














































































































































































































































Measurement of the inclusive branching fractions for B_s^0 decays into D mesons via hadronic tagging

The Belle and Belle II Collaborations

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ABSTRACT: We report measurements of the absolute branching fractions $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)$, $\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X)$, and $\mathcal{B}(B_s^0 \rightarrow D^\pm X)$, where the latter is measured for the first time. The results are based on a 121.4fb^{-1} data sample collected at the $\Upsilon(10860)$ resonance by the Belle detector at the KEKB asymmetric-energy e^+e^- collider. We reconstruct one B_s^0 meson in $e^+e^- \rightarrow \Upsilon(10860) \rightarrow B_s^* \bar{B}_s^*$ events and measure yields of D_s^+ , D^0 , and D^+ mesons in the rest of the event. We obtain $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X) = (68.6 \pm 7.2 \pm 4.0)\%$, $\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X) = (21.5 \pm 6.1 \pm 1.8)\%$, and $\mathcal{B}(B_s^0 \rightarrow D^\pm X) = (12.6 \pm 4.6 \pm 1.3)\%$, where the first uncertainty is statistical and the second is systematic. Averaging with previous Belle measurements gives $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X) = (63.4 \pm 4.5 \pm 2.2)\%$ and $\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X) = (23.9 \pm 4.1 \pm 1.8)\%$. For the B_s^0 production fraction at the $\Upsilon(10860)$, we find $f_s = (21.4^{+1.5}_{-1.7})\%$.

Contents

1	Introduction	1
2	Belle detector	2
3	Event selection	3
3.1	Reconstruction of B_s^0 tag candidates	3
3.2	Selection of signal D candidates	5
4	Yield of B_s^0 tags	6
5	Yields of $B_s^0 - D$ pairs	7
6	Branching fractions and systematic uncertainties	9
7	Average branching fractions and f_s	14
8	Conclusions	17
A	Channels used to reconstruct B and D mesons in the FEI	20
B	Determination of weights for simulated events	21
C	Broken signal and peaking background calibration	21
C.1	Loss of a photon from D_s^*	21
C.2	Misidentification of a kaon	22
C.3	Reconstruction of a B^0 meson as B_s^0	22

1 Introduction

Decays of B mesons provide a powerful tool for studying strong interactions at low energy, measuring parameters of the Standard Model, and searching for New Physics [1]. Absolute branching fractions of B^+ and B^0 decays have been precisely measured by the Belle, BaBar, and Belle II experiments using e^+e^- collisions at the $\Upsilon(4S)$ resonance. To study B_s^0 mesons, Belle collected data at the $\Upsilon(10860)$ resonance, which decays to $B_s^{(*)}\bar{B}_s^{(*)}$, $B^{(*)}\bar{B}^{(*)}(\pi)$, and final states with bottomonium and light hadrons. For brevity, in the following we refer to the $\Upsilon(10860)$ as the $\Upsilon(5S)$. The accuracy of absolute branching fractions of B_s^0 decays is limited by our knowledge of the B_s^0 production fraction at the $\Upsilon(5S)$ energy, f_s . To measure f_s , Belle used inclusive production of D_s^+ and D^0 mesons, which results in $f_s = (22.0_{-2.1}^{+2.0})\%$ [2]. The uncertainty in f_s is dominated by that of the inclusive branching fraction, $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)$, which was recently measured by Belle using semileptonic tagging

to be $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X) = (60.2 \pm 5.8 \pm 2.3)\%$ [3]. It is important to measure this branching fraction using hadronic tagging to improve its uncertainty. The sum of $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)$, $\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X)$, and $\mathcal{B}(B_s^0 \rightarrow D^\pm X)$ is expected to be above 100%, as the charm quark is produced both in the $b \rightarrow c$ and $W^- \rightarrow \bar{c}s$ parts of the B_s^0 decay diagram, and can be estimated based on similar sums for B^+ and B^0 mesons (see section 7). Thus, measurement of all three branching fractions will allow a consistency check of the results. Recently, Belle measured the ratio, $\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X)/\mathcal{B}(B_s^0 \rightarrow D_s^\pm X) = 0.416 \pm 0.018 \pm 0.092$ [2], from which we estimate $\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X) = (25.0 \pm 2.6 \pm 5.6)\%$. There is no information on $\mathcal{B}(B_s^0 \rightarrow D^\pm X)$.

The measured values of $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)$ and $\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X)$ are inconsistent with the widely quoted phenomenological estimates of $(92 \pm 11)\%$ and $(8 \pm 7)\%$, respectively [4]. We note that the fragmentation of the $c\bar{s}$ pair, where the c quark originates from a $b \rightarrow c$ transition and \bar{s} is a spectator quark, is not well understood. The fraction of $c\bar{s}$ pairs that hadronize into D_s^+ mesons, assumed in ref. [4] to be $(85 \pm 10)\%$, is probably an overestimate. Thus, measurements of the inclusive branching fractions of the B_s^0 meson provide information about dynamics of its decays.

In this paper, we report measurements of $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)$, $\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X)$, and $\mathcal{B}(B_s^0 \rightarrow D^\pm X)$. We use a data sample collected by the Belle experiment at the center-of-mass energy of the $\Upsilon(5S)$ resonance, 10.866 GeV, which has an integrated luminosity of 121.4 fb^{-1} . At the $\Upsilon(5S)$ resonance, B_s^0 mesons are produced in the processes $e^+e^- \rightarrow B_s^0 \bar{B}_s^0$, $B_s^0 \bar{B}_s^*$, and $B_s^* \bar{B}_s^0$, with $B_s^* \rightarrow B_s^0 \gamma$. We fully reconstruct one B_s^0 meson in many hadronic final states using a multivariate full event interpretation (FEI) algorithm [5]. We then reconstruct a D_s^+ , D^0 or D^+ meson in the rest of the event (ROE). The branching fraction is calculated as

$$\mathcal{B}(B_s^0 \rightarrow D/\bar{D} X) = \frac{N_{B_s-D}}{N_{B_s} \mathcal{B}_D \varepsilon_D^{\text{ROE}}}, \quad (1.1)$$

where D denotes D_s^+ , D^0 , and D^+ , \mathcal{B}_D is the branching fraction of the D reconstruction channel, and $\varepsilon_D^{\text{ROE}}$ is the reconstruction efficiency. The total number of B_s^0 tags, N_{B_s} , is determined from a fit to the $M(B_s)$ distribution. The number of $B_s^0 - D$ pairs, N_{B_s-D} , is determined from a two-dimensional fit to the distribution in $M(B_s)$ and $M(D)$, where $M(B_s)$ and $M(D)$ are the invariant masses of the B_s^0 and D candidates, respectively. To avoid bias, the data in the signal region were not examined until the selection criteria were fixed.

