^2} - \beta_t\,(\beta_t+1)\,(\beta_t+2)\,\frac{M_H'^2 M'}{g}H \\ &-\frac{1}{2}M_H'^2 H^2 - \frac{1}{4}M_H'^2\beta_t\,(\beta_t+2)\left(3H^2+\phi_0^2+2\phi_+\phi_-\right) \\ &-g\,(1+\beta_t)\,\frac{M_H'^2}{4M'}H\left(H^2+\phi_0^2+2\phi_+\phi_-\right) - g^2\frac{M_H'^2}{32M'^2}\left(H^2+\phi_0^2+2\phi_+\phi_-\right)^2, \end{aligned} \tag{23}$$

while the term of $\mathcal{L}_S$ involving $-(D_\mu K)^\dagger(D_\mu K)$, yields a (lengthy) β_t-*independent* expression (see refs. [18] and [19]), *plus* the following terms containing β_t:

$$\begin{aligned} \beta_t \times &\Bigg[igs_\theta M'\left(\phi^- W_\mu^+ - \phi^+ W_\mu^-\right)\left(A_\mu - \frac{s_\theta}{c_\theta}Z_\mu\right) - \frac{gM'}{2}H\left(2W_\mu^+W_\mu^- + \frac{Z_\mu Z_\mu}{c_\theta^2}\right) \\ &-\frac{M'^2}{2}(\beta_t+2)\left(2W_\mu^+W_\mu^- + \frac{Z_\mu Z_\mu}{c_\theta^2}\right) + \frac{M'}{c_\theta}Z_\mu\partial_\mu\phi_0 + M'W_\mu^+\partial_\mu\phi_- + M'W_\mu^-\partial_\mu\phi_+\Bigg], \end{aligned} \tag{24}$$

where, as usual, $W_\mu^\pm = (B_\mu^1 \mp iB_\mu^2)/\sqrt{2}$, and

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix}\begin{pmatrix} B_\mu^3 \\ B_\mu^0 \end{pmatrix}. \tag{25}$$

Where else, in the SM Lagrangian, does the parameter β_t appear? Wherever v does — as it can be readily seen from Eq.(21). Let us now quickly discuss the other sectors of the SM: Yang–Mills, fermionic, Faddeev–Popov (FP) and gauge-fixing. The pure Yang–Mills Lagrangian obviously contains no β_t terms.

The gauge-fixing part of the Lagrangian, $\mathcal{L}_{gf}$, cancels in the R_ξ gauges the gauge–scalar mixing terms Z–ϕ_0 and $W^\pm$–$\phi^\pm$ contained in the scalar Lagrangian $\mathcal{L}_S$. These terms are proportional to $gv/2$, i.e., to $M'(1+\beta_t)$ in the β_t scheme, and to M in the β_h scheme. The gauge-fixing Lagrangian $\mathcal{L}_{gf}$ is a matter of choice: we adopt the usual definition

$$\mathcal{L}_{gf} = -\mathcal{C}_+\mathcal{C}_- - \frac{1}{2}\mathcal{C}_Z^2 - \frac{1}{2}\mathcal{C}_A^2, \tag{26}$$

with

$$\mathcal{C}_A = -\frac{1}{\xi_A}\partial_\mu A_\mu, \quad \mathcal{C}_Z = -\frac{1}{\xi_Z}\partial_\mu Z_\mu + \xi_Z\frac{M'}{c_\theta}\phi_0, \quad \mathcal{C}_\pm = -\frac{1}{\xi_W}\partial_\mu W_\mu^\pm + \xi_W M'\phi_\pm \tag{27}$$

(note: no β_t terms), thus canceling the $\mathcal{L}_S$ g-independent gauge–scalar mixing terms proportional to M', but not those proportional to $M'\beta_t$ (appearing at the end of Eq.(24)), which are of $\mathcal{O}(g^2)$. Alternatively, one could choose $M'(1+\beta_t)$ instead of M' in Eq.(27), thus canceling all $\mathcal{L}_S$ gauge–scalar mixing terms, both proportional to M' and $M'\beta_t$, but introducing then new two-leg β_t vertices. In this latter case, as $M = M'(1+\beta_t)$, the gauge fixing Lagrangian would be identical to the one of the β_h scheme. We will not follow this latter approach. Of course it is only a matter of choice, but the explicit form of $\mathcal{L}_{gf}$ determines the FP ghost Lagrangian.

The parameter β_t shows up also in the FP ghost sector. The FP Lagrangian depends on the gauge variations of the chosen gauge-fixing functions $\mathcal{C}_A$, $\mathcal{C}_Z$ and $\mathcal{C}_\pm$. If, under gauge transformations, the functions $\mathcal{C}_i$ transform as

$$\mathcal{C}_i \to \mathcal{C}_i + (M_{ij} + gL_{ij})\,\Lambda_j, \tag{28}$$

with $i = (A, Z, \pm)$, then the FP ghost Lagrangian is given by

$$\mathcal{L}_{FP} = \overline{X}_i\,(M_{ij} + gL_{ij})\,X_j. \tag{29}$$

With the choice for $\mathcal{L}_{gf}$ given in Eq.(26) (and the relation $gv/2 = M'(1+\beta_t)$) it is easy to check that the FP ghost Lagrangian contains the β_t terms

$$\mathcal{L}_{FP} = -\left(M'\right)^2 \beta_t \left(\xi_W \overline{X}^+ X^+ + \xi_W \overline{X}^- X^- + \xi_Z \overline{X}_Z X_Z / c_\theta^2\right) + \cdots, \tag{30}$$

where the dots indicate the usual β_t–independent terms. Had we chosen $\mathcal{L}_{gf}$ with $M'(1+\beta_t)$ instead of M' in Eq.(27), additional β_t terms would now arise in the FP Lagrangian.

In the fermionic sector, the tadpole constant β_t appears in the mass terms:

$$\frac{v}{\sqrt{2}}\left(-\alpha\bar{u}u + \beta\bar{d}d\right) = -\left(1+\beta_t\right)\left(m_u\bar{u}u + m_d\bar{d}d\right) \tag{31}$$

($v = 2M'(1+\beta_t)/g$), where α and β are the Yukawa couplings, and m_u, m_d are the masses of the fermions. The rest of the fermion Lagrangian does not contain β_t, as it doesn't depend on v.

The Feynman rules for vertices involving a β_t factor ("β_t vertices") are listed in Appendix B, dropping the primes over M' and M'_H. In the β_t scheme, contrary to the β_h one, we have (many) two- and three-leg β_t vertices containing also non-scalar fields. Note that three-leg β_t vertices introduce a fourth irreducible topology for $\mathcal{O}(g^4)$ self-energy diagrams containing β_t vertices, namely: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2.3.2 β_t up to one loop

Define $\beta_t = \beta_{t_0} + \beta_{t_1} g^2 + \beta_{t_2} g^4 + \cdots$. As we did for β_h, we will now fix the parameter β_t such that the VEV of the Higgs field H remains zero order by order in perturbation theory. At the lowest order, the only diagram contributing to $\langle H\rangle_0$ is the same one depicted in Eq.(8), which origins from the term in $\mathcal{L}_S^I$ linear in H, $-\beta_t(\beta_t+1)(\beta_t+2)(M_H'^2 M'/g)H$. Therefore, at the lowest order we can simply set $\beta_t = 0$, i.e. $\beta_{t_0} = 0$.

Up to one loop, the diagrams T_0' and T_1' contributing to the Higgs VEV are analogous to T_0 and T_1 appearing in Eq.(9), so that β_{t_1} can be set in analogy with β_{h_1}:

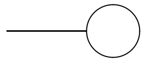
$$\beta_{t_1} = \frac{1}{(2\pi)^4 i}\left(\frac{T_1'}{2M'gM_H'^2}\right). \tag{32}$$

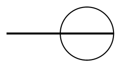
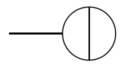
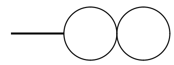
Note that T_1' and T_1 have the same functional form, but depend on different mass parameters.

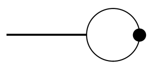
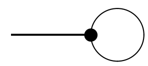
2.3.3 β_t up to two loops

The two-loop β_t fixing slightly differs from the β_h one. Up to terms of $\mathcal{O}(g^3)$, $\langle H \rangle_0$ gets contributions from the following diagrams:

T'_0 :  (1) +

T'_1 :  (1/2) +

T'_2 :  (1/6) +  (1/4) +  (1/4) +

T'_3 :  (1/2) +  (1/2),

plus reducible diagrams (analogous to those appearing in T_4–T_7 of section 2.4) which add up to zero because of our choice for β_{t_0} and β_{t_1}. Note the new diagrams in T'_3, with three-leg β_t vertices, not present in the β_h case (T_3). The parameter β_{t_2} can be set in the usual manner, requiring

$$\sum_{i=0}^{3} T'_i = 0, \qquad \Longrightarrow \qquad \beta_{t_2} = \frac{1}{(2\pi)^4 i}\left(\frac{T'_2 + T'_3}{2M' g^3 M'^2_H}\right) - \frac{3}{2}\beta_{t_1}^2. \tag{33}$$

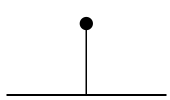
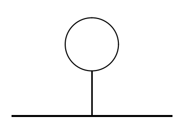
Note that $T'_{1,2}$ and $T_{1,2}$ have the same functional form (but depend on different mass parameters) while T'_3 and T_3 are different.

2.4 β_h and β_t: two comments

Consider the (doubly-contracted) WST identity relating the Z self-energy $\Pi_{\mu\nu,ZZ}(p)$, the ϕ_0 self-energy $\Pi_{\phi_o\phi_o}(p)$, and the Z–ϕ_0 transition $\Pi_{\mu,Z\phi_o}(p)$ (see Section 4):

$$p_\mu p_\nu \Pi_{\mu\nu,ZZ}(p) \; + \; M_0^2 \Pi_{\phi_o\phi_o}(p) \; + \; 2ip_\mu M_0 \Pi_{\mu,Z\phi_o}(p) \; = \; 0. \tag{34}$$

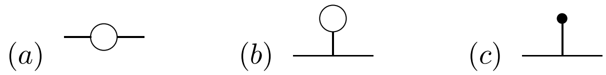
Both in the β_h and β_t schemes, each of the three terms in Eq.(34) contains contributions from the tadpole diagrams, but they add up to zero, within each term. For example, at the one-loop level, the first term in Eq.(34) contains the tadpole diagrams

Z  Z and Z  Z (35)

which cancel each other. In the β_h scheme at the one-loop level, only the second term of the l.h.s. of Eq.(34) includes a diagram with a two-leg β_h vertex (Eq.(15)), while in higher orders, two-leg β_h vertices appear in all three terms. In the β_t scheme, all three terms of Eq.(34) contain the two-leg β_t vertices already at the one-loop level. Similar comments are valid for the WST identity involving the W self-energy.

Concerning renormalization, the constraints imposed on β_h and β_t in the previous sections are the renormalization conditions to insure that $\langle 0|H|0\rangle = 0$, also in the presence of radiative corrections. In particular, the renormalized $\beta_{h,t}$ parameters are $\beta_{h,t}^{(R)} = \beta_{h,t} + \delta\beta_{h,t} = 0$. The equivalent of Eqs. (6) and

(21) for the renormalized parameters are just the same equations with the tadpole constants set to zero. In the β_h scheme, the one-loop renormalization of the W and Z masses involves the diagrams



. (36)

(Diagrams (a) have two possible loop topologies.) Both (a) and (b) are gauge-dependent, but their sum is gauge-independent on-shell. However, as we choose the β_h tadpole (c) to cancel (b), the mass counterterm contains only (a) and is therefore gauge-dependent. On the contrary, in the β_t scheme, the one-loop renormalization of the W and Z masses involves the diagrams

. (37)

Once again, both (a) and (b) diagrams are gauge-dependent, their sum is gauge-independent on-shell, and the β_t tadpole (d) is chosen to cancel (b). But, the mass counterterm is now gauge-independent, as it contains both (a) and the two-leg β_t vertex diagram (c) (which is missing in the β_h case).

3 Diagonalization of the neutral sector

3.1 New coupling constant in the β_h scheme

The Z–γ transition in the SM does not vanish at zero squared momentum transfer. Although this fact does not pose any serious problem, not even for the renormalization of the electric charge, it is preferable to use an alternative strategy. We will follow the treatment of Ref. [20]. Consider the new $SU(2)$ coupling constant $\bar{g}$, the new mixing angle $\bar{\theta}$ and the new W mass $\bar{M}$ in the β_h scheme:

$$\begin{aligned} g &= \bar{g}\,(1+\Gamma) \qquad g' = -(\sin\bar{\theta}/\cos\bar{\theta})\,\bar{g} \\ v &= 2\bar{M}/\bar{g} \quad \lambda = \left(\bar{g}M_{_H}/2\bar{M}\right)^2 \quad \mu^2 = \beta_h - \tfrac{1}{2}M_{_H}^2 \end{aligned} \tag{38}$$

(note: $g\sin\theta/\cos\theta = \bar{g}\sin\bar{\theta}/\cos\bar{\theta}$), where $\Gamma = \Gamma_1\,\bar{g}^2 + \Gamma_2\,\bar{g}^4 + \cdots$ is a new parameter yet to be specified. This change of parameters entails new $\overline{A}_\mu$ and $\overline{Z}_\mu$ fields related to B^3_μ and B^0_μ by

$$\begin{pmatrix} \overline{Z}_\mu \\ \overline{A}_\mu \end{pmatrix} = \begin{pmatrix} \cos\bar{\theta} & -\sin\bar{\theta} \\ \sin\bar{\theta} & \cos\bar{\theta} \end{pmatrix} \begin{pmatrix} B^3_\mu \\ B^0_\mu \end{pmatrix}. \tag{39}$$

The replacement $g \to \bar{g}(1+\Gamma)$ introduces in the SM Lagrangian several terms containing the new parameter Γ. In our approach Γ is fixed, order-by-order, by requiring that the Z–γ transition is zero at $p^2 = 0$ in the $\xi = 1$ gauge. Let us take a close look at these 'Γ terms' in each sector of the SM.

