Prepared for submission to JHEP DESY preprint number

Anomalous dimensions at small spins

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Abstract: Anomalous dimensions of twist-two operators, calculated in perturbation theory, may have poles when spin takes negative or small positive integer values, and therefore have to be resummed at these points. In the case of right-most singularity such resummation can be done using the special combination of anomalous dimensions that remains finite. Remarkably, this combination arises in different contexts. In conformal theory it originates from the mixing of leading Regge trajectory and its shadow. Moreover, in $O(N)$ vector φ^4 model this combination describes the corrections to masses of higher-spin fields in the context of the conjectured duality with the gauge theory on AdS_4. Finally, it appears in studies of double-logarithmic asymptotics of scattering amplitudes in QCD. In the paper we present the analysis of the small-spin limit of anomalous dimensions for all types of twist-two operators in the $O(N)$-symmetric φ^4 model at the four-loop level, in the complex φ^3 model at the three-loop level and the Gross-Neveu-Yukawa model at the two-loop level. In all cases we find perfect agreement with the predicted singular behaviour.

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1 Introduction

The scale dependence of the parton densities entering the theoretical description of the processes of Deep Inelastic Scattering [?] is governed by the anomalous dimensions of twist–two operators. At present, the latter are known with three–loop accuracy [1–3] and only partial results are available at four loops [4–6]. Higher order calculations remain extremely challenging and, despite all the advances in computational methods [?], can only provide anomalous dimensions for the operators with low values of spin, $s \leq 20 \div 30$. This data is usually insufficient to restore the full s dependence. Thus, any information about analytical properties of anomalous dimensions as a functions of spin can be very helpful, since it gives additional constraints. For example, the large s behaviour of the anomalous dimensions is determined by the so-called cusp anomalous dimension, [7, 8] currently known with four loop accuracy [9, 10]. The reciprocity relation (RR) [11–13] significantly reduces the number of possible harmonic sums [14] which can appear in the anomalous dimensions.

By Carlson's theorem anomalous dimensions continue to an analytic function in the right complex half-plane. In perturbation theory $\gamma(s) = \sum_k a^k \gamma_k(s)$, where a is a coupling constant and $\gamma_k(s)$ are meromorphic functions with the poles at integers values of spin $s = s_0$. As a rule, the series of poles is bounded from the right with the right-most pole close to $s = 0$ (one finds $s = -1, 0, 1$ in known examples). At the point $s = s_0$ we expect the behaviour $\gamma_k(s) \sim 1/(s - s_0)^{ak+b}$ with a, b being integers which depend on a model. The perturbative expansion for $\gamma(s)$ obviously fails whenever $a/(s - s_0) \sim 1$. Hence, in order to determine the correct analytic structure of $\gamma(s)$ near $s \sim s_0$ the perturbative series has to be resummed.

In QCD the first pole in the singlet sector occur at $s = 1$ and corresponds to the BFKL pomeron [15–18].

In this work we are interested the right-most singularity in the scalar theories in $d = 4, 6$. These singularities share the common feature with the right-most poles in the flavour nonsinglet anomalous dimensions in QCD, where the small$-s$ behavior is deduced from the double-logarithmic asymptotics of scattering amplitudes [19–21]. The leading singular contributions to the anomalous dimensions at $s = 0$ were obtained in [22, 23]. The generalization to subleading powers proposed by Velizhanin [24, 25] (the generalised double-logarithmic equation (GDLE)) states that the combination

$$\delta m^2(s) \equiv 2\left(s + \bar{\beta}(a) + \frac{1}{2}\gamma(s)\right)\gamma(s), \tag{1.1}$$

where $\bar{\beta}(a) = -\beta(a)/2a$, is regular at $s = 0$. It was checked that in the planar QCD and $\mathcal{N} = 4$ SYM theory this expression remains finite at $s \to 0$ up to three and seven loops, while the non-planar corrections possess $1/s$ poles starting from three and four loops, respectively [25, 26]. The leading and subleading poles, $s^{1-2\ell}$ and $s^{2-2\ell}$ in the non-planar contributions cancel out.

Recently, some progress has been made in understanding the structure of anomalous dimensions in conformal field theories (CFT) [27–29]. In CFTs the anomalous dimensions (critical dimensions) of operators play a role of observables and do not depend on the renormalization scheme as in the usual QFT. In the $d = 4-2\epsilon$ theory the critical point is determined by the equation $\bar{\beta}(a_*) = -\epsilon$. Thus changing $\gamma(s, a) \to \gamma(s, a_*)$ in (1.1) one gets a statement on the CFTs observables. It was argued in [29] that singularities in the anomalous dimensions arise at the points of intersection of two or more "Regge" trajectories. Specifically, the trajectory $\Delta(s)$ (or the anomalous dimension $\gamma(s)$) stops to be an analytic function of spin in the vicinity of an intersection point and acquire square root singularities*, which however cancel in the product $\Delta(s)\widetilde{\Delta}(s) = \Delta(s)(d - \Delta(s))$. It means that $\Delta(s)$ and $\widetilde{\Delta}(s)$ are different branches of the same analytic function. The equation (1.1) is equivalent to the statement that the product $\Delta(s)\widetilde{\Delta}(s)$ is analytic in the vicinity of intersection point ($s \sim 0$). It was checked that this is indeed true for the twist-two operators in the scalar φ^4 in $d = 4 - 2\epsilon$ dimensions where the anomalous dimensions are known with four loop accuracy [30]. For more details see ref. [29].

It is remarkable that the expression (1.1) arises in another context. It was conjectured by Klebanov and Polyakov [31] that the critical $O(N)$ vector φ^4 model in $d = 3$ is dual to the higher-spin theory on AdS_4, see also [32, 33]. According to [34–37] masses of the higher-spin fields on the AdS side are related to the scaling dimensions of the twist-two scalar operators (= higher-spin currents) as follows

$$m^2(s) = \Delta(s)\big(\Delta(s) - d\big) - s, \tag{1.2}$$

*This is similar to the situation with energy level crossing in ordinary quantum mechanics.

where masses are measured in the units of the cosmological constant. Writing $m^2(s) = m_0^2(s) + \delta m^2(s)$, and $\Delta(s) = \Delta_0(s) + \gamma(s) = d - 2 + s + \gamma(s)$ one gets

$$\delta m^2(s) = 2\left(s + d/2 - 2 + \frac{1}{2}\gamma(s)\right)\gamma(s)\,. \tag{1.3}$$

The critical exponents can be calculated as series in ϵ in $d = 4 - 2\epsilon$ dimensional theory [38]. Assuming that the radiative corrections to the mass of the scalar field ($s = 0$) in the dual model is finite one concludes that the r.h.s. of Eq. (1.3) has to be regular at $s \to 0$. Therefore, in the context of AdS/CFT correspondence one would expect pole cancellation in (1.3).

So there are various arguments leading to Eq. (1.3). The DLE technique [19–21] predicts cancellation of leading singularities in (1.1) in all orders and this is true in all known examples. The CFT approach [29] predicts cancellation of pole singularities. However, there are examples, in QCD and $N = 4$ SYM, which do not fit into this picture. It would be highly desirable, if only for practical purposes, to understand the origin of this discrepancy and the role CFT/AdS correspondence plays in the pole cancellation.

In this paper we analyze the relation (1.3) in several models: the $O(N)$ symmetric vector φ^4 model, the complex φ^3 and Gross-Neveu-Yukawa models. We calculated the anomalous dimensions for different types of twist two operators and study pole cancellation in the corresponding combination of anomalous dimensions.

The paper is organized as follows: In Sect. 2 we present four loop anomalous dimensions for the scalar, symmetric and antisymmetric twist-two operators in the $O(N)$ symmetric φ^4 model and verify that the mass correction (1.3) are free of $1/s$ poles in all cases. In Sect. 3 we carry out the same analysis in the complex φ^3 model in $d = 6 - 2\epsilon$ dimensions in the three-loop approximation. We again find that the mass corrections for odd-spin operators are finite at spin minus one. In Sect. 4 we analyze Gross-Neveu-Yukawa model in $d = 4 - 2\epsilon$ at two-loop level. We calculate anomalous dimensions of all twist-two operators and find the regularity of mass correction (1.3) for the multiplicatively renormalazible even-spin operators. We also discuss the critically equivalent Gross-Neveu model in $1/N$ expansion framework. We find the regularity of mass corrections (1.3) for the even-spin operators and discuss its connection to the intersection of Regge trajectories at the points $s = \pm(2 - \mu)$ and $s = 0$, where $\mu = d/2$. The last section contains our conclusions. The Appendix A contains explicit three loop expressions for the anomalous dimensions in the complex φ^3.

2 The $O(N)$ symmetric φ^4 model

The $O(N)$ symmetric φ^4 model played an important role in the development of the theory of critical phenomena and provides a basic example of a nontrivial CFT. It represents a natural testing ground for new approaches and methods, see e.g. [39–43]. The model describes dynamics of an N component scalar field φ^a, $a = 1, \ldots, N$, with an $O(N)$ invariant quartic interaction,

$$S_R = \int d^dx \left(\frac{Z_1}{2} (\partial\varphi)^2 + Z_3 \mu^{2\epsilon} \frac{g}{4!} \left(\varphi^2\right)^2 \right). \tag{2.1}$$

Here $Z_{1,2}$ are the renormalization constants in the $\overline{\text{MS}}$ scheme and μ is the renormalization scale. The renormalization group (RG) functions in this model are known with high-precision, see [44, 45] and [46] for recent developments. In particular the beta function and the field anomalous dimension, which we need later, take the form

$$\begin{aligned}
\beta(u) &= \mu\frac{du}{d\mu} = -2\epsilon u + \frac{N+8}{3}u^2 - \frac{3N+14}{3}u^3 \\
&\quad + \frac{u^4}{216}\left[33N^2 + 922N + 2960 + 96(5N+22)\zeta_3\right] + O(u^5) \equiv 2u(-\epsilon + \bar\beta(u)), \\
\gamma_\varphi(u) &= \frac{d\ln Z_1}{d\ln\mu^2} = (N+2)\Big(\frac{u^2}{36} - \frac{u^3(N+8)}{432} + \frac{5u^4}{5184}(-N^2+18N+100)\Big) + O(u^5),
\end{aligned} \tag{2.2}$$

where $u = g/(4\pi)^2$.