2 Belle detector

This analysis is based on data collected by the Belle detector at the KEKB asymmetric-energy e^+e^- collider [6, 7]. The Belle detector is a large-solid-angle magnetic spectrometer which consists of a four-layer silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter (ECL) composed of CsI(Tl) crystals located inside a superconducting solenoid coil that provides a magnetic field of 1.5 T. The

K_L^0 meson and muon detector (KLM), composed of resistive plate chambers, is located in the iron solenoid yoke. A detailed description of the detector can be found in refs. [8, 9].

Monte-Carlo (MC) simulation of $e^+e^- \rightarrow b\bar{b}$ and continuum $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) events uses EvtGen [10]. The $e^+e^- \rightarrow b\bar{b}$ events are generated from $\Upsilon(5S)$ decays, including $B_s^{(*)}\bar{B}_s^{(*)}$, $B^{(*)}\bar{B}^{(*)}(\pi)$, and final states with bottomonia. The MC sample size corresponds to an integrated luminosity six times larger than the data. The detector response is modeled using GEANT3 [11]. The MC simulation includes run-dependent variations in detector performance and background conditions.

3 Event selection

3.1 Reconstruction of B_s^0 tag candidates

We reconstruct B_s^0 mesons in the decay channels $D_s^{(*)-}\pi^+(\pi^0, \pi^+\pi^-)$, $D_s^{(*)+}D_s^{(*)-}$, $J/\psi K^+K^-(\pi^0)$, $\bar{D}^{(*)0}K^-\pi^+$, and $D_s^+K^-$.¹ The D^0 , D^+ , and D_s^+ mesons are reconstructed in final states with K^\pm , K_S^0 , π^\pm , η , η' , up to one π^0 , and up to five decay products. A list of the channels used in the reconstruction of B and D mesons is given in Appendix A. We reconstruct K_S^0 mesons in the $\pi^+\pi^-$ channel, π^0 mesons in the $\gamma\gamma$ channel, η mesons in the $\gamma\gamma$ and $\pi^+\pi^-\pi^0$ channels, η' mesons in the $\pi^+\pi^-\eta$ and $\pi^+\pi^-\gamma$ channels, D_s^{*+} mesons in the $D_s^+\gamma$ channel, D^{*0} mesons in the $D^0\pi^0$ and $D^0\gamma$ channels, D^{*+} mesons in the $D^+\pi^0$ and $D^0\pi^+$ channels, and the J/ψ in the $\mu^+\mu^-$ and e^+e^- channels.

We perform an initial loose selection of the final-state particles and decays, and then use a multivariate analysis for the final selection. We select tracks that originate from the vicinity of the interaction point (IP) by requiring $dr < 0.5$ cm and $dz < 3$ cm, where the z -axis is in the direction opposite to the e^+ beam, and dr and dz are transverse and longitudinal distances between the track and the IP, respectively. Charged particles are identified using ionization energy-loss measurements in the CDC, time-of-flight information from the TOF, and Cherenkov light yields in the ACC. Information from these subdetectors is combined into a likelihood $L(h)$ for a given hadron hypothesis h . In the initial selection, we apply the identification requirement only for kaon candidates, $L(K)/(L(K) + L(\pi)) > 0.1$. The efficiency of this requirement is 98% and the probability to misidentify a pion as a kaon is about 20%. For photons, we require the energy to be greater than 100, 50, and 150 MeV in the forward endcap ($12.4^\circ < \theta < 32.2^\circ$), barrel ($32.2^\circ < \theta < 128.7^\circ$), and backward endcap ($\theta > 128.7^\circ$) regions of the ECL, respectively, as these regions have different levels of background. For the π^0 , K_S^0 , η , η' , D , D^* , and J/ψ candidates, we apply a mass range requirement that corresponds to about ± 5 units of mass resolution. To improve momentum resolution, we apply a mass-constrained fit to π^0 , η , D^* , and J/ψ candidates; a mass-vertex-constrained fit to η' and D candidates; and a vertex-constrained fit to K_S^0 candidates.

In the FEI algorithm, a boosted decision tree (BDT) [12] is used with the following discriminating variables for various particle species.

¹Throughout this paper, charge-conjugate channels are always included.

- For charged pions, kaons, and leptons, we use the momentum, transverse momentum, and particle identification information.
- For photons, we use the energy, polar angle, number of crystals in the energy deposition (cluster), the ratio of the energy deposition in a 3×3 matrix of crystals to that in a 5×5 matrix, and cluster timing. These variables suppress hadronic showers and beam background.
- For $K_S^0 \rightarrow \pi^+\pi^-$ candidates, we use the invariant mass of the K_S^0 candidate and a set of parameters describing the displaced vertex of the K_S^0 . These are the distance of closest approach between the two daughter pions, the impact parameters of the daughter pions, the distance between the IP and the K_S^0 vertex, and the angle between the K_S^0 momentum and the direction from the IP to the K_S^0 vertex; the latter three variables are measured in the plane perpendicular to the beam direction.
- For $\pi^0 \rightarrow \gamma\gamma$ candidates, we use the two-photon invariant mass,² momentum and decay angle for the π^0 candidate, where the decay angle is defined as the angle between the photon momentum and the boost direction of the laboratory system in the π^0 rest frame.
- For $\eta \rightarrow \gamma\gamma$ candidates, we use the two-photon invariant mass and the decay angle. For $\eta \rightarrow \pi^+\pi^-\pi^0$ candidates, the $\pi^+\pi^-\pi^0$ invariant mass is used.
- For η' candidates, we use the invariant mass of the $\pi^+\pi^-\eta$ or $\pi^+\pi^-\gamma$ combination and the p-value of the mass-vertex-constrained fit.
- For D meson candidates, we use the invariant mass of the D candidate and the p-value of the mass-vertex-constrained fit. In three-body decays, we include the invariant masses of intermediate $\rho(\rightarrow \pi\pi)$, $K^*(\rightarrow K\pi)$, and $\phi(\rightarrow K^+K^-)$ resonance candidates.
- For J/ψ and D^* candidates, we use the invariant masses.
- For B_s^0 meson candidates, if there are several pions or kaons in the decay, we include the invariant masses of intermediate ρ , K^* , and $a_1(\rightarrow \pi\pi\pi)$ resonance candidates.
- To suppress continuum events, we use the event-shape variable R_2 (the ratio of the second to zeroth Fox-Wolfram moments [13]), the angle between the thrust axes of the B_s^0 candidate and that of the rest of the event [1]. All quantities are defined in the center-of-mass frame.

We train the BDT separately for each final-state particle species and for each decay of the unstable particle. The training result, the classifier output, is the probability (\mathcal{P}) that a given candidate is signal. In addition to the variables listed above, the training for each decay also uses the signal probabilities of all direct decay-products. To realize this,

²Here and below the invariant mass denotes the mass before the mass-constrained fit.

the training is carried out in stages, first to determine the signal probability for charged tracks, π^0 , and K_S^0 candidates, then for η and J/ψ candidates, next for η' candidates, then for D candidates, subsequently for D^* candidates, and finally for B_s^0 candidates.

