































































































































































































































Observation of $e^+e^- \rightarrow \eta\Upsilon(2S)$ and search for $e^+e^- \rightarrow \eta\Upsilon(1S)$, γX_b at \sqrt{s} near 10.75 GeV




















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(The Belle II Collaboration)

(Dated: September 3, 2025)

We present an analysis of the processes $e^+e^- \rightarrow \eta\mathcal{Y}(1S)$, $\eta\mathcal{Y}(2S)$, and γX_b with $X_b \rightarrow \pi^+\pi^-\chi_{bJ}$, $\chi_{bJ} \rightarrow \gamma\mathcal{Y}(1S)$ ($J = 1, 2$) reconstructed from $\gamma\gamma\pi^+\pi^-\ell^+\ell^-$ ($\ell = e, \mu$) final states in 19.6 fb^{-1} of Belle II data collected at four energy points near the peak of the $\mathcal{Y}(10753)$ resonance. Here, X_b is a hypothetical bottomonium-sector partner of the $X(3872)$. A signal of $e^+e^- \rightarrow \eta\mathcal{Y}(2S)$ is observed with a significance greater than 6.0σ . The central value of the Born cross section at 10.653 GeV is measured to be higher than that at 10.745 GeV, and we find evidence for a possible new state near $B^*\bar{B}^*$ threshold, with a significance of 3.2σ . No significant signal is observed for $e^+e^- \rightarrow \eta\mathcal{Y}(1S)$ or γX_b . Upper limits on the Born cross sections for the processes $e^+e^- \rightarrow \eta\mathcal{Y}(1S)$ and $e^+e^- \rightarrow \gamma X_b$ with $X_b \rightarrow \pi^+\pi^-\chi_{bJ}$ are determined.

In recent decades, a series of quarkonium-like states with unconventional characteristics have been observed at accelerator facilities [1], providing compelling evidence for the existence of exotic hadronic configurations. Among these, bottomonium states offer particularly valuable probes for investigating the non-perturbative regime of quantum chromodynamics due to their distinctive mass scale and the relatively clean theoretical description. In 2019, a new structure, known as $\Upsilon(10753)$, was identified in the cross-section measurements of $e^+e^- \rightarrow \pi^+\pi^-\Upsilon(mS)$, $m = 1, 2, 3$ [2]. With more data collected from the Belle II experiment, this structure was confirmed in the measurement of the cross section of $e^+e^- \rightarrow \omega\chi_{b1,2}$ [3], where its mass and width were determined with greater precision [4]. The small partial width of $\Upsilon(10753) \rightarrow \omega\eta_b$ [5] suggests that it may not be exotic; however, the investigation of $\Upsilon(10753) \rightarrow \pi^+\pi^-\Upsilon(mS)$ and $\omega\chi_{bJ}$, $J = 1, 2$ indicates that it is also challenging to classify $\Upsilon(10753)$ as a conventional bottomonium state [3, 4, 6]. Therefore, further investigation is warranted to fully understand the nature of the $\Upsilon(10753)$.

The measurement of hadronic transition of the $\Upsilon(10753)$ with emission of an η provides a complementary approach to understanding of this resonance. A relatively large branching fraction of $\Upsilon(10753) \rightarrow \eta\Upsilon(nS)$ with $n = 1, 2$ is expected given the possibility of $4S - 3D$ mixing [7]. The branching fraction for $\Upsilon(10753) \rightarrow \eta\Upsilon(1S)$ is predicted to be $(0.46 - 5.46) \times 10^{-3}$ within this scheme [7], which is comparable with $\Upsilon(10753) \rightarrow \omega\chi_{bJ}$ [3] observed at Belle II. Furthermore, the $e^+e^- \rightarrow \eta\Upsilon(nS)$ process provides a different approach to search for new bottomonium resonances. A measurement of the Born cross section of $e^+e^- \rightarrow B^{(*)}\bar{B}^{(*)}$ with $\Upsilon(10753)$ scan data [3] indicates that there might be a structure near the $B^*\bar{B}^*$ threshold. In the charmonium sector, a similar structure $\psi(4040)$ was found near the $D^*\bar{D}^*$ threshold and confirmed in $e^+e^- \rightarrow \eta J/\psi$ [1]. Analogously, we can search for the possible state near $B^*\bar{B}^*$ threshold in the $e^+e^- \rightarrow \eta\Upsilon(nS)$ process.

The charmonium-like state $Y(4230)$ [1], which is produced in e^+e^- annihilation, has been measured to have unexpectedly large branching fractions to $\pi^+\pi^-\psi(nS)$ and $\omega\chi_{cJ}$, and can also decay to the exotic state $X(3872)$ via radiative transition [8]. Analogously, it is possible that the $\Upsilon(10753)$ decays to a bottomonium partner of the $X(3872)$ [9], X_b , via radiative decay. The authors of Ref. [10] note that the X_b state would be likely to decay to $\pi^+\pi^-\chi_{bJ}$, where χ_{bJ} will predominantly decay to $\gamma\Upsilon(1S)$ [1].