We are interested in the anomalous dimensions of twist-2 operators

$$\mathcal{O}_s^{ab} = \mathcal{O}^{ab}_{\mu_1\ldots\mu_s}(x) = \varphi^a(x)\partial_{\mu_1}\ldots\partial_{\mu_s}\varphi^b(x) - \text{traces}. \tag{2.3}$$

It is convenient to single out irreducible components relative to the $O(N)$ group. Namely, we define the scalar, $\mathcal{O}_s = \sum_a \mathcal{O}_s^{aa}$, symmetric-traceless, $\mathcal{O}_s^+ = \frac{1}{2}(\mathcal{O}^{ab} + \mathcal{O}^{ba}) - N^{-1}\delta^{ab}\mathcal{O}_s$, and antisymmetric, $\mathcal{O}_s^- = \frac{1}{2}(\mathcal{O}^{ab} - \mathcal{O}^{ba})$, operators which do not mix under renormalization,

$$[\mathcal{O}_s^\alpha(x)]_{\overline{\text{MS}}} = Z_s^\alpha(u,\epsilon)\mathcal{O}_s^\alpha(x). \tag{2.4}$$

Here α denotes the type of operator (symmetric, antisymmetric or scalar) and $Z_s^\alpha = 1 + \sum_{k=1}^\infty \epsilon^{-k} z_k^\alpha(s,u)$ are the renormalization constants. The anomalous dimensions are given by the following expression

$$\gamma^\alpha(s) = 2\gamma_\varphi - \mu\frac{d}{d\mu}\ln Z_s^\alpha(u,\epsilon), \tag{2.5}$$

where $\gamma_\varphi = \frac{1}{2}\mu\partial_\mu \log Z_1$ is the anomalous dimension of the field φ. For $N = 1$ theory the anomalous dimensions, $\gamma(s)$, were calculated in the four loop approximation in ref. [30]. This result was generalised for the $O(N)$ model in the ref. [47] by recovering the symmetry coefficients for individual graphs

$$\begin{aligned}
\gamma(s) = 2\gamma_\varphi - u^2\frac{N+2}{9s(s+1)}\Bigg\{& 3 + \frac{u}{3}(N+8)\left(2S_1 - 5 + \frac{3}{2}\frac{2s+1}{s(s+1)}\right) + \frac{u^2}{9}\Bigg(\big(N+8\big)^2 S_2 \\
& + \big(N^2+11N+42\big)S_1^2 + \big(5N+22\big)\left(1 - \frac{4}{s(s+1)}\right)S_{-2} + \frac{3(N+2)}{2}\Bigg[S_1 - \frac{11}{2} \\
& + 3\frac{2s+1}{s^2(s+1)^2} + \frac{s-3}{s(s+1)}\Bigg] + \big(N^2+6N+20\big)\left[\left(\frac{1+2s}{s(s+1)} - 4\right)S_1 + \frac{3}{4s^2(s+1)^2}\right.
\end{aligned}$$

$$
-\frac{20s+1}{4s(s+1)}+2\Bigg]+\big(5N+22\big)\Bigg[S_1\bigg(2\frac{2s+1}{s(s+1)}-11\bigg)+\frac{3}{2s^2(s+1)^2}
-\frac{32s+7}{2s(s+1)}+21\Bigg]\Bigg)\Bigg\}+O(u^5)\,. \tag{2.6}
$$

Here $S_k = S_k(s)$ are harmonic sums [14]. Doing the same for the symmetric and antisymmetric operators we obtain the following anomalous dimensions

$$
\gamma^+(s) = 2\gamma_\varphi - \frac{u^2}{9s(s+1)}\Bigg\{N+6+\frac{u}{6}\Bigg(8\,(N+4)\,\bigg[2S_1+\frac{2s+1}{s(s+1)}-4\bigg]-\big(N^2+6N
+16\big)\bigg[2-\frac{2s+1}{s(s+1)}\bigg]\Bigg)+\frac{u^2}{9}\Bigg(4\big(N+4\big)\big(N+8\big)S_2+\big(N^2+28N+84\big)S_1^2
+\bigg(N^2+16N+44-8\frac{5N+22}{s(s+1)}\bigg)S_{-2}+\big(N^3+8N^2+24N+40\big)\bigg[\frac{1}{4s^2(s+1)^2}
-\frac{4s-1}{4s(s+1)}\bigg]+\big(N^2+16N+44\big)\bigg[\bigg(\frac{5}{3}\frac{2s+1}{s(s+1)}-\frac{32}{3}\bigg)S_1+\frac{5}{6s^2(s+1)^2}
-\frac{64s+17}{6s(s+1)}+\frac{52}{3}\bigg]+\frac{3N^2+20N+44}{3}\bigg[\bigg(\frac{2s+1}{s(s+1)}-1\bigg)S_1+\frac{2}{s^2(s+1)^2}
-2\frac{8s+1}{s(s+1)}+11\bigg]+\frac{N+6}{2}\bigg\{(N+2)\,\bigg[S_1+\frac{s}{s(s+1)}-\frac{11}{2}\bigg]+\big(N+6\big)\bigg[\frac{2s+1}{s^2(s+1)^2}
-\frac{1}{s(s+1)}\bigg]+\frac{8(N+5)}{3}\bigg[\bigg(\frac{2s+1}{s(s+1)}-4\bigg)S_1+\frac{1}{2s^2(s+1)^2}-\frac{8s+1}{2s(s+1)}
+2\bigg]\bigg\}\Bigg)\Bigg\}+O(u^5)\,, \tag{2.7}
$$

$$
\gamma^-(s) = 2\gamma_\varphi - u^2\frac{N+2}{9s(s+1)}\Bigg\{1+u\bigg(\frac{2}{3}\bigg[2S_1+\frac{2s+1}{s(s+1)}-4\bigg]-\frac{N+4}{6}\bigg[2-\frac{2s+1}{s(s+1)}\bigg]\bigg)
+\frac{u^2}{9}\Bigg(2\big(N+8\big)S_2-\big(N-6\big)S_1^2-\big(N-2\big)S_{-2}+\big(N^2+6N+12\big)\bigg(\frac{1}{4s^2(s+1)^2}
-\frac{4s-1}{4s(s+1)}\bigg)+\frac{4\big(6+N\big)}{3}\bigg[\bigg(\frac{2s+1}{s(s+1)}-7\bigg)S_1+\frac{1}{2s^2(s+1)^2}-\frac{7s+2}{s(s+1)}+\frac{25}{2}\bigg]
+\frac{10+3N}{3}\bigg[\bigg(\frac{2s+1}{s(s+1)}-1\bigg)S_1+\frac{2}{s^2(s+1)^2}-2\frac{8s+1}{s(s+1)}+11\bigg]
+\frac{14-N}{3}\bigg[\bigg(\frac{2s+1}{s(s+1)}-4\bigg)S_1+\frac{1}{2s^2(s+1)^2}-\frac{8s+1}{2s(s+1)}+2\bigg]+\frac{N+2}{2}\bigg[S_1
+\frac{2s+1}{s^2(s+1)^2}+\frac{s-1}{s(s+1)}-\frac{11}{2}\bigg]\Bigg)\Bigg\}+O(u^5)\,. \tag{2.8}
$$

One can easily check that the anomalous dimensions corresponding to the stress-energy tensor and the conserved $O(N)$ current vanish, $\gamma(2)=0$ and $\gamma^-(1)=0$.

We also checked that the anomalous dimensions $\gamma^\alpha(s)$ have the so-called the reciprocity respecting form [8, 12, 41]. Namely, the functions $f^\alpha(j)$ defined by the equation

$$\gamma^\alpha(s) = f^\alpha(j)\,, \tag{2.9}$$

where $j = s + \bar{\beta}(a) + \frac{1}{2}\gamma(s)$ is the conformal spin, have the reciprocity respecting form, for details see refs. [8, 12, 41]. For the operators in question we obtain

$$\begin{aligned} f(j) = 2\gamma_\varphi - u^2\frac{N+2}{9j(j+1)}\Bigg\{&3 + \frac{2u}{3}(N+8)\left(S_1 - \frac{5}{2}\right) + \frac{u^2}{9}\Bigg((22+5N)\left(1 - \frac{4}{j(j+1)}\right)S_{-2} \\ &+ \left(N^2+11N+42\right)S_1^2 - \frac{16N^2+310N+1276}{4}S_1 + \left(N+8\right)^2\zeta_2 - \frac{21(N+2)}{4j(j+1)} \\ &+ \frac{8N^2+435N+1942}{4}\Bigg)\Bigg\} + O(u^5), \end{aligned} \tag{2.10}$$

$$\begin{aligned} f^+(j) = 2\gamma_\varphi - u^2\frac{1}{9j(j+1)}\Bigg\{&N + 6 + \frac{u}{3}\Big(8(N+4)S_1 - N^2 - 22N - 80\Big) \\ &+ \frac{u^2}{9}\Bigg(\left(N^2+16N+44 - 8\frac{22+5N}{j(j+1)}\right)S_{-2} + \left(N^2+28N+84\right)S_1^2 \\ &- \frac{33N^2+464N+1276}{2}S_1 + 4\left(N+4\right)\left(N+8\right)\zeta_2 - \frac{3N^2+32N+84}{4j(j+1)} \\ &+ \frac{113N^2+1432N+3884}{4}\Bigg)\Bigg\} + O(u^5), \end{aligned} \tag{2.11}$$

$$\begin{aligned} f^-(j) = 2\gamma_\varphi - u^2\frac{N+2}{9j(j+1)}\Bigg\{&1 + \frac{u}{3}\Big(4S_1 - 12 - N\Big) - \frac{u^2}{9}\Bigg((N-2)S_{-2} + (N-6)S_1^2 \\ &+ \frac{17N+154}{2}S_1 - 2(N+8)\zeta_2 + \frac{3\,(N+2)}{4j(j+1)} - \frac{97N+562}{4}\Bigg)\Bigg\} + O(u^5). \end{aligned} \tag{2.12}$$

These expressions contain only reciprocity respecting harmonic sums [48, 49], while all rational terms are expressed through the RR invariant combination $j(j+1)$.

2.1 Small s limit

The anomalous dimensions of all operators are singular in the limit $s \to 0$. At ℓ loops one expects to find contributions which diverge as $s^{-\ell+1}$ when $s \to 0$. Using the results of the previous section it is easy to verify that the combination

$$\delta m_\alpha^2(s) = 2\left(s + \bar{\beta}(a) + \frac{1}{2}\gamma^\alpha(s)\right)\gamma^\alpha(s) \tag{2.13}$$

remains finite in the limit $s \to 0$ up to $O(u^5)$ terms for all operators†. Moreover, for the scalar and symmetric operators the Regge trajectories, as predicted in [29], pass through the point

† $N = 1$ case was considered in ref. [29].

corresponding the local operators, $\mathcal{O}_{s=0} = \varphi^2$, and $\mathcal{O}^+_{s=0} = \varphi^a\varphi^b - N^{-1}\delta^{ab}\varphi^2$. The anomalous dimensions, $\gamma_0 = \gamma_{\varphi^2}$ and γ_0^+, are known with high accuracy, see e.g. [45]

$$\gamma_0 = u\frac{N+2}{3}\left\{1 - \frac{5u}{6} + u^2\frac{(37+5N)}{12}\right\} + O(u^4), \tag{2.14}$$

$$\gamma_0^+ = \frac{u}{3}\left\{2 - u\frac{10+N}{6} - u^2\frac{5N^2-84N-444}{72}\right\} + O(u^4). \tag{2.15}$$

Defining

$$\begin{aligned}
\delta m_0^2 \equiv 2\left(\bar\beta(a) + \frac{1}{2}\gamma_0\right)\gamma_0 = 2u^2(N+2)\Bigg\{ -\frac{1}{3} + u\frac{13(N+8)}{108} \\
- u^2\left(\frac{2(22+5N)}{27}\zeta_3 + \frac{3N^2+950N+4588}{1296}\right)\Bigg\} + O(u^5), \\
\delta m_0^{2,+} \equiv 2\left(\bar\beta(a) + \frac{1}{2}\gamma_0^+\right)\gamma_0^+ = 2u^2\Bigg\{ -\frac{N+6}{9} + u\frac{N^2+50N+208}{108} \\
- u^2\left(\frac{4(22+5N)}{27}\zeta_3 - \frac{5N^3-164N^2-3112N-9176}{1296}\right)\Bigg\} + O(u^5)
\end{aligned} \tag{2.16}$$

we checked that with four loop accuracy

$$\lim_{s\to 0}\delta m^2(s) = \delta m_0^2\,, \qquad \lim_{s\to 0}\delta m_+^2(s) = \delta m_0^{2,+}\,. \tag{2.17}$$

The situation is similar for the antisymmetric operator, $\mathcal{O}_s^-$, although there is no local operator for $s=0$. We found that in the limit $s\to 0$ the mass correction remains finite

$$\lim_{s\to 0}\delta m_-^2(s) = 2u^2(N+2)\left\{-\frac{1}{9} + u\frac{24+N}{108} + u^2\left(\frac{5N^2-158N-1148}{1296}\right)\right\} + O(u^5). \tag{2.18}$$

As we will see, in the Sect. 3 the combination (1.3) remains singular at $s=0$ if anomalous dimensions are continued from *odd* values of s. Therefore, the finiteness of $\delta m_-^2(s)$ in the first four loops requires an additional explanation. We comment on this in sect. 4.1.