Not all branching fractions of the channels used for B_s^0 meson reconstruction are well measured. Therefore, not all relative yields agree between data and simulation. To mitigate this problem, we introduce weights for simulated events as described in Appendix B. The weights range from 0.35 for $D_s^{*-}\pi^+\pi^+\pi^-$ to 4.6 for $\bar{D}^{*0}K^-\pi^+$ channels. For well-measured channels the weights are close to unity.

We apply a requirement on the B_s momentum, $|p^*(B_s) - 0.42| < 0.09 \text{ GeV}/c$, and channel-dependent requirements on \mathcal{P}_{B_s} , which are given in table 1. The above requirements are optimized to reach maximal sensitivity to the yield of B_s – D pairs, as described in the next section. The B_s momentum requirement selects the dominant production channel $e^+e^- \rightarrow B_s^*\bar{B}_s^*$ with 96% efficiency, while the channels $e^+e^- \rightarrow B_s^0\bar{B}_s^0$ and $e^+e^- \rightarrow B_s^0\bar{B}_s^*$, which correspond to a total fraction of 15%, are not included. We select B_s^0 candidates with invariant mass $M(B_s)$ in the interval $(5.25, 5.51 \text{ GeV}/c^2)$, which is used for fitting as described below. In the case of multiple B_s^0 candidates, we select the one that has the highest signal probability.

Table 1. The requirements on \mathcal{P}_{B_s} optimized for the measurement of $\mathcal{B}(B_s^0 \rightarrow D/\bar{D}X)$, the number of selected B_s^0 candidates, and the shift and width scaling parameters of the $M(B_s)$ signal functions.

Decay	\mathcal{P}_{B_s} requirement	Number of tags, N_{B_s}
$B_s^0 \rightarrow D_s^\pm X$	> 0.0012	12500 ± 310
$B_s^0 \rightarrow D^0/\bar{D}^0 X$	> 0.0050	9610 ± 190
$B_s^0 \rightarrow D^\pm X$	> 0.0200	6485 ± 120

3.2 Selection of signal D candidates

To reconstruct D mesons in the ROE, we use the following channels: $D_s^+ \rightarrow \phi\pi^+$, $\bar{K}^{*0}K^+$, $K_S^0K^+$; $D^0 \rightarrow K^-\pi^+$; and $D^+ \rightarrow K^-\pi^+\pi^+$.

Charged kaons and pions, except those from K_S^0 decays, are required to originate from the IP region with $dr < 0.5 \text{ cm}$ and $dz < 2 \text{ cm}$. We require $R_{K/\pi} = L(K)/(L(K) + L(\pi)) > 0.1$ for kaons from D_s^+ decays and $R_{K/\pi} > 0.6$ for those from both D^0 and D^+ . The requirement for pions from D_s^+ and D^0 mesons is $R_{\pi/K} = L(\pi)/(L(K) + L(\pi)) > 0.1$ and for pions from D^+ mesons is $R_{\pi/K} > 0.6$.

For $\phi \rightarrow K^+K^-$ and $K^{*0} \rightarrow K^+\pi^-$ candidates, the invariant masses are required to be within $40 \text{ MeV}/c^2$ and $100 \text{ MeV}/c^2$ of the nominal ϕ and K^{*0} masses [14], respectively. These requirements select ϕ and K^{*0} mesons with an efficiency of 99% and 91%, respectively. The K_S^0 candidates are reconstructed via the decay $K_S^0 \rightarrow \pi^+\pi^-$, with the selection criteria listed in ref. [15], and are also required to have an invariant mass within $15 \text{ MeV}/c^2$ of the nominal K_S^0 mass; the efficiency of this requirement is 96%.

To reconstruct D_s^+ candidates, for both ϕ and K^{*0} resonances the selection requirement $|\cos \theta_{\text{hel}}| > 0.3$ is applied, where the helicity angle θ_{hel} is defined as the angle between the K^- and D_s^+ momenta in the resonance rest frame. The helicity angle distributions are expected to follow a $\cos^2 \theta_{\text{hel}}$ distribution.

The requirements on the B_s^0 and D variables described above are optimized using a two-dimensional distribution in $M(B_s)$ and $M(D)$ in simulation and maximizing the figure-of-merit, defined as $S/\sqrt{S+B}$, where S is the number of properly reconstructed signal $B_s - D$ pairs, and B is the number of all other candidates in the signal region. The average number of multiple D candidates is $1.04 - 1.10$ for D_s^+ channels, 1.03 for D^0 , and 1.17 for D^+ . All candidates are included in the analysis. We verify that there is no peaking behaviour in the $M(D)$ distribution from multiple D candidates.

4 Yield of B_s^0 tags

The mass distribution of B_s^0 tag candidates, selected with the requirements optimized for the measurement of the $B_s^0 \rightarrow D_s^\pm X$ branching fraction, is shown in figure 1. We perform a binned likelihood fit to this distribution with a bin size of $1 \text{ MeV}/c^2$. We fit this distribution to the sum of a correctly-reconstructed signal (CRS) component, a broken-signal component, peaking background from B^0 , and a smooth background component. The mass distribution of the CRS events has an RMS width from 9 MeV to 32 MeV depending on the B_s reconstruction channel. The determination of the CRS component shape is described below. The B_s^0 broken-signal components are due to signal decays with

- (1) secondary interactions of final-state particles with the detector material;
- (2) pions and kaons from a D decay being swapped with those produced directly in a decay of a B_s^0 ;
- (3) a low-momentum signal γ , π^0 or π^+ swapped with a background candidate;
- (4) loss of a photon from $D_s^* \rightarrow D_s \gamma$;
- (5) misidentification of a kaon as a pion.

The shapes of these contributions are determined from simulation; their yields are fixed relative to that of the CRS. Contributions 1–3 peak in the region of CRS but have larger widths. Their yields are added to that of CRS; thus, the sum of CRS and broken-signal components 1–3 is counted as the total signal yield. The fraction of broken signal in the above sum is 9 – 15% depending on the \mathcal{P}_{B_s} requirement. The B^0 peaking background is due to Cabibbo-suppressed decays or decays with pions misidentified as kaons. The calibration of the simulation of broken-signal components 4–5 and the B^0 peaking background is described in Appendix C. The broken signal and B^0 background contributions are represented in the fit as histograms. The smooth background is described by a second order polynomial.

To determine the overall shape of the CRS component, we perform the fits described above separately for each B_s^0 reconstruction channel. In these fits, the CRS component

is described by a sum of three ($D_s^{(*)-}\pi^+(\pi^0, \pi^+\pi^-)$, $J/\psi K^+K^-$) or two (other channels) Gaussian functions. We fix the relative normalizations, means, and widths of the Gaussians to the results from simulation and then introduce common parameters, representing a shift in means and broadening of the widths, which are floated to adjust for differences between data and simulation. We find good agreement between data and the fit function in each channel, which provides validation for our modelling of background. We note that the shift and the broadening factor applied to the CRS component also account for potential mismodelling of broken-signal components 1–3 that peak in the signal region. After fitting the distributions in each channel, we add all the signal components to determine the overall signal function.

