

Selection, Representation, and Execution in Sparse Fourier Neural Operators

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Abstract

Sparse representations are often expected to make models smaller and also reduce inference cost. For Fourier Neural Operators (FNOs), these objectives are not equivalent or do not always align: removing parts of the learned operator can leave the underlying transforms and dense computations unchanged, while changing the grid on which the model is evaluated can introduce overhead of its own. We therefore distinguish sparsity in the representation, in the stored parameters, in the theoretical operation count, and in measured runtime, and present an empirical study of several routes toward sparse FNOs that tests each transition between them separately. Coarsening the execution grid reduces the theoretical cost without reducing measured latency, and adding a correction term recovers accuracy at the cost of making the model slower. Even an 83% parameter reduction remains slower than the dense baseline under ordinary execution. These results motivate a stricter definition of useful sparsity: the deployed operator must preserve solution accuracy and map its reduced support to a genuinely cheaper execution path.

1 Introduction

Neural operators learn mappings between function spaces and provide a natural setting for representations that are not tied to a fixed discretization [Kovachki et al., 2023]. The Fourier neural operator (FNO) is a popular example: its learned interactions are represented in Fourier space, which makes spectral truncation, mode selection, and structured parameter reduction natural compression strategies [Li et al., 2021]. Wavelet operators provide a complementary representation that can capture localized scales and interfaces [Tripura and Chakraborty, 2023]. This motivates a simple question: can a dictionary containing both Fourier and wavelet components be used to identify which parts of an operator are actually needed?

We use such a Fourier–wavelet dictionary as a test-bed for support selection, rather than as a proposed replacement for the FNO. The purpose is to study whether different selection criteria identify removable operator groups, and whether this type of sparsity leads to reductions in model size or computation. A richer representation is therefore not assumed to be either more accurate or more efficient.

This distinction matters because different notions of sparsity need not lead to the same computational outcome. A model can be sparse in its basis representation while still evaluating a full transform. It can contain fewer stored parameters while relying on contractions that are slower than dense matrix multiplication. Likewise, a lower analytical FLOP count does not need to result in

lower runtime once restriction, interpolation, synchronization, or kernel-launch overhead is included. For FNOs, this issue is particularly relevant because Fourier transforms, inverse transforms, and pointwise channel mixing remain part of the computation even when only a subset of spectral coefficients is retained.

We study these effects through a sequence of controlled compression experiments for FNO-like operators. A sparsity mechanism is evaluated at three levels:

1. **Selection:** whether operator groups can be removed without substantially degrading the learned solution map;
2. **Representation:** whether this removal reduces stored parameters or an analytical operation count; and
3. **Execution:** whether the resulting model is faster in measured hardware execution at comparable accuracy.

Within a fixed Fourier–wavelet dictionary, we first compare support-selection rules based on their effect on the solution loss, parameter magnitude, and random removal. At tight budgets, loss-based selection gives lower mean error than the parameter-norm and random controls on both heterogeneous Darcy flow and one-dimensional Burgers flow. The result indicates that parameter magnitude alone is not always a reliable measure of the importance of an operator group.

The mixed representation itself does not outperform a conventional FNO. We therefore use the mixed model only to study selection within a heterogeneous representation, while the conventional FNO remains the main architectural reference throughout the paper. A spatial breakdown of the Darcy error refines this comparison: the FNO’s advantage is concentrated in the smooth bulk region, while near permeability interfaces a functionally selected mixed support is competitive with the independent FNO.

We also examine whether reductions obtained at the representation level carry over to actual execution. Neither resolution coupling nor a large reduction in stored parameters produces a faster model in our measurements, and a fine-scale residual recovers accuracy at the cost of the intended saving. In each case the reduction is real at the level it is claimed, but does not survive the transition to measured execution.

Throughout these experiments, we observed the same issue repeatedly: nominal sparsity does not guarantee computational sparsity. Reducing coefficients, parameters, or analytical operations is useful only when the resulting structure can be mapped to an execution path that is cheaper on the target hardware. This makes the implementation of the sparse operator, including its transforms, contractions, and kernels, part of the compression problem itself.

Contributions. The main contributions of this paper are:

1. a distinction between basis sparsity, parameter sparsity, analytical compute reduction, and measured computational sparsity;
2. a controlled comparison of loss-based, parameter-norm, and random support selection in a heterogeneous Fourier–wavelet operator;
3. a comparison with a conventional FNO that separates the support selection question from the architectural comparison, including a regional breakdown showing that the FNO’s advantage is concentrated in the bulk region rather than at permeability interfaces;

4. two resolution-based compression experiments, including a fine-scale residual correction, evaluated with synchronized hardware measurements;
5. an executable transformed-channel block prototype demonstrating that substantial parameter reduction need not reduce runtime without an appropriate sparse execution path; and
6. a reproducible experimental protocol that reports both successful and unsuccessful compression strategies.

2 Background and related work

In this section, we first review operator learning and the representation choices available to an FNO, then turn to sparsity and compression, where we introduce the four levels of sparsity used throughout the paper.

2.1 Neural operators and representation choice

An operator-learning model approximates a map such as

$$\mathcal{G}^\dagger : a \mapsto u, \quad (1)$$

where a may be a coefficient field or an initial condition and u is the corresponding PDE solution. Unlike a conventional finite-dimensional network, the intended object is a map between function spaces; in practice, the map is discretized on a grid. Neural operators and FNOs make this viewpoint explicit [Kovachki et al., 2023, Li et al., 2021], while DeepONet provides another important operator-learning construction [Lu et al., 2021].

For an input field v , a schematic FNO layer is

$$v \mapsto \sigma\left(\mathcal{F}^{-1}\left[\widehat{W}(k)\mathcal{F}(v)(k)\right] + W_{\text{loc}}v\right), \quad (2)$$

where $\mathcal{F}$ is the spatial Fourier transform, $\widehat{W}(k)$ is a learned channel-mixing matrix, and W_{loc} is a pointwise mixing term. Mode truncation reduces the number of retained spectral coefficients, but the complete FFT, inverse FFT, and pointwise paths may still execute.

Wavelet neural operators use localized multiscale representations and are therefore a natural complement to Fourier operators, whose modes are global. The wavelet literature provides the mathematical motivation for scale and orientation subbands [Daubechies, 1992, Mallat, 1999], while the WNO formulation applies that structure to operator learning [Tripura and Chakraborty, 2023]. Adaptive-basis methods and mixtures of neural operators provide related approaches to representation choice [Zhao and Aviles-Rivero, 2026, Kratsios et al., 2024].

2.2 Sparsity and compression

Classical parameter sparsity removes individual weights, filters, or blocks. Lasso and group-lasso penalties are standard tools for shrinkage and group selection [Tibshirani, 1996, Yuan and Lin, 2006], while pruning and compression methods distinguish between the support identified during training and the structure of the final exported network [Han et al., 2016, Molchanov et al., 2017]. Hard-thresholding and greedy methods make this selection step explicit [Blumensath and Davies, 2009, Binev et al., 2011].

In this work, we distinguish four forms of sparsity:

Basis sparsity: fewer active Fourier shells or wavelet subbands.

Parameter sparsity: fewer stored trainable scalars.

Analytical sparsity: a lower theoretical FLOP count or operation proxy.

Computational sparsity: lower measured memory use or wall-clock latency after inactive structure has been removed from execution.

A reduction at one level does not necessarily lead to a reduction at the next one. In the experiments below, we use these distinctions to separate support selection from reductions in model size and analytical cost, and from actual gains in execution.