Note also that for small s the equation (2.13) defines the anomalous dimension $\gamma(s)$ as an analytic function on a two-sheet Riemann surface, see also [25, 29],

$$\gamma(s) = -s - \bar\beta + \sqrt{(s+\bar\beta)^2 + \delta m^2(s)}. \tag{2.19}$$

The position of the branching points is determined by the expression under the square root. For the $O(N)$ model at the critical point, $\bar\beta = -\epsilon$, both branching points is of order ϵ and lay on the positive axis ($\delta m^2 < 0$ as we see in (2.16), (2.18)). For integer s one branch passes through the anomalous dimensions of the corresponding twist-two operators and another one through the anomalous dimensions of the shadow operators.

Regularity of the mass correction (2.13) can be used as a constraint on the expansion of anomalous dimensions at $s = 0$. Knowing anomalous dimensions at ℓ-th loop order one can find all singular terms except $1/s$ in the next order. The simple pole can be restored from the anomalous dimension of the local operator (2.16). In this way we obtain the five-loop singular part of the anomalous dimension of the scalar operator

$$\begin{aligned}\gamma^{(5)}(s) = (N+2)\Bigg\{&-\frac{(N+8)(N^2+34N+100)}{648s^4} + \frac{(N+8)(5N^2+236N+776)}{972s^3} \\ &- \left(\frac{(14+N)(22+5N)}{81}\zeta_3 + \frac{(8+N)(N^2+28N+88)}{486}\zeta_2\right. \\ &\left. + \frac{16N^3+2067N^2+24270N+63152}{3888}\right)\frac{1}{s^2} + \left(\frac{40N^2+1100N+3720}{243}\zeta_5\right. \\ &+ \frac{(32+N)(22+5N)}{810}\zeta_2^2 - \frac{5N^3-365N^2-5278N-16880}{972}\zeta_3 \\ &\left. + \frac{(N+8)(N+26)(8N+35)}{972}\zeta_2 + \frac{371N^2+6286N+18192}{648}\right)\frac{1}{s}\Bigg\} + O(1). \quad (2.20)\end{aligned}$$

In the case $N = 1$ this prediction is consistent with the one obtained in [29, Eq. (2.28)].

The $O(N)$ symmetric φ^4 model can be also analyzed in the $1/N$ expansion, see e.g. [44, 45] for a review. The corresponding anomalous dimensions for the symmetric and antisymmetric operators were calculated in ref. [50], and for the scalar operators in ref. [47]. We checked that our expressions (2.6), (2.7) and (2.8) agree with these results. In the $1/N$ expansion the anomalous dimensions are regular functions at $s = 0$ but they acquire a pole at the point $s = d/2-2$. This pole, however, cancels in the mass corrections, analogous to (1.3). Therefore $\delta m^2_\alpha(s)$ are regular functions for $s \geq 0$ and corresponding anomalous dimensions are defined analogous to (2.19) around point $s = d/2-2$. The explicit example for the scalar operator in $d = 3$ can be found in [47, Eq. (4.2)].

3 Complex cubic models in $d = 6 - 2\epsilon$

In this section we study the mass corrections (1.3) in two six-dimensional models. The first model describes the self-interacting complex field φ

$$S_I = \int d^dx \left(\partial\varphi\partial\bar{\varphi} + \frac{1}{6}g\left(\varphi^3 + \bar{\varphi}^3\right)\right) \quad (3.1)$$

and the second model involves N component field $\varphi = \{\varphi_1, \ldots, \varphi_N\}$ and the scalar field σ

$$S_{II} = \int d^dx \left(\partial\varphi\partial\bar{\varphi} + \partial\sigma\partial\bar{\sigma} + \frac{1}{2}g\left(\varphi^2\sigma + \bar{\varphi}^2\bar{\sigma}\right)\right), \quad (3.2)$$

where $\varphi^2 = \sum_a \varphi_a^2$. They are the special cases of the general cubic models considered in refs. [51–56]. The model I is invariant under the discrete symmetry transformations, $\varphi \mapsto$

$e^{\pm i2\pi/3}\varphi$, that guarantees its multiplicative renormalizability. The model II, in addition to $O(N)$ symmetry is invariant under U(1) rotations $\varphi \mapsto e^{i\alpha}\varphi$, $\sigma \mapsto e^{-2i\alpha}\sigma$. For $N=2$ the symmetry is enhanced to $\mathrm{U}(1)\times\mathrm{U}(1)\times S_3$ and the action can be written as

$$S = \int d^d x \left(\sum_{k=1}^{3} \partial\varphi_k \partial\bar{\varphi}_k + g\big(\varphi_1\varphi_2\varphi_2 + \mathrm{c.c}\big) \right). \tag{3.3}$$

The renormalization group functions read ($u = g^2/(16\pi)^3$)

$$\gamma_\varphi(u) = \frac{u}{12} - \frac{11}{3}\left(\frac{u}{12}\right)^2 + \frac{u^3}{24}\left(\frac{5357}{2592} - \zeta_3\right) + O(u^4)\,,$$
$$\beta(u) = -2\epsilon u + \frac{1}{2}u^2 - \frac{83}{72}u^3 + O(u^4) \tag{3.4}$$

for the model I and

$$\gamma_\sigma(u) = \frac{1}{12}Nu - \frac{11}{216}Nu^2 + \frac{1}{12}Nu^3\left(\frac{103N}{648} + \frac{2677}{1296} - \zeta_3\right) + \mathcal{O}(u^4)\,,$$
$$\gamma_\varphi(u) = \frac{1}{6}u - \frac{11(N+2)}{432}u^2 + \frac{u^3}{6}\left(-\frac{13N^2}{5184} + \frac{641N}{2592} + \frac{2461}{1296} - \zeta_3\right) + \mathcal{O}(u^4)\,,$$
$$\beta(u) = -2\epsilon u + \frac{1}{6}(N+4)u^2 - \frac{11N+119}{54}u^3 + \mathcal{O}(u^4) \equiv -2u(\epsilon + \bar{\beta}(u)) \tag{3.5}$$

for the model II. Note that for $N=2$ $\gamma_\sigma = \gamma_\varphi$ as it is required by the symmetry. Both models have an IR stable fixed point below $d=6$.

3.1 Leading twist operators: model I

In the model I we consider the following operators: "nonsinglet" operator (with respect to the symmetry group Z_3, $\varphi \to e^{\pm i2\pi/3}\varphi$)

$$\mathcal{O}_s^{\mathrm{ns}} = \varphi\partial_{\mu_1}\ldots\partial_{\mu_{s-1}}\varphi - \text{traces}, \tag{3.6}$$

and "singlet" operators

$$\mathcal{O}_s^{\pm} = \bar{\varphi}\partial_{\mu_1}\ldots\partial_{\mu_{s-1}}\varphi \mp (\varphi \leftrightarrow \bar{\varphi}) - \text{traces}. \tag{3.7}$$

Note here that the spin of the operator is equal to $s-1$. The nonsinglet operators are defined for odd s and the singlet operators – $\mathcal{O}_s^+$ and $\mathcal{O}_s^-$ – for even and odd s, respectively. We use such a definition because for this choice of the parameter s the mass correction formula, Eq. (1.3), keeps its form in $d=6$.

For nonsinglet operators the anomalous dimensions take the form

$$\gamma^{\mathrm{ns}}(s) = 2\gamma_\varphi - \frac{2u^2}{s^2(s+1)^2} + \mathcal{O}(u^3) \tag{3.8}$$

and the corresponding mass correction, $\delta m^2(s) = 2\Big(s + \bar\beta + \frac{1}{2}\gamma^{\rm ns}(s)\Big)\gamma^{\rm ns}(s)$ diverges at $s \to 0$.

For the singlet operators $\mathcal{O}_s^{\pm}$ we obtain

$$\begin{aligned}
\gamma^{\pm}(s) &= 2\gamma_\varphi \pm \frac{2u}{s(s+1)} \pm \frac{u^2}{3s(s+1)}\left(S_1 - \frac{1}{2s(s+1)} - 4 + \frac{2s+1}{s(s+1)}\right) \\
&+ \frac{2u^2}{s^2(s+1)^2}\left(1 - \frac{2s+1}{s(s+1)}\right) \\
&+ \frac{u^3}{3s^2(s+1)^2}\Bigg\{12S_3 + 24S_{1,-2} - 12S_{-3} - S_2 - 12\left(2 + \frac{1}{s(s+1)}\right)S_{-2} \\
&+ 2\left(7 - \frac{1+2s}{s(s+1)}\right)S_1 - 36\zeta_3 - \frac{24\,(S_{-2}+1)}{(s-1)(s+2)} + \frac{6s-3}{2s^2(s+1)^2} + \frac{20s-1}{s(s+1)} - 7\Bigg\} \\
&\pm \frac{u^3}{36s(s+1)}\Bigg\{2S_2 + S_1^2 + 144\left(\frac{1}{(s-1)(s+2)} - \frac{1}{s^2(s+1)^2} - \frac{1}{s(s+1)}\right)S_{-2} \\
&- 72\zeta_3 + \left(\frac{1+4s}{s(s+1)} - \frac{143}{3}\right)S_1 + \frac{144}{(s-1)(s+2)} + \frac{144}{s^4(s+1)^4} + 144\frac{2-3s}{s^3(s+1)^3} \\
&+ \frac{289-8s}{2s^2(s+1)^2} - \frac{538+286s}{3s(s+1)} + \frac{4927}{24}\Bigg\} + O(u^4). \qquad (3.9)
\end{aligned}$$

We checked that the anomalous dimensions have the reciprocity respecting form and that the anomalous dimension of the stress-energy tensor vanishes, $\gamma_-(3) = 0$. One can easily find that the correction $\delta m_+^2(s)$ is regular at $s = 0$,

$$\delta m_+^2(0) = 4u + \frac{41}{18}u^2 + \left(-\frac{16}{3}\zeta_3 - \frac{44}{5}\zeta_2^2 + 8\zeta_2 + \frac{19721}{1296}\right)u^3 + O(u^4), \qquad (3.10)$$

while $\delta m_-^2(s) \simeq 8u^3(2 - 3\zeta_3)/s$. Thus we see that the $\delta m^2(s)$ correction is finite at $s = 0$ only for the anomalous dimensions continued from *even* s (that corresponds to the odd spin operators in the cubic model).

3.2 Leading twist operators: model II

In this subsection we consider only operators of odd spin (which corresponds to the even s). Let us start with operators constructed of the holomorphic fields, σ and φ

$$\begin{aligned}
\mathcal{O}_s^a &= \sigma\partial_{\mu_1}\ldots\partial_{\mu_{s-1}}\varphi^a - \text{traces}, \\
\mathcal{O}_s^{[ab]} &= \varphi^a\partial_{\mu_1}\ldots\partial_{\mu_{s-1}}\varphi^b - (a \leftrightarrow b) - \text{traces}, \qquad (3.11)
\end{aligned}$$

The loop corrections for these operators vanish in the first two orders and the anomalous dimensions take the form[‡]

$$\gamma^a(s) = \gamma_\sigma + \gamma_\varphi - \frac{u^3}{6}(25N-2)\frac{S_1(s)}{s^2(s+1)^2} + O(u^4)\,,$$

[‡]Note that for $N = 2$ these anomalous dimensions coincide.

$$\gamma^{[ab]}(s) = 2\gamma_\varphi + \frac{u^3}{3}(N-26)\frac{S_1(s)}{s^2(s+1)^2} + O(u^4)\,. \tag{3.12}$$

Since $S_1(s) = \zeta_2 s + O(s^2)$, the corresponding mass corrections are finite at $s \to 0$.