To search for the $e^+e^- \rightarrow \eta\Upsilon(nS)$ and $X_b \rightarrow \pi^+\pi^-\chi_{bJ}$ events, we reconstruct $\gamma\gamma\pi^+\pi^-\ell^+\ell^-$, $\ell = e, \mu$ final states. For the $\eta\Upsilon(1S)$ channel, the η decays to $\pi^+\pi^-\pi^0$, where π^0 decays to $\gamma\gamma$. In the $\eta\Upsilon(2S)$ channel, the decay modes are either $\eta \rightarrow \gamma\gamma$ or $\eta \rightarrow \pi^+\pi^-\pi^0$, with $\Upsilon(2S) \rightarrow \pi^+\pi^-\Upsilon(1S)$ or $\Upsilon(2S) \rightarrow \ell^+\ell^-$. We reconstruct χ_{bJ} via the decay to $\gamma\Upsilon(1S)$. We always reconstruct $\Upsilon(1S)$ in the $\ell^+\ell^-$ final state. We exclude the decay chain $\eta \rightarrow \gamma\gamma$ with $\Upsilon(nS) \rightarrow \ell^+\ell^-$ due to the large background from quantum electrodynamics (QED) processes. The decay chain of $\eta \rightarrow \pi^+\pi^-\pi^0$ with $\Upsilon(2S) \rightarrow \pi^+\pi^-\Upsilon(1S)$ is also excluded due to its low signal sensitivity. We use data collected with the Belle II detector [11, 12] in November 2021 at center-of-mass (c.m.) energies of $\sqrt{s} = 10.653, 10.701, 10.746, \text{ and } 10.805$ GeV, which form the $\Upsilon(10753)$ scan data set.

The Belle II detector operates at the SuperKEKB asymmetric-energy electron-positron collider [13] at KEK. The Belle II detector is a nearly 4π spectrometer consisting of silicon-based vertexing and drift-chamber tracking systems, Cherenkov-light particle identification detectors, and an electromagnetic calorimeter (ECL), situated within a superconducting solenoid providing a 1.5 T axial magnetic field. The symmetry axis of these detectors, defined as the z axis, is almost coincident with the direction of the electron beam.

Monte Carlo (MC) simulated data are used to optimize event selection, determine reconstruction efficiencies, extraction of signal-resolution functions, and determine the fit models used to extract the signals. We simulate $\Upsilon(10753)$ decays with EVTGEN [14], using PHOTOS [15] to simulate final-state radiation. Initial-state radiation (ISR) at next-to-leading order accuracy in quantum electrodynamics is simulated with PHOKHARA [16]. The $e^+e^- \rightarrow \eta\Upsilon(nS)$ processes are simulated in P-wave, while $e^+e^- \rightarrow \gamma X_b$ is simulated uniformly in phase-space. Detector simulation is performed with Geant4 [17]. Reconstruction of events from simulation and collision data uses the Belle II analysis software [18, 19]. An inclusive background sample with four times the luminosity of data includes decays of pairs of B mesons, hadronic processes with light quarks (u, d, s, c), and low-multiplicity QED processes, *e.g.*, Bhabha scattering [20–24], $\mu^+\mu^-(\gamma)$ [20–24], ISR-produced hadron pairs [16], and four-track events with at least one lepton pair [25, 26]. These simulated processes are used to check for contamination from possible backgrounds.

Events are selected online by a hardware trigger that uses drift-chamber and calorimeter information with an efficiency greater than 97% for events containing at least three tracks according to simulation [27]. In the offline analysis, tracks reconstructed in the final state are required to originate from the vicinity of the interaction point (within 2 cm along the z axis, and 0.5 cm in the radial direction) to remove beam-related backgrounds and incorrectly reconstructed tracks. Tracks are required to be within the angular acceptance of the drift chamber, *i.e.*, the polar angle with respect to the z axis should lie in within $[17.0, 150.0]^\circ$. Tracks with momenta less than 1 GeV/ c are considered as pions, and those greater than 3 GeV/ c are considered as leptons. To suppress the background from e/π mis-identification, we require that at least one pion candidate should have a likelihood ratio $\mathcal{L}_e/(\mathcal{L}_e + \mathcal{L}_\pi)$ less than 0.1. To further separate leptons, the lepton candidates with a ratio of deposited energy in the ECL to track momentum greater than 0.7 are regarded as electrons, whereas those with a ratio less than 0.3 are regarded as muons.

Photons are reconstructed from the ECL clusters within the angular acceptance of the drift chamber. For photons reconstructed in the barrel and forward end-cap, the deposited energy in the ECL must be greater than 20 MeV. In contrast, for photons reconstructed from the backward end-cap, the energy threshold is set at 22.5 MeV. The barrel corresponds to a polar angle range of $[32.2, 128.7]^\circ$ while the forward and backward end-caps correspond to $[12.4, 31.4]^\circ$ and $[130.7, 155.1]^\circ$, respectively. In the reconstruction of $\Upsilon(nS) \rightarrow e^+e^-$ events, a bremsstrahlung correction is applied by adding the four-momentum of photons within 0.2 rad around the initial electron direction to the four-momentum of the electron. To suppress photon conversion background, we require $\cos(\Delta\theta) < 0.98$ for the polar angle between the two charged pions. A four-constraint kinematic fit is performed to the $\gamma\gamma\pi^+\pi^-\ell^+\ell^-$ candidates by constraining the four-momenta of the final-state system to the initial e^+e^- c.m. system, and to further suppress background, a requirement of $\chi^2 < 80$ is applied. On average, we find 1.07 candidates per event. In events with multiple candidates we select the one with the lowest value of χ^2 , which retains the correct candidate with an efficiency of 92.1%.