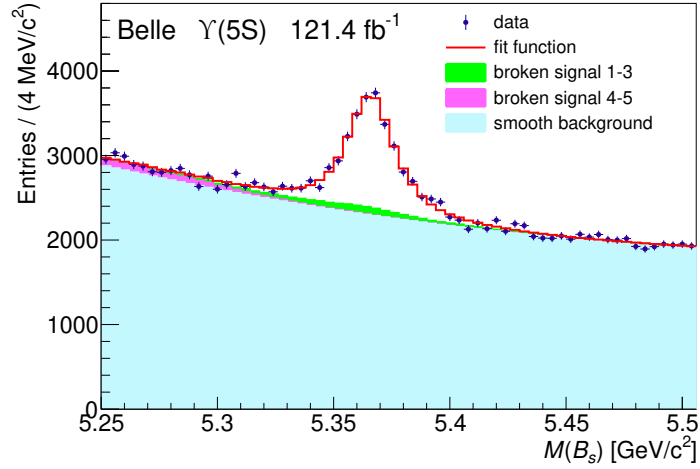


Figure 1. The mass distribution for the selected B_s^0 candidates. The points with error bars are data, the solid red histogram is the result of the fit, the filled green and magenta histograms are the broken-signal components, and the filled blue histogram is the smooth background component of the fit function

5 Yields of $B_s^0 - D$ pairs

To obtain the number of $B_s - D$ pairs, we perform a two-dimensional binned likelihood fit to the distribution in $M(B_s)$ and $M(D)$, with a bin size in $M(B_s)$ of $1 \text{ MeV}/c^2$ and in $M(D)$ of $0.5 \text{ MeV}/c^2$. The distribution for $D_s^+ \rightarrow \bar{K}^{*0}K^+$ in simulation is shown in figure 2. The boundaries of signal regions and sidebands are listed in table 2.

Our fit function has four components: both B_s and D candidates are signal (SS), the B_s candidate is signal and the D candidate is background (SB), the B_s candidate is background and the D candidate is signal (BS), and both candidates are background (BB), each being the product of one-dimensional signal or background mass functions. The B_s signal component is a sum of all peaking contributions: CRS, broken signal and B^0 peaking background. The shapes of all these contributions and their relative yields are the same as

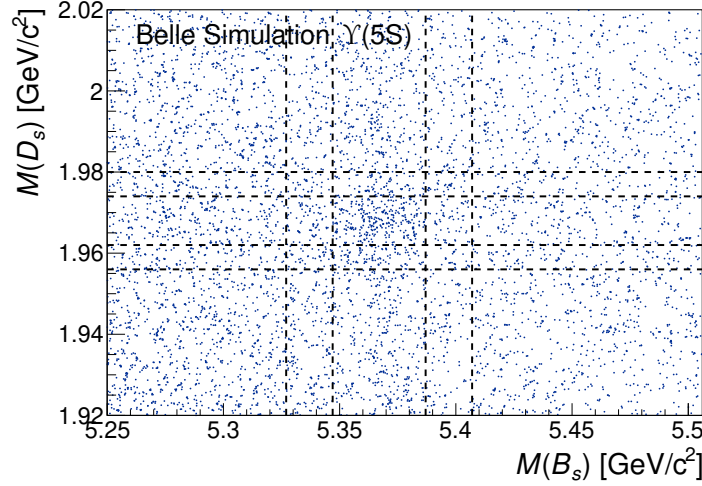


Figure 2. The distribution in $M(B_s)$ and $M(D_s(\rightarrow K^{*0}K))$ in simulation. Vertical and horizontal dashed lines indicate $M(B_s)$ and $M(D_s)$ sideband and signal regions.

Table 2. The boundaries of the signal region and sidebands along each axis in the two-dimensional distributions.

Region	$M(B_s)$	$M(D_s)$	$M(D^0)$	$M(D^+)$
Signal region	(5.347, 5.387)	(1.962, 1.974)	(1.849, 1.869)	(1.854, 1.874)
Left sideband	(5.250, 5.327)	(1.920, 1.956)	(1.800, 1.839)	(1.805, 1.844)
Right sideband	(5.407, 5.510)	(1.980, 2.020)	(1.879, 1.935)	(1.884, 1.940)

in figure 1. For the SS component, we fix the absolute yield of the B^0 peaking background taking into account the inclusive branching fraction $\mathcal{B}(B^0 \rightarrow D/\bar{D}X)$. The $M(D)$ signal function is described by a sum of three Gaussians. Its shift and width scaling factors are determined from a fit to the one-dimensional $M(D)$ distribution for inclusively produced D mesons that satisfy $p^*(D_s) < 2.7 \text{ GeV}/c$, $p^*(D^0) < 2.5 \text{ GeV}/c$, and $p^*(D^+) < 2.4 \text{ GeV}/c$. These requirements have close to 100% efficiency for D mesons produced in B_s^0 decays. The values of the shifts and width scaling factors for various D decay channels are listed in table 3; they are fixed in the two-dimensional fits. No peaking background is observed in the case of D meson reconstruction.

In the BS component the smooth background is described by an exponential function (D_s^+ or D^+ in the ROE) or a constant (D^+ in the ROE), and in the BB component by a first order polynomial. The background dependence in $M(D_s)$ in the SB component is constant ($\bar{K}^{*0}K^+$, $K_S^0K^+$) or linear ($\phi\pi^+$), and in the BB component it is linear for all channels. The background $M(D^0)$ and $M(D^+)$ functions in the SB and BB components are linear. The parameters of the background functions are free in the fit.

In the two-dimensional $M(B_s)$ and $M(D_s)$ distributions, the ratio of yields for two components, SS and BS, should not depend on the reconstructed channel of D_s^+ . Moreover,

Table 3. The shift and width scaling parameters of the $M(D)$ signal functions.

Channel	Shift, MeV/ c^2	Scaling
$D_s^+ \rightarrow \phi\pi^+$	-0.21 ± 0.02	0.978 ± 0.005
$D_s^+ \rightarrow \bar{K}^{*0}K^+$	-0.24 ± 0.05	1.004 ± 0.017
$D_s^+ \rightarrow K_S^0 K^+$	-0.25 ± 0.06	1.059 ± 0.022
$D^0 \rightarrow K^-\pi^+$	0.16 ± 0.01	0.982 ± 0.002
$D^+ \rightarrow K^-\pi^+\pi^+$	0.08 ± 0.02	0.968 ± 0.003

it is expected that the shape of the $M(B_s)$ background in the BS component does not depend on the D_s^+ channel. We thus perform a simultaneous two-dimensional fit to the distributions in $M(B_s)$ and $M(D_s)$ for all three D_s^+ channels with N_{BS}/N_{SS} and the slope of the exponential function in the BS components being common free parameters. Projections of the fit result on each of the axes in the signal and sideband regions defined in table 2 are shown in figures 3–7. The fit results for yields are given in table 4.

