

Measurement of the time-integrated CP asymmetry in $D^0 \rightarrow K_S^0 K_S^0$ decays using opposite-side flavor tagging at Belle and Belle II

I. Adachi[✉], Y. Ahn[✉], N. Akopov[✉], S. Alghamdi[✉], M. Alhakami[✉], A. Aloisio[✉], N. Althubiti[✉], K. Amos[✉], M. Angelsmark[✉], N. Anh Ky[✉], C. Antonioli[✉], D. M. Asner[✉], H. Atmacan[✉], T. Aushev[✉], M. Aversano[✉], R. Ayad[✉], V. Babu[✉], H. Bae[✉], N. K. Baghel[✉], S. Bahinipati[✉], P. Bambade[✉], Sw. Banerjee[✉], M. Barrett[✉], M. Bartl[✉], J. Baudot[✉], A. Baur[✉], A. Beaubien[✉], F. Becherer[✉], J. Becker[✉], J. V. Bennett[✉], F. U. Bernlochner[✉], V. Bertacchi[✉], M. Bertemes[✉], E. Bertholet[✉], M. Bessner[✉], S. Bettarini[✉], B. Bhuyan[✉], F. Bianchi[✉], T. Bilka[✉], D. Biswas[✉], A. Bobrov[✉], D. Bodrov[✉], A. Bondar[✉], J. Borah[✉], A. Boschetti[✉], A. Bozek[✉], M. Bračko[✉], P. Branchini[✉], R. A. Briere[✉], T. E. Browder[✉], A. Budano[✉], S. Bussino[✉], M. Campajola[✉], L. Cao[✉], G. Casarosa[✉], C. Cecchi[✉], P. Cheema[✉], B. G. Cheon[✉], K. Chilikin[✉], J. Chin[✉], K. Chirapatpimol[✉], H.-E. Cho[✉], K. Cho[✉], S.-J. Cho[✉], S.-K. Choi[✉], S. Choudhury[✉], I. Consigny[✉], L. Corona[✉], J. X. Cui[✉], E. De La Cruz-Burelo[✉], S. A. De La Motte[✉], G. de Marino[✉], G. De Pietro[✉], R. de Sangro[✉], M. Destefanis[✉], A. Di Canto[✉], J. Dingfelder[✉], Z. Doležal[✉], I. Domínguez Jiménez[✉], T. V. Dong[✉], M. Dorigo[✉], G. Dujany[✉], P. Ecker[✉], D. Epifanov[✉], R. Farkas[✉], P. Feichtinger[✉], T. Ferber[✉], T. Fillinger[✉], C. Finck[✉], G. Finocchiaro[✉], A. Fodor[✉], F. Forti[✉], B. G. Fulsom[✉], A. Gabrielli[✉], A. Gale[✉], E. Ganiev[✉], M. Garcia-Hernandez[✉], R. Garg[✉], G. Gaudino[✉], V. Gaur[✉], V. Gautam[✉], A. Gaz[✉], A. Gellrich[✉], D. Ghosh[✉], H. Ghumaryan[✉], G. Giakoustidis[✉], R. Giordano[✉], A. Giri[✉], P. Gironella Gironell[✉], B. Gobbo[✉], R. Godang[✉], O. Gogota[✉], P. Goldenzweig[✉], W. Gradl[✉], E. Graziani[✉], D. Greenwald[✉], Z. Gruberová[✉], Y. Guan[✉], K. Gudkova[✉], I. Haide[✉], Y. Han[✉], H. Hayashii[✉], S. Hazra[✉], C. Hearty[✉], M. T. Hedges[✉], A. Heidelberg[✉], G. Heine[✉], I. Heredia de la Cruz[✉], M. Hernández Villanueva[✉], T. Higuchi[✉], M. Hoek[✉], M. Hohmann[✉], P. Horak[✉], C.-L. Hsu[✉], T. Humair[✉], T. Iijima[✉], K. Inami[✉], G. Inguglia[✉], N. Ipsita[✉], A. Ishikawa[✉], R. Itoh[✉], M. Iwasaki[✉], P. Jackson[✉], D. Jacobi[✉], W. W. Jacobs[✉], D. E. Jaffe[✉], Q. P. Ji[✉], S. Jia[✉], Y. Jin[✉], A. Johnson[✉], J. Kandra[✉], K. H. Kang[✉], G. Karyan[✉], T. Kawasaki[✉], F. Keil[✉], C. Ketter[✉], M. Khan[✉], C. Kiesling[✉], D. Y. Kim[✉], J.-Y. Kim[✉], K.-H. Kim[✉], K. Kinoshita[✉], P. Kodyš[✉], T. Koga[✉], S. Kohani[✉], K. Kojima[✉], A. Korobov[✉], S. Korpar[✉], E. Kovalenko[✉], R. Kowalewski[✉], P. Križan[✉], P. Krokovny[✉], K. Kumara[✉], T. Kunigo[✉], A. Kuzmin[✉], Y.-J. Kwon[✉], K. Lalwani[✉], T. Lam[✉], J. S. Lange[✉], T. S. Lau[✉], M. Laurenza[✉], R. Leboucher[✉], F. R. Le Diberder[✉], M. J. Lee[✉], C. Lemettais[✉], P. Leo[✉], P. M. Lewis[✉], H.