In selecting $e^+e^- \rightarrow \eta\Upsilon(2S)$ with $\eta \rightarrow \pi^+\pi^-\pi^0$ and $\Upsilon(2S) \rightarrow \ell^+\ell^-$, we require $M(\gamma\gamma) \in [0.115, 0.165] \text{ GeV}/c^2$, and $M(\ell^+\ell^-) \in [10.00, 10.06] \text{ GeV}/c^2$. For $\eta \rightarrow \gamma\gamma$ and $\Upsilon(2S) \rightarrow \pi^+\pi^-\Upsilon(1S)$ candidates, we require $M(\ell^+\ell^-) \in [9.40, 9.52] \text{ GeV}/c^2$, and $M(\pi^+\pi^-\Upsilon(1S)) \in [10.015, 10.030] \text{ GeV}/c^2$, where $M(\pi^+\pi^-\Upsilon(1S)) \equiv M(\pi^+\pi^-\ell^+\ell^-) - M(\ell^+\ell^-) + m(\Upsilon(1S))$ is implemented to improve the mass resolution. The regions are chosen to cover approximately 95% of signal events. The $\Upsilon(2S)$ sidebands are defined so that each has the same size as the signal region; the gap between the signal region and the sideband is half the size of the signal region. To improve the mass resolution, corrected masses $M'(\gamma\gamma) \equiv M(\gamma\gamma) + M(\ell^+\ell^-) - m(\Upsilon(1S))$ and $M(\pi^+\pi^-\pi^0) \equiv M(\pi^+\pi^-\gamma\gamma) - M(\gamma\gamma) + m(\pi^0)$ are used instead of the invariant masses. Resolutions are improved by approximately a factor of two, and the methods for improving the resolution differ between $M'(\gamma\gamma)$ and $M(\pi^+\pi^-\pi^0)$ due to their distinct mass construction topologies. After applying the selection criteria, the two-dimensional distributions of $M(\ell^+\ell^-)$ and $M(\pi^+\pi^-\Upsilon(1S))$ versus $M'(\gamma\gamma)$ and $M(\pi^+\pi^-\pi^0)$ are shown in Fig. 1, summing over all c.m. energies in the $\Upsilon(10753)$ scan data. Nominal $\Upsilon(2S)$ signal and side-band regions are also displayed in the Fig. 4. A clear cluster can be seen in the $\Upsilon(2S)$ signal region. No peak is observed in the $M'(\gamma\gamma)$ and $M(\pi^+\pi^-\pi^0)$ distribution in either the $\Upsilon(2S)$ data sideband or the inclusive background MC sample.

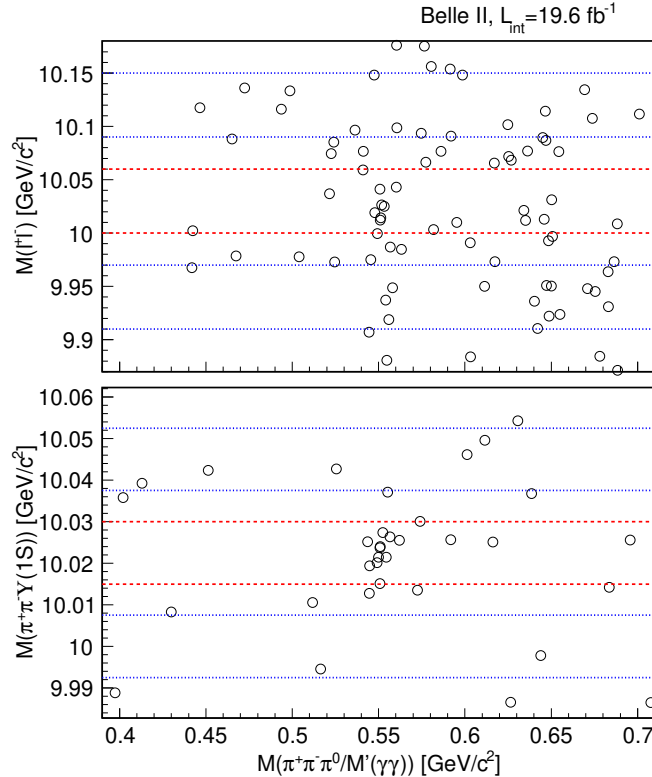


Figure 1: Two dimensional distribution of $M(\ell^+\ell^-)$ versus $M(\pi^+\pi^-\pi^0)$ (top) and $M(\pi^+\pi^-\Upsilon(1S))$ versus $M'(\gamma\gamma)$ (bottom) from data. Horizontal lines represents the $\Upsilon(2S)$ signal (red) and sideband boundaries (blue).