Table 4. The yields of the two-dimensional fit, the branching fractions of D mesons, the reconstruction efficiency, and calculated B_s^0 branching fractions.

Decay	N_{B_s-D}	$\mathcal{B}_D, \%$	$\epsilon_D^{\text{ROE}}, \%$	$\mathcal{B}(B_s^0 \rightarrow D/\bar{D}X), \%$
$B_s^0 \rightarrow D_s^\pm X$				
$\phi\pi^+$	85 ± 12	5.37 ± 0.10	17.3 ± 0.8	$73.0 \pm 10.6 \pm 5.2$
$\bar{K}^{*0}K^+$	53 ± 13	5.37 ± 0.10	17.3 ± 0.8	$54.1 \pm 11.7 \pm 3.7$
$K_S^0 K^+$	55 ± 10	1.450 ± 0.035	34.4 ± 1.9	$88.2 \pm 16.2 \pm 7.0$
$B_s^0 \rightarrow D^0/\bar{D}^0 X$	56 ± 16	3.947 ± 0.030	68.2 ± 5.1	$21.5 \pm 6.1 \pm 1.8$
$B_s^0 \rightarrow D^\pm X$	34 ± 12	9.38 ± 0.16	44.4 ± 4.0	$12.6 \pm 4.6 \pm 1.3$

6 Branching fractions and systematic uncertainties

We calculate the branching fractions using Eq. (1.1), where the yields of B_s^0 tags and $B_s - D$ pairs, the reconstruction efficiency for D mesons, and the D mesons branching fractions are given in table 4.

We determine the reconstruction efficiency for D mesons in the ROE using simulation. In the calculation of the reconstruction efficiency for the $D_s^+ \rightarrow \phi\pi^+$ and $D_s^+ \rightarrow \bar{K}^{*0}K^+$ channels, we use the $D_s^+ \rightarrow K^+K^-\pi^+$ branching fraction. Thus, the efficiency $\epsilon_{D_s}^{\text{ROE}}$ includes the selection efficiency for the corresponding $K^+K^-\pi^+$ Dalitz plot regions.

The values obtained for $\mathcal{B}(B_s^0 \rightarrow D/\bar{D}X)$ are listed in table 4, where the first uncertainty is statistical and the second is systematic. The statistical uncertainty includes

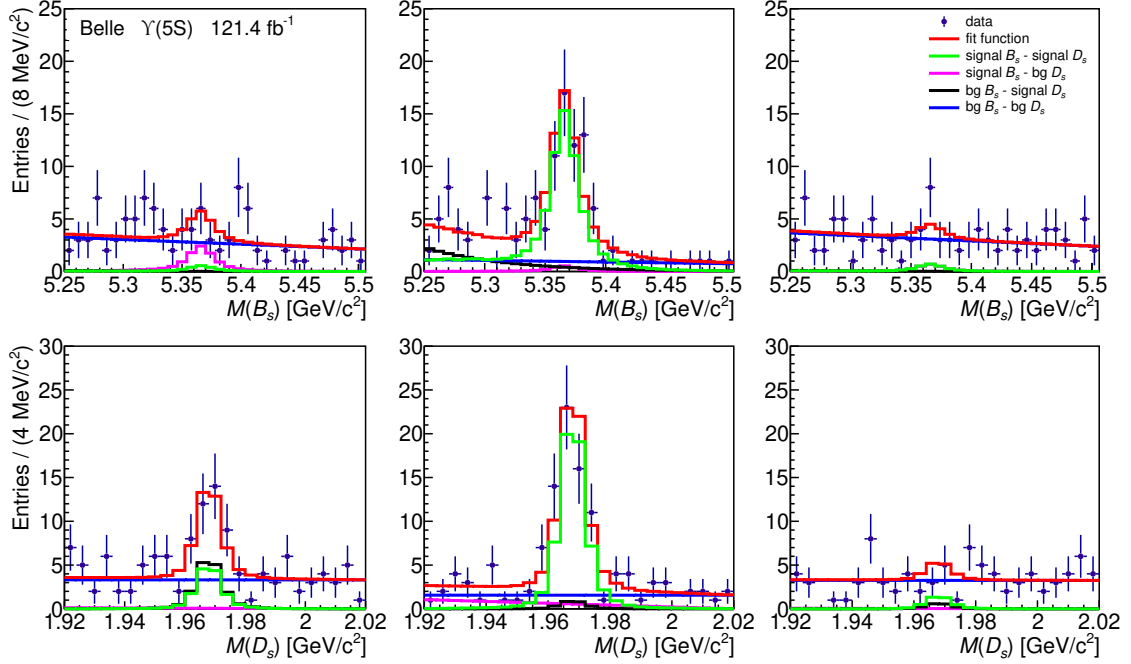


Figure 3. Projections of the two-dimensional fit to the distribution in $M(B_s)$ and $M(D_s)$ for the $D_s^+ \rightarrow \phi\pi^+$ channel onto $M(B_s)$ (top) and $M(D_s)$ (bottom) axis. The left and right figures correspond to the projections in the left and right sideband regions, and the central figures show projections in the signal region. Blue points with error bars represent the data. The solid red histograms show the total fit function, while the solid green, black, magenta, and blue histograms show the SS, BS, SB and BB components, respectively.

contributions from N_{B_s} and N_{B_s-D} .

Sources of systematic uncertainties are listed in tables 5 and 6 and are described below.

- To account for the uncertainty on the $M(B_s)$ signal shape, we vary the shift and width scaling parameters of the CRS component by $\pm 1\sigma$ in each reconstruction channel individually. Corresponding deviations in the measured branching fractions are added in quadrature for all channels to obtain the total uncertainty due to the description of the $M(B_s)$ signal shape. The uncertainty due to the description of the $M(D)$ signal shape is negligible.
- Variation of CRS shape parameters described in the previous item partly accounts for potential mismodelling of the broken-signal components that peak in the signal region. In addition, we multiply each of the above components by factors of 0.75 or 1.25 simultaneously in all channels. The broken-signal components in the left $M(B_s)$ sideband are calibrated as described in Appendix C; we vary the corresponding yields by $\pm 1\sigma$. Deviations in the measured branching fractions due to all variations are added in quadrature to obtain the total uncertainty due to possible mismodelling of the broken signal. The uncertainty due to B^0 peaking background is negligible.
- The uncertainty on the smooth background shape in the two-dimensional fit is ob-