-J. Li[✉], L. K. Li[✉], Q. M. Li[✉], W. Z. Li[✉], Y. Li[✉], Y. B. Li[✉], Y. P. Liao[✉], J. Libby[✉], J. Lin[✉], S. Lin[✉], V. Lisovskyi[✉], M. H. Liu[✉], Q. Y. Liu[✉], Y. Liu[✉], Z. Liu[✉], D. Liventsev[✉], S. Longo[✉], C. Lyu[✉], Y. Ma[✉], C. Madaan[✉], M. Maggiora[✉], S. P. Maharana[✉], R. Maiti[✉], G. Mancinelli[✉], R. Manfredi[✉], E. Manoni[✉], M. Mantovano[✉], D. Marcantonio[✉], S. Marcello[✉], C. Marinas[✉], C. Martellini[✉], A. Martens[✉], T. Martinov[✉], L. Massaccesi[✉], M. Masuda[✉], S. K. Maurya[✉], M. Maushart[✉], J. A. McKenna[✉], F. Meier[✉], M. Merola[✉], C. Miller[✉], M. Mirra[✉], S. Mitra[✉], K. Miyabayashi[✉], R. Mizuk[✉], G. B. Mohanty[✉], S. Moneta[✉], H.-G. Moser[✉], M. Nakao[✉], H. Nakazawa[✉], Y. Nakazawa[✉], M. Naruki[✉], Z. Natkaniec[✉], A. Natochii[✉], M. Nayak[✉], M. Neu[✉], S. Nishida[✉], S. Ogawa[✉], R. Okubo[✉], H. Ono[✉], E. R. Oxford[✉], G. Pakhlova[✉], S. Pardi[✉], K. Parham[✉], H. Park[✉], J. Park[✉], K. Park[✉], S.-H. Park[✉], A. Passeri[✉], S. Patra[✉], R. Pestotnik[✉], L. E. Piilonen[✉], P. L. M. Podesta-Lerma[✉], T. Podobnik[✉], A. Prakash[✉], C. Praz[✉], S. Prell[✉], E. Prencipe[✉], M. T. Prim[✉], S. Privalov[✉], H. Purwar[✉], P. Rados[✉], G. Raeuber[✉], S. Raiz[✉], V. Raj[✉], K. Ravindran[✉], J. U. Rehman[✉], M. Reif[✉], S. Reiter[✉], M. Remnev[✉], L. Reuter[✉], D. Ricalde Herrmann[✉], I. Ripp-Baudot[✉], G. Rizzo[✉], J. M. Roney[✉], A. Rostomyan[✉], N. Rout[✉], L. Salutati[✉], D. A. Sanders[✉], S. Sandilya[✉], L. Santelj[✉], V. Savinov[✉], B. Scavino[✉], C. Schmitt[✉], J. Schmitz[✉], S. Schneider[✉], G. Schnell[✉], M. Schnepf[✉], C. Schwanda[✉], A. J. Schwartz[✉], Y. Seino[✉], A. Selce[✉], K. Senyo[✉], J. Serrano[✉], M. E. Sevier[✉], C. Sfienti[✉], W. Shan[✉], X. D. Shi[✉], T. Shillington[✉], J.-G. Shiu[✉], D. Shtol[✉], B. Schwartz[✉], A. Sibidanov[✉], F. Simon[✉], J. Skorupa[✉], R. J. Sobie[✉], M. Sobotzik[✉], A. Soffer[✉], A. Sokolov[✉], E. Solovieva[✉], S. Spataro[✉], B. Spruck[✉], M. Starič[✉], P. Stavroulakis[✉], S. Stefkova[✉], L. Stoetzer[✉], R. Stroili[✉], Y. Sue[✉], M. Sumihama[✉], N. Suwonjandee[✉], H. Svidras[✉], M. Takizawa[✉], K. Tanida[✉], F. Tenchini[✉], F. Testa[✉], O. Tittel[✉], R. Tiwary[✉], E. Torassa[✉], K. Trabelsi[✉], F. F. Trantou[✉], I. Tsaklidis[✉], M. Uchida[✉], I. Ueda[✉], T. Uglov[✉], K. Unger[✉], Y. Unno[✉], K. Uno[✉], S. Uno[✉], Y. Ushiroda[✉], R. van Tonder[✉], K. E. Varvell[✉], M. Veronesi[✉], A. Vinokurova[✉], V. S. Vismaya[✉], L. Vitale[✉], R. Volpe[✉], A. Vossen[✉], S. Wallner[✉], M.-Z. Wang[✉], A. Warburton[✉], M. Watanabe[✉], S. Watanuki[✉], C. Wessel[✉], E. Won[✉], B. D. Yabsley[✉], S. Yamada[✉], W. Yan[✉], S. B. Yang[✉], J. Yelton[✉], J. H. Yin[✉], K. Yoshihara[✉], J. Yuan[✉], Y. Yusa[✉], L. Zani[✉], M. Zeyrek[✉], B. Zhang[✉], V. Zhilich[✉], J. S. Zhou[✉], Q. D. Zhou[✉], L. Zhu[✉], and R. Žlebčík[✉]