The distributions of $M'(\gamma\gamma)$ and $M(\pi^+\pi^-\pi^0)$ at the individual energy points are shown in Fig. 4. An extended unbinned

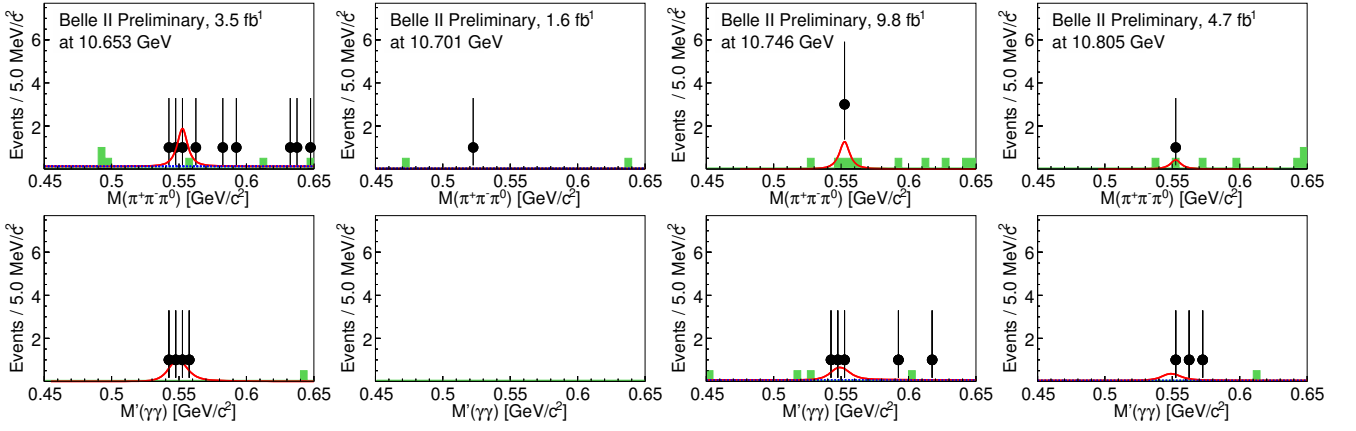


Figure 2: Simultaneous fit to $M(\pi^+\pi^-\pi^0)$ (top) and $M'(\gamma\gamma)$ (bottom) from each energy point. Plots in the same column correspond to the same scan energy. Dots with error bars are experimental data, the red solid lines represent the fit result, and the green histograms represent events from the $\Upsilon(2S)$ sideband.

maximum likelihood fit is performed simultaneously on $M'(\gamma\gamma)$ and $M(\pi^+\pi^-\pi^0)$ at each energy point, in which the number of $e^+e^- \rightarrow \eta\Upsilon(2S)$ (N_{prod}) events is the common free parameter, and the relative $e^+e^- \rightarrow \eta\Upsilon(2S)$ signal yields (N_{sig}) from the two spectra are computed according to the $\eta \rightarrow \gamma\gamma$ and $\eta \rightarrow \pi^+\pi^-\pi^0$ branching fractions (\mathcal{B}) and reconstruction efficiencies (ϵ), *i.e.*, $N_{\text{sig}} = N_{\text{prod}}\mathcal{B}\epsilon$. The background probability density functions (PDFs) are first-order polynomials, and the signal PDFs are the MC histograms. The fit results are shown in Table I as well as the statistical significance of the signal, which is determined by comparing the change of the likelihood and the number of degrees of freedom in the fit with and without the signal component in the fit. Profile likelihood distributions can be found in the Supplemental Material and on HEPData [28]. In a similar fit to the combined data sample, the significance is 6.4σ (see the Supplemental Material).

The Born cross-section at each energy point is calculated using $\sigma_B = \frac{N_{\text{prod}}|1-\Pi|^2}{\mathcal{L}(1+\delta)}$, where \mathcal{L} is the integrated luminosity of the individual dataset, $|1-\Pi|^2$ is the vacuum polarization factor [29], and $(1+\delta)$ is the radiative correction factor [30] calculated using the Born cross-section lineshape in the same way as in Ref. [4]. The numerical results of the Born cross-sections are listed in Table I and plotted in Fig. 3. Additional analysis with 711 fb^{-1} of data taken around the $\Upsilon(4S)$ resonant peak by Belle [31] is performed, and the cross section at the $\Upsilon(4S)$ peak is measured to be less than 0.01 pb (with statistical uncertainty only), indicating that the signal events at 10.653 GeV are not coming from the $\Upsilon(4S)$ tail. We fit the Born cross-sections together with the result from $\Upsilon(5S) \rightarrow \eta\Upsilon(2S)$ [32] as a function of center-of-mass energy \sqrt{s} with three coherent relativistic Breit-Wigner functions representing $\Upsilon(5S)$, $\Upsilon(10753)$, and a possible state near the $B^*\bar{B}^*$ threshold. The mass and width of $\Upsilon(5S)$ and $\Upsilon(10753)$ are fixed to the known values [1, 4]. The mass and width of the new state are taken from the fit to the $B^{(*)}\bar{B}^{(*)}$ cross section lineshape reported in Ref.[33]. The χ^2 of this fit is constructed from the Born cross-section-dependent profile likelihood distribution from the individual simultaneous fit. We also test the hypothesis with two resonances, $\Upsilon(5S)$ and $\Upsilon(10753)$, and one resonance, $\Upsilon(5S)$, as shown in Fig. 3. Other different hypotheses like replacing the $\Upsilon(10753)$ or the new state with a flat continuum contribution are also tested. The significance of $\Upsilon(10753)$ is less than 2.0σ whereas the significance of the new state is greater than 3.2σ .

We use the same $\gamma\gamma\pi^+\pi^-\ell^+\ell^-$ final state to reconstruct $e^+e^- \rightarrow \eta[\rightarrow \pi^+\pi^-\pi^0]\Upsilon(1S)$. To purify the signal, we apply the same selection criteria and mass selection on $\gamma\gamma$ and $\ell^+\ell^-$, which are reconstructed as π^0 and $\Upsilon(1S)$. Only two events are found in the η signal region $[0.52, 0.57]\text{ GeV}/c^2$ with the full data sample [34]. Therefore, upper limits at 90% credibility level (C.L.) of the signal yields at individual energies are estimated using method described in Ref. [35]. The results for the signal Born cross section, calculated assuming $1/s$ energy dependence, are shown in Table I.