Traceless and symmetric operators constructed from holomorphic and anti-holomorphic fields include two scalar operators,

$$\overline{\mathcal{O}}_s = \bar\varphi \partial_{\mu_1} \dots \partial_{\mu_{s-1}} \varphi, \qquad\qquad \Sigma_s = \bar\sigma \partial_{\mu_1} \dots \partial_{\mu_{s-1}} \sigma, \tag{3.13}$$

which mix under renormalization, and several $O(N)$ irreducible tensor operators[§]

$$\begin{aligned}
\overline{\mathcal{O}}^a_s &= \bar\sigma \partial_{\mu_1} \dots \partial_{\mu_{s-1}} \varphi^a, \\
\overline{\mathcal{O}}^{ab}_s &= \bar\varphi^a \partial_{\mu_1} \dots \partial_{\mu_{s-1}} \varphi^b + (a \leftrightarrow b) - N^{-1} \overline{\mathcal{O}}^{ab}_s, \\
\overline{\mathcal{O}}^{[ab]}_s &= \bar\varphi^a \partial_{\mu_1} \dots \partial_{\mu_{s-1}} \varphi^b - (a \leftrightarrow b).
\end{aligned} \tag{3.14}$$

The expressions for the anomalous dimensions are quite long and can be found in the Appendix A. Here we present only the corresponding small s expansion

$$\begin{aligned}
\bar\gamma^a(s) &= u\left\{\frac{2}{s} + \frac{N-22}{12} + 2s - 2s^2 + 2s^3 + O(s^4)\right\} + u^2\Bigg\{-\frac{2}{s^3} + \frac{13}{3s^2} - \frac{98+5N}{18s} \\
&\quad + \frac{(N+2)}{6}\zeta_2 + \frac{15N+1178}{432} + \left(-\frac{N+2}{6}\zeta_3 - \frac{N+2}{6}\zeta_2 + \frac{N+34}{18}\right)s + O(s^2)\Bigg\} \\
&\quad + u^3\Bigg\{\frac{4}{s^5} - \frac{13}{s^4} + \frac{10N+437}{18s^3} - \left(\frac{N+2}{3}\zeta_2 + \frac{189N+6938}{216}\right)\frac{1}{s^2} - \Bigg(4\zeta_3 \\
&\quad + \frac{22(N+1)}{5}\zeta_2^2 - \frac{169N+194}{36}\zeta_2 + \frac{3N^2-712N-25309}{648}\Bigg) + O(1)\Bigg\} + O(u^4),
\end{aligned} \tag{3.15}$$

and

$$\begin{aligned}
\bar\gamma^{(ab)}(s) &= u\left\{\frac{2}{s} - \frac{5}{3} + 2s - 2s^2 + 2s^3 + O(s^4)\right\} + u^2\Bigg\{-\frac{2}{s^3} + \frac{N+24}{6s^2} - \frac{2(2N+23)}{9s} \\
&\quad + \frac{2}{3}\zeta_2 + \frac{49N+506}{216} - \left(\frac{2}{3}\zeta_3 + \frac{2}{3}\zeta_2 + \frac{N-20}{9}\right)s + O(s^2)\Bigg\} + u^3\Bigg\{\frac{4}{s^5} - \frac{N+24}{2s^4} \\
&\quad + \frac{N^2+112N+1600}{72s^3} - \left(\frac{4}{3}\zeta_2 + \frac{4N^2+249N+3144}{108}\right)\frac{1}{s^2} + \Bigg(-4\zeta_3 - \frac{66}{5}\zeta_2^2 \\
&\quad + \frac{N+264}{18}\zeta_2 + \frac{12N^2+1621N+23431}{648}\Bigg)\frac{1}{s} + O(1)\Bigg\} + O(u^4),
\end{aligned} \tag{3.16}$$

[§]Note that $\overline{\mathcal{O}}^{[ab]}_2$ corresponds to the conserved $O(N)$ current.

$$
\begin{aligned}
\bar{\gamma}^{[ab]}(s) = u\left\{-\frac{2}{s}+\frac{7}{3}-2s+2s^2-2s^3+O(s^4)\right\} + u^2\Bigg\{-\frac{2}{s^3}-\frac{N-24}{6s^2}+\frac{2(2N-13)}{9s} \\
-\frac{2}{3}\zeta_2-\frac{71N-314}{216}+\left(\frac{2}{3}\zeta_3+\frac{2}{3}\zeta_2+\frac{N+16}{9}\right)s+O(s^2)\Bigg\} + u^3\Bigg\{-\frac{4}{s^5}-\frac{N-24}{s^4} \\
-\frac{N^2-112N+1280}{72s^3}+\left(-\frac{4}{3}\zeta_2+\frac{4N^2-207N+2472}{108}\right)\frac{1}{s^2}+\Bigg(4\zeta_3-\frac{22}{5}\zeta_2^2 \\
-\frac{N-120}{18}\zeta_2-\frac{12N^2-638N+21127}{648}\Bigg)\frac{1}{s}+O(1)\Bigg\}+O(u^4). \qquad (3.17)
\end{aligned}
$$

Again, the mass corrections for the above operators are finite in the limit $s \to 0$. Namely,

$$
\begin{aligned}
\delta m_a^2(0) &= 4u-\frac{N^2+88N-244}{144}u^2+\Bigg(-\frac{2(N+14)}{3}\zeta_3-\frac{44(N+1)}{5}\zeta_2^2 \\
&\quad +8(N+1)\zeta_2+\frac{10N^2+1255N+29422}{1296}\Bigg)u^3+O(u^4), \\
\delta m_{(ab)}^2(0) &= 4u+\frac{30-11N}{18}u^2+\Bigg(-\frac{32}{3}\zeta_3-\frac{132}{5}\zeta_2^2+24\zeta_2 \\
&\quad -\frac{N^2-618N-30740}{1296}\Bigg)u^3+O(u^4), \\
\delta m_{[ab]}^2(0) &= -4u+\frac{9N+110}{18}u^2+\Bigg(\frac{16}{3}\zeta_3-\frac{44}{5}\zeta_2^2+8\zeta_2 \\
&\quad +\frac{23N^2-2414N-33444}{1296}\Bigg)u^3+O(u^4). \qquad (3.18)
\end{aligned}
$$

For the "singlet" operators we present the expansion for the anomalous dimension matrix

$$
\begin{aligned}
\hat{\gamma}(s) = 2u\Bigg\{&\begin{pmatrix}1 & N\\ 1 & 0\end{pmatrix}\frac{1}{s}-\begin{pmatrix}\frac{5}{6} & N\\ 1 & \frac{N}{18}\end{pmatrix}+\begin{pmatrix}1 & N\\ 1 & 0\end{pmatrix}s-\begin{pmatrix}1 & N\\ 1 & 0\end{pmatrix}s^2+\begin{pmatrix}1 & N\\ 1 & 0\end{pmatrix}s^3+O(s^4)\Bigg\} \\
&+u^2\Bigg\{-\begin{pmatrix}2N+2 & 2N\\ 2 & 2N\end{pmatrix}\frac{1}{s^3}+\begin{pmatrix}\frac{25}{6}N+4 & \frac{13}{3}N\\ \frac{13}{3} & 4N\end{pmatrix}\frac{1}{s^2}-\begin{pmatrix}\frac{40}{9}N+\frac{46}{9} & 6N\\ \frac{5}{9}N+\frac{44}{9} & 4N\end{pmatrix}\frac{1}{s} \\
&+\begin{pmatrix}\frac{2}{3} & \frac{2}{3}N\\ \frac{1}{3}N & 0\end{pmatrix}\zeta_2+\begin{pmatrix}\frac{481}{216}N+\frac{253}{108} & 3N\\ \frac{2}{9}N+\frac{23}{9} & \frac{205}{108}N\end{pmatrix}+\Bigg[-\begin{pmatrix}\frac{2}{3} & \frac{2}{3}N\\ \frac{N}{3} & 0\end{pmatrix}\zeta_3 \\
&-\begin{pmatrix}\frac{2}{3} & \frac{2}{3}N\\ \frac{1}{3}N & 0\end{pmatrix}\zeta_2+\begin{pmatrix}\frac{17}{9}N+\frac{20}{9} & 2N\\ \frac{1}{9}N+\frac{16}{9} & 2N\end{pmatrix}\Bigg]s+O(s^2)\Bigg\} \\
&+u^3\begin{pmatrix}\gamma^{(3)}_{\varphi\varphi}(s) & \gamma^{(3)}_{\varphi\sigma}(s)\\ \gamma^{(3)}_{\sigma\varphi}(s) & \gamma^{(3)}_{\sigma\sigma}(s)\end{pmatrix}+O(u^4), \qquad (3.19)
\end{aligned}
$$

where

$$\gamma^{(3)}_{\varphi\varphi}(s) = 4\frac{2N+1}{s^5} - \frac{N^2+151N+72}{6s^4} + \frac{65N^2+3248N+1600}{72s^3} - \left(\left(N^2+2N+4\right)\zeta_2 + \frac{198N^2+5837N+3144}{36}\right)\frac{1}{3s^2} + \left(\frac{(N-6)(N+4)}{6}\zeta_3 - \frac{66}{5}\zeta_2^2 + \frac{17N^2+15N+264}{18}\zeta_2 + \frac{1116N^2+33973N+23431}{648}\right)\frac{1}{s} + O(1), \tag{3.20}$$

$$\gamma^{(3)}_{\varphi\sigma}(s) = N\Bigg\{\frac{4(N+1)}{s^5} - \frac{73N+76}{6s^4} + \frac{1475N+1758}{72s^3} - \left(4\zeta_2 + \frac{5297N+7090}{72}\right)\frac{1}{3s^2} \left(-4\zeta_3 - \frac{22}{5}\zeta_2^2 + \frac{61}{9}\zeta_2 + 5\frac{4283N+6854}{864}\right)\frac{1}{s} + O(1)\Bigg\}, \tag{3.21}$$

$$\gamma^{(3)}_{\sigma\varphi}(s) = \frac{4(N+1)}{s^5} - \frac{73N+76}{6s^4} + \frac{1533N+1642}{72s^3} - \left((N+2)\zeta_2 + \frac{5517N+6650}{72}\right)\frac{1}{3s^2} \left(-\frac{N+22}{6}\zeta_3 - \frac{22}{5}\zeta_2^2 + \frac{4N+53}{9}\zeta_2 - \frac{36N^2-66107N-99230}{2592}\right)\frac{1}{s} + O(1), \tag{3.22}$$

$$\gamma^{(3)}_{\sigma\sigma}(s) = N\Bigg\{\frac{4}{s^5} + \frac{N-80}{6s^4} + \frac{2(N+112)}{9s^3} - \left((N+2)\zeta_2 - \frac{13N-1718}{18}\right)\frac{1}{3s^2} \left(-\frac{N-2}{6}\zeta_3 - \frac{22}{5}\zeta_2^2 + \frac{7N+106}{18}\zeta_2 - 22\frac{N-38}{27}\right)\frac{1}{s} + O(1)\Bigg\}. \tag{3.23}$$