Recent work on neural-operator compression shows why these distinctions matter. DINOZAUR replaces the dense FNO spectral multiplier with a diffusion-based parametrization that substantially reduces the parameter count and memory footprint. Its profiling also shows that factorized or gradient-augmented variants can remain slower because FFTs and additional tensor operations still account for much of the execution cost [Matveev et al., 2025]. Operator boosting constructs stacked FNO, DeepONet, and CNO surrogates with 72–95% fewer trainable parameters and mainly reports improvements in the accuracy–parameter trade-off [Shikhman, 2026]. The spectrally-sparsified FNO (SS-FNO) augments each spectral layer with a lightweight diagonal gating selector whose implicit bias under stochastic gradient descent drives many frequency weights toward zero, and reports reduced active-mode counts together with lower memory footprint and computation cost [Bacsá et al., 2025]. λ -FNO likewise uses a learned pruning matrix, and reports little change in inference time in its transfer-learning experiments [Xu et al., 2025]. A particularly close systems-level observation is reported for variable-spiking wavelet neural operators. Despite substantial spike sparsity, VS-WNO remains slower and more energy intensive than its dense WNO counterpart on a Jetson GPU because the deployed execution path continues to perform dense work [Yoo et al., 2026].

These studies measure different aspects of compression. DINOZAUR includes execution profiling, while the remaining methods report reductions in parameters or active modes and, in some cases, aggregate runtime or memory figures. Here we focus on a more specific question: for a fixed exported model, does the proposed reduction lower single-sample inference latency under ordinary execution? Parameter count, analytical cost, and measured runtime are related, but they are not equivalent. More generally, hardware-aware sparsity results show that commodity dense hardware benefits most from regular block or tile structure rather than arbitrary zero patterns [Guo et al., 2020]. We therefore test directly whether the reductions considered here lead to measurable speedups for FNO-like operators.

3 PDE benchmarks

We use two standard benchmarks from the FNO literature, chosen so that the two representations in the mixed dictionary are tested against different kinds of structure. Darcy flow provides a two-dimensional problem with sharp coefficient contrasts and localized solution features, while Burgers flow provides a one-dimensional problem with nonlinear transport and gradients that evolve in time. Both use the public benchmark release of Li et al. [2021].

3.1 Heterogeneous Darcy flow

Darcy flow models steady flow through a porous medium:

$$-\nabla \cdot (a(x) \nabla u(x)) = f(x), \quad x \in (0, 1)^2, \quad u|_{\partial\Omega} = 0. \quad (3)$$

Here $a(x) > 0$ is the heterogeneous permeability or diffusion coefficient, f is a forcing field, and u is the resulting pressure-like solution. The coefficient field is piecewise constant with a sharp contrast between two values, so the solution is smooth over most of the domain but develops localized gradients where the coefficient jumps. This makes Darcy a useful test of whether a representation preserves both global structure and fine-scale features. We use the data distribution and public benchmark release introduced with the FNO study [Li et al., 2021]: fields are generated at high resolution and subsampled to 141×141 for the experiments. Figure 1 (top) shows a representative coefficient field together with the corresponding solution.

3.2 One-dimensional viscous Burgers flow

The periodic one-dimensional Burgers equation is

$$\partial_t u(x, t) + u(x, t) \partial_x u(x, t) = \nu \partial_{xx} u(x, t), \quad x \in (0, 1), \quad t \in (0, 1], \quad (4)$$

with a random periodic initial condition $u(x, 0) = u_0(x)$. We use the FNO benchmark release [Li et al., 2021] at $\nu = 0.01$ and resolution 256. The nonlinear transport term produces evolving gradients across different spatial scales, making Burgers a complementary test to Darcy; Figure 1 (bottom) shows a representative initial condition and the solution at $t = 1$.

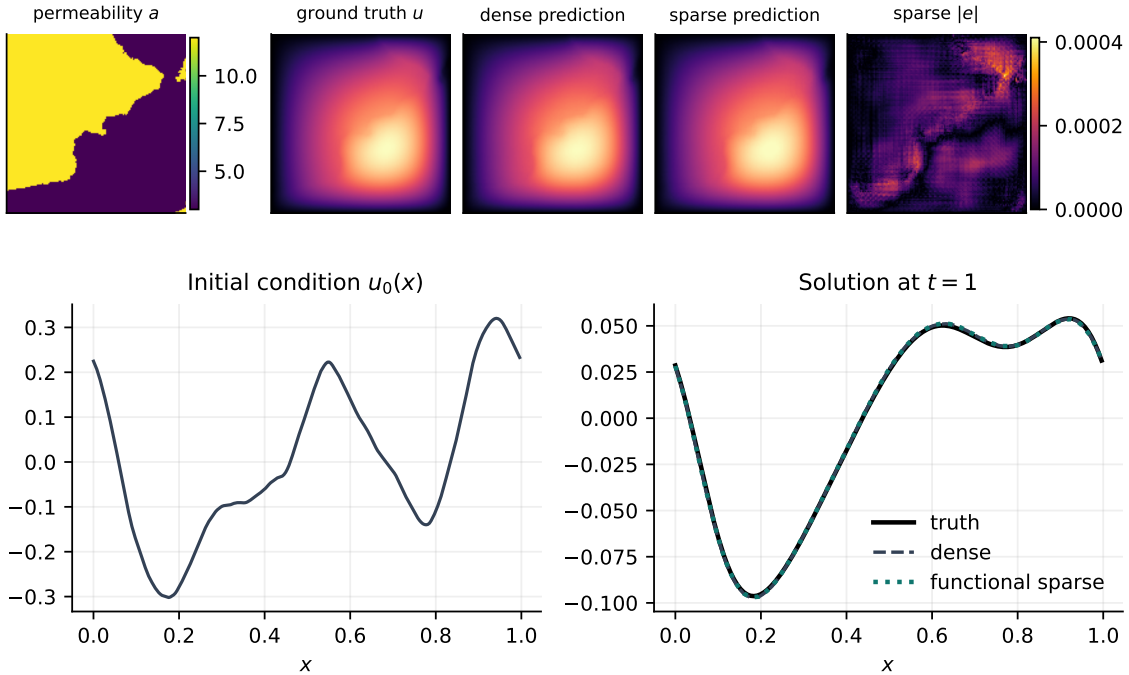


Figure 1: Representative held-out fields from the two primary benchmarks. Darcy (top) contains heterogeneous coefficient structure and localized solution features; Burgers (bottom) contains nonlinear transport and evolving gradients. The dense and sparse predictions shown for reference are from the mixed dictionary of Section 4 and its functionally selected support; both are visually indistinguishable from the ground truth at this scale.

4 Methods

This section describes the models and procedures used in the experiments. We first define the mixed Fourier–wavelet dictionary that serves as the test-bed for support selection, together with the structured groups that act as the unit of removal. We then give the functional selection rule and the parameter-norm and random controls against which it is compared. The remaining subsections describe the three execution routes: resolution coupling, a coarse model with a fine-scale residual, and the transformed-channel block prototype.

4.1 Mixed Fourier–wavelet operator dictionary

The mixed model uses a lift–mix–project architecture. Each block contains a Fourier expert, a wavelet expert, and group gates. The Fourier branch groups global modes into shells, while the wavelet branch groups localized coefficients into approximation and detail subbands. The operator contribution is therefore expressed as a sum of structured operator components rather than individual scalar weights. For Fourier groups $\mathcal{S}_j$ and wavelet groups $\mathcal{W}_r$, a schematic block is

$$z_{\ell+1} = z_{\ell} + \mathcal{P}_{\ell} \left[\sum_j g_{\ell,j}^F \Phi_{\ell,j}^F(z_{\ell}) + \sum_r g_{\ell,r}^W \Phi_{\ell,r}^W(z_{\ell}) \right], \quad (5)$$

where Φ^F and Φ^W denote the Fourier and wavelet contributions, respectively, g are group gates, and $\mathcal{P}_{\ell}$ denotes the local mixing and projection operations. During dense training, the gates are scalar parameters initialized and held fixed at one. During support selection, they are set exactly to zero or one. Since the gates are not learned during dense training, their values cannot be compensated by rescaling the corresponding expert parameters.

For 2-D Darcy, shell j contains retained frequencies satisfying $\max(k_x, k_y) = j$. Each of the four blocks contains 12 Fourier shells. For each of Haar, db2, and db4, a two-level 2-D wavelet decomposition contributes one approximation group and three orientation groups (LH, HL, HH) at each level, giving seven groups per wavelet family and block. The total number of structured groups is therefore

$$N_{\text{group}} = 4[12 + 3(1 + 3 \cdot 2)] = 132. \quad (6)$$

For 1-D Burgers, each block contains 16 individual Fourier modes. Each of the three wavelet families contributes one approximation group and three level-detail groups, giving

$$N_{\text{group}} = 4[16 + 3(1 + 3)] = 112. \quad (7)$$

The wavelet transforms use periodization boundary handling, while the Fourier transforms use orthonormal normalization.