(Belle and Belle II Collaborations)



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We measure the time-integrated CP asymmetry in $D^0 \rightarrow K_S^0 K_S^0$ decays reconstructed in $e^+e^- \rightarrow c\bar{c}$ events collected by the Belle and Belle II experiments. The corresponding data samples have integrated luminosities of 980 and 428 fb $^{-1}$, respectively. To infer the flavor of the D^0 meson, we exploit the correlation between the flavor of the reconstructed decay and the electric charges of particles reconstructed in the rest of the $e^+e^- \rightarrow c\bar{c}$ event. This results in a sample which is independent from any other previously used at Belle or Belle II. The result, $A_{CP}(D^0 \rightarrow K_S^0 K_S^0) = (1.3 \pm 2.0 \pm 0.2)\%$, where the first uncertainty is statistical and the second systematic, is consistent with previous determinations and with CP symmetry.

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I. INTRODUCTION

Decays of charm hadrons offer a unique avenue for exploring flavor and charge-parity (CP) violation in the sector of up-type quarks, which is complementary to the searches performed with strange and beauty hadrons. The dynamics of charm decays is complicated by the presence of nonperturbative QCD effects that are difficult to calculate, making it a test case for both the electroweak and strong interactions. After being first observed in 2019 [1], CP violation in charm decays has gained renewed attention. The nature of the observed CP violation has yet to be fully understood, and could be due to enhanced nonperturbative QCD effects or to physics beyond the standard model [2–13]. Flavor and isospin symmetries can be used to relate measurements from different decay modes, helping to constrain nonperturbative QCD effects and identify possible new physics contributions [5,14,15]. Hence, searches in additional channels and improved measurements of CP asymmetries in already explored decay modes are important.

In this paper, we report a measurement of the time-integrated CP asymmetry in $D^0 \rightarrow K_S^0 K_S^0$ decays using a combination of Belle and Belle II data, which have integrated luminosities of 980 and 428 fb $^{-1}$, respectively [16,17]. The time-integrated CP asymmetry is defined as

$$A_{CP}(D^0 \rightarrow K_S^0 K_S^0) = \frac{\Gamma(D^0 \rightarrow K_S^0 K_S^0) - \Gamma(\bar{D}^0 \rightarrow K_S^0 K_S^0)}{\Gamma(D^0 \rightarrow K_S^0 K_S^0) + \Gamma(\bar{D}^0 \rightarrow K_S^0 K_S^0)}, \quad (1)$$

where Γ indicates the decay rate integrated over decay time, which includes effects due to D^0 - \bar{D}^0 mixing. Experimental measurements of $A_{CP}(D^0 \rightarrow K_S^0 K_S^0)$ [18–22] are consistent with CP symmetry within about 1%, putting them at the upper limit of that predicted by the standard model

and in the range where new-physics contributions might be seen [15,23–26].

An important part in the measurement of $A_{CP}(D^0 \rightarrow K_S^0 K_S^0)$ is the ability to determine the production flavor of the neutral D meson, which is referred to as “tagging.” All measurements performed so far use neutral D decays originating from the strong-interaction decay $D^{*+} \rightarrow D^0 \pi^+$, where the charge of the accompanying pion can be used for tagging. (Charge-conjugate modes are implied throughout the paper, unless stated otherwise.) In this measurement, instead, we use the charm flavor tagger (CFT) described in Ref. [27]. The CFT identifies the flavor of a reconstructed neutral D meson by exploiting correlations with the electric charges of particles reconstructed in the rest of the $e^+e^- \rightarrow c\bar{c}$ event. These include those originating from the decay of the other charm hadron produced in the event, as well as those possibly produced in association with the reconstructed D meson, such as in $D^{*+} \rightarrow D^0 \pi^+$ decays. To avoid correlations with the recently published D^{*+} -tagged measurement of Ref. [22], we use an independent dataset where all the candidates previously used are removed. As a consequence, the CFT in this measurement acts as an “opposite-side” tagger using information from the other charm hadron in the event. The CFT is calibrated on large data samples of $D^0 \rightarrow K^- \pi^+$ decays, following the procedure described in Ref. [27]. The time-integrated CP asymmetry is then determined from an unbinned maximum-likelihood fit to the two-dimensional distribution of the $K_S^0 K_S^0$ mass and the CFT output. To avoid potential bias, an arbitrary and undisclosed offset was added to the measured value of $A_{CP}(D^0 \rightarrow K_S^0 K_S^0)$ when fitting to the data. The offset remained undisclosed until the entire analysis procedure was completed and the determination of all uncertainties was finalized.