• The pure Yang–Mills Lagrangian

$$\mathcal{L}_{_{YM}} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - \frac{1}{4}F^0_{\mu\nu}F^0_{\mu\nu}, \tag{40}$$

with $F^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g\epsilon^{abc}B^b_\mu B^c_\nu$ and $F^0_{\mu\nu} = \partial_\mu B^0_\nu - \partial_\nu B^0_\mu$, contains the following new Γ terms when we replace g by $\bar{g}(1+\Gamma)$:

$$\Delta\mathcal{L}_{_{YM}} = -i\bar{g}\Gamma\bar{c}_\theta \left[\partial_\nu\overline{Z}_\mu\left(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu\right) - \overline{Z}_\nu\left(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu\right) + \right.$$

$$
\begin{aligned}
&+ \overline{Z}_\mu \left(W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+\right)\Big] - i\bar{g}\Gamma\bar{s}_\theta \left[\partial_\nu \overline{A}_\mu \left(W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-\right)\right. \\
&- \overline{A}_\nu \left(W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+\right) + \overline{A}_\mu \left(W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+\right)\Big] \\
+\ & \bar{g}^2 \Gamma \left(2+\Gamma\right) \left[\frac{1}{2}\left(W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- - W_\mu^+ W_\mu^- W_\nu^+ W_\nu^-\right)\right. \\
&+ \bar{c}_\theta^2 \left(\overline{Z}_\mu W_\mu^+ \overline{Z}_\nu W_\nu^- - \overline{Z}_\mu \overline{Z}_\mu W_\nu^+ W_\nu^-\right) \quad + \bar{s}_\theta^2 \left(\overline{A}_\mu W_\mu^+ \overline{A}_\nu W_\nu^- - \overline{A}_\mu \overline{A}_\mu W_\nu^+ W_\nu^-\right) \\
&\left. + \bar{s}_\theta \bar{c}_\theta \left(\overline{A}_\mu \overline{Z}_\nu (W_\mu^+ W_\nu^- + W_\nu^+ W_\mu^-) - 2\overline{A}_\mu \overline{Z}_\mu W_\nu^+ W_\nu^-\right)\right],
\end{aligned}
\tag{41}
$$

where $\bar{s}_\theta = \sin\bar{\theta}$ and $\bar{c}_\theta = \cos\bar{\theta}$. As these terms are of $\mathcal{O}(\bar{g}^3)$ or $\mathcal{O}(\bar{g}^4)$, they do not contribute to the calculation of self-energies at the one-loop level, but they do beyond it.

• The Lagrangian $\mathcal{L}_S$, Eq.(1), contains several new Γ terms when we employ the relation $g = \bar{g}(1+\Gamma)$ and the β_h scheme of Eqs. (38). They can be arranged in the following three classes

$$
\Delta\mathcal{L}_{S,h} = \Delta\mathcal{L}_{S,h}^{(n_f=2)} + \Delta\mathcal{L}_{S,h}^{(n_f=3)} + \Delta\mathcal{L}_{S,h}^{(n_f=4)},
\tag{42}
$$

according to the number of fields (n_f) appearing in each interaction term (indicated by the superscript in parentheses). The explicit expressions, up to terms of $\mathcal{O}(\bar{g}^4)$, are

$$
\begin{aligned}
\Delta\mathcal{L}_{S,h}^{(n_f=2)} &= \bar{M}\Gamma \left[-\frac{1}{2}\bar{M}\bar{s}_\theta^2 \Gamma \overline{A}_\mu \overline{A}_\mu - \frac{1}{2}\bar{M}\left(2+\Gamma\bar{c}_\theta^2\right)\overline{Z}_\mu \overline{Z}_\mu\right. \\
&\quad -\bar{M}\,\frac{\bar{s}_\theta}{\bar{c}_\theta}\left(1+\Gamma\bar{c}_\theta^2\right)\overline{A}_\mu \overline{Z}_\mu + \partial_\mu \phi_0 \left(\bar{s}_\theta \overline{A}_\mu + \bar{c}_\theta \overline{Z}_\mu\right) \\
&\quad \left. -\bar{M}\left(2+\Gamma\right) W_\mu^+ W_\mu^- + W_\mu^- \partial_\mu \phi^+ + W_\mu^+ \partial_\mu \phi^-\right],
\end{aligned}
\tag{43}
$$

$$
\begin{aligned}
\Delta\mathcal{L}_{S,h}^{(n_f=3)} &= \bar{g}\Gamma \left[-\bar{M}H\left(\overline{Z}_\mu \overline{Z}_\mu + \frac{\bar{s}_\theta}{\bar{c}_\theta}\overline{A}_\mu \overline{Z}_\mu + 2W_\mu^+ W_\mu^-\right)\right. \\
&\quad +\frac{1}{2}\left(\bar{s}_\theta \overline{A}_\mu + \bar{c}_\theta \overline{Z}_\mu\right)\left(H\partial_\mu \phi^0 - \phi^0 \partial_\mu H + i\phi^+ \partial_\mu \phi^- - i\phi^- \partial_\mu \phi^+\right) \\
&\quad +i\left(\phi^- W_\mu^+ - \phi^+ W_\mu^-\right)\left(\bar{s}_\theta \bar{M}\overline{A}_\mu - (\bar{s}_\theta^2/\bar{c}_\theta)\bar{M}\overline{Z}_\mu + \frac{1}{2}\partial_\mu \phi^0\right) \\
&\quad \left. +\frac{1}{2}W_\mu^- \partial_\mu \phi^+ \left(H + i\phi^0\right) + \frac{1}{2}W_\mu^+ \partial_\mu \phi^- \left(H - i\phi^0\right) - \frac{1}{2}\partial_\mu H\left(\phi^+ W_\mu^- + \phi^- W_\mu^+\right)\right],
\end{aligned}
\tag{44}
$$

$$
\begin{aligned}
\Delta\mathcal{L}_{S,h}^{(n_f=4)} &= \frac{\bar{g}^2}{2}\Gamma \left\{-\frac{1}{2}\left(H^2 + \phi_0^2\right)\left(\overline{Z}_\mu \overline{Z}_\mu + \frac{\bar{s}_\theta}{\bar{c}_\theta}\overline{A}_\mu \overline{Z}_\mu + 2W_\mu^+ W_\mu^-\right)\right. \\
&\quad +\phi^+\phi^-\left(-2\bar{s}_\theta^2 \overline{A}_\mu \overline{A}_\mu + (1-2\bar{c}_\theta^2)\overline{Z}_\mu \overline{Z}_\mu + \left(\bar{s}_\theta/\bar{c}_\theta - 4\bar{s}_\theta \bar{c}_\theta\right)\overline{A}_\mu \overline{Z}_\mu\right) \\
&\quad -2W_\mu^+ W_\mu^- \phi^+ \phi^- + \left(\bar{s}_\theta \overline{A}_\mu - (\bar{s}_\theta^2/\bar{c}_\theta)\overline{Z}_\mu\right) \times \\
&\quad \left. \times \left[\phi_0 \left(\phi^+ W_\mu^- + \phi^- W_\mu^+\right) - iH\left(\phi^+ W_\mu^- - \phi^- W_\mu^+\right)\right]\right\}.
\end{aligned}
\tag{45}
$$

The interaction part of the scalar Lagrangian, $\mathcal{L}_S^I = -\mu^2 K^\dagger K - (\lambda/2)(K^\dagger K)^2$, does not induce Γ terms; these are only originated by the term involving the covariant derivatives, $-(D_\mu K)^\dagger(D_\mu K)$. On the other hand, as $M/g = \bar{M}/\bar{g}$, the β_h terms induced by $\mathcal{L}_S^I$ are given by Eq.(7) expressed in terms of the ratio $\bar{M}/\bar{g}$ of the barred parameters.

• We choose the gauge-fixing Lagrangian $\mathcal{L}_{gf}$ of Eq.(26) with the following gauge functions:

$$\mathcal{C}_A = -\frac{1}{\xi_A}\partial_\mu \overline{A}_\mu, \quad \mathcal{C}_Z = -\frac{1}{\xi_Z}\partial_\mu \overline{Z}_\mu + \xi_Z \frac{\bar{M}}{\bar{c}_\theta}\phi_0, \quad \mathcal{C}_\pm = -\frac{1}{\xi_W}\partial_\mu W^\pm_\mu + \xi_W \bar{M}\phi_\pm. \tag{46}$$

This R_ξ gauge Γ-independent $\mathcal{L}_{gf}$ cancels the zeroth order (in $\bar{g}$) gauge–scalar mixing terms introduced by $\mathcal{L}_S$, but not those proportional to Γ. Had one chosen gauge-fixing functions Eqs. (46) with unbarred quantities, all the gauge–scalar mixing terms of $\mathcal{L}_S$ would be canceled, including those proportional to Γ, but additional new Γ vertices would also be introduced.

• New Γ terms are also originated in the Faddeev–Popov ghost sector. Studying the gauge transformations (Eq.(28)) of the gauge-fixing functions $\mathcal{C}_A$, $\mathcal{C}_Z$ and $\mathcal{C}_\pm$ defined in Eqs. (46), the additional new Γ terms of the FP Lagrangian (which is defined in Eq.(29)) in the β_h scheme are:

$$\Delta\mathcal{L}_{FP,h} = \Delta\mathcal{L}^{(n_f=2)}_{FP,h} + \Delta\mathcal{L}^{(n_f=3)}_{FP,h}, \tag{47}$$

where the two-field terms are,

$$\Delta\mathcal{L}^{(n_f=2)}_{FP,h} = -\Gamma\bar{M}^2\left[\xi_Z\overline{X}_Z\left(X_Z + \frac{\bar{s}_\theta}{\bar{c}_\theta}X_A\right) + \xi_W\left(\overline{X}_+X_+ + \overline{X}_-X_-\right)\right], \tag{48}$$

and the three-field terms are

$$\begin{aligned}
\Delta\mathcal{L}^{(n_f=3)}_{FP,h} = \;& \Gamma\bar{g}\Big\{ i\bar{c}_\theta W^+_\mu\Big((\partial_\mu\overline{X}_Z/\xi_Z)X_- - (\partial_\mu\overline{X}_+/\xi_W)X_Z\Big) \\
+ \;& i\bar{s}_\theta W^+_\mu\Big((\partial_\mu\overline{X}_A/\xi_A)X_- - (\partial_\mu\overline{X}_+/\xi_W)X_A\Big) \\
+ \;& i\bar{c}_\theta W^-_\mu\Big((\partial_\mu\overline{X}_-/\xi_W)X_Z - (\partial_\mu\overline{X}_Z/\xi_Z)X_+\Big) \\
+ \;& i\bar{s}_\theta W^-_\mu\Big((\partial_\mu\overline{X}_-/\xi_W)X_A - (\partial_\mu\overline{X}_A/\xi_A)X_+\Big) \\
+ \;& i\bar{c}_\theta \overline{Z}_\mu\Big((\partial_\mu\overline{X}_+/\xi_W)X_+ - (\partial_\mu\overline{X}_-/\xi_W)X_-\Big) \\
+ \;& i\bar{s}_\theta \overline{A}_\mu\Big((\partial_\mu\overline{X}_+/\xi_W)X_+ - (\partial_\mu\overline{X}_-/\xi_W)X_-\Big) \\
+ \;& \frac{1}{2}\xi_W\bar{M}\Big[i\phi_0\left(\overline{X}_+X_+ - \overline{X}_-X_-\right) - H\left(\overline{X}_+X_+ + \overline{X}_-X_-\right)\Big] \\
+ \;& \frac{1}{2\bar{c}_\theta}\xi_Z\bar{M}\overline{X}_Z\Big[iX_-\phi_+ - iX_+\phi_- - \bar{s}_\theta HX_A - \bar{c}_\theta HX_Z\Big] \\
+ \;& \frac{i}{2}\xi_W\bar{M}\Big[\overline{X}_-\phi_-\left(\bar{c}_\theta X_Z + \bar{s}_\theta X_A\right) - \overline{X}_+\phi_+\left(\bar{c}_\theta X_Z + \bar{s}_\theta X_A\right)\Big]\Big\}.
\end{aligned} \tag{49}$$

(The bars over the FP ghost fields indicate conjugation. Obviously, the new FP fields X_A and X_Z should also be denoted with the bar indicating the field rediagonalization, just like the new fields $\overline{A}_\mu$ and $\overline{Z}_\mu$. However, this notation would be confusing and we will leave this point understood.) Note that the FP ghost – gauge boson vertices are simply the usual ones with g replaced by $\bar{g}\Gamma$. This is not the case, in general, for the FP ghost – scalar terms.

• Finally, the fermionic sector. The fermion – gauge boson Lagrangian,

$$\begin{aligned}
\mathcal{L}_{fG} = \;& \frac{i}{2\sqrt{2}}\,g\big[W^+_\mu\bar{u}\gamma_\mu\,(1+\gamma_5)\,d \;+ W^-_\mu\bar{d}\gamma_\mu\,(1+\gamma_5)\,u\big] \\
& + \frac{i}{2c}\,g\,Z_\mu\,\bar{f}\gamma_\mu\big(I_3 - 2Q_f s^2 + I_3\gamma_5\big)f \;+ i\,gs\,Q_f A_\mu\bar{f}\gamma_\mu f,
\end{aligned} \tag{50}$$

(where $I_3 = \pm 1/2$ is the third component of the weak isospin of the fermion f, and Q_f its charge in units of $|e|$) becomes, under the replacement $g \to \bar{g}(1+\Gamma)$ and the θ, A_μ and Z_μ redefinitions,

$$
\begin{aligned}
\mathcal{L}_{fG} &= \frac{i}{2\sqrt{2}}\,\bar{g}\,(1+\Gamma)\left[W_\mu^+ \bar{u}\gamma_\mu\,(1+\gamma_5)\,d + W_\mu^- \bar{d}\gamma_\mu\,(1+\gamma_5)\,u\right] \\
&\quad + \frac{i}{2\bar{c}_\theta}\,\bar{g}\,\overline{Z}_\mu\,\bar{f}\gamma_\mu\left(I_3 - 2Q_f\bar{s}_\theta^2 + I_3\gamma_5\right)f \;+\; i\,\bar{g}\bar{s}_\theta\,Q_f\overline{A}_\mu\bar{f}\gamma_\mu f \\
&\quad + \frac{i}{2}\,\bar{g}\,\Gamma\left(\bar{s}_\theta\overline{A}_\mu + \bar{c}_\theta\overline{Z}_\mu\right) I_3\,\bar{f}\gamma_\mu\,(1+\gamma_5)\,f. \qquad (51)
\end{aligned}
$$

The new neutral and charged current Γ vertices are immediately recognizable. The CKM matrix has been set to unity.