We also search for the X_b with $X_b \rightarrow \pi\pi\chi_{bJ}$ in $e^+e^- \rightarrow \gamma X_b$ with the same final state. We do not separate χ_{b1} and χ_{b2} because of their very close invariant masses. The search is performed in the mass window $M(\gamma\Upsilon(1S)) \in [9.84, 9.96]\text{ GeV}/c^2$. The signal PDFs are the MC histograms. The dominant backgrounds for these channels are $e^+e^- \rightarrow \eta\Upsilon(1,2S)$, $e^+e^- \rightarrow \omega[\rightarrow \pi^+\pi^-\pi^0]\chi_{b1,2}$, and $e^+e^- \rightarrow \pi^+\pi^-\Upsilon(2S)[\rightarrow \gamma\chi_{bJ}]$, where subsequent decay final states are shown in square brackets. The cross section of $e^+e^- \rightarrow \eta\Upsilon(1,2S)$ is reported in Table I and the contribution to $\pi\pi\chi_{bJ}$ is small. The other two backgrounds are difficult to suppress because of the similarity in phase-space, thus their contributions are constrained during the fit according to the measured Born cross-sections [3, 4], branching fractions into the final state $\gamma\gamma\pi^+\pi^-\ell^+\ell^-$ [1], and reconstruction efficiencies

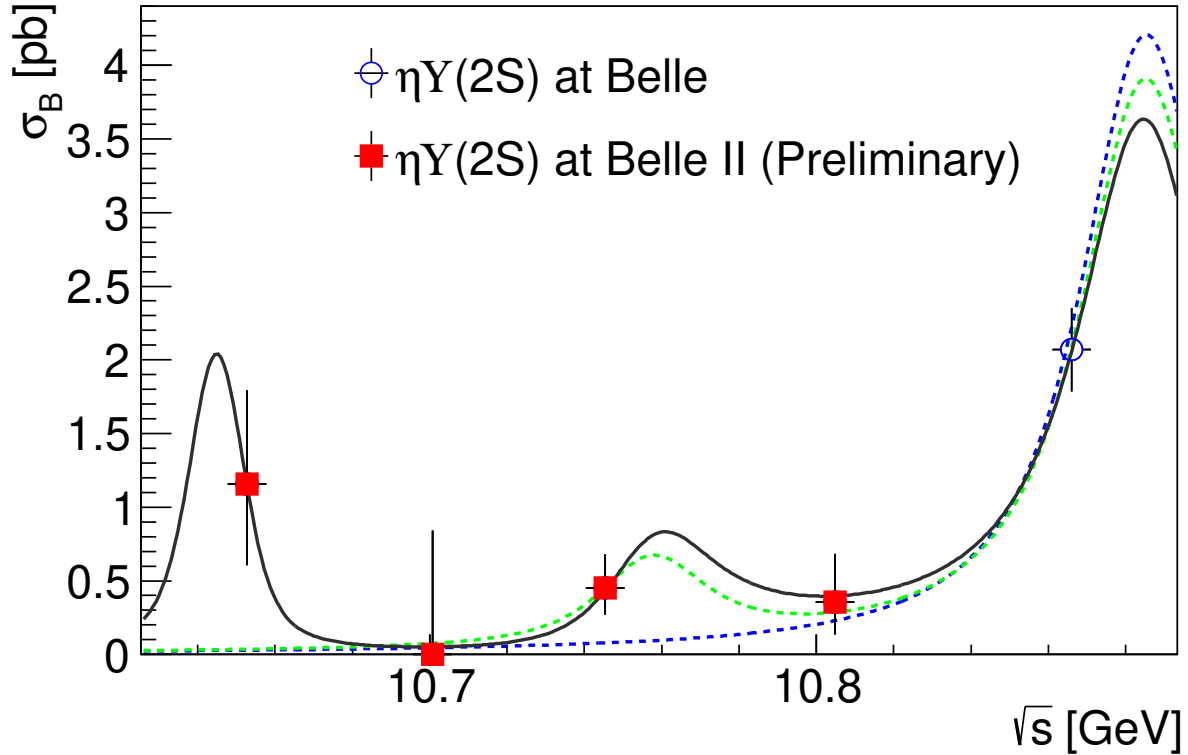


Figure 3: Born cross-sections for $e^+e^- \rightarrow \eta\Upsilon(2S)$ with fit results overlaid. Points with error bars show measured cross sections, the solid curve is the nominal fit result with $\Upsilon(5S)$, $\Upsilon(10753)$ and a state near $B^*\bar{B}^*$ threshold, the green dashed curve is the fit result with $\Upsilon(5S)$ and $\Upsilon(10753)$, and the blue dashed curve is the fit result with only $\Upsilon(5S)$. The new state near the $B^*\bar{B}^*$ threshold is decoupled from the other two states, with only its amplitude contributing to the fit.

obtained from the MC study. An unbinned maximum likelihood fit is performed to the invariant mass $M(\pi^+\pi^-\chi_{b1})$ [34]. Different hypotheses of the mass of X_b are evaluated, and the maximum probability is found when $m(X_b) = 10.50 \text{ GeV}/c^2$. No evident signal of X_b is found. The upper limits of $\sigma(e^+e^- \rightarrow \gamma X_b) \times \mathcal{B}(X_b \rightarrow \pi^+\pi^-\chi_{bJ})$ at 90% C.L. are estimated using a Bayesian method as described in Ref. [4] with the assumption that $M(X_b) = 10.50 \text{ GeV}/c^2$, as shown in Table I.