The paper is organized as follows. Sec. II provides an overview of the Belle and Belle II detectors. Sec. III details the simulation samples used in the measurement. The reconstruction and selection of the signal $D^0 \rightarrow K_S^0 K_S^0$ decays is presented in Sec. IV. Determination of the asymmetry is covered in Sec. V, followed by a discussion of the systematic uncertainties affecting the measurement in Sec. VI. The final results, and a combination with the D^{*+} -tagged measurement of Ref. [22], are presented in Sec. VII.

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II. BELLE AND BELLE II DETECTORS

The Belle experiment [16,28] operated at the KEKB asymmetric-energy e^+e^- collider [29,30] between 1999 and 2010. The detector consisted of a large-solid-angle spectrometer, which included a double-sided silicon-strip vertex detector, a 50-layer central drift chamber, an array of aerogel threshold Cherenkov counters, a barrel-like arrangement of time-of-flight scintillation counters, and an electromagnetic calorimeter comprised of CsI(Tl) crystals. All subdetectors were located inside a superconducting solenoid coil that provided a 1.5 T magnetic field. An iron flux-return yoke, placed outside the coil, was instrumented with resistive-plate chambers to detect K_L^0 mesons and identify muons. Two inner detector configurations were used: a 2.0 cm radius beam pipe and a three-layer silicon vertex detector; and, from October 2003, a 1.5 cm radius beam pipe, a four-layer silicon vertex detector, and a small-inner-cell drift chamber [31].

The Belle II detector [32,33] is an upgrade with several new subdetectors designed to handle the significantly larger beam-related backgrounds of the new SuperKEKB e^+e^- collider [34]. It consists of a silicon vertex detector wrapped around a 1 cm radius beam pipe and comprising two inner layers of pixel detectors and four outer layers of double-sided strip detectors, a 56-layer central drift chamber, a time-of-propagation detector, an aerogel ring-imaging Cherenkov detector, and an electromagnetic calorimeter, all located inside the same solenoid as used for Belle. A flux return outside the solenoid is instrumented with resistive-plate chambers, plastic scintillator modules, and an upgraded read-out system to detect muons and K_L^0 mesons. For the data used in this paper, collected between 2019 and 2022, only part of the second layer of the pixel detector, covering 15% of the azimuthal angle, was installed.

For both experiments, the z axis of the laboratory frame is defined as the central axis of the solenoid, with its positive direction determined by the direction of the electron beam.

III. SIMULATION

We use simulated event samples to identify sources of background, optimize selection criteria, and validate the analysis procedure. We generate $e^+e^- \rightarrow \Upsilon(nS)$ ($n = 4, 5$) events and simulate particle decays with EvtGen [35]; we generate continuum $e^+e^- \rightarrow q\bar{q}$ (where q is a u, d, c , or s quark) with Pythia6 [36] for Belle, and with KKMC [37] and Pythia8 [38] for Belle II; we simulate final-state radiation with Photos [39,40]; we simulate detector response using Geant3 [41] for Belle and Geant4 [42] for Belle II. Beam backgrounds are taken into account by overlaying random trigger data.

IV. RECONSTRUCTION AND EVENT SELECTION

We use the Belle II analysis software framework (basf2) to reconstruct both Belle and Belle II data [43,44]. The Belle data are converted to the Belle II format for basf2 compatibility using the B2BII framework [45].

Events are selected by a trigger based on either the total energy deposited in the electromagnetic calorimeter or the number of charged-particle tracks reconstructed in the central drift chamber. The efficiency of the trigger is found to be close to 100% for signal decays.

Candidate $K_S^0 \rightarrow \pi^+\pi^-$ decays are reconstructed from combinations of oppositely charged particles that are constrained to originate from a common vertex. These particles are assumed to be pions and the resulting dipion mass is required to be in the $[0.45, 0.55]$ GeV/c^2 range. Pairs of K_S^0 candidates are combined to form candidate $D^0 \rightarrow K_S^0 K_S^0$ decays. We perform a kinematic fit [46] to the D^0 candidates by constraining their momentum directions to point back to the measured position of the beam interaction point, and constraining the masses of the two K_S^0 candidates to the nominal K_S^0 mass [47]. Only candidates whose kinematic fits converge are retained for further analysis. The mass of the D^0 candidate, $m(K_S^0 K_S^0)$, is required to be in the range $[1.84, 2.00]$ GeV/c^2 to exclude partially reconstructed $D_s^+ \rightarrow K_S^0 K_S^0 \pi^+$ decays, which peak at lower mass values. The D^0 momentum in the e^+e^- center-of-mass system is required to be greater than 2.2 GeV/c to suppress candidates in which the D^0 meson arises from the decay of a B meson, as the CFT is trained and calibrated for $e^+e^- \rightarrow c\bar{c}$ events. Candidates that are also reconstructed in the D^{*+} -tagged analysis of Ref. [22] are removed.