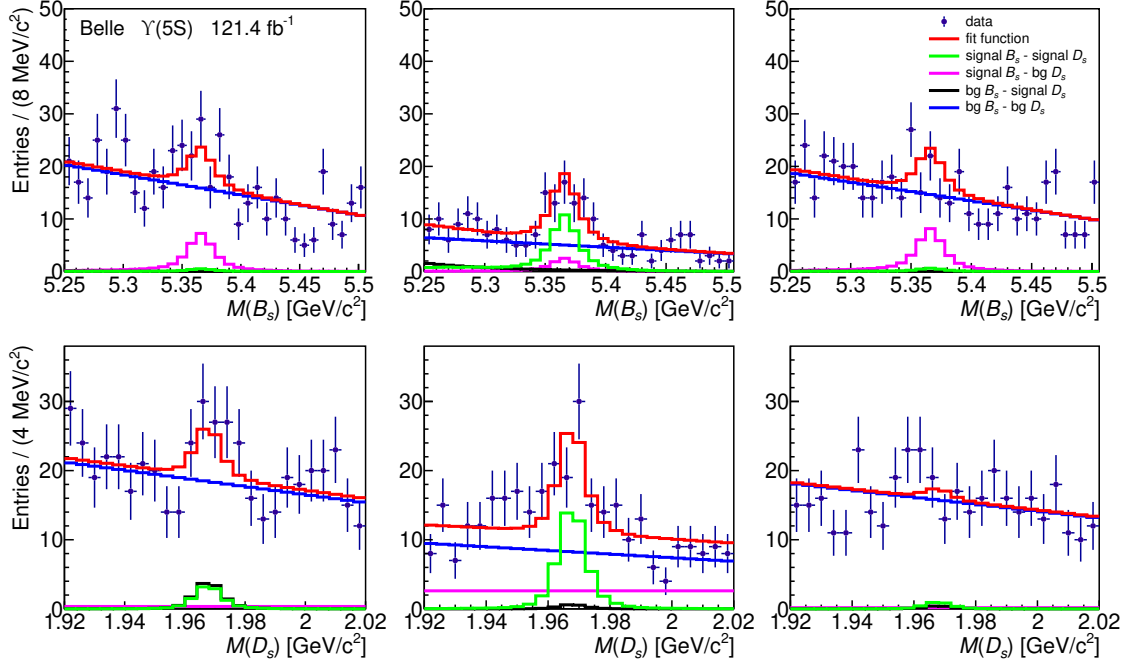


Figure 4. Projections of the two-dimensional fit to the distribution in $M(B_s)$ and $M(D_s)$ for the $D_s^+ \rightarrow \bar{K}^{*0} K^+$ channel onto $M(B_s)$ (top) and $M(D_s)$ (bottom) axis. The left and right figures correspond to the projections in the left and right sideband regions, and the central figures show projections in the signal region. The legend is the same as in Fig. 3.

tained by varying its shape parameters: we change all constant and linear functions into exponential functions, and also add a cross term that does not arise as a result of multiplying one-dimensional functions. The deviations in the fit results are added in quadrature, and the sum is assigned as the systematic uncertainty.

- The systematic uncertainty from the track reconstruction efficiency, estimated using partially reconstructed $D^{*+} \rightarrow D^0 \pi^+$, $D^0 \rightarrow \pi^+ \pi^- K_S^0$ and $K_S^0 \rightarrow \pi^+ \pi^-$ events [16], is 0.35% per track. We take 1.1% as the associated systematic uncertainty for the D_s^+ and D^+ channels, and 0.7% for the D^0 channel.
- The uncertainty from the K/π identification efficiency due to a possible difference between MC and data is studied using $D^{*+} \rightarrow D^0(K^-\pi^+)\pi^+$ decays [16]. The uncertainty is 2.1% for $D_s^+ \rightarrow \phi\pi^+$, 1.9% for $D_s^+ \rightarrow \bar{K}^{*0}K^+$, and 0.7% for $D_s^+ \rightarrow K_S^0 K^+$; 1.2% for $D^0 \rightarrow K^-\pi^+$ and 3.0% for $D^+ \rightarrow K^-\pi^+\pi^+$.
- The uncertainty from the K_S^0 reconstruction efficiency, which is studied using $D^{*+} \rightarrow D^0(\pi^+\pi^-K_S^0)\pi^+$ decays [16], is found to be 2.3%.
- We account for the uncertainty due to the difference between the $D_s^+ \rightarrow K^+K^-\pi^+$ Dalitz plot in simulation and data. This difference is studied for inclusive D_s^+ mesons, and the following correction factors for the ϕ and K^{*0} reconstruction efficiencies are

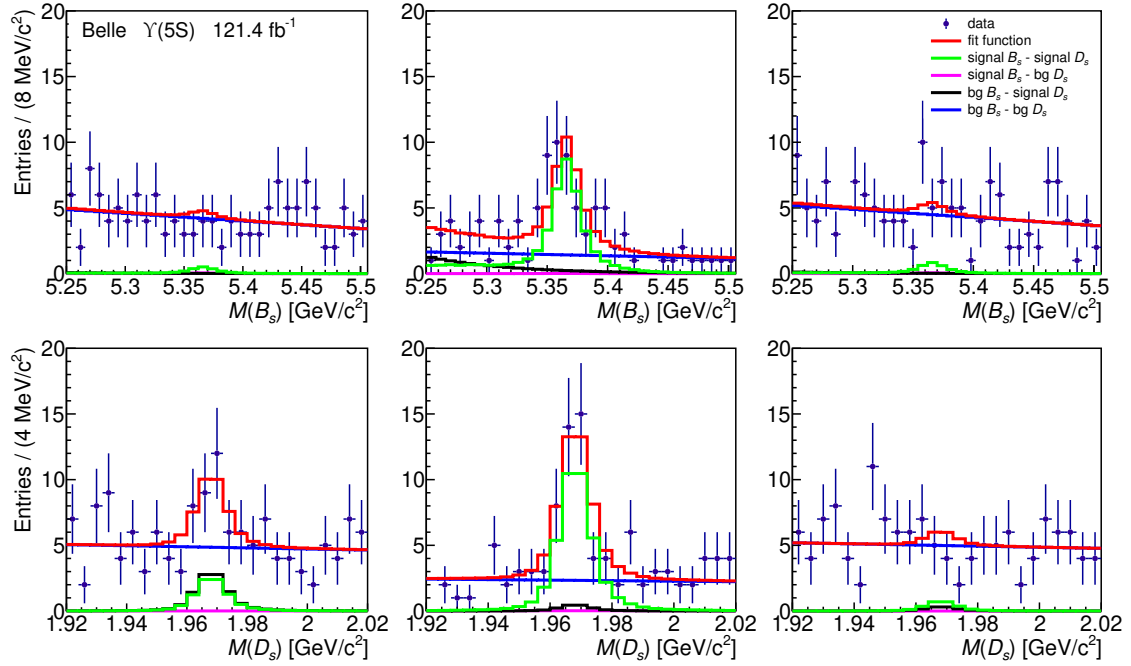


Figure 5. Projections of the two-dimensional fit to the distribution in $M(B_s)$ and $M(D_s)$ for the $D_s^+ \rightarrow K_S^0 K^+$ channel onto $M(B_s)$ (top) and $M(D_s)$ (bottom) axis. The left and right figures correspond to the projections in the left and right sideband regions, and the central figures show projections in the signal region.