The fermion–scalar Lagrangian does not induce Γ terms. Indeed, the Yukawa couplings α and β in

$$
\mathcal{L}_{fS} = -\alpha\bar{\psi}_L K u_R \;-\; \beta\bar{\psi}_L K^c d_R \;+\; \text{h.c.} \qquad (52)
$$

(where $K^c = i\tau_2 K^\star$ is the conjugate Higgs doublet) are set by $\alpha v/\sqrt{2} = m_u$ and $\beta v/\sqrt{2} = -m_d$. As $v = 2\bar{M}/\bar{g}$, it is $\alpha = \bar{g}m_u/\sqrt{2}\;\bar{M}$ and $\beta = -\bar{g}m_d/\sqrt{2}\;\bar{M}$, and no Γ appears in Eq.(52).

The Feynman rules for all these new Γ vertices are listed in Appendix C, up to terms of $\mathcal{O}(\bar{g}^4)$. Those corresponding to the pure Yang–Mills Lagrangian (Eq.(41)) are not listed, as they are identical to the usual Yang–Mills ones, except for the replacement $g \to \bar{g}\Gamma$ in the three-leg vertices, and $g^2 \to \bar{g}^2\Gamma(2+\Gamma)$ in the four-leg ones. In Appendix C, all bars over the symbols (indicating rediagonalization) have been dropped, except over $\bar{g}$.

3.2 New coupling constant in the β_t scheme

The β_t scheme equations corresponding to Eqs. (38) are the following

$$
\begin{aligned}
g &= \bar{g}\,(1+\Gamma) \qquad g' = -(\sin\bar{\theta}/\cos\bar{\theta})\,\bar{g} \\
v &= 2\bar{M}'(1+\beta_t)/\bar{g} \quad \lambda = \left(\bar{g}M'_{_H}/2\bar{M}'\right)^2 \quad \mu^2 = -\tfrac{1}{2}(M'_{_H})^2.
\end{aligned} \qquad (53)
$$

(Note: $g\sin\theta/\cos\theta = \bar{g}\sin\bar{\theta}/\cos\bar{\theta}$.) The analysis of the Γ terms presented in the previous section for the β_h scheme can be repeated for the β_t scheme using Eqs. (53) instead of Eqs. (38). The new fields $\overline{A}_\mu$ and $\overline{Z}_\mu$ are related to B_μ^3 and B_μ^0 by Eq.(39). Thus, we obtain the following results:

- The replacement $g \to \bar{g}(1+\Gamma)$ in the pure Yang–Mills sector introduces new Γ vertices collected in $\Delta\mathcal{L}_{_{YM}}$, which does not depend on the parameters of the $\beta_{h,t}$ schemes. $\Delta\mathcal{L}_{_{YM}}$ has already been given in Eq.(41).
- The new Γ terms introduced in $\mathcal{L}_S$ by Eqs. (53) can be arranged once again in the three classes

$$
\Delta\mathcal{L}_{S,t} = \Delta\mathcal{L}_{S,t}^{(n_f=2)} + \Delta\mathcal{L}_{S,t}^{(n_f=3)} + \Delta\mathcal{L}_{S,t}^{(n_f=4)}, \qquad (54)
$$

according to the number of fields appearing in the Γ terms. The explicit expression for $\Delta\mathcal{L}_{S,t}^{(2)}$ is, up to terms of $\mathcal{O}(\bar{g}^4)$,

$$
\begin{aligned}
\Delta\mathcal{L}_{S,t}^{(n_f=2)} &= \bar{M}'\Gamma\Bigg[-\frac{1}{2}\bar{M}'\bar{s}_\theta^2\Gamma\overline{A}_\mu\overline{A}_\mu - \frac{1}{2}\bar{M}'\left(2+\Gamma\bar{c}_\theta^2+4\beta_t\right)\overline{Z}_\mu\overline{Z}_\mu \qquad (55) \\
&\quad -\bar{M}'\,\frac{\bar{s}_\theta}{\bar{c}_\theta}\left(1+\Gamma\bar{c}_\theta^2+2\beta_t\right)\overline{A}_\mu\overline{Z}_\mu + \partial_\mu\phi_0\left(\bar{s}_\theta\overline{A}_\mu + \bar{c}_\theta\overline{Z}_\mu\right)(1+\beta_t) \\
&\quad -\bar{M}'\,(2+\Gamma+4\beta_t)\,W_\mu^+W_\mu^- + \left(W_\mu^-\partial_\mu\phi^+ + W_\mu^+\partial_\mu\phi^-\right)(1+\beta_t)\Bigg]
\end{aligned}
$$

with $\bar{s}_\theta = \sin\bar{\theta}$ and $\bar{c}_\theta = \cos\bar{\theta}$, while, up to the same $\mathcal{O}(\bar{g}^4)$,

$$\Delta\mathcal{L}_{S,t}^{(n_f=3,4)} = \Delta\mathcal{L}_{S,h}^{(n_f=3,4)}\left(\bar{M}\to\bar{M}'\right) \tag{56}$$

($\Delta\mathcal{L}_{S,h}^{(n_f=3)}$ and $\Delta\mathcal{L}_{S,h}^{(n_f=4)}$ are given in Eq.(44) and Eq.(45)). The subscripts t and h indicate the β_t and β_h schemes. Note the presence of β_t factors in the new Γ terms of Eq.(55). We will comment on this in Section 3.3.

• Our recipe for gauge-fixing is the same as in the previous sections: we choose the R_ξ gauge $\mathcal{L}_{gf}$ to cancel the zeroth order (in $\bar{g}$) gauge–scalar mixing terms introduced by $\mathcal{L}_S$, but not those of higher orders (see discussions in Sections 2.3.1 and 3.1). Here, this prescription is realized by $\mathcal{L}_{gf}$ (Eq.(26)) with

$$\mathcal{C}_A = -\frac{1}{\xi_A}\partial_\mu\overline{A}_\mu, \quad \mathcal{C}_Z = -\frac{1}{\xi_Z}\partial_\mu\overline{Z}_\mu + \xi_Z\frac{\bar{M}'}{\bar{c}_\theta}\phi_0, \quad \mathcal{C}_\pm = -\frac{1}{\xi_W}\partial_\mu W_\mu^\pm + \xi_W\bar{M}'\phi_\pm, \tag{57}$$

clearly Γ-independent. The new Γ terms of the FP ghost Lagrangian in the β_t scheme are:

$$\Delta\mathcal{L}_{FP,t} = \Delta\mathcal{L}_{FP,t}^{(n_f=2)} + \Delta\mathcal{L}_{FP,t}^{(n_f=3)}\ , \tag{58}$$

where the two-field terms are

$$\Delta\mathcal{L}_{FP,t}^{(n_f=2)} = -\left(1+\beta_t\right)\Gamma\bar{M}'^2\left[\xi_Z\overline{X}_Z\left(X_Z + \frac{\bar{s}_\theta}{\bar{c}_\theta}X_A\right) + \xi_W\left(\overline{X}_+X_+ + \overline{X}_-X_-\right)\right], \tag{59}$$

and the three-field terms are the same as in the β_h scheme, with $\bar{M}$ replaced by $\bar{M}'$: $\Delta\mathcal{L}_{FP,t}^{(n_f=3)} = \Delta\mathcal{L}_{FP,h}^{(n_f=3)}(\bar{M}\to\bar{M}')$ (Eq.(49)). Like in the scalar sector, the Γ and β_t factors are entangled.

• We conclude this analysis with the fermionic sector. As in the Yang–Mills case, the fermion – gauge boson Lagrangian $\mathcal{L}_{fG}$ does not depend on the parameters of the β_h or β_t schemes. Its expression in terms of the new coupling constant $\bar{g}$ contains new Γ terms and is given in Eq.(51). The neutral sector rediagonalization induces no Γ terms in the fermion–scalar Lagrangian $\mathcal{L}_{fS}$ (Eq.(52)), which contains, however, the β_t vertices discussed in Section 2.3 (Eq.(31)) (the ratio M'/g is now replaced by the identical ratio $\bar{M}'/\bar{g}$).

The Feynman rules for all Γ vertices are listed in Appendix C, up to terms of $\mathcal{O}(\bar{g}^4)$. All primes and bars over A_μ, Z_μ, M, M_H and θ have been dropped (but not over $\bar{g}$). As we mentioned at the end of the previous section, the Γ vertices of the pure Yang–Mills sector need not be listed.

3.3 The Γ–β_t mixing

A comment on the presence of β_t factors in the new Γ vertices is now appropriate. Consider the Lagrangian $\mathcal{L}_S$. As we already pointed out in Section 3.1, the interaction part $\mathcal{L}_S^I = -\mu^2K^\dagger K - (\lambda/2)(K^\dagger K)^2$ does not induce Γ terms, but gives rise to β_t terms: as $M'/g = \bar{M}'/\bar{g}$, these β_t terms are simply given by Eq.(23) expressed in terms of $\bar{M}'/\bar{g}$ instead of M'/g. On the other hand, the derivative part of $\mathcal{L}_S$, $-(D_\mu K)^\dagger(D_\mu K)$, induces both Γ and β_t vertices, plus mixed ones which we still call Γ vertices (see the β_t factors in the two-leg Γ terms of $\Delta\mathcal{L}_{S,t}^{(n_f=2)}$). It works like this: first, we replace $g\to\bar{g}(1+\Gamma)$ and $g'\to-\bar{g}(\bar{s}_\theta/\bar{c}_\theta)$ in $-(D_\mu K)^\dagger(D_\mu K)$, splitting the result in two classes of terms, both written in terms of $\bar{g}$, with or without Γ. Then we substitute in both classes $v\to 2\bar{M}'(1+\beta_t)/\bar{g}$: the class containing Γ is, up to terms of $\mathcal{O}(\bar{g}^4)$, $\Delta\mathcal{L}_{S,t}$ (Eq.(54)), and includes also β_t factors, while the class free of Γ has the same β_t vertices as Eq.(24) with g, θ, M', A_μ and Z_μ replaced by $\bar{g}$, $\bar{\theta}$, $\bar{M}'$, $\overline{A}_\mu$ and $\overline{Z}_\mu$. The Γ and β_t terms of the Faddeev–Popov sector are intertwined just as in the case of the scalar Lagrangian.

3.4 Summary of the special vertices

The upshot of this first part of the paper lies in the Appendices. There the readers find the full set of SM Γ (up to $\mathcal{O}(\bar{g}^4)$) and $\beta_{h,t}$ special vertices in the R_ξ gauges. All primes and bars over A_μ, Z_μ, M, M_H and θ have been dropped, but not over $\bar{g}$, the $SU(2)$ coupling constant of the rediagonalized neutral sector. The readers can pick their preferred tadpole scheme, β_h or β_t, and compute their Feynman diagrams including the $\beta_{h,t}$ vertices of Appendix A or B, respectively. If they prefer to work with the rediagonalized neutral sector, they should simply replace g by $\bar{g}$ in the $\beta_{h,t}$ vertices, and add to them the Γ ones of Appendix C. There, Γ vertices are listed for the β_t scheme (note that Γ and β_t terms are intertwined — see Section 3.3); just set $\beta_t = 0$ to use the β_h scheme instead.

Finally, Tab. 1 graphically summarizes which of the SM sectors provides each type of special vertex. Note the overlap of Γ and β_t terms in the scalar and Faddeev–Popov sectors.

SECTOR	β_h	β_t	Γ
Scalar: $(D_\mu K)^\dagger (D_\mu K)$		•	•
Scalar: $\mu^2 K^\dagger K + (\lambda/2)(K^\dagger K)^2$	•	•	
Yang–Mills			•
Gauge-Fixing			
Faddeev–Popov		•	•
Fermion – gauge boson			•
Fermion – Higgs		•	

Table 1: Special vertices in the Standard Model.

4 WST identities for two-loop gauge boson self-energies

The purpose of this section is to discuss in detail the structure of the (doubly-contracted) Ward-Slavnov-Taylor identities (WSTI) for the two-loop gauge boson self-energies in the SM, focusing in particular on the role played by the reducible diagrams. This analysis is performed in the 't Hooft–Feynman gauge.

4.1 Definitions and WST identities

Let Π_{ij} be the sum of all diagrams (both one-particle reducible and irreducible) with two external boson fields, i and j, to all orders in perturbation theory (as usual, the external Born propagators are not to be included in the expression for Π_{ij})

$$\Pi_{ij} = \sum_{n=1}^{\infty} \frac{g^{2n}}{(16\pi^2)^n} \Pi_{ij}^{(n)} \,. \tag{60}$$

In the subscripts of the quantities $\Pi_{ij}^{(n)}$ we will also explicitly indicate, when necessary, the appropriate Lorentz indices with Greek letters. At each order in the perturbative expansion it is convenient to make explicit the tensor structure of these functions by employing the following definitions:

$$\Pi^{(n)}_{\mu\nu,VV} = D^{(n)}_{VV}\,\delta_{\mu\nu} + P^{(n)}_{VV}\,p_\mu\,p_\nu \qquad \Pi^{(n)}_{\mu,VS} = -ip_\mu\,M_S\,G^{(n)}_{VS} \qquad \Pi^{(n)}_{SS} = R^{(n)}_{SS}\,, \tag{61}$$

where the subscripts V and S indicate vector and scalar fields, M_S is the mass of the Higgs-Kibble scalar S, and p is the incoming momentum of the vector boson (note: $\Pi^{(n)}_{\mu,SV} = -\Pi^{(n)}_{\mu,VS}$). The quantities D_{ij}, P_{ij}, G_{ij}, and R_{ij} depend only on the squared four-momentum and are symmetric in i and j. Furthermore, D and R have the dimensions of a mass squared, while G and P are dimensionless.