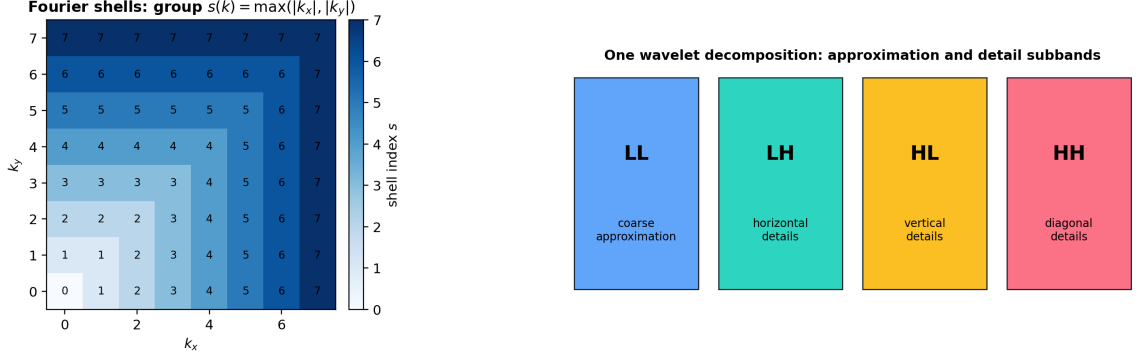


Figure 2: The structured groups used by the mixed dictionary: global Fourier shells (left) and localized wavelet subbands (right). Groups are the unit of selection; an active group is either retained or removed as a complete operator component.

4.2 Functional support selection

Let $\mathcal{I}$ be the set of active groups and let $\mathcal{L}_{\text{cal}}$ denote the calibration loss. For each candidate group i , we remove it from the current model and measure the change in loss,

$$\Delta_i(\mathcal{I}) = \mathcal{L}_{\text{cal}}(\mathcal{I} \setminus \{i\}) - \mathcal{L}_{\text{cal}}(\mathcal{I}). \quad (8)$$

The greedy functional selector removes the group with the smallest effect and recomputes the scores after each removal. Once the desired number of groups is reached, the support is fixed and the model is retrained from the dense checkpoint. This is an offline selection procedure, and its computational cost is reported separately from the training cost.

The parameter-norm control does not rank the fixed gates. For group i , let $\theta_i \in \mathbb{R}^{p_i}$ collect the trainable scalar parameters of the corresponding expert component $\Phi_{\ell,j}^F$ or $\Phi_{\ell,r}^W$ in Eq. (5), counting real and imaginary parts separately for the complex spectral weights. These are the parameters that are removed from the model when the group is deactivated, and we refer to them as the *controlled* parameters of group i ; parameters shared across groups, such as the lifting, projection, and local mixing weights, are excluded. We rank groups by the size-normalized score

$$m_i = \frac{\|\theta_i\|_2}{\sqrt{p_i}}. \quad (9)$$

Groups with the largest scores are retained until the specified group-count or parameter budget is reached. The normalization reduces the tendency to favor groups simply because they contain more parameters, although group size is not included directly in the ranking. As a result, a parameter-matched support can contain fewer but larger groups than the support obtained by functional selection. Parameter norms are also not invariant under general reparameterizations within an expert. Random selection uses predeclared masks matched either by group count or controlled parameter count. Together, these controls separate the effect of support identity from group count, parameter count, and fixed-support retraining.

4.3 Resolution-coupled execution

To test whether representational reductions translate into computational savings, we couple the retained Fourier prefix K_ℓ at layer ℓ to a latent grid of size N_ℓ . The spectral, pointwise, and

activation operations are performed on this grid, with Fourier restriction before the layer and interpolation afterward. This reduces the intended FFT and pointwise workload, while the measured runtime also includes the cost of the grid transitions and synchronization.

4.4 Hybrid fine-scale residual

Because coarsening can remove information needed to represent heterogeneous coefficients, we also test a coarse branch with a lightweight fine-grid residual:

$$u_{\text{hybrid}} = u_{\text{coarse}} + \rho u_{\text{fine residual}}. \quad (10)$$

The residual gate is a single global scalar $\rho = \text{sigmoid}(\alpha) \in (0, 1)$, where the unconstrained logit α is learned jointly with the model rather than fixed or annealed; it is not per-channel or spatially varying. The logit is initialized to -1 , so that ρ starts at $\text{sigmoid}(-1) \approx 0.269$. Although the implementation supports an optional penalty on ρ , its coefficient is zero in the reported experiment, so the gate is optimized only through the data loss. This experiment tests whether the accuracy lost through coarsening can be recovered without restoring the full computational cost.

4.5 Executable transformed-channel basis

The transformed-channel prototype acts on latent channels rather than spatial frequencies. For a Fourier coefficient vector $\hat{v}(k) \in \mathbb{C}^C$, the spectral multiplier is written as

$$W(k) = B_{\text{out}}^{-1} S(k) B_{\text{in}}, \quad (11)$$

where $S(k)$ contains explicit channel blocks. In the general construction, B_{in} and B_{out} are separate real orthogonal butterfly transforms acting on the C latent channels, with $B_{\text{out}}^{-1} = B_{\text{out}}^T$. The spectral coefficients and $S(k)$ are complex.

The phase-0 experiment fixes both transforms to the identity. We use $C = 64$, 16 Fourier modes, and four layers, with both channel axes partitioned into eight blocks of size 8. The dense-block model stores all $8 \times 8 = 64$ channel-block pairs per layer, whereas the diagonal model stores only the eight pairs (q, q) . Thus, “diagonal” refers to a block-diagonal structure in channel space rather than diagonal matrices within each 8×8 block. Inactive blocks are omitted from the module rather than stored as zeros in a dense tensor. All three models retain the same channel and mode dimensions.

The standard FNO, dense-block model, and diagonal model contain 549,569, 551,105, and 92,353 real scalar parameters, respectively. The diagonal representation therefore reduces the parameter count relative to the standard FNO by

$$1 - \frac{92,353}{549,569} = 83.2\%.$$

We test the identity basis first because learning an additional transform is only useful if the corresponding sparse execution path is already competitive. This separates the effect of the representation from limitations of its execution.

4.6 Experimental protocol

All canonical selection experiments use 1000 training, 200 validation, and 200 test samples. The mixed Darcy model has width 32, four blocks, 12 Fourier shells, two wavelet levels, and 132 groups. The mixed Burgers model has width 32, four blocks, 16 Fourier modes, three wavelet levels, and 112

groups. Dense models are trained first, after which the selected supports are fixed and retrained from the corresponding dense checkpoint. The operating-point budgets were selected using one development seed and then held fixed. Results are reported over three seeds, with the remaining two serving as held-out confirmations.

Table 1: Optimization and implementation details for the canonical accuracy experiments. “Best val.” denotes the checkpoint with the lowest validation relative L^2 error in physical units; the test split is evaluated once using that checkpoint.

Item	Dense mixed dictionary	Fixed-support retraining	Conventional FNO
Optimizer	AdamW	AdamW	AdamW
Learning rate / weight decay	10^{-3} / 10^{-4}	10^{-4} / 10^{-4}	10^{-3} / 10^{-4}
Schedule	StepLR, 25 epochs, $\gamma = 0.5$	constant learning rate	StepLR, 25 epochs, $\gamma = 0.5$
Epochs; batch (Darcy/Burgers)	100; 10/20	100; 10/20	100; 10/20
Loss and checkpoint rule	physical relative L^2 ; best val.	physical relative L^2 ; best val.	physical relative L^2 ; best val.
Initialization	$1/C$ Gaussian spectral; final correction conv. $\mathcal{N}(0, (10^{-3})^2)$	dense checkpoint	$(C_{\text{in}}C_{\text{out}})^{-1}$ complex uniform spectral
Precision	FP32	FP32	FP32
FFT normalization	orthonormal	orthonormal	PyTorch default (backward)
Wavelet boundary	periodization	periodization	not applicable
Gates	direct, fixed at 1	exact binary mask, fixed	not applicable

For the conventional FNO comparison, the implementation concatenates spatial coordinates, uses four spectral-plus-pointwise layers, retains both signed vertical-frequency corners in two dimensions, and uses the canonical Darcy padding. Widths were selected using a predeclared validation sweep. The primary parameter-matched controls use width 32 for Darcy and width 56 for Burgers.