Sources of systematic uncertainty for the cross-section measurements include tracking and photon detection efficiencies, the integrated luminosity of the sample (\mathcal{L}), the choice of simulated-event generators, ISR factors, the chosen generator decay model, trigger efficiency, the cascade branching fractions (\mathcal{B}), and the fit procedure.

In the simulation of $e^+e^- \rightarrow \gamma X_b$, we change the uniformly-distributed radiative transition to alternative models. The change of detector efficiency due to variation of the decay model is taken as the systematic uncertainty. This uncertainty is not considered for the process $e^+e^- \rightarrow \eta\Upsilon(nS)$, which is known to have a P-wave distribution. For the ISR factors in $e^+e^- \rightarrow \eta\Upsilon(2S)$, we take the changes of the Born cross-sections in different lineshape hypotheses as the systematic uncertainties. To evaluate the systematic uncertainty due to fit procedure, we expand the fit ranges and change the background parameterization from a linear function to a uniform or a quadratic function, taking the differences in the signal yields between the nominal fit and the alternative fit as the systematic uncertainty. We find that the significance of the observation of $e^+e^- \rightarrow \eta\Upsilon(2S)$ is always greater than 6.0σ in all fit ranges and background parameterizations. For $e^+e^- \rightarrow \gamma X_b$, to be conservative, the largest upper limits are taken as the nominal results from fits with various fitting ranges.

From studies of $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$, we assign a systematic uncertainty of 1% to the trigger efficiency simulation. Tracking uncertainty of 0.3% per track is obtained from $\bar{B}^0 \rightarrow D^{*+}[\rightarrow D^0\pi^+]\pi^-$ and $e^+e^- \rightarrow \tau^+\tau^-$ control samples in data. The uncertainty on the photon detection efficiency, studied with $e^+e^- \rightarrow \mu^+\mu^-\gamma_{\text{ISR}}$, is 3.5% per photon. The uncertainty on the reconstruction efficiency for the π^0 produced in $\eta \rightarrow \pi^+\pi^-\pi^0$ is 4.3% based on studies of $B^+ \rightarrow \pi^+\bar{D}^{*0}$, $\bar{D}^{*0} \rightarrow \bar{D}^0\pi^0$. The uncertainty of the integrated luminosity is 0.7%, and the uncertainty from the choice of generator is estimated to be 1.5% [36]. The uncertainties on pion and electron identification efficiencies are negligible compared

Table I: Summary of the c.m. energy-dependent produced signal yield (N_{prod}) and its statistical significance, ISR correction factors ($1 + \delta$), weighted efficiencies (ϵ), and Born cross-sections or upper limits at 90% C.L. ($\sigma_{\text{B}}^{\text{UL}}$). There are two efficiencies for $\eta\Upsilon(2S)$ channel: the $\eta \rightarrow \gamma\gamma$ mode (left) and $\eta \rightarrow \pi^+\pi^-\pi^0$ mode (right). For γX_b channel, $\sigma_{\text{B}}^{\text{UL}}$ is the upper limit of $\sigma(e^+e^- \rightarrow \gamma X_b) \times \mathcal{B}(X_b \rightarrow \pi^+\pi^-\chi_{bJ})$ at 90% C.L. Uncertainty values for the signal yields are statistical only. For the Born cross sections, the first uncertainty is statistical and the second systematic.

Mode	$N_{\text{prod}} (\times 10^3)$	$(1 + \delta)$	$\epsilon(\%)$	$\sigma_{\text{B}}^{\text{UL}} (\text{pb})$
$\sqrt{s} = (10653.30 \pm 1.14) \text{ MeV}, \quad \mathcal{L} = 3.512 \text{ fb}^{-1}$				
$\eta\Upsilon(2S)$	$(3.7_{-1.3}^{+1.6}), 4.2\sigma$	0.843	19.2/15.1	$1.16_{-0.41}^{+0.51} \pm 0.38$
$\eta\Upsilon(1S)$	< 0.4	0.895	23.9	< 0.10
γX_b	< 0.3	0.784	32.0	< 0.14
$\sqrt{s} = (10700.90 \pm 0.63) \text{ MeV}, \quad \mathcal{L} = 1.632 \text{ fb}^{-1}$				
$\eta\Upsilon(2S)$	$(0.0_{-0.0}^{+1.0})$	1.691	13.2/7.6	$0.00_{-0.00}^{+0.34} \pm 0.50$
$\eta\Upsilon(1S)$	< 0.4	0.901	24.0	< 0.22
γX_b	< 0.1	0.803	31.3	< 0.09
$\sqrt{s} = (10746.30 \pm 0.48) \text{ MeV}, \quad \mathcal{L} = 9.818 \text{ fb}^{-1}$				
$\eta\Upsilon(2S)$	$(3.2_{-1.2}^{+1.6}), 4.8\sigma$	0.673	17.4/14.2	$0.45_{-0.17}^{+0.23} \pm 0.05$
$\eta\Upsilon(1S)$	< 0.9	0.906	23.8	< 0.09
γX_b	< 1.4	0.817	29.8	< 0.17
$\sqrt{s} = (10804.50 \pm 0.70) \text{ MeV}, \quad \mathcal{L} = 4.689 \text{ fb}^{-1}$				
$\eta\Upsilon(2S)$	$(1.5_{-0.9}^{+1.3}), 2.8\sigma$	0.822	17.1/15.2	$0.36_{-0.21}^{+0.33} \pm 0.04$
$\eta\Upsilon(1S)$	< 0.4	0.912	24.6	< 0.08
γX_b	< 1.3	0.833	28.2	< 0.32

to other systematic uncertainties. The uncertainties in all the branching fractions used in this analysis are taken from Ref. [1]. All systematic uncertainties are combined in quadrature [34]. The Born cross sections or their upper limits after considering the systematic uncertainties are summarized in Table I.