The WST identities require that, at each perturbative order, the gauge-boson self-energies satisfy the equations

$$\begin{aligned} p_\mu\,p_\nu\,\Pi^{(n)}_{\mu\nu,AA} &= 0 \\ p_\mu\,p_\nu\,\Pi^{(n)}_{\mu\nu,AZ} + ip_\mu\,M_0\,\Pi^{(n)}_{\mu,A\phi_o} &= 0 \\ p_\mu\,p_\nu\,\Pi^{(n)}_{\mu\nu,ZZ} + M_0^2\,\Pi^{(n)}_{\phi_o\phi_o} + 2\,ip_\mu\,M_0\,\Pi^{(n)}_{\mu,Z\phi_o} &= 0 \\ p_\mu\,p_\nu\,\Pi^{(n)}_{\mu\nu,WW} + M^2\,\Pi^{(n)}_{\phi\phi} + 2\,ip_\mu\,M\,\Pi^{(n)}_{\mu,W\phi} &= 0\,, \end{aligned} \tag{62}$$

which imply the following relations among the form factors D, P, G, and R

$$D^{(n)}_{AA} + p^2\,P^{(n)}_{AA} = 0 \tag{63}$$

$$D^{(n)}_{AZ} + p^2\,P^{(n)}_{AZ} + M_0^2\,G^{(n)}_{A\phi_o} = 0 \tag{64}$$

$$p^2\,D^{(n)}_{ZZ} + p^4\,P^{(n)}_{ZZ} + M_0^2\,R^{(n)}_{\phi_o\phi_o} = -2\,M_0^2\,p^2 G^{(n)}_{Z\phi_o} \tag{65}$$

$$p^2\,D^{(n)}_{WW} + p^4\,P^{(n)}_{WW} + M^2\,R^{(n)}_{\phi\phi} = -2\,M^2\,p^2 G^{(n)}_{W\phi}\,. \tag{66}$$

The subscripts A, Z, W, ϕ and ϕ_0 clearly indicate the SM fields. We have verified these WST Identities at the two-loop level (i.e. $n = 2$) with our code $\mathcal{G}raph\mathcal{S}hot$ [13].

4.2 WST identities at two loops: the role of reducible diagrams

At any given order in the coupling constant expansion, the SM gauge boson self-energies satisfy the WSTI (62). For $n \geq 2$, the quantities $\Pi^{(n)}_{ij}$ contain both one-particle irreducible (1PI) and reducible (1PR) contributions. At $\mathcal{O}(g^4)$, the SM $\Pi^{(n)}_{ij}$ functions contain the following irreducible topologies: eight two-loop topologies, three one-loop topologies with a β_{t_1} vertex, four one-loop topologies with a Γ_1 vertex, and one tree-level diagram with a two-leg $\mathcal{O}(g^4)$ β_t or Γ vertex (see figure at the end of Section 2.3.1). Reducible $\mathcal{O}(g^4)$ graphs involve the product of two $\mathcal{O}(g^2)$ ones: two one-loop diagrams, one one-loop diagram and a tree-level diagram with a $\mathcal{O}(g^2)$ two-leg vertex insertion, or two tree-level diagrams, each with a $\mathcal{O}(g^2)$ two-leg vertex insertion. There are also $\mathcal{O}(g^4)$ topologies containing tadpoles but, as we discussed in previous sections, their contributions add up to zero as a consequence of our choice for β_t.

In the following we analyze the structure of the $\mathcal{O}(g^4)$ WSTI for photon, Z, and W self-energies, as well as for the photon–Z mixing, emphasizing the role played by the reducible diagrams.

4.2.1 The photon self-energy

The contribution of the 1PR diagrams to the photon self-energy at $\mathcal{O}(g^4)$ is given, in the 't Hooft–Feynman gauge, by (with obvious notation)

$$\Pi^{(2)R}_{\mu\nu,AA} = \frac{1}{(2\pi)^4 i}\left[\frac{1}{p^2}\,\tilde{\Pi}^{(2)R}_{\mu\nu,AA} + \frac{1}{p^2 + M_0^2}\,\hat{\Pi}^{(2)R}_{\mu\nu,AA}\right], \tag{67}$$

where

$$\tilde{\Pi}^{(2)R}_{\mu\nu,AA} \;=\; \Pi^{(1)}_{\mu\alpha,AA}\,\Pi^{(1)}_{\alpha\nu,AA} \qquad \hat{\Pi}^{(2)R}_{\mu\nu,AA} = \Pi^{(1)}_{\mu\alpha,AZ}\,\Pi^{(1)}_{\alpha\nu,ZA} + \Pi^{(1)}_{\mu,A\phi_o}\,\Pi^{(1)}_{\nu,\phi_o A}\,.$$

It is interesting to consider separately the reducible diagrams that involve an intermediate photon propagator ($\tilde{\Pi}^{(2)R}_{\mu\nu,AA}$) and those including an intermediate Z or ϕ_0 propagator ($\hat{\Pi}^{(2)R}_{\mu\nu,AA}$). By employing the definitions given in the previous subsection and Eq.(63) with $n=1$, one verifies that $\tilde{\Pi}^{2R}_{\mu\nu,AA}$ obeys the photon WSTI by itself,

$$p_\mu\, p_\nu\, \tilde{\Pi}^{(2)R}_{\mu\nu,AA} = p^2\,\left[D^{(1)}_{AA} + p^2\,P^{(1)}_{AA}\right]^2 = 0\,. \tag{68}$$

This is not the case for $\hat{\Pi}^{(2)R}_{\mu\nu,AA}$, although most of its contributions cancel when contracted by $p_\mu p_\nu$ as a consequence of Eq.(64) ($n=1$),

$$p_\mu\, p_\nu \hat{\Pi}^{(2)R}_{\mu\nu,AA} = p^2\, M_0^2\,\left(p^2 + M_0^2\right)\left[G^{(1)}_{A\phi_o}\right]^2. \tag{69}$$

The only diagrams contributing to the A–ϕ_0 mixing up to $\mathcal{O}(g^2)$ are those with a W–ϕ or FP ghosts loop, and the tree-level diagram with a Γ insertion. Their contribution, in the 't Hooft–Feynman gauge, is

$$G^{(1)}_{A\phi_0} = (2\pi)^4 i\, s_\theta c_\theta\,\left[2B_0(p^2, M, M) + 16\pi^2\Gamma_1\right]. \tag{70}$$

A direct calculation (e.g. with $\mathcal{G}raph\mathcal{S}hot$) shows that this residual contribution of the reducible diagrams to the $\mathcal{O}(g^4)$ photon WSTI, Eq.(69), is exactly canceled by the contribution of the $\mathcal{O}(g^4)$ irreducible diagrams, which include two-loop diagrams as well as one-loop graphs with a two-leg vertex insertion.

4.2.2 The photon–Z mixing

We now consider the second of Eqs. (62) for $n=2$. Reducible diagrams contribute to both A–Z and A–ϕ_0 transitions. Following the example of Eq.(67), we divide these contributions in two classes: the diagrams that include an intermediate photon propagator and those mediated by a Z or a ϕ_0, namely, for the photon–Z transition in the 't Hooft–Feynman gauge,

$$\begin{aligned}
\Pi^{(2)R}_{\mu\nu,AZ} &= \frac{1}{(2\pi)^4 i}\left[\frac{1}{p^2}\,\tilde{\Pi}^{(2)R}_{\mu\nu,AZ} + \frac{1}{p^2+M_0^2}\,\hat{\Pi}^{(2)R}_{\mu\nu,AZ}\right] \\
\tilde{\Pi}^{(2)R}_{\mu\nu,AZ} &= \Pi^{(1)}_{\mu\alpha,AA}\,\Pi^{(1)}_{\alpha\nu,AZ} \\
\hat{\Pi}^{(2)R}_{\mu\nu,AZ} &= \Pi^{(1)}_{\mu\alpha,AZ}\,\Pi^{(1)}_{\alpha\nu,ZZ} + \Pi^{(1)}_{\mu,A\phi_o}\,\Pi^{(1)}_{\nu,\phi_o Z}\,,
\end{aligned} \tag{71}$$

and, for the photon–ϕ_0 transition in the same gauge,

$$\begin{aligned}
\Pi^{(2)R}_{\mu,A\phi_o} &= \frac{1}{(2\pi)^4 i}\left[\frac{1}{p^2}\,\tilde{\Pi}^{(2)R}_{\mu,A\phi_o} + \frac{1}{p^2+M_0^2}\,\hat{\Pi}^{(2)R}_{\mu,A\phi_o}\right] \\
\tilde{\Pi}^{(2)R}_{\mu,A\phi_o} &= \Pi^{(1)}_{\mu\alpha,AA}\,\Pi^{(1)}_{\alpha,A\phi_o} \\
\hat{\Pi}^{(2)R}_{\mu,A\phi_o} &= \Pi^{(1)}_{\mu\alpha,AZ}\,\Pi^{(1)}_{\alpha,Z\phi_o} + \Pi^{(1)}_{\mu,A\phi_o}\,\Pi^{(1)}_{\phi_o\phi_o}\,.
\end{aligned} \tag{72}$$

The reducible diagrams with an intermediate photon propagator satisfy the WSTI by themselves. Indeed,

$$p_\mu\, p_\nu\, \tilde{\Pi}^{(2)R}_{\mu\nu,AZ} + iM_0 p_\mu \tilde{\Pi}^{(2)R}_{\mu,A\phi_o} = 0\,, \tag{73}$$

as it can be easily checked using Eq.(63) with $n = 1$. On the contrary, the remaining reducible diagrams must be added to the irreducible $\mathcal{O}(g^4)$ contributions in order to satisfy the WSTI for the photon–Z mixing:

$$p_\mu p_\nu \left[\frac{\hat{\Pi}^{(2)R}_{\mu\nu,AZ}}{(2\pi)^4 i (p^2 + M_0^2)} + \Pi^{(2)I}_{\mu\nu,AZ} \right] + i M_0 p_\mu \left[\frac{\hat{\Pi}^{(2)R}_{\mu,A\phi_0}}{(2\pi)^4 i (p^2 + M_0^2)} + \Pi^{(2)I}_{\mu,A\phi_0} \right] = 0. \tag{74}$$

4.2.3 The Z self-energy

Also in the case of the WSTI for the $\mathcal{O}(g^4)$ Z self-energy it is convenient to separate the reducible contributions mediated by a photon propagator from the rest of the reducible diagrams. In the 't Hooft–Feynman gauge it is

$$\begin{aligned}
\Pi^{(2)R}_{\mu\nu,ZZ} &= \frac{1}{(2\pi)^4 i} \left[\frac{1}{p^2} \tilde{\Pi}^{(2)R}_{\mu\nu,ZZ} + \frac{1}{p^2 + M_0^2} \hat{\Pi}^{(2)R}_{\mu\nu,ZZ} \right] \\
\tilde{\Pi}^{(2)R}_{\mu\nu,ZZ} &= \Pi^{(1)}_{\mu\alpha,ZA} \Pi^{(1)}_{\alpha\nu,AZ} \\
\hat{\Pi}^{(2)R}_{\mu\nu,ZZ} &= \Pi^{(1)}_{\mu\alpha,ZZ} \Pi^{(1)}_{\alpha\nu,ZZ} + \Pi^{(1)}_{\mu,Z\phi_o} \Pi^{(1)}_{\nu,\phi_o Z} \,,
\end{aligned} \tag{75}$$

$$\begin{aligned}
\Pi^{(2)R}_{\mu,Z\phi_o} &= \frac{1}{(2\pi)^4 i} \left[\frac{1}{p^2} \tilde{\Pi}^{(2)R}_{\mu,Z\phi_o} + \frac{1}{p^2 + M_0^2} \hat{\Pi}^{(2)R}_{\mu,Z\phi_o} \right] \\
\tilde{\Pi}^{(2)R}_{\mu,Z\phi_o} &= \Pi^{(1)}_{\mu\alpha,ZA} \Pi^{(1)}_{\alpha,A\phi_o} \\
\hat{\Pi}^{(2)R}_{\mu,Z\phi_o} &= \Pi^{(1)}_{\mu\alpha,ZZ} \Pi^{(1)}_{\alpha,Z\phi_o} + \Pi^{(1)}_{\mu,Z\phi_o} \Pi^{(1)}_{\phi_o\phi_o} \,,
\end{aligned} \tag{76}$$

$$\begin{aligned}
\Pi^{(2)R}_{\phi_o\phi_o} &= \frac{1}{(2\pi)^4 i} \left[\frac{1}{p^2} \tilde{\Pi}^{(2)R}_{\phi_o\phi_o} + \frac{1}{p^2 + M_0^2} \hat{\Pi}^{(2)R}_{\phi_o\phi_o} \right] \\
\tilde{\Pi}^{(2)R}_{\phi_o\phi_o} &= \Pi^{(1)}_{\alpha,\phi_o A} \Pi^{(1)}_{\alpha,A\phi_o} \\
\hat{\Pi}^{(2)R}_{\phi_o\phi_o} &= \Pi^{(1)}_{\alpha,\phi_o Z} \Pi^{(1)}_{\alpha,Z\phi_o} + \Pi^{(1)}_{\phi_o\phi_o} \Pi^{(1)}_{\phi_o\phi_o},
\end{aligned} \tag{77}$$

and, once again, the reducible diagrams mediated by a photon propagator satisfy the WSTI by themselves, i.e.

$$p_\mu \, p_\nu \, \tilde{\Pi}^{(2)R}_{\mu\nu,ZZ} + M_0^2 \, \tilde{\Pi}^{(2)R}_{\phi_o\phi_o} + 2 \, i \, p_\mu \, M_0 \, \tilde{\Pi}^{(2)R}_{\mu,Z\phi_o} = 0 \,, \tag{78}$$

as it can be easily checked using the one-loop WSTI for the photon–Z mixing (Eq.(64) with $n = 1$).