The mass correction matrix

$$\delta\widehat{m}^2(s) = 2\left(\left(s+\bar{\beta}(u)\right)\mathbb{1} + \frac{1}{2}\widehat{\gamma}(s)\right)\widehat{\gamma}(s), \tag{3.24}$$

where $\mathbb{1}$ is an identity matrix, is regular at $s=0$ in the first two orders,

$$\delta\widehat{m}^2(0) = 4u\begin{pmatrix} 1 & N \\ 1 & 0 \end{pmatrix} + \frac{u^2}{18}\begin{pmatrix} 61N+30 & 12N \\ -20N+52 & 70N \end{pmatrix} + O(u^3)\,. \tag{3.25}$$

The three loop matrix elements retain $1/s^2$ and $1/s$ poles which however cancel in the expressions for the sum and product of the mass eigenvalues, δm_k^2, $k=1,2$. Namely

$$\delta m_1^2 + \delta m_2^2 = \mathrm{tr}\left[\delta\widehat{m}^2\right](0) = 4u + \frac{131N+30}{18}u^2 + \left(-\frac{4\left(N^2+2N+8\right)}{3}\zeta_3 - \frac{44}{5}(N+3)\zeta_2^2 + 8(N+3)\zeta_2 - \frac{1131N^2-25950N-30740}{1296}\right)u^3 + O(u^4), \tag{3.26}$$

$$\delta m_1^2 \cdot \delta m_2^2 = \det\left[\delta\widehat{m}^2\right](0) = N\Bigg\{-16u^2 + \frac{4(10N+3)}{9}u^3 + \Bigg(\frac{224}{3}\zeta_3 + \frac{176}{5}\zeta_2^2 - 32\zeta_2 - \frac{28N^2+4421N+12504}{81}\Bigg)u^4 + O(u^5)\Bigg\}. \tag{3.27}$$

that implies regularity of δm_k^2 at $s=0$. Note also that

$$\delta m_{1,2}^2(0) = 2u\left(1 \pm i\sqrt{4N-1}\right) + O(u^2), \tag{3.28}$$

are complex numbers conjugated to each other $(\delta m_2^2(0))^* = \delta m_1^2(0)$.

4 Gross-Neveu-Yukawa model in $d = 4 - 2\epsilon$ dimesions

In this section we consider mass-corrections in the Gross-Neveu-Yukawa model [44],

$$S = \int d^d x \left\{ \bar{q}\partial\!\!\!/ q + \frac{1}{2}(\partial\sigma)^2 + g\bar{q}\sigma q + \frac{\bar{\lambda}}{4!}\sigma^4 \right\}, \tag{4.1}$$

where $q(x)$ is N component quark field and $\sigma(x)$ is a scalar field. The RG functions in this model are known with four loop accuracy [57]

$$\begin{aligned} \beta_u(u,\lambda) &= -2u\epsilon + 2(3+2N)u^2 + O(u^3,\lambda^3), \\ \beta_\lambda(u,\lambda) &= -2\lambda\epsilon + 3\lambda^2 + 8Nu\lambda - 24Nu^2 + O(u^3,\lambda^3), \end{aligned} \tag{4.2}$$

$$\begin{aligned} \gamma_q(u,\lambda) &= \frac{u}{2} - \frac{u^2}{8}(12N+1) + O(u^3,\lambda^3), \\ \gamma_\sigma(u,\lambda) &= 2Nu + \frac{1}{3}\lambda^2 - 5Nu^2 + O(u^3,\lambda^3), \end{aligned} \tag{4.3}$$

where

$$u = \frac{g^2}{(4\pi)^2}, \qquad \lambda = \frac{\bar{\lambda}}{(4\pi)^2}. \tag{4.4}$$

The model posses an IR-stable fixed point (see for details refs. [44, 45, 57])

$$\begin{aligned} u_* &= u_1\epsilon + u_2\epsilon^2 + \ldots = \frac{\epsilon}{3+2N} \\ &\quad - \frac{8N^2 - 1032N - 441 - (4N+66)\sqrt{4N^2+132N+9}}{108(3+2N)^3}\epsilon^2 + O(\epsilon^3), \\ \lambda_* &= \lambda_1\epsilon + \lambda_2\epsilon^2 + \ldots = \frac{3 - 2N + \sqrt{4N^2+132N+9}}{9+6N}\epsilon + O(\epsilon^2). \end{aligned} \tag{4.5}$$

We consider the following twist-two operators:

- the flavor nonsinglet operator

$$\mathcal{O}_s^{q,a} = \bar{q}_i\gamma_{\mu_1}\partial_{\mu_2}\ldots\partial_{\mu_s}t_{ij}^a q_j, \tag{4.6}$$

where t_{ik}^a $(a = 1,\ldots,N^2-1)$ are the generators of $SU(N)$ flavor group.

- the flavor singlet operators

$$\mathcal{O}_s^q = -s\,\bar{q}_i \gamma_{\mu_1}\partial_{\mu_2}\dots\partial_{\mu_s} q_i, \qquad\qquad \mathcal{O}_s^\sigma = \sigma\partial_{\mu_1}\dots\partial_{\mu_s}\sigma. \tag{4.7}$$

  Note that operator $\mathcal{O}_s^\sigma$ does not exist for odd s.

We calculate anomalous dimensions of twist-two operators in the two-loop approximation. For the nonsiglet operator we get

$$\gamma^q(s) = 2\gamma_q - \frac{2u}{s(s+1)} - \frac{2u^2}{s(s+1)}\Bigg(3S_1 + \frac{1+2s}{s^2(s+1)^2} + \frac{1+5s+2N(1+2s)}{s(1+s)} \\ - 8 - 6N + (1-(-1)^s)\left(1 - \frac{1}{s(s+1)}\right)\Bigg) + O(u^3,\lambda^3). \tag{4.8}$$

One can check that $\gamma^q(s=1)=0$ which corresponds to the conserved current. The singlet anomalous dimension matrix, defined for *even* s only, takes the form

$$\widehat{\gamma}(s) = \begin{pmatrix} 2\gamma_q & 0 \\ 0 & 2\gamma_\sigma \end{pmatrix} - 2u\begin{pmatrix} \frac{1}{s(s+1)} & 4N \\ \frac{2}{s(s+1)} & 0 \end{pmatrix} + \begin{pmatrix} \gamma^{(2)}_{qq}(s) & \gamma^{(2)}_{q\sigma}(s) \\ \gamma^{(2)}_{\sigma q}(s) & \gamma^{(2)}_{\sigma\sigma}(s) \end{pmatrix} + O(u^3,\lambda^3), \tag{4.9}$$

where

$$\gamma^{(2)}_{qq}(s) = -\frac{2u^2}{s(s+1)}\left(3S_1 + \frac{1+2s}{s^2(s+1)^2} + \frac{1+5s+2N(3+2s)}{s(1+s)} - 8 - 10N\right), \\ \gamma^{(2)}_{q\sigma}(s) = -8Nu^2\left(3S_1 + \frac{1+2s}{s^2(s+1)^2} + \frac{3}{2s(s+1)} - 7\right), \\ \gamma^{(2)}_{\sigma q}(s) = -\frac{4u^2}{s(s+1)}\left((1+2N)S_1 + \frac{1+2s}{2s^2(s+1)^2} + \frac{7+2(5+4N)s}{4s(s+1)} - 4N - \frac{5}{2}\right), \\ \gamma^{(2)}_{\sigma\sigma}(s) = -\frac{8Nu^2}{s(s+1)}\left(\frac{1+2s}{s(s+1)} - 2\right) - \frac{\lambda^2}{s(s+1)}. \tag{4.10}$$

One can check that

$$\begin{pmatrix} 1, & 2 \end{pmatrix}\hat{\gamma}(s=2) = 0. \tag{4.11}$$

which corresponds to the conserved stress-energy tensor.

We consider the mass correction (1.3) at the critical point (4.5). For the nonsinglet operators we define

$$\delta m^2_\pm(s) = 2\left(s - \epsilon + \frac{1}{2}\gamma^q_*(s)\right)\gamma^q_*(s), \tag{4.12}$$

where $\pm$ functions are defined for the even (odd) values of s and $\gamma^q_*(s) \equiv \gamma^q(s,u_*,\lambda_*)$. The function $\delta m^2_+(s)$ is regular at $s=0$

$$\delta m^2_+(0) = -4u_1\epsilon + \left((24N+37)u_1^2 - 6u_1 - 4u_2\right)\epsilon^2 + O(\epsilon^3), \tag{4.13}$$

while δm_-^2 is singular, $\delta m_-^2(s) \simeq 8u_1^2/s$.

For the singlet operators we define the mass correction as

$$\delta\widehat{m}^2(s) = 2\left(s - \epsilon + \frac{1}{2}\widehat{\gamma}_*(s)\right)\widehat{\gamma}_*(s). \tag{4.14}$$

The eigenvalues of the matrix $\delta\widehat{m}^2(s)$ are finite at $s \to 0$,

$$\begin{aligned}\delta\widehat{m}_1^2(0) &= -4u_1\epsilon + \Big((16N-6)u_1 - (32N^2+24N-37)u_1^2 - 4u_2\Big)\epsilon^2 + O(\epsilon^3),\\ \delta\widehat{m}_2^2(0) &= \Big(48N(N+2)u_1^2 - 24Nu_1 - 2\lambda_1^2\Big)\epsilon^2 + O(\epsilon^3).\end{aligned} \tag{4.15}$$

Moreover, with the help of Eqs.(4.5), one can check

$$\delta\widehat{m}_2^2(0) = \left(-2\epsilon + \gamma_{\sigma^2}^*\right)\gamma_{\sigma^2}^* = (\lambda_1 + 4Nu_1)(\lambda_1 + 4Nu_1 - 2)\,\epsilon^2 + O(\epsilon^3), \tag{4.16}$$

where $\gamma_{\sigma^2}^* = (\lambda_1 + 4Nu_1)\,\epsilon + O(\epsilon^2)$ is the anomalous dimension of the operator σ^2.

Using analiticity of (4.15) we define two "trajectories" around $s = 0$ (note that $\epsilon \ll 1$)

$$\gamma_{1,2}(s) = \epsilon - s + \sqrt{(\epsilon - s)^2 + \delta\widehat{m}_{1,2}^2(s)}. \tag{4.17}$$

For the large s they correspond to the eigenvalues of the anomalous dimension matrix (4.9) and remain analytical around $s = 0$. Branching points of these trajectories are defined as

$$s_1^{\pm} = \pm 2\sqrt{\epsilon/(2N+3)} + O(\epsilon), \qquad s_2^{\pm} = \left(\frac{9-30N}{9+6N} \pm a_N\right)\epsilon + O(\epsilon^{3/2}), \tag{4.18}$$

where $a_N \in \mathbb{R}$ for $N > 1$. All of these points lie on the real axis.

4.1 Gross-Neveu model at large N

The GNY model in d dimensions is critically equivalent to the GN model [58]. The later can be analysed in the $1/N$ expansion framework. The index η determining the critical dimension of the basic fermion field[¶] ($\Delta_q = \mu - 1 + \eta/2$) is known with an accuracy $1/N^3$ [59, 60] and the critical dimensions of the auxiliary σ field ($\Delta_\sigma = 2 + \gamma_\sigma$) and the operator σ^2 ($\Delta_{\sigma^2} = 4 + \gamma_{\sigma^2}$) with an accuracy $1/N^2$ [61–63].