In summary, we observe the process $e^+e^- \rightarrow \eta\Upsilon(2S)$ using four samples of e^+e^- collision data taken at c.m. energies near the $\Upsilon(10753)$ with a significance greater than 6.0σ . The measured Born cross sections suggest that the signal is unlikely to originate entirely from the $\Upsilon(5S)$ or $\Upsilon(10753)$ states, favoring a contribution from a possible new state near 10.60 GeV. Although we cannot extract the resonance parameters due to an insufficient number of scan data points, the evidence for this new state is greater than 3.2σ . We also search for the process $e^+e^- \rightarrow \eta\Upsilon(1S)$, as well as for $e^+e^- \rightarrow \gamma X_b$, where X_b is the bottomonium-sector partner of the $X(3872)$. No evidence of a $e^+e^- \rightarrow \eta\Upsilon(1S)$ or $e^+e^- \rightarrow \gamma X_b$ signal is found. The upper limits of $e^+e^- \rightarrow \eta\Upsilon(1S)$ and γX_b with $X_b \rightarrow \pi^+\pi^-\chi_{bJ, J=1,2}$ are estimated at 90% C.L.

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PROJECTION OF THE η DISTRIBUTION MASS DISTRIBUTION FOR THE COMBINED DATA SAMPLE

Projections of invariant masses $M'(\gamma\gamma)$ and $M(\pi^+\pi^-\pi^0)$ are shown in Fig. 1, with data from all four energy points combined. The spectra are fitted with the same method as the individual fit.

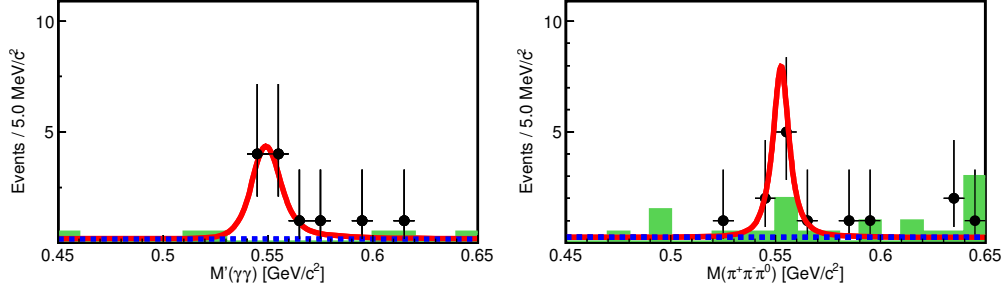


Figure 4: Simultaneous fit to $M(\gamma\gamma)$ and $M(\pi^+\pi^-\pi^0)$ in the data sample where all four energy points are combined; the green histogram shows events from the $\Upsilon(2S)$ sidebands.

PROFILE LIKELIHOOD DISTRIBUTIONS IN $e^+e^- \rightarrow \eta\Upsilon(2S)$

We provide the profiled likelihood distributions as a function of the Born cross sections at each energy point. The likelihoods are obtained from the simultaneous fit to $M'(\gamma\gamma)$ and $M(\pi^+\pi^-\pi^0)$ for the $e^+e^- \rightarrow \eta\Upsilon(2S)$ yield. These profiled likelihoods are used as input in the Born cross section fit.

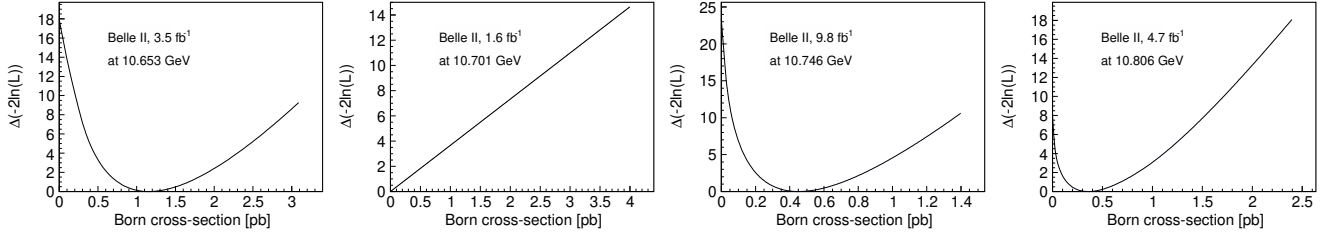


Figure 5: $\Delta(-2\ln(L))$ as a function of the Born cross-section in the fit to the $e^+e^- \rightarrow \eta\Upsilon(2S)$ signal yield.