4.2.4 The W self-energy

All the $\mathcal{O}(g^4)$ 1PR contributions to the WSTI for the W self-energy are mediated, in the 't Hooft–Feynman gauge, by a charged particle of mass M. A separate analysis of their contribution does not lead, in this case, to particularly significant simplifications of the structure of the WSTI. However, some cancellations among the reducible terms occur, allowing to obtain a relation that will be useful in the discussion of the Dyson resummation of the W propagator. The 1PR quantities that contribute to the $\mathcal{O}(g^4)$ WSTI for the W self-energy have the following form:

$$\Pi^{(2)R}_{\mu\nu,WW} = \frac{1}{(2\pi)^4 i \; (p^2 + M^2)} \left\{ \left(D^{(1)}_{WW} \right)^2 \delta_{\mu\nu} + p_\mu \, p_\nu \left[2 \, D^{(1)}_{WW} \, P^{(1)}_{WW} + p^2 \left(P^{(1)}_{WW} \right)^2 + M^2 \left(G^{(1)}_{W\phi} \right)^2 \right] \right\}$$

$$\Pi^{(2)R}_{\mu,W\phi} = \frac{-i\,p_\mu\,M}{(2\pi)^4 i\,(p^2+M^2)}\,G^{(1)}_{W\phi}\left[D^{(1)}_{WW} + p^2\,P^{(1)}_{WW} + R^{(1)}_{\phi\phi}\right]$$

$$\Pi^{(2)R}_{\phi\phi} = \frac{1}{(2\pi)^4 i\,(p^2+M^2)}\left[p^2\,M^2\,\left(G^{(1)}_{W\phi}\right)^2 + \left(R^{(1)}_{\phi\phi}\right)^2\right]. \tag{79}$$

Contracting the free indices with the corresponding external momenta, summing the three contributions and employing Eq.(66) with $n = 1$, we obtain

$$(2\pi)^4 i\left[p_\mu\,p_\nu\,\Pi^{(2)R}_{\mu\nu,WW} + M^2\,\Pi^{(2)R}_{\phi\phi} + 2\,i\,p_\mu\,M\,\Pi^{(2)R}_{\mu,W\phi}\right] = p^2\,M^2\,\left(G^{(1)}_{W\phi}\right)^2 - R^{(1)}_{\phi\phi}\left[D^{(1)}_{WW} + p^2\,P^{(1)}_{WW}\right]. \tag{80}$$

5 Dyson resummed propagators and their WST identities

We will now present the Dyson resummed propagators for the electroweak gauge bosons. We will then employ the results of Section 4 to show explicitly, up to terms of $\mathcal{O}(g^4)$, that the resummed propagators satisfy the WST identities.

Following definition Eq.(60) for Π_{ij}, the function Π^I_{ij} represents the sum of all 1PI diagrams with two external boson fields, i and j, to all orders in perturbation theory (as usual, the external Born propagators are not to be included in the expression for Π^I_{ij}). As we did in Eqs. (61), we write explicitly its Lorentz structure,

$$\Pi^I_{\mu\nu,VV} = D^I_{VV}\,\delta_{\mu\nu} + P^I_{VV}\,p_\mu\,p_\nu \quad \Pi^I_{\mu,VS} = -ip_\mu\,M_S\,G^I_{VS} \quad \Pi^I_{SS} = R^I_{SS}\,, \tag{81}$$

where V and S indicate SM vector and scalar fields, and p_μ is the incoming momentum of the vector boson (note: $\Pi^I_{\mu,SV} = -\Pi^I_{\mu,VS}$). We also introduce the transverse and longitudinal projectors

$$\begin{gathered} t^{\mu\nu} = \delta_{\mu\nu} - \tfrac{p_\mu p_\nu}{p^2}\,, \qquad l^{\mu\nu} = \tfrac{p_\mu p_\nu}{p^2}\,, \\ t^{\mu\alpha}\,t^{\alpha\nu} = t^{\mu\nu}\,, \quad l^{\mu\alpha}\,l^{\alpha\nu} = l^{\mu\nu}\,, \quad t^{\mu\alpha}\,l^{\alpha\nu} = 0\,, \\ \Pi^I_{\mu\nu,VV} = D^I_{VV}\,t_{\mu\nu} + L^I_{VV}\,l_{\mu\nu}, \quad L^I_{VV} = D^I_{VV} + p^2\,P^I_{VV}\,. \end{gathered} \tag{82}$$

The full propagator for a field i which mixes with a field j via the function Π^I_{ij} is given by the perturbative series

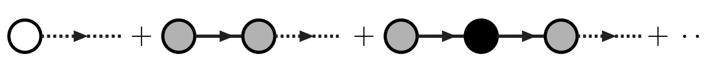
$$\begin{aligned} \bar\Delta_{ii} &= \Delta_{ii} + \Delta_{ii}\sum_{n=0}^{\infty}\prod_{l=1}^{n+1}\sum_{k_l}\Pi^I_{k_{l-1}k_l}\Delta_{k_lk_l} \qquad (83) \\ &= \Delta_{ii} + \Delta_{ii}\,\Pi^I_{ii}\,\Delta_{ii} + \Delta_{ii}\sum_{k_1=i,j}\Pi^I_{ik_1}\Delta_{k_1k_1}\Pi^I_{k_1i}\Delta_{ii} + \cdots\,, \end{aligned}$$

where $k_0 = k_{n+1} = i$, while for $l \neq n+1$, k_l can be i or j. Δ_{ii} is the Born propagator of the field i. We rewrite Eq.(83) as

$$\bar\Delta_{ii} = \Delta_{ii}\,\left[1 - (\Pi\,\Delta)_{ii}\right]^{-1}, \tag{84}$$

and refer to $\bar\Delta_{ii}$ as the resummed propagator. The quantity $(\Pi\,\Delta)_{ii}$ is the sum of all the possible products of Born propagators and self-energies, starting with a 1PI self-energy Π^I_{ii}, or transition Π^I_{ij}, and ending with a propagator Δ_{ii}, such that each element of the sum cannot be obtained as a product of other elements in the sum. A diagrammatic representation of $(\Pi\,\Delta)_{ii}$ is the following,

$(\Pi\,\Delta)_{ii}$ =

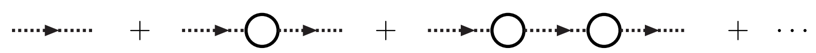


where the Born propagator of the field i (j) is represented by a dotted (solid) line, the white blob is the i 1PI self-energy, and the dots at the end indicate a sum running over an infinite number of 1PI j self-energies (black blobs) inserted between two 1PI i–j transitions (gray blobs).

It is also useful to define, as an auxiliary quantity, the *partially resummed* propagator for the field i, $\hat{\Delta}_{ii}$, in which we resum only the proper 1PI self-energy insertions Π^I_{ii}, namely,

$$\hat{\Delta}_{ii} = \Delta_{ii}\,\left[1 - \Pi^I_{ii}\,\Delta_{ii}\right]^{-1}. \tag{85}$$

If the particle i were not mixing with j through loops or two-leg vertex insertions, $\hat{\Delta}_{ii}$ would coincide with the resummed propagator $\bar{\Delta}_{ii}$. $\hat{\Delta}_{ii}$ can be graphically depicted as

$\hat{\Delta}_{ii} =$ 

Partially resummed propagators allow for a compact expression for $(\Pi\,\Delta)_{ii}$,

$$(\Pi\,\Delta)_{ii} = \Pi^I_{ii}\Delta_{ii} + \Pi^I_{ij}\hat{\Delta}_{jj}\Pi^I_{ji}\Delta_{ii}, \tag{86}$$

so that the resummed propagator of the field i can be cast in the form

$$\bar{\Delta}_{ii} = \Delta_{ii}\,\left[1 - \left(\Pi^I_{ii} + \Pi^I_{ij}\hat{\Delta}_{jj}\Pi^I_{ji}\right)\Delta_{ii}\right]^{-1}. \tag{87}$$

We can also define a resummed propagator for the i–j transition. In this case there is no corresponding Born propagator, and the resummed one is given by the sum of all possible products of 1PI i and j self-energies, transitions, and Born propagators starting with Δ_{ii} and ending with Δ_{jj}. This sum can be simply expressed in the following compact form,

$$\bar{\Delta}_{ij} = \bar{\Delta}_{ii}\,\Pi^I_{ij}\,\hat{\Delta}_{jj}. \tag{88}$$

5.1 The charged sector

We now apply Eqs. (85, 87, 88) to W and charged Goldstone boson fields. The *partially* resummed propagator of the charged Goldstone scalar follows immediately from Eq.(85). The Born W and ϕ propagators in the 't Hooft–Feynman gauge are

$$\Delta^{\mu\nu}_{WW} = \frac{\delta_{\mu\nu}}{p^2 + M^2}, \quad \Delta_{\phi\phi} = \frac{1}{p^2 + M^2}, \tag{89}$$

where, for simplicity of notation, we have dropped the coefficients $(2\pi)^4 i$. In the same gauge, the partially resummed ϕ and W propagators are

$$\hat{\Delta}_{\phi\phi} = \Delta_{\phi\phi}\,\left[1 - \Pi^I_{\phi\phi}\,\Delta_{\phi\phi}\right]^{-1} = \left[p^2 + M^2 - R^I_{\phi\phi}\right]^{-1} \tag{90}$$

$$\hat{\Delta}^{\mu\nu}_{WW} = \frac{1}{p^2 + M^2 - D^I_{WW}}\left(\delta_{\mu\nu} + \frac{p_\mu p_\nu P^I_{WW}}{p^2 + M^2 - D^I_{WW} - p^2 P^I_{WW}}\right). \tag{91}$$

Eq.(91) assumes a more compact form when expressed in terms of the transverse and longitudinal projectors $t_{\mu\nu}$ and $l_{\mu\nu}$,

$$\hat{\Delta}^{\mu\nu}_{WW} = \frac{t^{\mu\nu}}{p^2 + M^2 - D^I_{WW}} + \frac{l^{\mu\nu}}{p^2 + M^2 - L^I_{WW}}. \tag{92}$$

The resummed W and ϕ propagators can be then derived from Eq.(87),

$$\bar{\Delta}_{\phi\phi} = \left[p^2 + M^2 - R^I_{\phi\phi} - \frac{p^2\, M^2\, (G^I_{W\phi})^2}{p^2 + M^2 - L^I_{WW}}\right]^{-1} \tag{93}$$

$$\bar{\Delta}^{\mu\nu}_{WW} = \frac{t^{\mu\nu}}{p^2 + M^2 - D^I_{WW}} + l^{\mu\nu}\left[p^2 + M^2 - L^I_{WW} - \frac{p^2 M^2 (G^I_{W\phi})^2}{p^2 + M^2 - R^I_{\phi\phi}}\right]^{-1}. \tag{94}$$

The resummed propagator for the W–ϕ transition is provided by Eq.(88),

$$\bar{\Delta}^{\mu}_{W\phi} = \frac{-ip_\mu M G^I_{\phi W}}{p^2 + M^2 - R^I_{\phi\phi}} \left[p^2 + M^2 - L^I_{WW} - \frac{p^2 M^2 (G^I_{W\phi})^2}{p^2 + M^2 - R^I_{\phi\phi}}\right]^{-1}. \tag{95}$$

We will now show explicitly, up to terms of $\mathcal{O}(g^4)$, that the resummed propagators defined above satisfy the following WST identity:

$$p_\mu\, p_\nu\, \bar{\Delta}^{\mu\nu}_{WW} \; + i\, p_\mu\, M\, \bar{\Delta}^{\mu}_{W\phi} - i\, p_\nu\, M\, \bar{\Delta}^{\nu}_{\phi W} + M^2\, \bar{\Delta}_{\phi\phi} = 1\,, \tag{96}$$

which, in turn, is satisfied if

$$p^2 M^2 \left(G^I_{W\phi}\right)^2 + M^2 R^I_{\phi\phi} + p^2 L^I_{WW} - R^I_{\phi\phi} L^I_{WW} + 2p^2 M^2 G^I_{W\phi} = 0\,. \tag{97}$$

This equation can be verified explicitly, up to terms of $\mathcal{O}(g^4)$, using the WSTI for the W self-energy: at $\mathcal{O}(g^2)$ Eq.(97) becomes simply

$$M^2 R^{(1)}_{\phi\phi} + p^2 L^{(1)}_{WW} + 2p^2 M^2 G^{(1)}_{W\phi} = 0\,, \tag{98}$$

which coincides with Eq.(66) for $n = 1$. To prove Eq.(97) at $\mathcal{O}(g^4)$ we can combine the last of Eqs. (62) with $n = 2$ and Eq.(80) to get [1]

$$p^2 M^2 \left(G^{(1)}_{W\phi}\right)^2 + M^2 R^{(2)I}_{\phi\phi} + p^2 L^{(2)I}_{WW} - R^{(1)}_{\phi\phi} L^{(1)}_{WW} + 2p^2 M^2 G^{(2)I}_{W\phi} = 0\,. \tag{99}$$

5.2 The neutral sector

The SM neutral sector involves the mixing of three boson fields, A_μ, Z_μ and ϕ_0. As the definitions for the resummed propagators presented at the beginning of Section 5 refer to the mixing of only two boson fields, we will now discuss their generalization to the three-field case.