Latency is measured on one NVIDIA RTX A6000 using PyTorch 2.4.1 with CUDA 12.1, FP32 tensors, eager execution, `model.eval()`, and `torch.inference_mode()`. We do not use `torch.compile` or CUDA graphs. The benchmark code does not override the default TF32 settings.

For batch-1 resolution measurements, we use 20 warm-up iterations followed by seven measurement blocks. Each block averages 100 forward passes for Burgers and 50 for Darcy, with `torch.cuda.synchronize()` before and after each block. We report the median of the seven block means together with the sample standard deviation of those means. Analytical FLOP ratios are reported only as proxies and are not interpreted as measured speedups.

5 Results

5.1 Functional selection versus parameter norm and random support

Table 2 summarizes the main within-dictionary selection results. At the same group count, functional selection achieves lower mean test error than parameter-norm selection on both Darcy and Burgers. The difference is modest on Darcy but substantially larger on Burgers. Matching the controlled parameter count gives a clearer separation on Darcy, where parameter-norm selection retains fewer groups but produces considerably higher error. The random controls show the same general pattern: support identity matters in addition to the number of retained groups or parameters.

Table 2: Canonical within-dictionary selection results. Test relative L^2 error is reported as mean $\pm$ SD. Dense, functional, and parameter-norm rows use three runs; the group-matched random controls use six runs and the parameter-matched random controls use four.

Benchmark	Method	Active groups	Controlled params.	Test error
<i>Group-matched comparison</i>				
Darcy	Dense	132	2.733M	0.013942 ± 0.000268
	Functional	79	1.307M	0.012619 ± 0.000283
	Parameter norm	79	1.835M	0.013438 ± 0.000720
	Random	79	1.513–1.701M	0.016986 ± 0.002233
Burgers	Dense	112	0.429M	0.003143 ± 0.000116
	Functional	84	0.355M	0.003132 ± 0.000061
	Parameter norm	84	0.350M	0.010046 ± 0.003027
	Random	84	0.313–0.334M	0.004538 ± 0.000654
<i>Parameter-matched comparison</i>				
Darcy	Functional	79	1.307M	0.012619 ± 0.000283
	Parameter norm	50–52	1.306M	0.051999 ± 0.009393
	Random	57–65	1.298–1.303M	0.039454 ± 0.010100
Burgers	Functional	84	0.355M	0.003132 ± 0.000061
	Parameter norm	86	0.355M	0.009083 ± 0.001664
	Random	92–96	0.351–0.359M	0.003409 ± 0.000253

Figure 3 shows the group-matched comparison at the main operating points. In Darcy, functional and parameter-norm selection are relatively close in mean error, although the parameter-norm support uses substantially more controlled parameters at the same group count. In Burgers, the separation between the two selection rules is much larger.

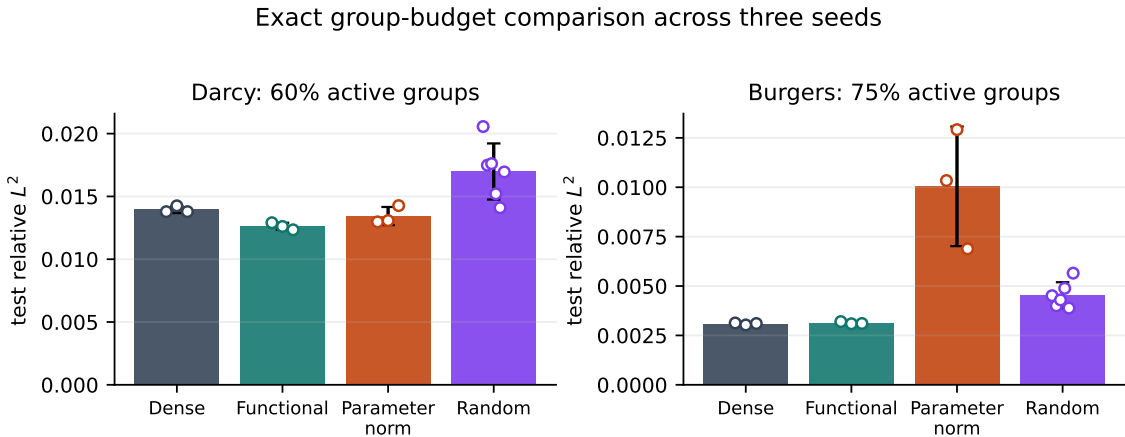


Figure 3: Group-matched comparison at 79/132 active groups for Darcy and 84/112 for Burgers. “Parameter norm” refers to the size-normalized score in Eq. (9). These are the group-matched results from Table 2. The functional result is a within-dictionary selection result rather than an architectural comparison with a conventional FNO.

Across the full exact-cardinality control, functional selection gives lower test error in 34 of 36 comparisons with independently sampled random supports. The operating-point budgets were

chosen using the development run and then held fixed for the confirmation runs. Additional support distributions and retraining diagnostics are reported in Appendix A.

Functional selection is an offline procedure and is more expensive than parameter-norm ranking. The complete Darcy selection path takes approximately 22–25 minutes per run, while the corresponding Burgers path takes approximately 4–5 minutes on the recorded GPU. These costs are incurred during support discovery and are not included in inference latency. We therefore interpret the advantage of functional selection as better support quality, rather than lower overall training cost. Figure 7 in Appendix A provides additional diagnostics. The retraining curves show the optimization behaviour after fixing the selected support, while the random-support distributions show how much the result varies with support identity.

5.2 The mixed dictionary does not outperform a conventional FNO

Table 3 compares the conventional FNO with the dense mixed-dictionary model over three runs. The Darcy FNO is both smaller and more accurate than the corresponding mixed model. The parameter-matched Burgers FNO is also more accurate while using fewer parameters. These results show that the mixed dictionary should not be interpreted as an architectural improvement over a conventional FNO on either benchmark.

Table 3: Comparison of the conventional FNO and the dense mixed-dictionary model. Test relative L^2 error is reported as mean $\pm$ SD over three runs. The mixed rows correspond to the pre-selection dense checkpoints.

Benchmark	Model	Width	Parameters	Test error
Darcy	FNO	32	2,368,001	0.012088 ± 0.000042
	Mixed dictionary	32	2,742,209	0.014795 ± 0.000252
Burgers	FNO	56	421,769	0.002611 ± 0.000103
	Mixed dictionary	32	437,953	0.004074 ± 0.000106

The mixed-model values in Table 3 correspond to the initial pre-selection checkpoints, whereas the dense values in Table 2 are obtained after an additional period of training. They therefore represent different checkpoints of the same architecture and should not be interpreted as repeated estimates of the same quantity.

5.2.1 Where does the FNO’s advantage come from?

The global comparison in Table 3 does not say whether the conventional FNO is uniformly more accurate or whether its advantage is concentrated in a particular part of the domain. To check this for Darcy, we split the test-set relative L^2 error into a smooth bulk region and a band around permeability interfaces, using the same masked metric for the FNO and for the mixed dictionary. Table 4 reports this split for the independent FNO alongside the post-continuation dense checkpoint and the 79/132 functional support from Table 2; it therefore uses a different mixed-model checkpoint than Table 3 for the reasons given above.

Table 4: Darcy interface/bulk error breakdown, using the checkpoints from Table 2 (dense and 79/132 functional) and the independent conventional FNO from Table 3. Test relative L^2 error is mean $\pm$ SD over three seeds. Bold marks the lowest error in each column.