The singlet operators $\mathcal{O}_s^q$ and $\mathcal{O}_s^\sigma$ (defined as in (4.7)) have different canonical dimensions, $d - 2 + s$ and $4 + s$, and therefore do not mix under renormalization. The anomalous dimensions of the quark operator were calculated in the order of $1/N$ in [64] and in the order of $1/N^2$ in [65]. For the operator $\mathcal{O}_s^\sigma$ the anomalous dimensions are known only in the order $1/N$ [66]. The corresponding $1/N$ results are quite compact,

$$\gamma_\sigma(s) = -\frac{\eta_1}{n}\frac{2(2\mu-1)}{(\mu-1)}\left(1 - \frac{\mu}{2\mu-1}\frac{\Gamma(\mu)}{\Gamma(s+\mu)}\frac{\Gamma(s+2-\mu)}{\Gamma(3-\mu)}\right) + O(1/n^2)\,, \tag{4.19a}$$

[¶] In this subsection we use the standard notation $\mu \equiv d/2$.

$$\gamma_q(s) = \frac{\eta_1}{n}\left(1 - \frac{\mu(\mu-1)}{(s+\mu-1)(s+\mu-2)}\left(1 + \frac{\Gamma(2\mu-1)\Gamma(s+1)}{(\mu-1)\Gamma(2\mu-3+s)}\right)\right) + O(1/n^2)\,, \quad (4.19b)$$

where $n = N \times \mathrm{tr}\,\mathbb{1} \equiv 4N$ and the index η_1 is

$$\eta_1 = -\frac{2\Gamma(2\mu-1)}{\Gamma(\mu+1)\Gamma(\mu)\Gamma(\mu-1)\Gamma(1-\mu)}\,. \quad (4.20)$$

Analyzing the singularities we see that the anomalous dimensions, $\gamma_\sigma(s)$ and $\gamma_q(s)$, have poles at $s = \mu - 2$ and $s = 2 - \mu$, respectively. Next, the leading order anomalous dimensions are finite at $s = 0$, however, as was noted in [66], $\gamma_\sigma(s=0) \neq \gamma_{\sigma^2}$. Given the analogous situation with φ^4 model, one would expect $1/s$ poles to appear at higher orders. This is indeed the case for the quark anomalous dimension,

$$\gamma_q(s) = \left(-\frac{\eta_1^2}{n^2}\frac{2\mu(2\mu-1)(2\mu-3)}{(\mu-2)^2} \times \frac{1}{s} + O(s^0)\right) + O(1/n^3)\,. \quad (4.21)$$

Thus the anomalous dimensions become singular at the points, $s = \pm(2-\mu)$ and $s = 0$, which, according to ref. [29], can be identified with the intersection points of the following trajectories defined by scaling dimensions:

- the quark trajectory $\Delta_q(s)$ intersects with its shadow $\widetilde{\Delta}_q(s) = d - \Delta_q(s)$ at $s = 2 - \mu$.
- the trajectory $\Delta_\sigma(s)$ intersects with $\widetilde{\Delta}_\sigma(s)$ at $s = \mu - 2$.
- the trajectory $\Delta_q(s)$ intersects with $\widetilde{\Delta}_\sigma(s)$, and $\Delta_\sigma(s)$ intersects with $\widetilde{\Delta}_q(s)$ at $s = 0$.

Note, that in the $1/N$ framework all four points of intersections are well separated since one has not to assume that ϵ is small. The natural choice of the dimension for the theory in $1/N$ expansion is $2 < d < 4$, so we consider these limits in what follows.

In the first two cases assuming the analyticity of the products $\Delta_q(s)\widetilde{\Delta}_q(s)$ and $\Delta_\sigma(s)\widetilde{\Delta}_\sigma(s)$ at the corresponding intersection points $s_{q,\sigma} = \pm(2-\mu)$ one derives that the following combination of the anomalous dimensions

$$\delta m_q^2(s) = 2\left(s + \mu - 2 + \frac{1}{2}\gamma_q(s)\right)\gamma_q(s)\,, \quad (4.22a)$$

$$\delta m_\sigma^2(s) = 2\left(s + 2 - \mu + \frac{1}{2}\gamma_\sigma(s)\right)\gamma_\sigma(s), \quad (4.22b)$$

are regular functions of s at these points. In the order $1/N$ it follows immediately from Eqs. (4.19) and for $\delta m_q^2(s)$ in the order $1/N^2$ can be checked using the results of ref. [65].

$$\delta m_q^2(s_q) = -\frac{\eta_1}{n}\mu(\mu-1)\left(1 + \frac{\Gamma(2\mu-1)\Gamma(3-\mu)}{\Gamma(\mu)}\right) + O\left(1/n^2\right),$$
$$\delta m_\sigma^2(s_\sigma) = \frac{\eta_1}{n}\frac{8\Gamma(\mu+1)}{\Gamma(2\mu-1)\Gamma(3-\mu)} + O\left(1/n^2\right). \quad (4.23)$$

Here we have tacitly assumed that the anomalous dimensions $\gamma_\sigma(s)$ and $\gamma_q(s)$ are determined by analytic continuation from *even* integer s. However the mass correction $\delta m_q^2(s)$ is also regular at the point $s = 2 - \mu$ for the anomalous dimensions $\gamma_q(s)$ continued from *odd* spins as well as for the nonsinglet anomalous dimensions of both signatures. Thus in the vicinity of the points $s_{q,\sigma}$, the anomalous dimensions acquire square root branching points

$$\gamma_\alpha(s) = s_\alpha - s + \sqrt{(s - s_\alpha)^2 + \delta m_\alpha^2(s)}, \tag{4.24}$$

where $\alpha = q, \sigma$. Using (4.23) one finds that in the region $2 < d < 4$ mass corrections have different sign, $\delta m_q^2(s_q) < 0$, $\delta m_\sigma^2(s_\sigma) > 0$. Therefore branching points of the square root lie on the real axis for the $\gamma_q(s)$ and become imaginary for the $\gamma_\sigma(s)$. The trajectories (4.24) around points $s = s_\alpha$ are shown in Figure 1. Note the different type of behaviour when branching points lie on the real axis (a) and have the non-zero imaginary part (b).

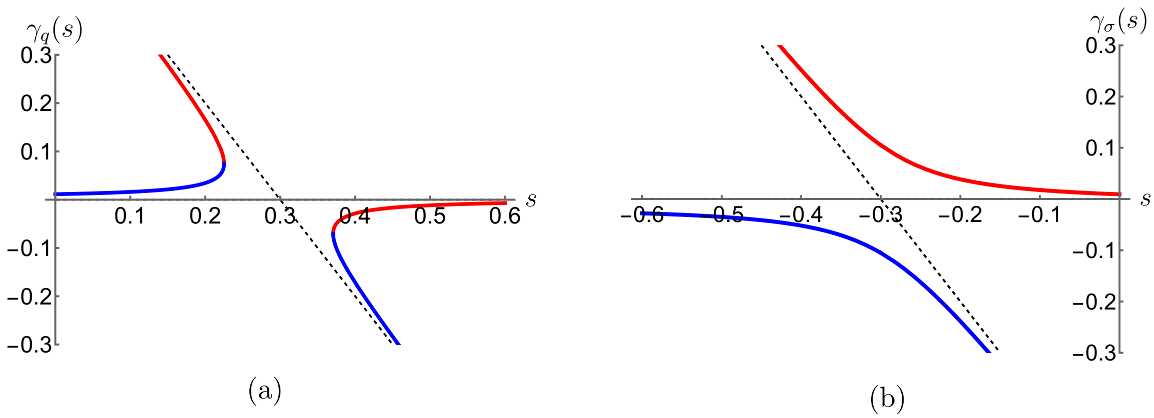


Figure 1: Anomalous dimensions defined as (4.24) around points $s = s_\alpha$. Graph (a) shows the $\gamma_q(s)$ and (b) shows $\gamma_\sigma(s)$. Red and blue trajectories correspond to the two branches of square root, while dashed black line represents the trajectories in free theory $N \to \infty$. The plot is made for $\epsilon = 0.3$, $N = 40$.

At the point $s = 0$ the quark trajectory intersects with the shadow $\sigma-$trajectory, and vice versa. The analyticity of the products $\Delta_q(s)\widetilde{\Delta}_\sigma(s)$ and $\Delta_\sigma(s)\widetilde{\Delta}_q(s)$ is equivalent to the analyticity of the following functions at $s = 0$

$$\omega_1(s) = \frac{1}{2}\left(\gamma_q(s) - \gamma_\sigma(s)\right), \tag{4.25a}$$

$$\omega_2(s) = s(\gamma_q(s) + \gamma_\sigma(s)) + \gamma_q(s)\gamma_\sigma(s). \tag{4.25b}$$

The first of these equation states that poles of the anomalous dimensions of the operators $\mathcal{O}_s^q$ and $\mathcal{O}_s^\sigma$ are exactly the same in all orders of the $1/N$ expansion. In particular, we predict then that the singular structure of $\gamma_\sigma(s)$ coincides with (4.21) in $1/N^2$ order. Unfortunately, the validity of general statement can be checked so far only in the order $1/N$, where both function are regular, since $\gamma_\sigma(s)$ is not known beyond this order.

Resolving equations (4.25) one derives for the anomalous dimensions in the vicinity of the point $s = 0$

$$\gamma_{q,\sigma}(s) = \pm\omega_1(s) - s + \sqrt{s^2 + \omega_2(s) + \omega_1^2(s)}. \tag{4.26}$$

Taking into account that

$$\omega_1(0) = -\frac{\eta_1}{n}\frac{(2\mu-3)(\mu-1)}{\mu-2} + O\left(1/n^2\right), \tag{4.27}$$

and

$$\omega_2(0) = -\left(\frac{\eta_1}{n}\right)^2 \frac{4(2\mu-1)}{\mu-2} + O\left(1/n^3\right), \tag{4.28}$$

one finds that the anomalous dimension of the operator σ^2 lays to the trajectory $\gamma_\sigma(s)$ on the first Riemann sheet,

$$\lim_{s\to 0}\gamma_\sigma(s) = \gamma_{\sigma^2} = \frac{\eta_1}{n}2(2\mu-1) + O\left(1/n^2\right)\,, \tag{4.29}$$

while

$$\lim_{s\to 0}\gamma_q(s) = \frac{\eta_1}{n}\frac{2}{2-\mu} + O\left(1/n^2\right)\,. \tag{4.30}$$

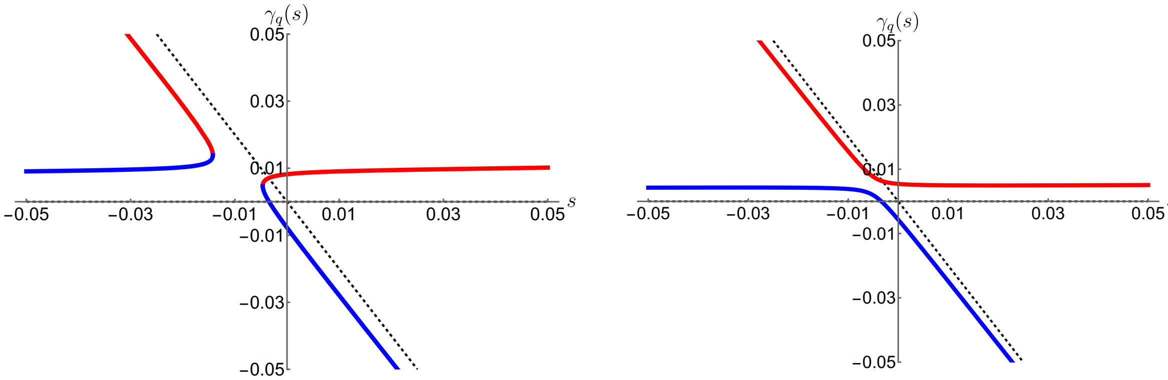
The branching points of the trajectory (4.26) are located at

$$s^\pm = \frac{\eta_1}{n}\frac{(\mu-1)(2\mu+1) \pm 2\sqrt{\mu(2\mu-1)(2\mu-3)}}{\mu-2} + O(1/n^2), \tag{4.31}$$

with the $1/N$ accuracy. In the region $2 < d < 3$ they lie on the real axis and acquire non-zero imaginary part in $3 < d < 4$. At the $d = 3$ the difference between two branching points loses, so one has to take into account the $1/N^2$ order. We plot the function $\gamma_q(s)$ around point $s = 0$ in Figure 2. Note the change of the behaviour for $d < 3$ (a) and $d > 3$ (b). The branching points (4.31) are the same for quark and σ anomalous dimensions, so the trajectory $\gamma_\sigma(s)$ looks the same as shown in Figure 2 modulo small corrections of the order $1/N$.