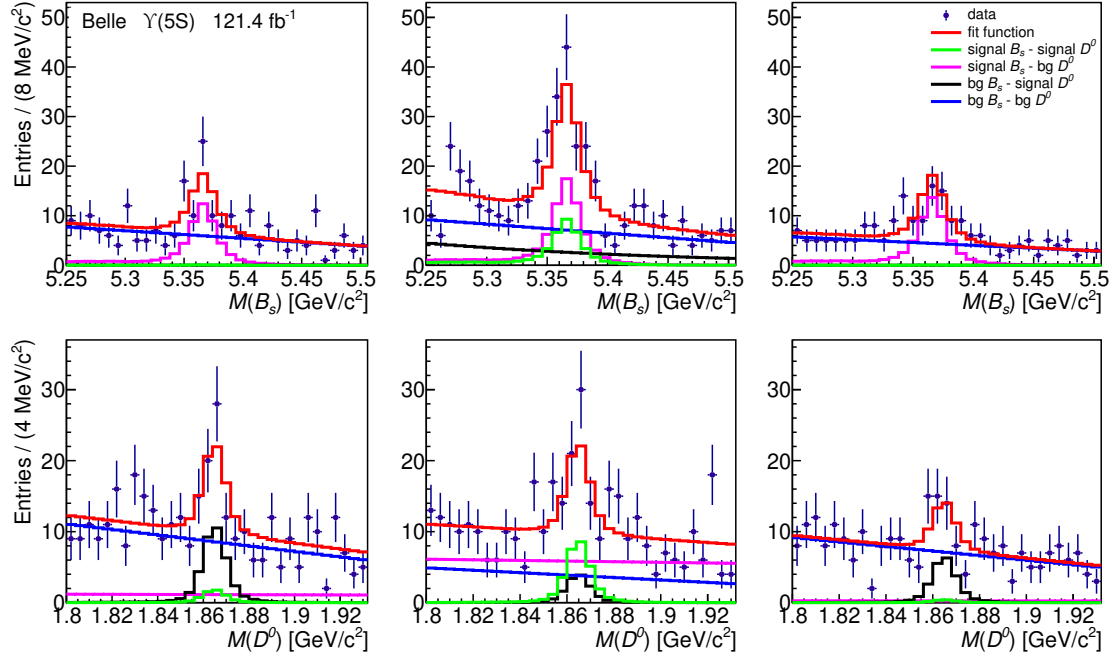


Figure 6. Projections of the two-dimensional fit to the distribution in $M(B_s)$ and $M(D^0)$ onto $M(B_s)$ (top) and $M(D^0)$ (bottom) axis. The left and right figures correspond to the projections in the left and right sideband regions, and the central figures show projections in the signal region.

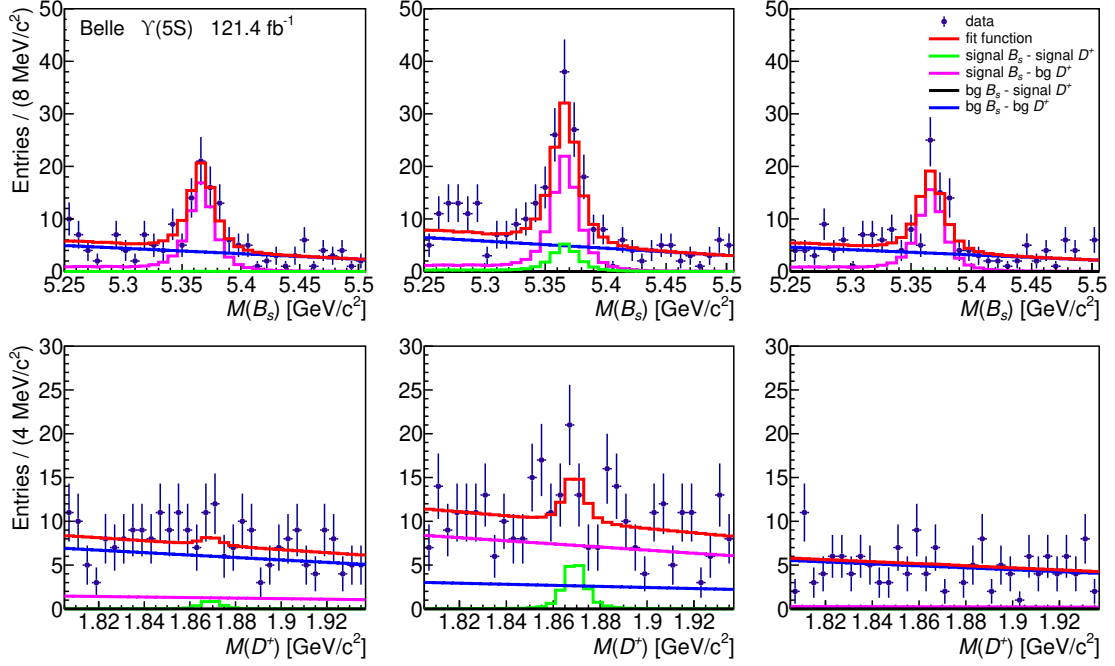


Figure 7. Projections of the two-dimensional fit to the distribution $M(B_s)$ and $M(D^+)$ onto $M(B_s)$ (top) and $M(D^+)$ (bottom) axis. The left and right figures correspond to the projections in the left and right sideband regions, and the central figures show projections in the signal region.

applied

$$\begin{aligned} r_\phi &= \varepsilon_\phi^{\text{data}} / \varepsilon_\phi^{\text{MC}} = 0.947 \pm 0.008; \\ r_{K^*} &= \varepsilon_{K^*}^{\text{data}} / \varepsilon_{K^*}^{\text{MC}} = 1.042 \pm 0.008. \end{aligned} \quad (6.1)$$

The statistical uncertainty for this factor is included as a systematic uncertainty, which is estimated to be 0.8%.