Consider three boson fields i, j and k mixing up through radiative corrections. For each of them we can define a *partially resummed* propagator $\hat{\Delta}_{ll}$ ($l = i, j,$ or k) according to Eq.(85). For each pair of the three fields, say (j, k), we can also define the following *intermediate* propagators

$$\tilde{\Delta}_{jj}(j,k) = \Delta_{jj} \left[1 - \left(\Pi^I_{jj} + \Pi^I_{jk} \hat{\Delta}_{kk} \Pi^I_{kj}\right) \Delta_{jj}\right]^{-1} \tag{100}$$

$$\tilde{\Delta}_{jk}(j,k) = \tilde{\Delta}_{jj}(j,k)\, \Pi^I_{jk}\, \hat{\Delta}_{kk}\,, \tag{101}$$

where the parentheses on the l.h.s. indicate the chosen pair of fields. [$\tilde{\Delta}_{kk}(j,k)$ and $\tilde{\Delta}_{kj}(j,k)$ can be simply derived from the above definitions by exchanging $j \leftrightarrow k$.] The reader will immediately note

[1] For simplicity of notation, in this section we dropped the coefficients $(2\pi)^4 i$.

that the r.h.s. of the above Eqs. (100, 101) are almost identical to those of Eqs. (87, 88), with the appropriate renaming of the fields. Equations (100, 101), introduced in the context of three-field mixing, define however only *intermediate* propagators (denoted by the tilde), while Eqs. (87, 88), presented in the analysis of the two-field mixing case, define the complete resummed propagators (denoted by the bar). Indeed, the definition of full resummed propagator in the three-field mixing scenario requires one further step: the resummed propagator for a field i mixing with the fields j and k via the functions Π^I_{ij}, Π^I_{ik} and Π^I_{jk} can be cast in the following form

$$\bar{\Delta}_{ii} = \Delta_{ii}\left[1 - \left(\Pi^I_{ii} + \sum_{l,m} \Pi^I_{il}\,\tilde{\Delta}_{lm}(j,k)\,\Pi^I_{mi}\right)\Delta_{ii}\right]^{-1}, \tag{102}$$

where l and m can be j or k, while the resummed propagator for the transition between the fields i and k is

$$\bar{\Delta}_{ik} = \bar{\Delta}_{ii} \sum_{l=j,k} \Pi^I_{il}\tilde{\Delta}_{lk}(j,k)\,. \tag{103}$$

Armed with Eqs. (100)–(103), we can now present the A_μ, Z_μ and A_μ–Z_μ propagators. First of all, the Born A_μ, Z_μ and ϕ_0 propagators in the 't Hooft–Feynman gauge are

$$\Delta^{\mu\nu}_{AA} = \frac{\delta_{\mu\nu}}{p^2}, \quad \Delta^{\mu\nu}_{ZZ} = \frac{\delta_{\mu\nu}}{p^2+M_0^2}, \quad \Delta_{\phi_0\phi_0} = \frac{1}{p^2+M_0^2}, \tag{104}$$

where, for simplicity of notation, we have dropped once again the coefficients $(2\pi)^4 i$. The *partially resummed* propagators (three) can be immediately computed via Eq.(85) and the *intermediate* ones (twelve) via Eqs. (100) and (101). Finally, after some algebra, Eqs. (102) and (103) provide us with the fully resummed propagators: $\bar{\Delta}_{VV} = t_{\mu\nu}\bar{\Delta}^T_{VV} + l_{\mu\nu}\bar{\Delta}^L_{VV}$, with $V = A, Z$ and

$$\bar{\Delta}^T_{AA} = \left[p^2 - D^I_{AA} - \frac{(D^I_{AZ})^2}{p^2+M_0^2-D^I_{ZZ}}\right]^{-1} \tag{105}$$

$$\bar{\Delta}^T_{ZZ} = \left[p^2 + M_0^2 - D^I_{ZZ} - \frac{(D^I_{AZ})^2}{p^2-D^I_{AA}}\right]^{-1} \tag{106}$$

$$\bar{\Delta}^T_{AZ} = D^I_{AZ}\left[\left(p^2-D^I_{AA}\right)\left(p^2+M_0^2-D^I_{ZZ}\right) - (D^I_{AZ})^2\right]^{-1}. \tag{107}$$

The expressions of the longitudinal components of these propagators are more lengthy and we will only present them up to terms of $\mathcal{O}(g^4)$:

$$\bar{\Delta}^L_{AA} = \left[p^2 + \mathcal{O}(g^6)\right]^{-1} \tag{108}$$

$$\bar{\Delta}^L_{ZZ} = \left[p^2 + M_0^2 - L^I_{ZZ} - \frac{(L^I_{AZ})^2}{p^2} - \frac{p^2 M_0^2 (G^I_{Z\phi_o})^2}{p^2+M_0^2} + \mathcal{O}(g^6)\right]^{-1} \tag{109}$$

$$\bar{\Delta}^L_{AZ} = \frac{L^I_{AZ}}{p^2\left(p^2+M_0^2-L^I_{ZZ}\right)} + \frac{M_0^2}{\left(p^2+M_0^2\right)^2} G^I_{A\phi_o} G^I_{Z\phi_o} + \mathcal{O}(g^6). \tag{110}$$

Equation (108) achieves its compact form due to the use of the WSTI (63) and (64) with $n = 1, 2$. Also Eq.(110) has been simplified using $L^{(1)}_{AA} = 0$ (i.e. Eq.(63) with $n = 1$). We point out that if we use the one-loop WSTI for the photon self-energy, Eq.(63), the transverse part of the resummed A–Z propagator becomes, up to terms of $\mathcal{O}(g^4)$,

$$\bar{\Delta}^T_{AZ} = D^I_{AZ}\left[p^2\left(1+P^I_{AA}\right)\left(p^2+M_0^2-D^I_{ZZ}\right)\right]^{-1}+\mathcal{O}(g^6), \tag{111}$$

thus showing a pole at $p^2=0$ if $D^I_{AZ}(p^2=0)$ were not vanishing because of the rediagonalization of the neutral sector.

In order to show explicitly, up to terms of $\mathcal{O}(g^4)$, that the above resummed propagators satisfy their WSTI, we also present the resummed propagators involving the neutral Higgs-Kibble scalar ϕ_0:

$$\bar{\Delta}^\mu_{A\phi_o} = -ip_\mu\frac{M_0}{p^2}\left[\frac{G^I_{Z\phi_o}L^I_{AZ}}{\left(p^2+M_0^2\right)^2}+\frac{G^I_{A\phi_o}}{p^2+M_0^2-R^I_{\phi_o\phi_o}}\right]+\mathcal{O}(g^6) \tag{112}$$

$$\bar{\Delta}^\mu_{Z\phi_o} = \frac{-ip_\mu M_0}{p^2+M_0^2-L^I_{ZZ}}\left[\frac{G^I_{A\phi_o}L^I_{AZ}}{p^2\left(p^2+M_0^2\right)}+\frac{G^I_{Z\phi_o}}{p^2+M_0^2-R^I_{\phi_o\phi_o}}\right]+\mathcal{O}(g^6) \tag{113}$$

$$\bar{\Delta}_{\phi_o\phi_o} = \left[p^2+M_0^2-R^I_{\phi_o\phi_o}-M_0^2\left(G^I_{A\phi_o}\right)^2-\frac{p^2M_0^2}{p^2+M_0^2}\left(G^I_{Z\phi_o}\right)^2\right]^{-1}+\mathcal{O}(g^6). \tag{114}$$

With these results, and with the WSTI (63)–(65), (Eq.(74)) and (Eq.(78)), we can finally prove, up to $\mathcal{O}(g^4)$, the following WSTI for the resummed A, Z and A–Z propagators,

$$p_\mu\,p_\nu\,\bar{\Delta}^{\mu\nu}_{AA} = 1 \tag{115}$$

$$p_\mu\,p_\nu\,\bar{\Delta}^{\mu\nu}_{AZ} + ip_\mu M_0\,\bar{\Delta}^\mu_{A\phi_o} = 0 \tag{116}$$

$$p_\mu\,p_\nu\,\bar{\Delta}^{\mu\nu}_{ZZ} + M_0^2\,\bar{\Delta}_{\phi_o\phi_o} + 2ip_\mu M_0\,\bar{\Delta}^\mu_{Z\phi_o} = 1\,. \tag{117}$$

6 The LQ basis

For the purpose of the renormalization, it is more convenient to extract from the quantities defined in the previous sections the factors involving the weak mixing angle θ. To achieve this goal, we employ the LQ basis [18], which relates the photon and Z fields to a new pair of fields, L and Q:

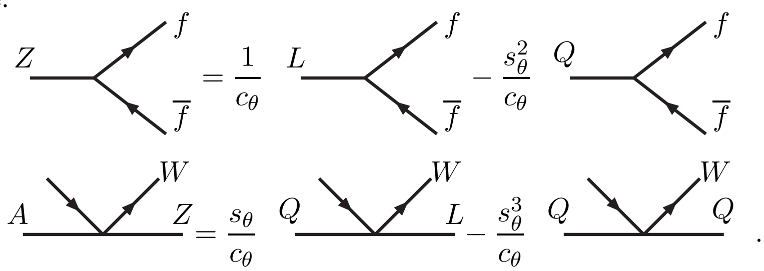
$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_\theta & 0 \\ s_\theta & 1/s_\theta \end{pmatrix}\begin{pmatrix} L_\mu \\ Q_\mu \end{pmatrix}. \tag{118}$$

Consider the fermion currents j^μ_A and j^μ_Z coupling to the photon and to the Z. As the Lagrangian must be left unchanged under this transformation, namely $j^\mu_Z\,Z_\mu + j^\mu_A\,A_\mu = j^\mu_L\,L_\mu + j^\mu_Q\,Q_\mu$, the currents transform as

$$\begin{pmatrix} j^\mu_Z \\ j^\mu_A \end{pmatrix} = \begin{pmatrix} 1/c_\theta & -s_\theta^2/c_\theta \\ 0 & s_\theta \end{pmatrix}\begin{pmatrix} j^\mu_L \\ j^\mu_Q \end{pmatrix}. \tag{119}$$

If we rewrite the SM Lagrangian in terms of the fields L and Q, and perform the same transformation (Eq.(118)) on the FP ghosts fields (from (X_A,X_Z) to (X_L,X_Q)), then all the interaction terms of the SM Lagrangian are independent of θ. Note that this is true only if the relation $M/c_\theta = M_0$ is employed, wherever necessary, to remove the remaining dependence on θ. In this way the dependence on the weak mixing angle is moved to the kinetic terms of the L and Q fields which, clearly, are not mass eigenstates.

The relevant fact for our discussion is that the couplings of Z, photon, X_Z and X_A are related to those of the fields L and Q, X_L and X_Q by identities like the ones described, in a diagrammatic way, in the following figure:



As the couplings of the fields L, Q, X_L and X_Q do not depend on θ, all the dependence on this parameter is factored out in the coefficients in the r.h.s. of these identities.

Since θ appears only in the couplings of the fields A, Z, X_A and X_Z (once again, the relation $M/c_\theta = M_0$ must also be employed, wherever necessary), it is possible to single out this parameter in the two-loop self-energies of the vector bosons. Consider, for example, the transverse part of the photon two-loop self-energy $D^{(2)}_{AA}$ (which includes the contribution of both irreducible and reducible diagrams). All diagrams contributing to $D^{(2)}_{AA}$ can be classified in two classes: those including (i) one internal A, Z, X_A or X_Z field, and (ii) those not containing any of these fields. The complete dependence on θ can be factored out by expressing the external photon couplings and the internal A, Z X_A or X_Z couplings of the diagrams of class (i) in terms of the couplings of the fields L, Q, X_L and X_Q, namely

$$D^{(2)}_{AA} = s^2_\theta \left[\frac{1}{c^2_\theta} f^{AA}_1 + f^{AA}_2 + s^2_\theta f^{AA}_3 \right], \tag{120}$$

where the functions f^{AA}_i $(i = 1, 2, 3)$ are θ-independent. Similarly, we can factor out the θ dependence of the transverse part of the two-loop photon–Z mixing and Z self-energy,

$$D^{(2)}_{AZ} = \frac{s_\theta}{c_\theta} \left[\frac{1}{c^2_\theta} f^{AZ}_1 + f^{AZ}_2 + s^2_\theta f^{AZ}_3 + s^4_\theta f^{AZ}_4 \right], \tag{121}$$

$$D^{(2)}_{ZZ} = \frac{1}{c^2_\theta} \left[\frac{1}{c^2_\theta} f^{ZZ}_1 + f^{ZZ}_2 + s^2_\theta f^{ZZ}_3 + s^4_\theta f^{ZZ}_4 + s^6_\theta f^{ZZ}_5 \right], \tag{122}$$

where, once again, the functions f^{AZ}_i and f^{ZZ}_i $(i = 1, \ldots, 5)$ do not depend on θ. Analogous relations hold for the longitudinal components of the two-loop self-energies.

We note that $D^{(2)}_{AZ}$ and $D^{(2)}_{ZZ}$ also contain a third class of diagrams containing more than one internal Z (or X_Z) field (up to three, in $D^{(2)}_{ZZ}$). However, the diagrams of this class involve the trilinear vertex ZHZ (or $\overline{X}_Z H X_Z$), which does not induce any new θ dependence.