To suppress $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays, we use the large distance (L) between the K_S^0 and D^0 decay vertices resulting from the long K_S^0 lifetime. We introduce the variable $S_{\min}(K_S^0) = \log[\min(L_1/\sigma_{L_1}, L_2/\sigma_{L_2})]$, where $L_{1(2)}$ and $\sigma_{L_{1(2)}}$ are the distance and its uncertainty for the first (second) K_S^0 candidate, respectively [22]. Candidates satisfying $\min(L_1/\sigma_{L_1}, L_2/\sigma_{L_2}) \leq 0$ are removed. We require $S_{\min}(K_S^0)$ to be larger than 1.75 and 2.05 for the Belle and Belle II samples, respectively.

Combinatorial background from two unrelated K_S^0 candidates is suppressed using the output of a boosted decision tree (BDT) trained to discriminate such background from signal decays [48,49]. The BDT is trained using simulated $D^0 \rightarrow K_S^0 K_S^0$ decays as signal, and data candidates populating the $m(K_S^0 K_S^0)$ sideband $[1.90, 2.00]$ GeV/c^2 as background. The input variables to the BDT are selected to effectively separate the signal from the background, while minimizing any correlations with $m(K_S^0 K_S^0)$ and $S_{\min}(K_S^0)$. The input variables are the logarithm of the minimum of the transverse impact parameters of the four

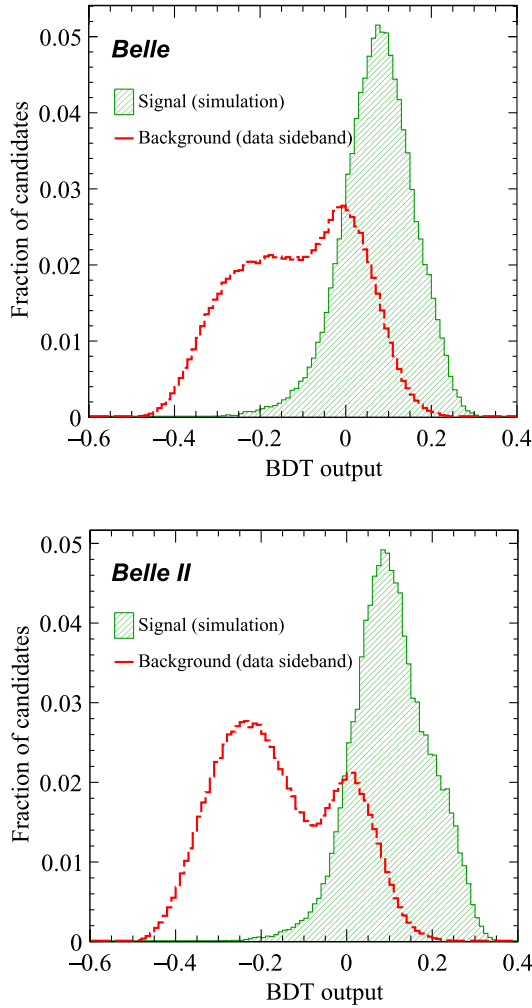


FIG. 1. Distributions of the BDT output for the training samples of (green) signal and (red) background candidates in (top) Belle and (bottom) Belle II.

final-state pions, the logarithm of the minimum of the longitudinal impact parameters of the four final-state pions, the maximum of the momenta of the final-state pions of the K_S^0 candidate with lower momentum, the minimum of the momenta of the final-state pions of the K_S^0 candidate with lower momentum, the absolute value of the polar angle difference between the final-state pions of the K_S^0 candidate with lower momentum, the invariant masses of the two K_S^0 candidates, the scalar sum of the momenta of the two K_S^0 candidates, the absolute value of the asymmetry between the momenta of the two K_S^0 candidates, the flight-distance of the D^0 candidate divided by its uncertainty, and the logarithm of the χ^2 probability of the vertex fit of the D^0 candidate. The BDT output distributions for the training samples of signal and background candidates are shown in Fig. 1. The background distribution shows a two-peak structure, arising from candidates formed with either correctly reconstructed K_S^0 candidates or random pion

combinations. Due to the additional pixel detector layers and the improved reconstruction performance, the Belle II sample has a better signal-to-background separation as compared to Belle. We require the BDT output to exceed 0.037 and 0.075 for Belle and Belle II, respectively.

The requirements on $S_{\min}(K_S^0)$ and on the BDT output are optimized simultaneously by maximizing the quantity $S/\sqrt{S+B}$, where S and B are simulated signal and background yields in a $3\sigma m(K_S^0 K_S^0)$ range around the signal peak (i.e., the signal region shown in Fig. 3), while keeping the residual $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ background below 3% of the $D^0 \rightarrow K_S^0 K_S^0$ yield. The 3% threshold ensures that the measured asymmetry cannot be biased by more than about one tenth of the expected statistical uncertainty, as discussed in Sec. VI. The optimization is done separately for the Belle and Belle II samples. The optimized requirements have signal efficiencies of 55% in Belle and 58% in Belle II, and background rejections of 87% for Belle and 94% for Belle II.

We use the CFT to predict the flavor q of the selected candidates ($q = +1$ for D^0 , $q = -1$ for \bar{D}^0). The tagger also outputs an associated dilution factor r , which is related to the per-candidate mistag probability ω by $r = 1 - 2\omega$. We remove untagged candidates, i.e., candidates for which the CFT does not produce any flavor prediction ($qr = 0$), which amount to 0.2% and 0.6% of the total in Belle and Belle II, respectively. We accept all candidates in events where multiple D^0 candidates are reconstructed, which amount to 1% and 0.3% of the total in Belle and Belle II, respectively.