Taking into account information about anomalous dimensions around points $s = \pm(2-\mu)$ and $s = 0$, mentioned above, we can now plot the approximate graph of the scaling dimensions of operators $\mathcal{O}_s^q$ and $\mathcal{O}_s^\sigma$. Figure 3 represents the dependence of four scaling dimensions, $\Delta_q(s)$, $\Delta_s(s)$ and their shadows, $\widetilde{\Delta}_s(s)$ and $\widetilde{\Delta}_q(s)$, in the axes $\Delta(s) - d/2$ and s. Unlike the results from the naive perturbation theory, resummation, done by the using (4.24) and (4.26), gives us the analytical behaviour around points $s = \pm(2-\mu)$ and $s = 0$.

Before closing this section we make a remark concerning the limit $d \to 3$, in which case the GN model describes a real physical system in three dimensions. The pole contribution (4.21) vanishes at $d = 3$. Therefore, the anomalous dimensions, $\gamma_q(s), \gamma_\sigma(s)$ and mass correction



(a) (b)

Figure 2: Anomalous dimension of the quark operator, $\gamma_q(s)$, defined as (4.26) around point $s = 0$. Plot (a) is made for $\epsilon = 0.45$, $N = 40$, while plot (b) is made for $\epsilon = 0.55$, $N = 40$. Red and blue trajectories correspond to the two branches of square root, while dashed black line represents the trajectories in free theory $N \to \infty$.

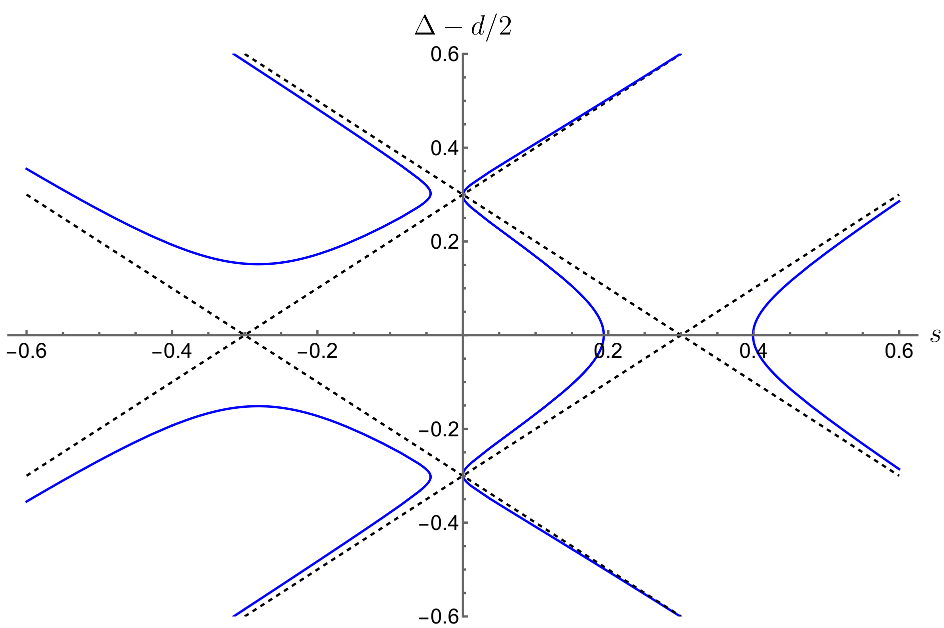


Figure 3: Blue solid line represents the scaling dimensions of quark and σ operators and their shadows. Black dashed line shows the free theory $N \to \infty$. The plot is made for $\epsilon = 0.3$, $N = 40$.

$\delta m_q^2(s)$, Eq. (4.22a), are analytic functions at $s = 0$, and the anomalous dimensions $\gamma_q(s)$, and $\gamma_\sigma(s)$ cross each other at $s = 0$ without branching.

Second, we have observed that the absence of a pole at $s = 2 - \mu$ in $\delta m_q^2(s)$ does not depend on the signature and the flavor structure of the operators. In the GN model the anomalous dimensions of the negative signature operators have additional poles at $s = 0$, so that $\delta m_q^2(s)$ is singular at this point. In the ϵ expansion these poles merge so that $\delta m_q^2(s)$ has a pole at $s = 0$ for the negative signature operators. The situation is a bit different for the $O(N)$ symmetric φ^4 model. It can be checked that in this model the anomalous dimensions do not have pole at $s = 0$ in the $1/N$ expansion [47]. Therefore $\delta m_q^2(s)$ for all types of the twist-two quark operators are regular both at $s = 2 - \mu$ and at $s = 0$ and hence should be regular at $s = 0$ in the ϵ expansion. That is indeed the case as we saw in Sect. 2.

5 Summary

The anomalous dimensions of the leading twist operators become, as a rule, singular at small positive or negative values of spin. These singular contributions can be resummed using technique developed in refs. [19–21]. The resummed anomalous dimensions have a square root singularities in the vicinity of these points [24, 67]. It was argued in ref. [29] that such structure naturally arises in CFT at the intersection points of different trajectories which are the scaling dimensions of the operators as function of spins. Technically, it results in a prediction that a certain quadratic combination of the scaling (anomalous) dimensions of operators is free from singularities in all orders in the perturbation theory. In particular it means that the poles at $s = 0$ in the anomalous dimensions at high orders are determined from the data at low orders. This knowledge can be very helpful in the ongoing calculation of four-loop anomalous dimensions of twist-two operators in QCD.

We studied the behaviour of the anomalous dimensions of leading twist operators in several models, the $O(N)$ symmetric φ^4 model and the φ^3 model in the ϵ expansion near $d = 4$ and $d = 6$, respectively, and the Gross-Neveu-Yukawa model in the ϵ and $1/N$ expansions. We found that in the φ^4 model the quadratic combination of the anomalous dimensions (1.3) remains finite for all type of twist two operators in the four loop approximation. In the φ^3 model and GNY model it holds only for the positive signature anomalous dimensions, while the mass corrections (1.3) for the negative signature operators are singular at $s = 0$ that agrees with the results of refs. [19–21].

In the $1/N$ approach instead of one singular point at $s = 0$ one finds three singular point at $s = \epsilon, 0, -\epsilon$ which in the ϵ expansions merge together. Each of these points corresponds in the language of ref. [29] to the intersection of the Regge trajectories of certain operators. We constructed the corresponding trajectories in the vicinity of the singular points and showed that the anomalous dimension of the local operator σ^2 lies on the trajectory of the operators $\sigma\partial_+^s\sigma$.

In the gauge theories, in QCD and $N = 4$ SYM, the mass corrections (1.3) remains finite at two and three loops, respectively. Starting from three and four loops poles cancel only in the planar sector. It would be quite interesting to understand reasons of this phenomenon.

Acknowledgments

We are grateful to Vitaly Velizhanin for bringing this topic to our attention. We would also like to thank him and Vladimir Braun, Sergey Derkachov, Gregory Korchemsky and Petr Kravchuk for numerous and illuminating discussions. This work was supported by Deutsche Forschungsgemeinschaft (DFG) through the Research Unit FOR 2926, "Next Generation pQCD for Hadron Structure: Preparing for the EIC", project number 40824754.

Appendices

A Anomalous dimensions in the $O(N)$-symmetric φ^3 theory

We present the three-loop results for the anomalous dimensions of the the operators considered in the Section 3.2. For the operators $\overline{\mathcal{O}}_s^{(ab)}$ and $\overline{\mathcal{O}}_s^{[ab]}$ we obtained

$$
\begin{aligned}
\bar{\gamma}^{(ab)}(s) = 2\gamma_\varphi &+ \frac{2u(-1)^s}{s(s+1)} + \frac{2u^2}{s^2(s+1)^2}\left(1 - \frac{1+2s}{s(s+1)}\right) \\
&+ \frac{2u^2(-1)^s}{3s(s+1)}\left(S_1 + \frac{2(N+2)s+N}{4s(s+1)} - \frac{2N+8}{3}\right) \\
&+ \frac{u^3}{s^2(s+1)^2}\Bigg\{12S_3 + 24S_{1,-2} - 12S_{-3} - \frac{2}{3}S_2 - 12\left(\frac{1}{s(s+1)} + 2\right)S_{-2} - 36\zeta_3 \\
&+ \frac{4}{3}\left(10 - \frac{1+2s}{s(s+1)}\right)S_1 - \frac{24\,(S_{-2}+1)}{(s-1)(s+2)} + \frac{4s-N}{2s^2(s+1)^2} + \frac{(22N+76)s+2-4N}{9s(s+1)} \\
&- \frac{2(4N+13)}{9}\Bigg\} \\
&+ \frac{u^3(-1)^s}{s(s+1)}\Bigg\{\frac{N+2}{18}S_2 + \frac{1}{9}S_1^2 - 12\left(\frac{1}{s^2(s+1)^2} + \frac{1}{s(s+1)}\right)S_{-2} - 4\zeta_3 \\
&+ \left(\frac{N+(2N+4)s}{18s(s+1)} - \frac{302+27N}{108}\right)S_1 + \frac{12(S_{-2}+1)}{(s-1)(s+2)} + \frac{4}{s^4(s+1)^4} \\
&+ \frac{N^2-8(N+2)s+288}{72s^2(s+1)^2} + \frac{N^2(1-16s)-18N(1+9s)-4(259s+744)}{216s(s+1)} \\
&+ \frac{8-12s}{s^3(s+1)^3} + \frac{12N^2+631N+7099}{648}\Bigg\} + O(u^4),
\end{aligned}
\tag{A.1}
$$

$$
\begin{aligned}
\bar{\gamma}^{[ab]}(s) = 2\gamma_\varphi &- \frac{2u(-1)^s}{s(s+1)} + \frac{2u^2}{s^2(s+1)^2}\left(1 - \frac{1+2s}{s(s+1)}\right) \\
&- \frac{2u^2(-1)^s}{3s(s+1)}\left(S_1 - \frac{2(N+2)s+N}{4s(s+1)} + \frac{2N+8}{3}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{u^3}{s^2(s+1)^2}\Bigg\{4S_3+8S_{1,-2}-4S_{-3}-\frac{2}{3}S_2-4\left(\frac{1}{s(s+1)}+2\right)S_{-2}\\
&-12\zeta_3+\frac{4}{3}\left(4-\frac{1+2s}{s(s+1)}\right)S_1-\frac{8\left(S_{-2}+1\right)}{(s-1)(s+2)}+\frac{4s-N}{2s^2(s+1)^2}\\
&+\frac{(22N+76)s+2-4N}{9s(s+1)}-\frac{2(4N+13)}{9}\Bigg\}\\
&-\frac{u^3(-1)^s}{s(s+1)}\Bigg\{\frac{N+2}{18}S_2+\frac{1}{9}S_1^2+4\left(\frac{1}{s^2(s+1)^2}+\frac{1}{s(s+1)}\right)S_{-2}\\
&-4\zeta_3+\left(\frac{N+(2N+4)s}{18s(s+1)}-\frac{302+27N}{108}\right)S_1-\frac{4\left(S_{-2}+1\right)}{(s-1)(s+2)}+\frac{4}{s^4(s+1)^4}\\
&+\frac{N^2-8(N+2)s+288}{72s^2(s+1)^2}+\frac{N^2(1-16s)-18N(1+9s)-4(259s-120)}{216s(s+1)}\\
&+\frac{8-12s}{s^3(s+1)^3}+\frac{12N^2+631N+7099}{648}\Bigg\}+O(u^4).
\end{aligned}
\tag{A.2}
$$