INVARIANT MASS DISTRIBUTIONS OF $\pi^+\pi^-\pi^0$ IN $e^+e^- \rightarrow \eta\Upsilon(1S)$

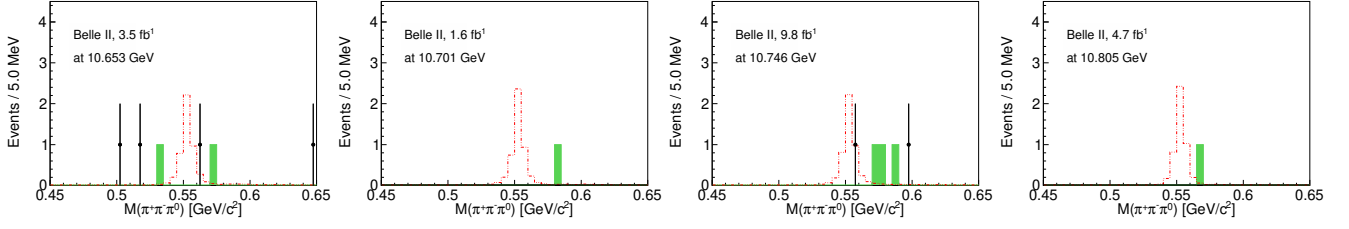


Figure 6: Invariant mass distributions of $\pi^+\pi^-\pi^0$ in the $e^+e^- \rightarrow \eta\Upsilon(1S)$ channel. Dots with error bars are from data in the $\Upsilon(1S)$ signal region, green shaded histograms are from events in $\Upsilon(1S)$ sideband region, and the red histograms are from the signal MC simulation (with arbitrary normalization).

INVARIANT MASS DISTRIBUTIONS OF $M(\pi^+\pi^-\chi_{bJ})$ IN $e^+e^- \rightarrow \gamma X_b$

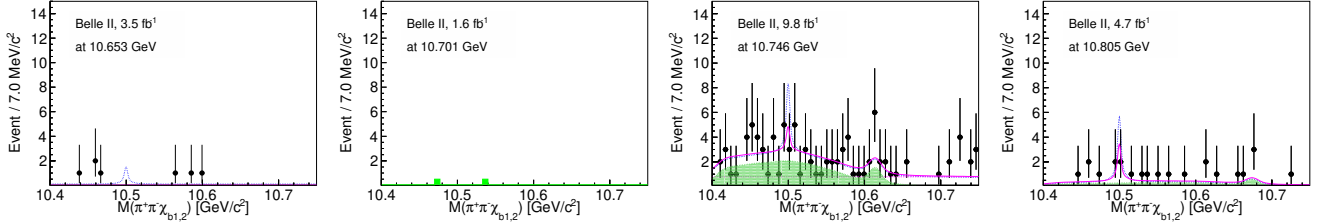


Figure 7: Invariant mass distributions of $M(\pi^+\pi^-\chi_{bJ})$ in the $e^+e^- \rightarrow \gamma X_b$ channel superimposed with the fit results. Dots with error bars are from data, while the solid line presents the nominal fit result, and the shaded histograms represent the background from $\omega\chi_{bJ}$ and $\pi^+\pi^-\Upsilon(2S)[\gamma\chi_{bJ}]$. Blue dashed lines represent the fit results with the contribution of X_b set to the upper limits at 90% credibility level.

SUMMARY OF SYSTEMATIC UNCERTAINTY

Mode	\mathcal{B}	\mathcal{L}	Tracking	Photon(π^0)	Generator	Trigger	Fit	ISR	Model	Sum
10.653 GeV										
$\eta\Upsilon(2S)$	8.9	0.7	2.6	3.7	1.4	1.0	1.2	31.5	< 0.1	33.1
$\eta\Upsilon(1S)$	1.9	0.7	1.0	4.0	1.4	1.0	-	2.0	< 0.1	5.3
γX_b	5.9	0.7	1.7	3.8	1.4	1.0	-	4.0	8.2	11.1
10.701 GeV										
$\eta\Upsilon(2S)$	8.9	0.7	2.4	3.7	1.4	1.0	-	-	< 0.1	10.4
$\eta\Upsilon(1S)$	1.9	0.7	1.0	4.1	1.4	1.0	-	-	< 0.1	4.5
γX_b	5.9	0.7	1.7	3.8	1.4	1.0	-	-	9.1	11.8
10.746 GeV										
$\eta\Upsilon(2S)$	8.9	0.7	2.2	3.7	1.4	1.0	1.2	8.9	< 0.1	13.5
$\eta\Upsilon(1S)$	1.9	0.7	1.0	4.1	1.4	1.0	-	2.3	< 0.1	5.5
γX_b	5.9	0.7	1.7	3.8	1.4	1.0	-	1.7	12.2	14.4
10.805 GeV										
$\eta\Upsilon(2S)$	8.9	0.7	2.0	3.7	1.4	1.0	6.0	8.3	< 0.1	14.3
$\eta\Upsilon(1S)$	1.9	0.7	1.0	4.1	1.4	1.0	-	2.3	< 0.1	5.5
γX_b	5.9	0.7	1.7	3.7	1.4	1.0	-	1.3	9.7	12.3

Table II: Summary of the systematic uncertainties in percent for the cross section measurement of $e^+e^- \rightarrow \pi^+\pi^-\Upsilon(nS)$ and $e^+e^- \rightarrow \gamma X_b$. The symbol “-” denotes an uncertainty value which is not applicable.