V. DETERMINATION OF THE CP ASYMMETRY

The CP asymmetry is determined using an unbinned maximum-likelihood fit to the $m(K_S^0 K_S^0)$ and r distributions of the two flavors $q = \pm 1$. The $m(K_S^0 K_S^0)$ distribution discriminates the $D^0 \rightarrow K_S^0 K_S^0$ signal component from a smooth combinatorial background. The qr distribution determines the asymmetry between D^0 and \bar{D}^0 candidates. The two-dimensional probability density functions (PDFs) of each component can be factorized into the product of one-dimensional PDFs. The signal $m(K_S^0 K_S^0)$ PDF, $P_s(m)$, is modeled using the sum of two Gaussian distributions. The background PDF, $P_b(m)$, is modeled with an exponential distribution. We model the distributions of r , $P_{s,b}(r)$, using histogram templates extracted directly from the data. For background, we use candidates populating the $m(K_S^0 K_S^0)$ sideband $[1.91, 2.00]$ GeV/ c^2 ; for signal, we subtract the background distribution from that of the candidates in the $m(K_S^0 K_S^0)$ signal region. Simulation shows that the sideband data describe well the r distribution of the background candidates in the signal region.

The symbolic expression of the PDF of a single candidate is

$$P(m, q, r|A_{CP}, A_b, \dots) = f_b(1 + qrA_b)P_b(m|\dots)P_b(r) + (1 - f_b)[1 + qd(r|\dots)A_{CP} + q\Delta_d(r|\dots)]P_s(m|\dots)P_s(r), \quad (2)$$

where f_b is the fraction of background candidates, A_b is the observed background asymmetry, d and Δ_d are third-order polynomials used to calibrate the per-candidate dilution, and the ellipses (...) indicates other fit parameters omitted here for brevity. All parameters are floated in the fit together with the signal asymmetry A_{CP} .

The true dilution, r^{true} , is expressed as a function of the predicted flavor q and per-candidate dilution r using

$$r^{\text{true}}(q, r|p_1, p_2, p_3, \Delta_{p_1}, \Delta_{p_2}, \Delta_{p_3}) = d(r|p_1, p_2, p_3) + q\Delta_d(r|\Delta_{p_1}, \Delta_{p_2}, \Delta_{p_3}), \quad (3)$$

where the coefficients of the polynomials ($p_1, \dots, \Delta_{p_1}, \dots$) are determined by comparing predicted and true dilutions in high-yield data samples of $D^0 \rightarrow K^-\pi^+$ decays, where the neutral D flavor is inferred from the charge of the final-state kaon [27] (Fig. 2). The tagging power (or effective tagging efficiency), computed from the calibrated per-candidate

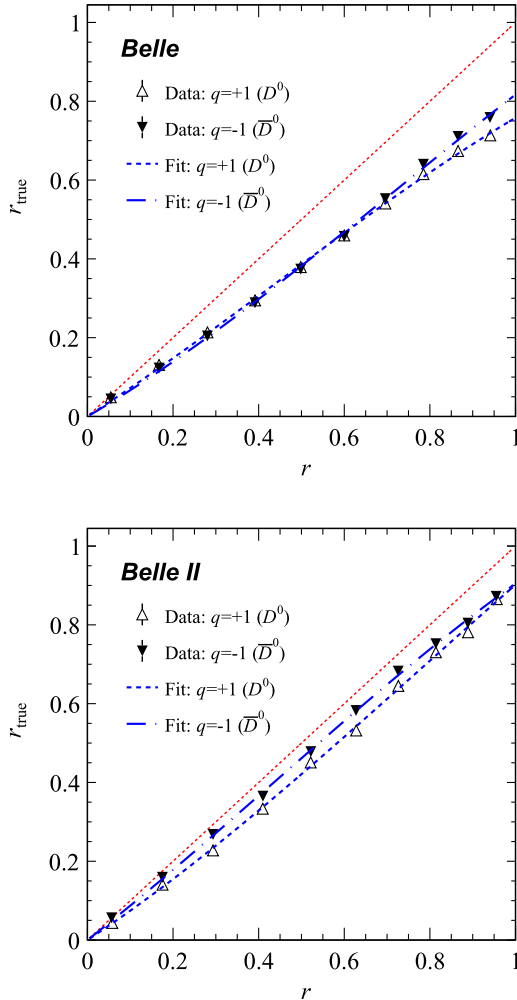


FIG. 2. True dilution as a function of the predicted dilution for $D^0 \rightarrow K^-\pi^+$ and $\bar{D}^0 \rightarrow K^+\pi^-$ decays in (top) Belle and (bottom) Belle II data with projections of the calibration fit overlaid. The bisector of the plane (red dotted line) represents the expected relation for perfectly calibrated predicted dilution.

dilution, is $(23.52 \pm 0.01)\%$ in Belle and $(32.71 \pm 0.05)\%$ in Belle II. The tagging power is lower than reported in Ref. [27] due to the removal of the same-side D^{*+} -tagged candidates. To account for the uncertainties in the calibration parameters, a term constructed from their covariance matrix is included in the likelihood, such that the parameters are Gaussian constrained to their measured values. Thus, the systematic uncertainty associated with the knowledge of the calibration parameters is already taken into account in the statistical uncertainty returned by the fit.