We checked that the anomalous dimension of the conserved $O(N)$ current vanishes, $\bar{\gamma}^{[ab]}(s=2)=0$. For the operator $\overline{\mathcal{O}}_s^a$ (see Eq. (3.14)) we obtained

$$
\begin{aligned}
\bar{\gamma}^a(s)=\;&\gamma_\varphi+\gamma_\sigma+(-1)^s\frac{2u}{s(s+1)}+\frac{2u^2}{s^2(s+1)^2}\left(1-\frac{1+2s}{s(s+1)}\right)\\
&+(-1)^s\frac{u^2}{6s(s+1)}\left((N+2)S_1+\frac{2+s(6+N)}{s(s+1)}-\frac{8(N+4)}{3}\right)\\
&+\frac{u^3}{s^2(s+1)^2}\Bigg\{(N+1)\left(4S_3-4S_{-3}+8S_{1,-2}-12\zeta_3-4\left(\frac{1}{s(s+1)}+2\right)S_{-2}\right.\\
&\left.-\frac{8\left(S_{-2}+1\right)}{(s-1)(s+2)}\right)-\frac{1}{3}\left(\frac{N+(4+2N)s+2}{s(s+1)}-(13N+14)\right)S_1-\frac{N+2}{6}S_2\\
&+\frac{(N+2)s-2}{2s^2(s+1)^2}+\frac{N+2s(19N+82)-14}{18s(s+1)}-\frac{13N+58}{18}\Bigg\}\\
&+(-1)^s\frac{u^3}{s(s+1)}\Bigg\{\frac{(N+2)(N+6)}{144}S_2+\frac{(N+2)^2}{144}S_1^2-4(N+1)\left(+\frac{1}{s^2(s+1)^2}\right.\\
&\left.+\frac{1}{s(s+1)}\right)S_{-2}-4\zeta_3+\left((N+2)\frac{(N+6)s+2}{72s(s+1)}-\frac{8N^2+81N+518}{216}\right)S_1\\
&+\frac{4(N+1)\left(S_{-2}+1\right)}{(s-1)(s+2)}+\frac{4}{s^4(s+1)^4}+\frac{4(2-3s)}{s^3(s+1)^3}-\frac{(N^2+8N+12)s-292}{72s^2(s+1)^2}\\
&-\frac{(8N^2+135N+1122)s-3N^2+861N+1298}{216s(s+1)}+\frac{12N^2+631N+7099}{648}\Bigg\}+O(u^4).
\end{aligned}
\tag{A.3}
$$

For $N = 2$ we have checked that $\bar{\gamma}^a(s) = \bar{\gamma}^{(ab)}(s)$ as it is required by the symmetry.

For the singlet operators $\overline{\mathcal{O}}_s$ and Σ_s the anomalous dimensions matrix takes the form

$$\hat{\gamma}(s) = \begin{pmatrix} \gamma_{\varphi\varphi}(s) & \gamma_{\varphi\sigma}(s) \\ \gamma_{\sigma\varphi}(s) & \gamma_{\sigma\sigma}(s) \end{pmatrix}, \tag{A.4}$$

where

$$\begin{aligned}
\gamma_{\varphi\varphi}(s) = {} & 2\gamma_\varphi + (-1)^s \frac{2u}{s(s+1)} + \frac{2(N+1)u^2}{s^2(s+1)^2}\left(1 - \frac{1+2s}{s(s+1)}\right) \\
& + (-1)^s \frac{2u^2}{3s(s+1)}\left(S_1 - \frac{2N+8}{3} + \frac{2(N+2)s+N}{4s(s+1)}\right) \\
& + \frac{u^3}{s^2(s+1)^2}\Bigg\{12S_3 + 24S_{1,-2} - 12S_{-3} - \frac{(N+2)(N+4)}{12}S_2 \\
& - 12\left(\frac{1}{s(s+1)} + 2\right)S_{-2} - 36\zeta_3 - \frac{N(N-2)}{12}S_1^2 + \left(\frac{7N^2 - 2N + 120}{9}\right. \\
& \left. - \frac{(5N^2 + 6N + 16)s + 2(N^2 + 2N + 4)}{6s(s+1)}\right)S_1 - \frac{24\,(S_{-2}+1)}{(s-1)(s+2)} \\
& + \frac{3(N^2 + 2N + 4)s - N(N+7)}{6s^2(s+1)^2} + \frac{4(N+2)(13N+19)s - 5N^2 - 10N + 4}{18s(s+1)} - \\
& \frac{2(25N^2 + 25N + 39)}{27}\Bigg\} \\
& + (-1)^s \frac{u^3}{s(s+1)}\Bigg\{\frac{N+2}{18}S_2 + \frac{1}{9}S_1^2 - 12\left(\frac{1}{s^2(s+1)^2} + \frac{1}{s(s+1)}\right)S_{-2} \\
& - 4\zeta_3 + \left(\frac{N + (2N+4)s}{18s(s+1)} - \frac{302 + 27N}{108}\right)S_1 + \frac{12(S_{-2}+1)}{(s-1)(s+2)} + \frac{4(2N+1)}{s^4(s+1)^4} \\
& - 4\frac{(2N+1)(3s-2)}{s^3(s+1)^3} + \frac{N^2 + 576N + 288 - 8(N+2)s}{72s^2(s+1)^2} \\
& + \frac{N^2(1-16s) - 18N(1+9s) - 4(259s+744)}{216s(s+1)} + \frac{12N^2 + 631N + 7099}{648}\Bigg\} + O(u^4),
\end{aligned} \tag{A.5}$$

$$\begin{aligned}
\gamma_{\varphi\sigma}(s) = {} & (-1)^s \frac{2uN}{s(s+1)} + \frac{2u^2N}{s^2(s+1)^2}\left(1 - \frac{1+2s}{s(s+1)}\right) \\
& + (-1)^s \frac{2u^2N}{3s(s+1)}\left(S_1 + \frac{1+4s}{2s(s+1)} - 4\right) \\
& + \frac{u^3N}{s^2(s+1)^2}\Bigg\{4S_3 + 8S_{1,-2} - 4S_{-3} - \frac{2}{3}S_2 - 4\left(\frac{1}{s(s+1)} + 2\right)S_{-2} - 12\zeta_3
\end{aligned}$$

$$
+\frac{4}{3}\left(4-\frac{1+2s}{s(s+1)}\right)S_1-\frac{8\left(S_{-2}+1\right)}{(s-1)(s+2)}+\frac{12s-N-4}{6s^2(s+1)^2}+\frac{N(22s-7)+436s-10}{36s(s+1)}
$$

$$
\left.+\frac{N-128}{27}\right\}
$$

$$
+(-1)^s\frac{u^3N}{s(s+1)}\left\{\frac{N+14}{72}S_2+\frac{N+6}{72}S_1^2-4\left(\frac{1}{s^2(s+1)^2}+\frac{1}{s(s+1)}\right)S_{-2}\right.
$$

$$
-4\zeta_3+\left(\frac{(N+14)s+4}{36s(s+1)}-\frac{19N+318}{108}\right)S_1+\frac{4\left(S_{-2}+1\right)}{(s-1)(s+2)}+\frac{4(N+1)}{s^4(s+1)^4}
$$

$$
-\frac{4(N+1)(3s-2)}{s^3(s+1)^3}+\frac{N(289-2s)-28s+290}{72s^2(s+1)^2}-\frac{(19N+318)s+N+318}{54s(s+1)}
$$

$$
\left.+\frac{583N+10046}{864}\right\}+O(u^4), \tag{A.6}
$$

$$
\gamma_{\sigma\varphi}(s)=(-1)^s\frac{2u}{s(s+1)}+\frac{2u^2}{s^2(s+1)^2}\left(1-\frac{1+2s}{s(s+1)}\right)
$$

$$
+(-1)^s\frac{u^2}{3s(s+1)}\left(NS_1+\frac{(N+2)s+1}{s(s+1)}-\frac{8(N+1)}{3}\right)
$$

$$
+\frac{u^3}{s^2(s+1)^2}\left\{4S_3+8S_{1,-2}-4S_{-3}+\frac{(N-2)}{12}S_1^2-\frac{3N+2}{12}S_2-12\zeta_3\right.
$$

$$
-4\left(\frac{1}{s(s+1)}+2\right)S_{-2}-\left(\frac{N(3s+2)+10s+4}{6s(s+1)}+\frac{N-50}{9}\right)S_1
$$

$$
\left.-\frac{8\left(S_{-2}+1\right)}{(s-1)(s+2)}+\frac{(3s-1)N+6s-4}{6s^2(s+1)^2}+\frac{N(114s+5)+252s-34}{36s(s+1)}-\frac{14(N+7)}{27}\right\}
$$

$$
+(-1)^s\frac{u^3}{s(s+1)}\left\{\frac{N(3N+10)}{144}S_2+\frac{N(3N+2)}{144}S_1^2-4\left(\frac{1}{s^2(s+1)^2}\right.\right.
$$

$$
\left.+\frac{1}{s(s+1)}\right)S_{-2}-4\zeta_3+\left(\frac{N((3N+10)s+4)}{72s(s+1)}-\frac{6N^2+23N+108}{54}\right)S_1
$$

$$
+\frac{4\left(S_{-2}+1\right)}{(s-1)(s+2)}+\frac{4(N+1)}{s^4(s+1)^4}-\frac{4(N+1)(3s-2)}{s^3(s+1)^3}-\frac{3N^2s-N(287-10s)-294}{72s^2(s+1)^2}
$$

$$
+\frac{N^2(9-24s)-2N(81s+1)-4(251s+328)}{216s(s+1)}
$$

$$
\left.+\frac{144N^2+3107N+26846}{2592}\right\}+O(u^4), \tag{A.7}
$$

$$
\gamma_{\sigma\sigma}(s)=2\gamma_\sigma+\frac{2u^2N}{s^2(s+1)^2}\left(1-\frac{1+2s}{s(s+1)}\right)+\frac{4u^3N}{s^2(s+1)^2}\left\{S_3+2S_{1,-2}-S_{-3}-\frac{3N+2}{48}S_2\right.
$$

$$
\begin{aligned}
&-\left(\frac{1}{s(s+1)}+2\right)S_{-2}-3\zeta_3+\frac{N-2}{48}S_1^2-\left(\frac{(3N+10)s+2N+4}{24s(s+1)}+\frac{N-50}{36}\right)S_1\\
&-\frac{2\left(S_{-2}+1\right)}{(s-1)(s+2)}+\frac{3s(N+2)+N-8}{24s^2(s+1)^2}+\frac{24(N+8)s+7N-26}{72s(s+1)}+\frac{11N-148}{108}\Bigg\}\\
&+(-1)^s\frac{4u^3N}{s^3(s+1)^3}\left\{S_{-2}+\frac{4\left(S_{-2}+1\right)}{(s-1)(s+2)}+\frac{1}{s^2(s+1)^2}+\frac{2-3s}{s(s+1)}+3\right\}. \qquad \text{(A.8)}
\end{aligned}
$$

We checked that the anomalous dimensions of the conserved $U(1)$ current and stress-energy tensor vanish,

$$
\begin{pmatrix}1, & -2\end{pmatrix}\hat{\gamma}(s=2)=0, \qquad \begin{pmatrix}1, & 1\end{pmatrix}\hat{\gamma}(s=3)=0. \qquad \text{(A.9)}
$$

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