Model	Global error	Interface error	Bulk error
Conventional FNO	0.012088 $\pm$ 0.000042	0.017402 $\pm$ 0.000075	0.010831 $\pm$ 0.000026
Dense mixed (132)	0.013942 $\pm$ 0.000268	0.018667 $\pm$ 0.000574	0.012846 $\pm$ 0.000204
Functional (79/132)	0.012619 $\pm$ 0.000283	0.017045 $\pm$ 0.000567	0.011599 $\pm$ 0.000222

The FNO’s global advantage is driven almost entirely by the bulk region, where its error is 10–16% lower than either mixed-dictionary row. Near interfaces the ordering changes: the 79/132 functional support reaches slightly lower interface error than the independent FNO, and even the untouched dense mixed model is within 7% of it. Figure 5 in Appendix A extends this comparison across the full functional and parameter-norm budget sweep, with the FNO shown as a reference band. One plausible explanation is that the wavelet-derived families in the dictionary are suited to localized, non-smooth structure (Section 1), while the FNO’s global Fourier basis spends its representational budget on the smooth bulk region, where locality is not needed.

The parameter-matched controls in Table 2 provide a separate test of the support-selection result. Because the structured groups have different parameter costs, matching the number of active groups does not necessarily match the number of parameters. On Darcy, the parameter-matched random and parameter-norm supports both produce substantially higher error than functional selection. On Burgers, the parameter-matched random control is much closer to the functional result, while parameter-norm selection remains worse. We therefore report group-matched and parameter-matched comparisons separately.

The conclusion is limited but clear. Functional selection can identify useful support within the heterogeneous dictionary, but this does not imply that the dictionary itself is a better architecture. On the benchmarks considered here, the conventional FNO provides the stronger dense baseline.

5.3 Resolution coupling reduces a proxy, not measured latency

On Burgers, the resolution-coupled $K = 12, N = 128$ model reduces the analytical cost proxy to 50.9% of the dense value, but its latency increases to 1.474 ± 0.033 ms compared with 1.194 ± 0.020 ms for the dense model (median $\pm$ repeat-block SD). A compact width-40 FNO achieves a validation error of 0.002471 with a latency of 1.159 ms, reported as a point estimate.

The same pattern is more pronounced on Darcy. The $K = 10, N = 71$ model reduces the analytical cost proxy to 27.2% of the dense value, but its latency increases to 2.070 ± 0.023 ms compared with 1.631 ± 0.047 ms for the dense control. Its validation error also rises to 0.109249, compared with 0.011561 for the dense model, indicating substantial loss of solution accuracy. A compact width-26 FNO achieves a validation error of 0.011557 at a latency of 1.623 ms. Because this latency is available only as a point estimate, the 0.008 ms difference from the dense median is not interpreted as a measured speedup.

Overall, reducing the execution grid lowers the analytical cost proxy but does not reduce measured latency in these experiments. The additional grid transitions offset the intended computational savings, while aggressive coarsening can also remove information needed for the Darcy solution. Figure 4 compares this result with the other tested execution routes.

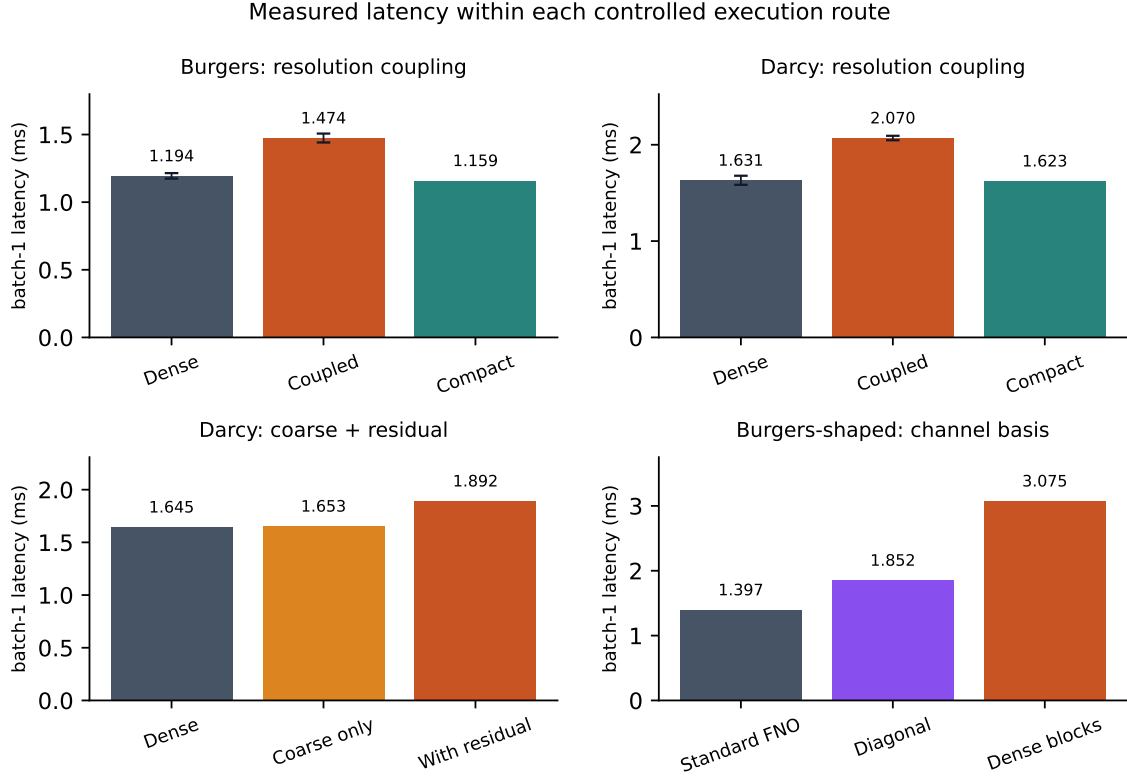


Figure 4: Batch-1 latency comparisons for four controlled execution routes. Each panel uses its own benchmark and implementation protocol, so absolute latency values should not be compared across panels. Error bars show the sample SD of seven repeat-block means where the raw repeats were retained; results without error bars are reported as point estimates. Resolution coupling and the residual route do not outperform their dense controls, and the transformed-channel implementations remain slower than the standard FNO.

5.4 The fine-scale residual restores accuracy and cost

The hybrid experiment makes the trade-off clear. Coarse-only execution has a test error of 0.021322 and a latency of 1.653 ms. Adding the fine-scale residual reduces the test error to 0.011888, close to the dense development checkpoint’s 0.012107, but increases latency to 1.892 ms. The corresponding dense latency is 1.645 ms; the 0.008 ms difference from the coarse-only result is treated as parity rather than a measured difference. At the reported seed-42 checkpoint, the residual gate is 0.2677, essentially unchanged from its initial value of approximately 0.269. Because this run uses no explicit gate penalty, the near-constant value should not be interpreted as evidence of learned gate selection. Instead, the test-error comparison shows that the jointly trained fine-scale branch contributes while operating at approximately its initialized scale. Thus, the residual restores accuracy but not the intended computational saving.

Figure 8 in Appendix A therefore represents a parameter–error trade-off rather than evidence of improved latency.

5.5 Fast-basis parameters are not fast execution

The fixed identity-basis phase-0 experiment provides a direct test of the transformed-channel execution path. The diagonal representation reduces the number of real scalar parameters by 83%, but its batch-1 latency is 1.852 ms compared with 1.397 ms for the standard FNO. The dense-block transformed representation is slower still, at 3.075 ms. We therefore do not proceed to learning the channel transform, since the fixed identity-basis implementation is already slower than the dense baseline. Adding a learned transform would also not address the underlying execution cost. Achieving a speedup would instead require a more efficient implementation, such as a fused block-sparse kernel.

6 Discussion

6.1 Summary of the compression routes

Table 5: Summary of the compression routes evaluated in this study.