However, from the point of view of renormalization it is more convenient to distinguish between the θ dependence originating from external legs and the one introduced by external legs. We define, to all orders,

$$D_{AA} = s^2_\theta \, \Pi_{QQ\,;\,\mathrm{ext}}\, p^2 = s^2_\theta \sum_{n=1}^{\infty} \left(\frac{g^2}{16\,\pi^2} \right)^n \Pi^{(n)}_{QQ\,;\,\mathrm{ext}}\, p^2,$$

$$D_{AZ} = \frac{s_\theta}{c_\theta} \, \Sigma_{AZ\,;\,\mathrm{ext}} = \frac{s_\theta}{c_\theta} \sum_{n=1}^{\infty} \left(\frac{g^2}{16\,\pi^2} \right)^n \Sigma^{(n)}_{AZ\,;\,\mathrm{ext}},$$

$$D_{ZZ} = \frac{1}{c^2_\theta} \, \Sigma_{ZZ\,;\,\mathrm{ext}} = \frac{1}{c^2_\theta} \sum_{n=1}^{\infty} \left(\frac{g^2}{16\,\pi^2} \right)^n \Sigma^{(n)}_{ZZ\,;\,\mathrm{ext}}, \tag{123}$$

$$\Sigma^{(n)}_{AZ\,;\,\rm ext} = \Sigma^{(n)}_{3Q\,;\,\rm ext} - s^2_\theta\,\Pi^{(n)}_{QQ\,;\,\rm ext}\,p^2, \quad \Sigma^{(n)}_{ZZ\,;\,\rm ext} = \Sigma^{(n)}_{33\,;\,\rm ext} - 2\,s^2_\theta\,\Sigma^{(n)}_{3Q\,;\,\rm ext} + s^4_\theta\,\Pi^{(n)}_{QQ\,;\,\rm ext}\,p^2. \tag{124}$$

Furthermore, our procedure is such that

$$\Sigma^{(n)}_{3Q\,;\,\rm ext} = \Pi^{(n)}_{3Q\,;\,\rm ext}\,p^2, \tag{125}$$

with $\Pi^{(n)}_{3Q\,;\,\rm ext}$ regular at $p^2 = 0$. At $\mathcal{O}\left(g^2\right)$ the external quantities are θ-independent while, at $\mathcal{O}\left(g^4\right)$ the relation with the coefficients of Eqs.(120)–(122) is

$$\begin{aligned}
\Pi^{(2)}_{QQ\,;\,\rm ext}\,p^2 &= \frac{1}{c^2_\theta}\,f^{AA}_1 + f^{AA}_2 + f^{AA}_3\,s^2_\theta, \\
\Sigma^{(2)}_{3Q\,;\,\rm ext} &= \frac{1}{c^2_\theta}\,(f^{AA}_1 + f^{AZ}_1) - f^{AA}_1 + f^{AZ}_2 + s^2_\theta\,(f^{AA}_2 + f^{AZ}_3) + s^4_\theta\,(f^{AA}_3 + f^{AZ}_4) \\
\Sigma^{(2)}_{33\,;\,\rm ext} &= \frac{1}{c^2_\theta}\,(f^{AA}_1 + 2\,f^{AZ}_1 + f^{ZZ}_1) - f^{AA}_1 - 2\,f^{AZ}_1 + f^{ZZ}_2 + s^2_\theta\,(-f^{AA}_1 + 2\,f^{AZ}_2 + f^{ZZ}_3) \\
&\quad + s^4_\theta\,(f^{AA}_2 + 2\,f^{AZ}_3 + f^{ZZ}_4) + s^6_\theta\,(f^{AA}_3 + 2\,f^{AZ}_4 + f^{ZZ}_5),
\end{aligned} \tag{126}$$

and s_θ, c_θ in Eq.(126) should be evaluated at $\mathcal{O}\left(g^0\right)$, in any renormalization scheme, for two-loop accuracy.

Consider the process $\overline{f}f \to \overline{h}h$; taking into account Dyson resummed propagators and neglecting, for the moment, vertices and boxes we write

$$\begin{aligned}
\mathcal{M}(\overline{f}f \to \overline{h}h) = -\,(2\pi)^4\,i\,\Big[&e^2\,Q_f Q_h\,\gamma^\mu \otimes \gamma^\mu\,\bar{\Delta}^T_{AA} + \frac{eg}{2\,c_\theta}\,Q_f\,\gamma^\mu \otimes \gamma^\mu\,(v_h + a_h\,\gamma_5)\,\bar{\Delta}^T_{ZA} \\
&+ \frac{eg}{2\,c_\theta}\,Q_h\gamma^\mu\,(v_f + a_f\,\gamma_5) \otimes \gamma^\mu\,\bar{\Delta}^T_{ZA} + \frac{g^2}{4c^2_\theta}\gamma^\mu(v_f + a_f\,\gamma_5) \otimes \gamma^\mu(v_h + a_h\gamma_5)\bar{\Delta}^T_{ZZ}\Big]
\end{aligned} \tag{127}$$

where f and h are fermions with quantum numbers Q_I, I_{3i}, $i = f, h$; furthermore we have introduced

$$v_f = I_{3f} - 2\,Q_f\,s^2_\theta, \qquad a_f = I_{3f}, \tag{128}$$

with $e^2 = g^2\,s^2_\theta$. Always neglecting terms proportional to fermion masses it is useful to introduce an effective weak-mixing angle as follows:

$$s^2_{\rm eff} = s^2_\theta\left[1 - \frac{\Pi_{AZ\,;\,\rm ext}}{1 - s^2_\theta\,\Pi_{AA\,;\,\rm ext}}\right], \quad V_f = I_{3f} - 2\,Q_f\,s^2_{\rm eff}. \tag{129}$$

The amplitude of Eq.(127) can be cast into the following form:

$$\begin{aligned}
\mathcal{M}(\overline{f}f \to \overline{h}h) = -\,(2\pi)^4\,i\,\Big[&\gamma^\mu \otimes \gamma^\mu\,\frac{1}{1 - s^2_\theta\,\Pi_{AA\,;\,\rm ext}}\,\frac{e^2\,Q_f Q_h}{p^2} \\
&\frac{g^2}{4\,c^2_\theta}\,\gamma^\mu\,(V_f + a_f\,\gamma_5) \otimes \gamma^\mu\,(V_h + a_h\,\gamma_5)\,\bar{\Delta}^T_{ZZ}\Big].
\end{aligned} \tag{130}$$

The functions $\Pi_{AA\,;\,\rm ext}, \Pi_{AZ\,;\,\rm ext}$ and $\Sigma_{ZZ\,;\,\rm ext}$ start at $\mathcal{O}\left(g^2\right)$ in perturbation theory. Equation (130) shows the nice effect of absorbing – to all orders – non-diagonal transitions into a redefinition of s^2_θ and forms the basis for introducing renormalization equations in the neutral sector, e.g. the one associated with the fine-structure constant α. Questions related to gauge-parameter independence of Dyson resummation, e.g. in Eq.(129), are not addressed here, but we will present a detailed discussion in Part III, where their relevance will be investigated.

7 Conclusions

In this paper we prepared the ground to perform a comprehensive renormalization procedure of the Standard Model at the two-loop level; with minor changes our results can be extended to an arbitrary gauge theory with spontaneously broken symmetry.

The same set of problems that we encountered in this paper may receive different answers; for instance, one could decide to work in the background-field method and treat differently the problem of diagonalization of the neutral sector in the SM. Our solution has been extended beyond one-loop and it is an integral part of a renormalization procedure which goes from fundamentals to applications. The whole set of *new* Feynman rules of our Appendices has been coded in $\mathcal{G}raph\mathcal{S}hot$ and has proven its value in several applications, including the proof of the WST identities.

In this paper we outlined peculiar aspects of tadpoles in a spontaneously broken gauge theory and extended beyond one-loop a strategy to diagonalize the neutral sector of the SM, order-by-order in perturbation theory. The obtained results have been used as the starting point in the construction of the renormalized Lagrangian of the SM and in the computation of (pseudo-)observables up to two loops.

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A Appendix: Feynman rules for β_h vertices

In this appendix we present the new set of diagrammatic rules induced by our approach. The Feynman rules for the β_h vertices are extremely simple and can be immediately derived from Eq.(7):

H	——•		$-\,2M\beta_h/g$
H	——•——	H	$-\,\beta_h$
ϕ_0	——•——	ϕ_0	$-\,\beta_h$
ϕ_+	——•——	ϕ_-	$-\,\beta_h,$

where $\beta_h = \beta_{h_1} g^2 + \beta_{h_2} g^4 + \cdots$ and M is the bare W mass. If working with the rediagonalized neutral sector, simply replace g by $\bar{g}$. Multiply each vertex by a factor $(2\pi)^4 i$. As usual, we have included the combinatorial factors for identical fields (see Appendix D of ref. [19]).

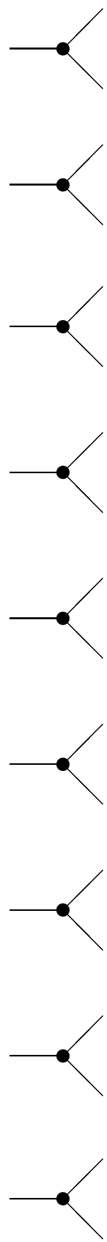
B Appendix: Feynman rules for β_t vertices

In this appendix we present the β_t vertices. They can be read off the Lagrangian terms of Eqs. (23), (24), (30) and (31), including the combinatorial factors for identical fields. Also, $\beta_t = \beta_{t_1} g^2 + \beta_{t_2} g^4 + \cdots$. Simply replace g by $\bar{g}$ if working with the rediagonalized neutral sector. The two-leg β_t vertices are:

H	——•——	H	$-\left(3M_H^2/2\right)\beta_t\left(\beta_t+2\right)$
ϕ_0	——•——	ϕ_0	$-\left(M_H^2/2\right)\beta_t\left(\beta_t+2\right)$
ϕ_+	——•——	ϕ_-	$-\left(M_H^2/2\right)\beta_t\left(\beta_t+2\right)$
Z_μ	——•——	Z_ν	$-M_0^2\beta_t\left(\beta_t+2\right)\,\delta_{\mu\nu}$
W_μ^+	——•——	W_ν^-	$-M^2\beta_t\left(\beta_t+2\right)\,\delta_{\mu\nu}$
Z_μ	——•—— $\leftarrow p$	ϕ_0	$ip_\mu M_0\beta_t$
W_μ^+	——•—— $\leftarrow p$	ϕ_-	$ip_\mu M\beta_t$
W_μ^-	——•—— $\leftarrow p$	ϕ_+	$ip_\mu M\beta_t$
$\bar{f}$	——•——	f	$-m_f\beta_t$
$\overline{X}^+$	——•——	X^+	$-\xi_W M^2\beta_t$
$\overline{X}^-$	——•——	X^-	$-\xi_W M^2\beta_t$
$\overline{X}_Z$	——•——	X_Z	$-\xi_Z M_0^2\beta_t$

where $M_0 = M/\cos_\theta$ and $\beta_t\left(\beta_t+2\right) = 2\beta_{t_1} g^2 + \left(\beta_{t_1}^2 + 2\beta_{t_2}\right) g^4 + O\left(g^6\right)$. Each vertex must be multiplied by the usual factor $(2\pi)^4 i$.

– The three-leg β_t vertices are:



H	H, H	$-g\beta_t\left(3M_{_H}^2/2M\right)$
H	ϕ_0, ϕ_0	$-g\beta_t\left(M_{_H}^2/2M\right)$
H	ϕ_+, ϕ_-	$-g\beta_t\left(M_{_H}^2/2M\right)$
A_μ	W_ν^+, ϕ_-	$+ig\beta_t s_\theta M\,\delta_{\mu\nu}$
A_μ	W_ν^-, ϕ_+	$-ig\beta_t s_\theta M\,\delta_{\mu\nu}$
Z_μ	W_ν^+, ϕ_-	$-ig\beta_t s_\theta^2 M_0\,\delta_{\mu\nu}$
Z_μ	W_ν^-, ϕ_+	$+ig\beta_t s_\theta^2 M_0\,\delta_{\mu\nu}$
H	W_μ^+, W_ν^-	$-g\beta_t M\,\delta_{\mu\nu}$
H	Z_μ, Z_ν	$-g\beta_t\left(M/c_\theta^2\right)\delta_{\mu\nu}$

where $s_\theta = \sin\theta$, $c_\theta = \cos\theta$ and, once again, each vertex must be multiplied by the factor $(2\pi)^4 i$. The β_t tadpole H ——● is:

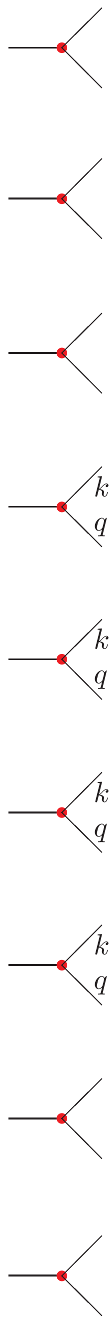
$$(2\pi)^4 i\left(M_{_H}^2 M\right)\left[-\frac{1}{g}\beta_t\left(\beta_t+1\right)\left(\beta_t+2\right)\right] =$$
$$(2\pi)^4 i\left(M_{_H}^2 M\right)\left[-2\beta_{t_1} g - \left(3\beta_{t_1}^2 + 2\beta_{t_2}\right) g^3 + O\left(g^5\right)\right].$$

C Appendix: Feynman rules for Γ vertices

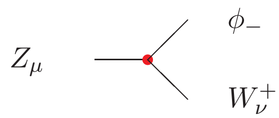
In this appendix we present the Γ vertices. The new Γ vertices introduced by the replacement $g \to \bar{g}(1+\Gamma)$ in the SM Lagrangian are listed here up to terms of $\mathcal{O}(\bar{g}^4)$ in the R_ξ gauges. All primes and bars over A_μ, Z_μ, M, M_H and θ have been dropped, except over $\bar{g}$. Also, $\Gamma = \Gamma_1\,\bar{g}^2 + \Gamma_2\,\bar{g}^4 + \cdots$. As usual, each vertex must be multiplied by the factor $(2\pi)^4 i$. The following two-leg Γ vertices are in the β_t scheme. For the β_h scheme, just set $\beta_t = 0$.