Fits to the simulation and to pseudoexperiments generated by sampling from the PDF show no evidence of a bias in the determinations of the signal yield and asymmetry, or in their uncertainties.

We perform independent fits to Belle and Belle II data. The fit model describes the data well, as shown in Fig. 3. The measured signal yields are 14490 ± 340 in Belle, and 5180 ± 120 in Belle II. The asymmetry is measured to be $(2.5 \pm 2.7)\%$ in Belle, and $(-0.1 \pm 3.0)\%$ in Belle II. The uncertainties are statistical only. The two results are in agreement.

VI. SYSTEMATIC UNCERTAINTIES

We consider the following sources of systematic uncertainties: fit modeling, residual contamination from $D^0 \rightarrow K_S^0\pi^+\pi^-$ decays, and effects due to the forward-backward asymmetry in $e^+e^- \rightarrow c\bar{c}$ production.

We estimate the first using pseudoexperiments generated with the default fit model, and fitted with alternative models derived from data where one of the fit shapes is changed. As alternative models for the mass shapes we use a Johnson S_U distribution [50] for signal and a second-order polynomial for background. For the r distributions, we fill alternative histogram templates by varying the definition of the mass sideband. The alternative models give an equally good description of the data as the default models. The observed average differences between measured and generated asymmetries, 0.35% for Belle and 0.10% for Belle II, are assigned as systematic uncertainties.

The residual $D^0 \rightarrow K_S^0\pi^+\pi^-$ background, which is indistinguishable from the signal in $m(K_S^0K_S^0)$, is neglected in the fit and counted as part of the signal component. This introduces a bias on the measured asymmetry, which can be estimated as the product of the contamination fraction and