Route	Main result	Limitation
Functional group selection	Preserves accuracy at tight budgets and outperforms parameter-norm selection within the mixed dictionary	Does not by itself establish computational speedup
Parameter-norm selection	Reduces the number of active groups and stored parameters	Can remove groups that are important for solution accuracy
Resolution coupling	Reduces the analytical cost proxy	Grid transitions add overhead and coarsening can remove important fine-scale information
Coarse + fine residual	Recovers much of the accuracy lost through coarsening	The additional fine-scale computation removes the expected latency benefit
Transformed-channel blocks	Reduces the stored parameter count by 83%	The tested execution path remains slower than the dense FNO and would require a more efficient sparse implementation

Table 5 summarizes a common pattern across the experiments. Selecting a reduced support can preserve solution accuracy, but this does not necessarily produce a cheaper representation. Likewise, a cheaper representation does not necessarily lead to faster execution. Computational compression therefore requires all three steps: preserving the solution map, reducing the representation or analytical cost, and realizing that reduction through an efficient execution path.

6.2 Interpreting the results

Functional ablation can identify useful groups within a heterogeneous dictionary, but this result is conditional on the chosen representation and does not imply an architectural advantage over a conventional FNO. In our experiments, the conventional FNO provides a stronger dense baseline. The main result is therefore about support selection and execution rather than replacing the FNO with a mixed Fourier-wavelet architecture. The regional breakdown in Table 4 qualifies this conclusion: the FNO’s advantage is not uniform across the domain but concentrated in the smooth bulk region, with the gap largely closing near permeability interfaces. A stronger global baseline can therefore still leave room for a heterogeneous representation to be locally competitive.

6.3 From analytical cost to measured latency

For an FNO layer on a grid of N points, the nominal spectral cost is often described by an $O(N \log N)$ transform together with channel mixing. Actual latency also depends on memory movement, kernel fusion, batch size, transform implementation, and whether inactive structures are physically removed from execution. Hardware-aware sparsity studies similarly show that regular block or tile structures are easier for dense hardware to exploit than arbitrary sparse patterns [Guo et al., 2020, Han et al., 2016]. Our batch-1 measurements focus on single-sample inference, where kernel-launch and transition overheads are visible. In the resolution-coupled experiment, additional restriction and interpolation operations offset the reduction in analytical cost. In the transformed-channel experiment, the sparse representation replaces an optimized dense contraction with smaller block operations that are slower under the tested implementation. In both cases, a lower analytical cost does not translate into lower measured latency. This is important when claiming computational sparsity. A zero coefficient or masked parameter reduces the representation, but it reduces execution cost only if the corresponding operation is also removed or replaced by a cheaper implementation.

6.4 Relation to adaptive-basis and mixture approaches

Methods based on learned Fourier frames, adaptive bases, or mixtures of operators are motivated by the fact that a single representation may not be optimal for every problem [Zhao and Aviles-Rivero, 2026, Kratsios et al., 2024]. Our results are consistent with this motivation, but they add an additional requirement: the learned or selected structure must also admit an efficient implementation. This helps explain why effective support selection within the mixed dictionary can coexist with a stronger conventional FNO baseline. The first concerns which components are useful within a fixed representation, while the second concerns the choice of representation and architecture itself.

6.5 Limitations and future work

Several comparisons remain limited. The WNO experiment is restricted to a Burgers diagnostic, while the Darcy run did not produce a valid final result. We therefore make no matched comparison with a pure wavelet operator. Some latency measurements also use development checkpoints and are reported separately from the canonical accuracy results.

The parameter-norm baseline is more informative than raw gate magnitude: the gates are fixed at one, and each group norm is normalized by the square root of its parameter count. However, this score is not invariant to all possible reparameterizations within a multi-layer expert. Our conclusions therefore concern this particular norm-based control and do not imply that functional selection outperforms every possible parameter-importance criterion.

Other limitations include the small number of independent runs, the limited number of random-control trials, the use of a single NVIDIA A6000 GPU, and ordinary PyTorch eager execution rather than a specialized sparse implementation. The latency results therefore apply to the tested model sizes, software stack, and hardware configuration. Different resolutions, hardware, or sparse implementations may lead to different execution characteristics. The interface/bulk breakdown in Table 4 is likewise based on three seeds per model with no formal significance test, uses a fixed band definition around permeability interfaces, and is reported only for Darcy; we treat it as a descriptive regional trend rather than a general claim that heterogeneous dictionaries are more accurate near interfaces.

The transformed-channel experiment is also limited by its current implementation. A fused block-sparse kernel could change the latency result, but this would require a separate implementation

and profiling study. Larger spatial grids or three-dimensional problems may likewise change the balance between transform overhead and arithmetic cost.

Natural extensions of this work include:

1. a resource-matched comparison with a pure wavelet operator;
2. synchronized profiling of the canonical exported models; and
3. evaluation of specialized sparse kernels if the resulting execution path can exploit the selected structure.

7 Conclusion

We studied several approaches to reducing the representation and computational cost of Fourier neural operators. Within a fixed heterogeneous Fourier–wavelet dictionary, functional support selection preserves accuracy and outperforms size-normalized parameter-norm selection at the tested operating points. However, the mixed dictionary does not outperform the conventional FNO baseline overall. Moreover, neither resolution-based reduction nor an 83% reduction in stored parameters leads to lower measured latency under the tested execution paths. A fine-scale residual recovers much of the accuracy lost through coarsening, but also removes the expected computational saving.

These results show that parameter reduction and analytical cost reduction are not sufficient measures of computational sparsity. A useful sparse neural-operator representation must preserve the solution map and translate its reduced structure into a cheaper execution path on the target hardware. This requires considering the operator representation and its implementation together, including transforms, memory movement, and the kernels used to execute the retained structure.

Data and code availability

Code is available at https://github.com/aqibrahimbt/sparsity_is_not_speed. The version corresponding to this manuscript is archived under the tag `arxiv-v1`. Large datasets, model checkpoints, and raw training outputs are not included in the repository; the corresponding experimental protocols and artifact names are documented there.

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References

- Nikola Kovachki, Zongyi Li, Burigede Liu, Kamyar Azizzadenesheli, Kaushik Bhattacharya, Andrew Stuart, and Anima Anandkumar. Neural operator: Learning maps between function spaces with applications to PDEs. *Journal of Machine Learning Research*, 24(89):1–97, 2023.
- Zongyi Li, Nikola Kovachki, Kamyar Azizzadenesheli, Burigede Liu, Kaushik Bhattacharya, Andrew Stuart, and Anima Anandkumar. Fourier neural operator for parametric partial differential equations. In *International Conference on Learning Representations*, 2021.