A_μ		A_ν	$-\delta_{\mu\nu}[\,\bar{g}^4 s_\theta^2 M^2 \Gamma_1^2]$
Z_μ		Z_ν	$-2\delta_{\mu\nu}[\,\bar{g}^2 M^2 \Gamma_1 + \bar{g}^4 M^2 (\Gamma_2 + 2\Gamma_1 \beta_{t_1} + c_\theta^2 \Gamma_1^2/2)]$
A_μ		Z_ν	$-\delta_{\mu\nu}(s_\theta/c_\theta)[\,\bar{g}^2 M^2 \Gamma_1 + \bar{g}^4 M^2 (\Gamma_2 + 2\Gamma_1 \beta_{t_1} + c_\theta^2 \Gamma_1^2)]$
W_μ^+		W_ν^-	$-2\delta_{\mu\nu}[\,\bar{g}^2 M^2 \Gamma_1 + \bar{g}^4 M^2 (\Gamma_2 + 2\Gamma_1 \beta_{t_1} + \Gamma_1^2/2)]$
A_μ	$\leftarrow p$	ϕ_0	$i p_\mu s_\theta M[\bar{g}^2 \Gamma_1 + \bar{g}^4 (\Gamma_2 + \Gamma_1 \beta_{t_1})]$
Z_μ	$\leftarrow p$	ϕ_0	$i p_\mu c_\theta M[\bar{g}^2 \Gamma_1 + \bar{g}^4 (\Gamma_2 + \Gamma_1 \beta_{t_1})]$
W_μ^+	$\leftarrow p$	ϕ_-	$i p_\mu M[\bar{g}^2 \Gamma_1 + \bar{g}^4 (\Gamma_2 + \Gamma_1 \beta_{t_1})]$
W_μ^-	$\leftarrow p$	ϕ_+	$i p_\mu M[\bar{g}^2 \Gamma_1 + \bar{g}^4 (\Gamma_2 + \Gamma_1 \beta_{t_1})]$
$\overline{X}^+$		X^+	$-\xi_W M^2[\bar{g}^2 \Gamma_1 + \bar{g}^4 (\Gamma_2 + \Gamma_1 \beta_{t_1})]$
$\overline{X}^-$		X^-	$-\xi_W M^2[\bar{g}^2 \Gamma_1 + \bar{g}^4 (\Gamma_2 + \Gamma_1 \beta_{t_1})]$
$\overline{X}_Z$		X_Z	$-\xi_Z M^2[\bar{g}^2 \Gamma_1 + \bar{g}^4 (\Gamma_2 + \Gamma_1 \beta_{t_1})]$
$\overline{X}_Z$		X_A	$-\xi_Z (s_\theta/c_\theta) M^2[\bar{g}^2 \Gamma_1 + \bar{g}^4 (\Gamma_2 + \Gamma_1 \beta_{t_1})]$

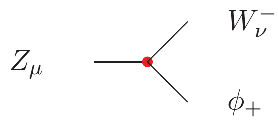
– The three-leg Γ vertices are (all momenta are flowing inwards):



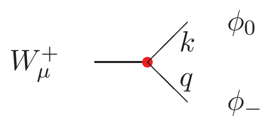
Leg	Legs (momenta)	Vertex
H	Z_μ, Z_ν	$\bar{g}^3\Gamma_1\,[-2M\delta_{\mu\nu}]$
H	A_μ, Z_ν	$\bar{g}^3\Gamma_1\,[-(s_\theta/c_\theta)M\delta_{\mu\nu}]$
H	W^+_μ, W^-_ν	$\bar{g}^3\Gamma_1\,[-2M\delta_{\mu\nu}]$
A_μ	H (k), ϕ_0 (q)	$\bar{g}^3\Gamma_1\,(is_\theta/2)(q_\mu - k_\mu)$
A_μ	ϕ_- (k), ϕ_+ (q)	$\bar{g}^3\Gamma_1\,(s_\theta/2)(q_\mu - k_\mu)$
Z_μ	H (k), ϕ_0 (q)	$\bar{g}^3\Gamma_1\,(ic_\theta/2)(q_\mu - k_\mu)$
Z_μ	ϕ_- (k), ϕ_+ (q)	$\bar{g}^3\Gamma_1\,(c_\theta/2)(q_\mu - k_\mu)$
A_μ	ϕ_-, W^+_ν	$\bar{g}^3\Gamma_1\,[is_\theta M\delta_{\mu\nu}]$
A_μ	W^-_ν, ϕ_+	$\bar{g}^3\Gamma_1\,[-is_\theta M\delta_{\mu\nu}]$



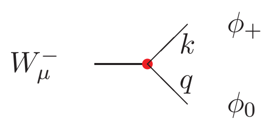
$$\bar{g}^3\Gamma_1\,(-is^2_\theta M/c)\,\delta_{\mu\nu}$$



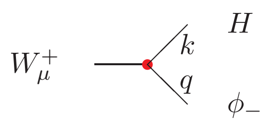
$$\bar{g}^3\Gamma_1\,(is^2_\theta M/c)\,\delta_{\mu\nu}$$



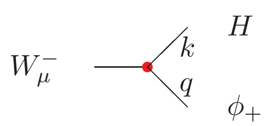
$$\bar{g}^3\Gamma_1\,(q_\mu - k_\mu)/2$$



$$\bar{g}^3\Gamma_1\,(q_\mu - k_\mu)/2$$

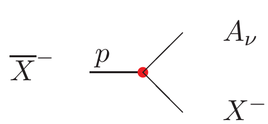


$$\bar{g}^3\Gamma_1\,i(q_\mu - k_\mu)/2$$

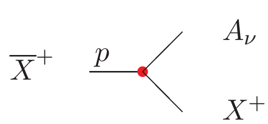


$$\bar{g}^3\Gamma_1\,i(q_\mu - k_\mu)/2$$

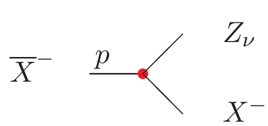
– The trilinear Γ vertices with FP ghosts are:



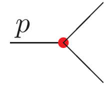
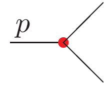
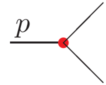
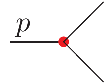
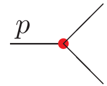
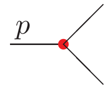
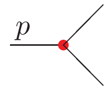
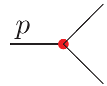
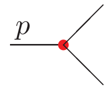
$$\bar{g}^3\Gamma_1\,s_\theta p_\nu/\xi_W$$

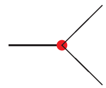


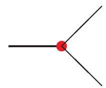
$$\bar{g}^3\Gamma_1\,(-s_\theta p_\nu/\xi_W)$$

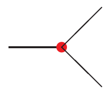


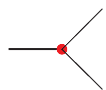
$$\bar{g}^3\Gamma_1\,c_\theta p_\nu/\xi_W$$

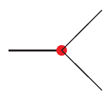
$\overline{X}^+$		Z_ν X^+	$\bar{g}^3\Gamma_1\,(-c_\theta p_\nu/\xi_W)$
$\overline{X}^-$		W^-_ν X_Z	$\bar{g}^3\Gamma_1\,(-c_\theta p_\nu/\xi_W)$
$\overline{X}^-$		W^-_ν X_A	$\bar{g}^3\Gamma_1\,(-s_\theta p_\nu/\xi_W)$
$\overline{X}_Z$		W^+_ν X^-	$\bar{g}^3\Gamma_1\,(-c_\theta p_\nu/\xi_Z)$
$\overline{X}_A$		W^+_ν X^-	$\bar{g}^3\Gamma_1\,(-s_\theta p_\nu/\xi_A)$
$\overline{X}_Z$		W^-_ν X^+	$\bar{g}^3\Gamma_1\,c_\theta p_\nu/\xi_Z$
$\overline{X}^+$		W^+_ν X_Z	$\bar{g}^3\Gamma_1\,c_\theta p_\nu/\xi_W$
$\overline{X}_A$		W^-_ν X^+	$\bar{g}^3\Gamma_1\,s_\theta p_\nu/\xi_A$
$\overline{X}^+$		W^+_ν X_A	$\bar{g}^3\Gamma_1\,s_\theta p_\nu/\xi_W$

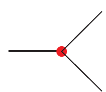
$\overline{X}^{+}$  ϕ_0, X^{+} $\quad \bar{g}^3\Gamma_1\,(iM\xi_W/2)$

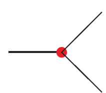
$\overline{X}^{-}$  ϕ_0, X^{-} $\quad \bar{g}^3\Gamma_1\,(-iM\xi_W/2)$

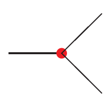
$\overline{X}^{+}$  H, X^{+} $\quad \bar{g}^3\Gamma_1\,(-M\xi_W/2)$

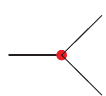
$\overline{X}^{-}$  H, X^{-} $\quad \bar{g}^3\Gamma_1\,(-M\xi_W/2)$

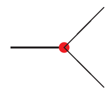
$\overline{X}_Z$  ϕ_+, X^{-} $\quad \bar{g}^3\Gamma_1\,(iM\xi_Z/2c_\theta)$

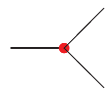
$\overline{X}_Z$  ϕ_-, X^{+} $\quad \bar{g}^3\Gamma_1\,(-iM\xi_Z/2c_\theta)$

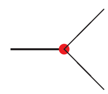
$\overline{X}_Z$  H, X_A $\quad \bar{g}^3\Gamma_1\,(-s_\theta M\xi_Z/2c_\theta)$

$\overline{X}_Z$  H, X_Z $\quad \bar{g}^3\Gamma_1\,(-M\xi_Z/2)$

$\overline{X}^{-}$  ϕ_-, X_Z $\quad \bar{g}^3\Gamma_1\,(ic_\theta M\xi_W/2)$

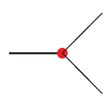
$\overline{X}^-$  ϕ_-, X_A $\quad \bar{g}^3\Gamma_1\,(is_\theta M\xi_W/2)$

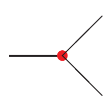
$\overline{X}^+$  ϕ_+, X_Z $\quad \bar{g}^3\Gamma_1\,(-ic_\theta M\xi_W/2)$

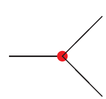
$\overline{X}^+$  ϕ_+, X_A $\quad \bar{g}^3\Gamma_1\,(-is_\theta M\xi_W/2)$

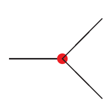
The three-leg Γ vertices introduced by the pure Yang–Mills Lagrangian are not listed here as they can be immediately derived from the usual Yang–Mills vertices (see, e.g., the appendix D of ref. [19]) by simply replacing $g \to \bar{g}\Gamma$.

– The trilinear Γ vertices with fermions are:

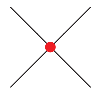
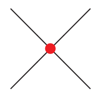
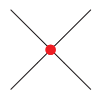
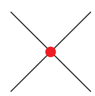
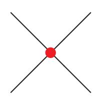
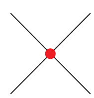
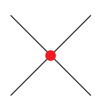
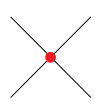
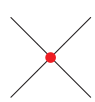
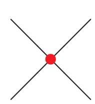
A_μ  $\bar{f}$, f $\quad \bar{g}^3\Gamma_1\,(is_\theta I_3/2)\,\gamma_\mu(1+\gamma_5)$

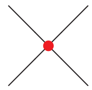
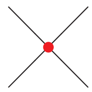
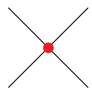
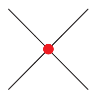
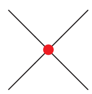
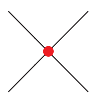
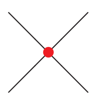
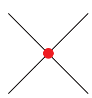
Z_μ  $\bar{f}$, f $\quad \bar{g}^3\Gamma_1\,(ic_\theta I_3/2)\,\gamma_\mu(1+\gamma_5)$

W^+_μ  $\bar{u}$, d $\quad \bar{g}^3\Gamma_1\,(i/2\sqrt{2})\,\gamma_\mu(1+\gamma_5)$

W^-_μ  $\bar{d}$, u $\quad \bar{g}^3\Gamma_1\,(i/2\sqrt{2})\,\gamma_\mu(1+\gamma_5)$

– The four-leg Γ vertices are:

H H		Z_μ Z_ν	$-\,\bar{g}^4\Gamma_1\,\delta_{\mu\nu}$
H H		A_μ Z_ν	$\bar{g}^4\Gamma_1\,(-s_\theta/2c_\theta)\,\delta_{\mu\nu}$
H H		W^+_μ W^-_ν	$-\,\bar{g}^4\Gamma_1\,\delta_{\mu\nu}$
ϕ_0 ϕ_0		Z_μ Z_ν	$-\,\bar{g}^4\Gamma_1\,\delta_{\mu\nu}$
ϕ_0 ϕ_0		A_μ Z_ν	$\bar{g}^4\Gamma_1\,(-s_\theta/2c_\theta)\,\delta_{\mu\nu}$
ϕ_0 ϕ_0		W^+_μ W^-_ν	$-\,\bar{g}^4\Gamma_1\,\delta_{\mu\nu}$
ϕ_+ ϕ_-		A_μ A_ν	$\bar{g}^4\Gamma_1\,(-2s^2_\theta)\,\delta_{\mu\nu}$
ϕ_+ ϕ_-		Z_μ Z_ν	$\bar{g}^4\Gamma_1\,(1-2c^2_\theta)\,\delta_{\mu\nu}$
ϕ_+ ϕ_-		A_μ Z_ν	$\bar{g}^4\Gamma_1\,(s_\theta/2c_\theta-2s_\theta c_\theta)\,\delta_{\mu\nu}$
ϕ_+ ϕ_-		W^+_μ W^-_ν	$-\,\bar{g}^4\Gamma_1\,\delta_{\mu\nu}$

A_μ, ϕ_0		ϕ_+, W^-_ν	$\bar{g}^4\Gamma_1\,(s_\theta/2)\,\delta_{\mu\nu}$
A_μ, ϕ_0		ϕ_-, W^+_ν	$\bar{g}^4\Gamma_1\,(s_\theta/2)\,\delta_{\mu\nu}$
A_μ, H		ϕ_+, W^-_ν	$\bar{g}^4\Gamma_1\,(-is_\theta/2)\,\delta_{\mu\nu}$
A_μ, H		ϕ_-, W^+_ν	$\bar{g}^4\Gamma_1\,(is_\theta/2)\,\delta_{\mu\nu}$
Z_μ, ϕ_0		ϕ_+, W^-_ν	$\bar{g}^4\Gamma_1\,(-s^2_\theta/2c_\theta)\,\delta_{\mu\nu}$
Z_μ, ϕ_0		ϕ_-, W^+_ν	$\bar{g}^4\Gamma_1\,(-s^2_\theta/2c_\theta)\,\delta_{\mu\nu}$
Z_μ, H		ϕ_+, W^-_ν	$\bar{g}^4\Gamma_1\,(is^2_\theta/2c_\theta)\,\delta_{\mu\nu}$
Z_μ, H		ϕ_-, W^+_ν	$\bar{g}^4\Gamma_1\,(-is^2_\theta/2c_\theta)\,\delta_{\mu\nu}$

The four-leg Γ vertices introduced by the pure Yang–Mills Lagrangian are not listed here as they can be immediately derived from the usual Yang–Mills vertices (see, e.g., the Appendix D of ref. [19]) by simply replacing $g^2 \to \bar{g}^2\Gamma(2+\Gamma)$.

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