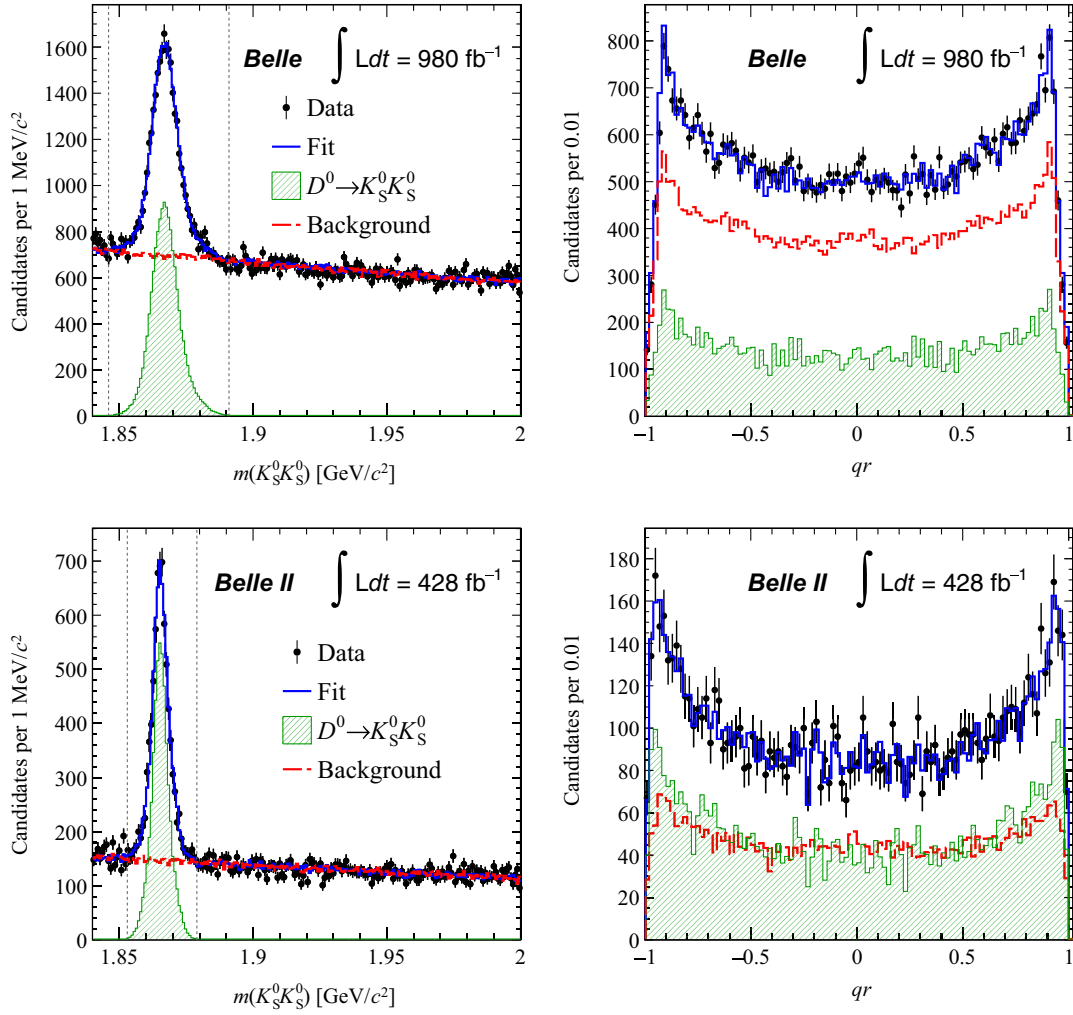


FIG. 3. Distributions of (left) $m(K_S^0 K_S^0)$ and (right) qr for $D^0 \rightarrow K_S^0 K_S^0$ candidates in (top) Belle and (bottom) Belle II data, with fit projections overlaid. The qr distributions are only for candidates in the $m(K_S^0 K_S^0)$ signal regions indicated by the vertical lines.

the difference between the CP asymmetries in $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ and $D^0 \rightarrow K_S^0 K_S^0$ decays. The contamination fraction is estimated in simulation to be 2.5% for Belle, and 2.3% for Belle II. Given that $A_{CP}(D^0 \rightarrow K_S^0 \pi^+ \pi^-) - A_{CP}(D^0 \rightarrow K_S^0 K_S^0) < 10\%$ [47], the bias can be conservatively bounded to be smaller than 0.25% for Belle, and 0.23% for Belle II. These values are assigned as systematic uncertainties due to neglecting $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ contamination, and are conservative enough to also cover possible detection asymmetries in the $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decay.

In $e^+e^- \rightarrow c\bar{c}$ events, charmed hadrons are produced with a forward-backward asymmetry due to γ - Z^0 interference and higher order effects [51–53]. The forward-backward asymmetry is an odd function of the cosine of the polar angle in the center of mass system, $\cos\theta^*$. Since the acceptances of the Belle and Belle II detectors are not the same for $\cos\theta^* > 0$ and $\cos\theta^* < 0$, a charge asymmetry in the production of a given species of charmed

hadrons remains. In our measurement, however, we effectively count pairs of charmed hadrons: the signal $D^0 \rightarrow K_S^0 K_S^0$ and the other (oppositely flavored) charmed hadron of the event, which provides the tagging information. As a result, we expect a negligible effect from the forward-backward asymmetry. To verify this, we weight the reconstructed candidates so that the $|\cos\theta^*|$ distribution of the signal is the same for candidates with $\cos\theta^* > 0$ and $\cos\theta^* < 0$ and redetermine the values of A_{CP} . As expected, we find variations in A_{CP} consistent with zero and do not assign any systematic uncertainty due to the forward-backward asymmetry.

Finally, as a cross-check we fit to subsamples of the data defined according to data-taking conditions and find no significant variation of the measured asymmetry.

The total systematic uncertainties, 0.43% for Belle and 0.25% for Belle II, are evaluated as the sums in quadrature of the components due to the fit modeling and the $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ contamination.

VII. FINAL RESULTS AND CONCLUSIONS

We measure the time-integrated CP asymmetry in $D^0 \rightarrow K_S^0 K_S^0$ decays using a charm-flavor tagging algorithm that exploits the correlation between the flavor of the reconstructed neutral D meson and the electric charges of particles reconstructed in the rest of the $e^+e^- \rightarrow c\bar{c}$ event. Using 980 fb^{-1} of data collected by Belle and 428 fb^{-1} of data collected by Belle II, we obtain

$$A_{CP}(D^0 \rightarrow K_S^0 K_S^0) = (2.5 \pm 2.7 \pm 0.4)\% \quad (4)$$

and

$$A_{CP}(D^0 \rightarrow K_S^0 K_S^0) = (-0.1 \pm 3.0 \pm 0.3)\%, \quad (5)$$

respectively. The first uncertainties are statistical and the second systematic. The two results are in agreement and combined, using the best linear unbiased estimator [54], into

$$A_{CP}(D^0 \rightarrow K_S^0 K_S^0) = (1.3 \pm 2.0 \pm 0.2)\%. \quad (6)$$

In the combination, the systematic uncertainties due to the fit modeling are considered uncorrelated, while those due to the $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ contamination are considered fully correlated.

The results are also consistent with previous Belle and Belle II determinations based on the independent sample of D^{*+} -tagged $D^0 \rightarrow K_S^0 K_S^0$ decays [22]. A combination of the result of this paper with that of Ref. [22], $(-1.4 \pm 1.3 \pm 0.1)\%$, yields

$$A_{CP}(D^0 \rightarrow K_S^0 K_S^0) = (-0.6 \pm 1.1 \pm 0.1)\%. \quad (7)$$

This is the most precise determination of $A_{CP}(D^0 \rightarrow K_S^0 K_S^0)$ to date. It agrees with CP symmetry and with results from other experiments [18–21].

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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