- Tapas Tripura and Souvik Chakraborty. Wavelet neural operator for solving parametric partial differential equations in computational mechanics problems. *Computer Methods in Applied Mechanics and Engineering*, 404:115783, 2023. doi: 10.1016/j.cma.2022.115783.
- Lu Lu, Pengzhan Jin, Guofei Pang, Zhongqiang Zhang, and George Em Karniadakis. Learning nonlinear operators via DeepONet based on the universal approximation theorem of operators. *Nature Machine Intelligence*, 3(3):218–229, 2021. doi: 10.1038/s42256-021-00302-5.
- Ingrid Daubechies. *Ten Lectures on Wavelets*. SIAM, 1992.
- Stéphane Mallat. *A Wavelet Tour of Signal Processing*. Academic Press, 2 edition, 1999.
- Xuxiang Zhao and Angelica I. Aviles-Rivero. Don’t fix the basis – learn it: Spectral representation with adaptive basis learning for PDEs. arXiv:2605.10451, 2026.
- Anastasis Kratsios, Takashi Furuya, Jose Antonio Lara Benitez, Matti Lassas, and Maarten de Hoop. Mixtures of neural operators reduce active complexity in operator learning, 2024. Revised 2026; originally titled “Mixture of Experts Soften the Curse of Dimensionality in Operator Learning”.
- Robert Tibshirani. Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society: Series B*, 58(1):267–288, 1996.
- Ming Yuan and Yi Lin. Model selection and estimation in regression with grouped variables. *Journal of the Royal Statistical Society: Series B*, 68(1):49–67, 2006.
- Song Han, Huizi Mao, and William J. Dally. Deep compression: Compressing deep neural networks with pruning, trained quantization and huffman coding. In *International Conference on Learning Representations*, 2016.
- Pavlo Molchanov, Stephen Tyree, Tero Karras, Timo Aila, and Jan Kautz. Pruning convolutional neural networks for resource efficient inference. In *International Conference on Learning Representations*, 2017.
- Thomas Blumensath and Mike E. Davies. Iterative hard thresholding for compressed sensing. *Applied and Computational Harmonic Analysis*, 27(3):265–274, 2009.
- Peter Binev, Albert Cohen, Wolfgang Dahmen, Ronald DeVore, Guergana Petrova, and Przemyslaw Wojtaszczyk. Convergence rates for greedy algorithms in reduced basis methods. *SIAM Journal on Mathematical Analysis*, 43(3):1457–1472, 2011.
- Albert Matveev, Sanmitra Ghosh, Aamal Hussain, James-Michael Leahy, and Michalis Michaelides. Light-weight diffusion multiplier and uncertainty quantification for Fourier neural operators, 2025.
- Lennon J. Shikhman. Operator boosting produces pareto-efficient PDE surrogates. arXiv:2606.17460, 2026. Preprint.
- Kiran Bacsá, Wei Liu, Xudong Jian, Huangbin Liang, Zhilu Lai, and Eleni Chatzi. Selective sparsity in Fourier neural operator networks to accelerate partial differential equation solving. OpenReview, <https://openreview.net/forum?id=b6dGZCfIX3>, 2025. ICLR 2026 submission; rejected. Accessed 17 August 2026.
- Jinghong Xu, Yuqian Zhou, Qian Liu, Kebin Li, and Haolin Yang. Transfer learning of neural operators for partial differential equations based on sparse network λ -FNO. *PLOS ONE*, 20(5):e0321154, 2025. doi: 10.1371/journal.pone.0321154.

Jason Yoo, Shailesh Garg, Souvik Chakraborty, and Syed Bahauddin Alam. When spike sparsity does not translate to deployed cost: Vs-wno on jetson orin nano, 2026. URL <https://arxiv.org/abs/2604.17040>.

Cong Guo, Bo-Yang Hsueh, Jingwen Leng, Yuxian Qiu, Yue Guan, Zehuan Wang, Xiaoying Jia, Xipeng Li, Minyi Guo, and Yuhao Zhu. Accelerating sparse dnn models without hardware-support via tile-wise sparsity. In *SC20: International Conference for High Performance Computing, Networking, Storage and Analysis*, pages 1–15, 2020. doi: 10.1109/SC41405.2020.00020.

A Supporting figures

The figures in this appendix provide additional diagnostics for the selection and compression experiments reported in the main text. They are included here to keep the main results focused on the primary accuracy and latency comparisons.

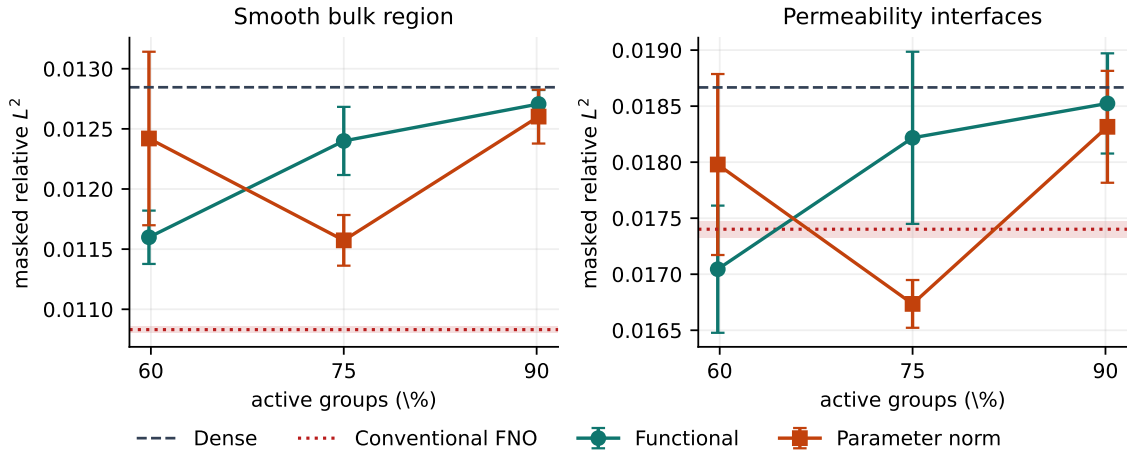


Figure 5: Regional error diagnostics for Darcy. The interface and bulk regions show how the remaining error is distributed spatially and provide information that is not visible from the global relative L^2 error alone. The dotted red line and shaded band show the mean $\pm$ SD of the independent conventional FNO (width 32, seeds 42–44, Table 3) under the same masked metric. The FNO is well below the mixed-dictionary curves in the bulk panel but sits inside their range in the interface panel, matching the summary in Table 4.

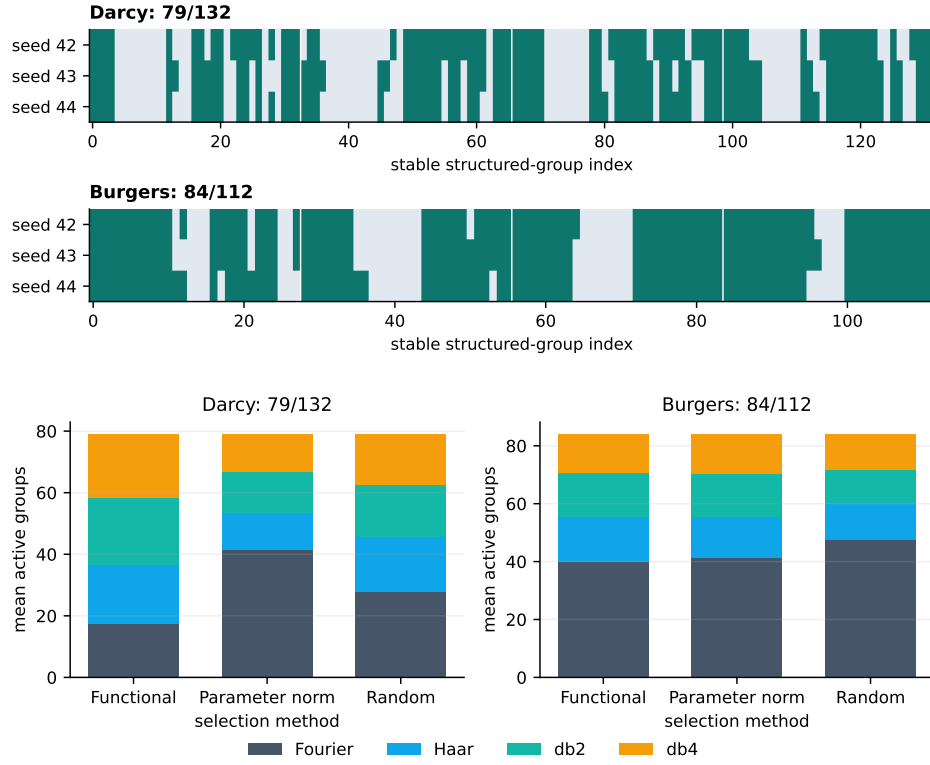


Figure 6: Diagnostics of the selected support. The top panel shows agreement between supports obtained in different runs, while the bottom panel shows the distribution of retained groups across Fourier and wavelet families. The comparison is made at the level of structured groups rather than individual scalar parameters.

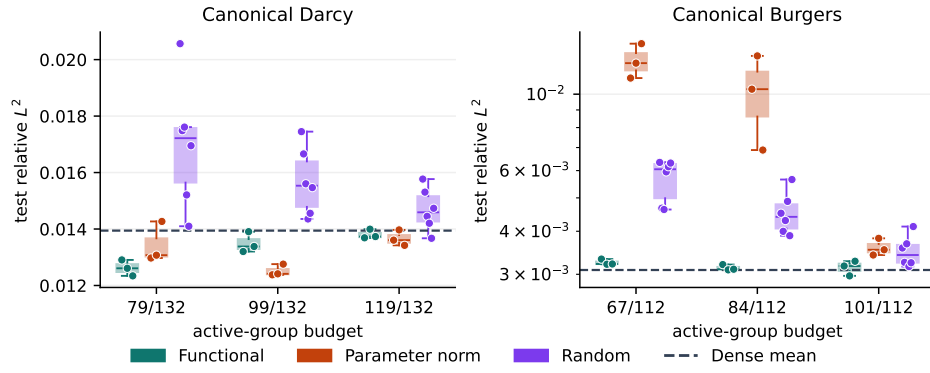
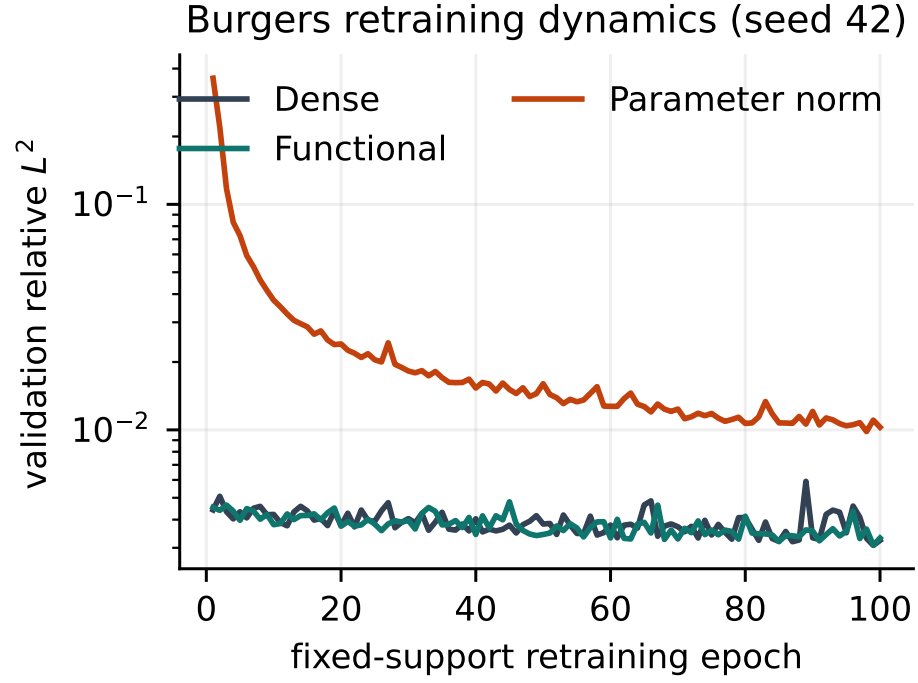


Figure 7: Additional checks on the support-selection experiment. The top panel shows retraining after the support is fixed, while the bottom panel shows the distribution of outcomes for the random-support controls.

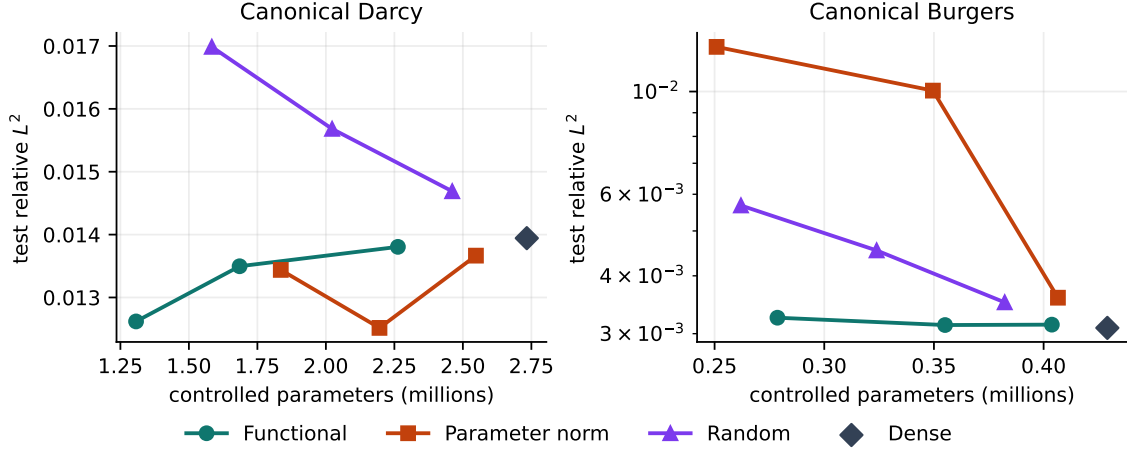


Figure 8: Parameter-error trade-offs for functional selection. The figure shows that parameter reduction can coexist with preserved accuracy, but does not by itself imply lower inference latency.

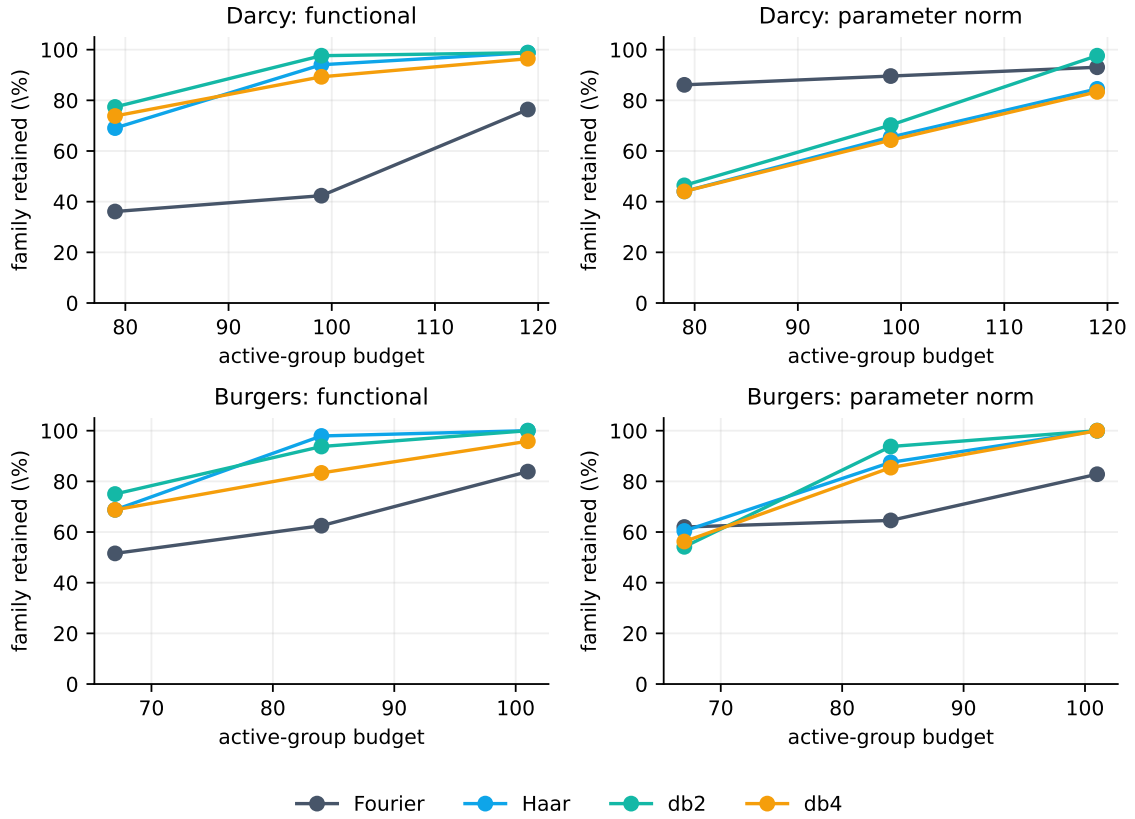


Figure 9: Distribution of retained operator families across sparsity budgets. Functional selection changes both the number and the type of groups retained, whereas parameter-norm selection produces a different allocation across Fourier and wavelet families.