- Since the momentum spectra of D mesons from B_s^0 decays may differ between simulation and data, we estimate the uncertainty due to the dependence of reconstruction efficiency on the momentum. We examine D_s^+ mesons in the ROE and measure their yields for center-of-mass momenta below and above $1.5 \text{ GeV}/c$. The yield ratio in data, 1.15 ± 0.22 , is in good agreement with that in simulation, 1.17. We introduce weights for simulated events according to the above uncertainty in data and include the corresponding deviations in the efficiency as systematic uncertainties. We find $0.2 - 0.8\%$ for D_s^+ , 0.2% for D^0 , and below 0.1% for D^+ .
- The reconstruction efficiency of the B_s^0 tag depends on multiplicity in the ROE, primarily because of the requirement of one B_s^0 candidate per event. If there is a reconstructed D in the ROE, then the multiplicity is lower than the average value and the B_s^0 reconstruction efficiency is expected to be slightly higher. According to simulation, the effect is at the 5% level. To estimate the corresponding systematic uncertainty, we measure the inclusive branching fractions of B^+ and B^0 mesons using part of the $\Upsilon(4S)$ data with an integrated luminosity of 571 fb^{-1} corresponding to

the same inner detector configuration as the $\Upsilon(5S)$ data. The analysis procedure is the same as for B_s^0 mesons. The B^+ and B^0 decay channels used in the FEI are listed in table 9 in Appendix A. We find good agreement with the previous Belle measurement [2] and assign a systematic uncertainty of 3.6% for D_s^+ and 2.1% for D^0 and D^+ based on the uncertainty of the results.

- The contributions from the limited size of the MC samples are estimated to be 4.4% – 5.7% for D_s^+ , 7.5% for D^0 , and 9.0% for D^+ .
- The uncertainty on the world averages $\mathcal{B}(D_s^+ \rightarrow K^+ K^- \pi^+)$, $\mathcal{B}(D_s^+ \rightarrow K_S^0 K^+)$, and $\mathcal{B}(D^0 \rightarrow K^- \pi^+)$ are 1.9%, 2.5%, and 1.7%, respectively [14].

The total systematic uncertainty is obtained by adding the various contributions in quadrature.

Table 5. Systematic uncertainties in the measurement of $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)$.

Source	Channel			Combined
	$\phi\pi^+$	$\bar{K}^{*0}K^+$	$K_S^0 K^+$	
Signal shape	2.3	1.8	1.6	2.0
Broken signal	0.9	0.9	0.9	0.9
Smooth background	1.6	1.0	1.1	1.4
Tracking	1.1	1.1	1.1	1.1
K/π identification	2.1	1.9	0.7	1.7
K_S^0 reconstruction	–	–	2.3	0.6
D_s momentum	0.8	0.6	0.2	0.6
Dalitz plot	0.8	0.8	–	0.6
FEI efficiency	3.6	3.6	3.6	3.6
MC statistics	4.4	4.5	5.7	2.7
$\mathcal{B}(D_s \rightarrow KK\pi)$	1.9	1.9	–	1.4
$\mathcal{B}(D_s \rightarrow K_S K)$	–	–	2.4	0.6
$\mathcal{B}(K_S^0 \rightarrow \pi^+ \pi^-)$	–	–	< 0.1	–
Total	7.2	6.9	7.9	5.9

7 Average branching fractions and f_s

Averaging the $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)$ over three D_s^+ channels, we obtain

$$\mathcal{B}(B_s^0 \rightarrow D_s^\pm X) = (68.6 \pm 7.2 \pm 4.0)\%, \quad (7.1)$$

where the correlations in statistical uncertainty on the number of B_s^0 tags and systematic uncertainties shown in table 5 are taken into account using the fitting method described in ref. [17]. The p-value of this fit is 28%.

Table 6. Systematic uncertainties in the measurements of $\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X)$ and $\mathcal{B}(B_s^0 \rightarrow D^\pm X)$.

Source	$B_s^0 \rightarrow D^0/\bar{D}^0 X$	$B_s^0 \rightarrow D^\pm X$
Signal shape	2.0	0.6
Broken signal	1.1	2.9
Smooth background	0.3	0.9
Tracking	0.7	1.1
K/π identification	1.2	3.0
D momentum	0.2	<0.1
FEI efficiency	2.1	2.1
MC statistics	7.5	9.0
$\mathcal{B}(D \rightarrow K\pi(\pi))$	0.8	1.7
Total	8.3	10.4

The measurement of $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)$ is in agreement with the previous Belle measurement using semileptonic tagging, $(60.2 \pm 5.8 \pm 2.3)\%$ [3], which when rescaled with the most recent values of D_s^+ branching fractions gives $(60.5 \pm 5.8 \pm 2.2)\%$. Averaging our measurement with the previous result after rescaling, and taking into account correlated uncertainties as shown in table 7, gives

$$\mathcal{B}(B_s^0 \rightarrow D_s^\pm X) = (63.4 \pm 4.5 \pm 2.2)\%. \quad (7.2)$$

Table 7. Systematic uncertainties in the $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)$ measurement in ref. [3] and in this work.

Source	Ref. [3]	This work	Combined
Uncorrelated	3.0	5.3	2.6
Tracking	1.1	1.1	1.1
K/π identification	1.3	1.7	1.5
$\mathcal{B}(D_s \rightarrow KK\pi)$	1.5	1.4	1.4
$\mathcal{B}(D_s \rightarrow K_S K)$	0.4	0.6	0.5
Total			3.5

Using this value and the ratio $\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X)/\mathcal{B}(B_s^0 \rightarrow D_s^\pm X) = 0.416 \pm 0.018 \pm 0.092$ measured in ref. [2], we obtain

$$\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X) = \mathcal{B}(B_s^0 \rightarrow D_s^\pm X) \frac{\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X)}{\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)} = (26.4 \pm 2.2 \pm 5.9)\%, \quad (7.3)$$

where the uncertainty is similar to that of the direct measurement shown in table 4. We average the two values taking into account their correlation to obtain

$$\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X) = (23.9 \pm 4.1 \pm 1.8)\%. \quad (7.4)$$

Systematic uncertainties for the direct measurement, the calculated value, and their average are presented in table 8.

Table 8. Systematic uncertainties in the direct $\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X)$ measurement and in the value of the product $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X) \times \frac{\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X)}{\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)}$.

Source	Direct measurement	$\mathcal{B}(B_s^0 \rightarrow D_s^\pm X) \frac{\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X)}{\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)}$	Combined
Uncorrelated	7.5	22.4	7.1
Signal shape	2.0	0.8	1.3
Broken signal	1.1	0.3	0.6
Tracking	0.7	0.7	0.7
K/π identification	1.2	1.6	1.4
D momentum	0.2	0.5	0.4
FEI efficiency	2.1	1.4	1.7
$\mathcal{B}(D^0 \rightarrow K^- \pi^+)$	0.8	0.8	0.8
Total			7.7

The sum of three branching fractions, $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)$, $\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X)$, and $\mathcal{B}(B_s^0 \rightarrow D^\pm X)$, is equal to $(99.9 \pm 7.6 \pm 3.8)\%$, where the correlations in systematic uncertainties are taken into account. The corresponding sums for B^+ and B^0 are $(109.0 \pm 4.5)\%$ and $(106.5 \pm 5.2)\%$, respectively [14]. These sums are not expected to depend upon the flavor of the spectator quark. Thus, averaging the results for B^+ and B^0 , we find $(107.9 \pm 3.4)\%$; the sum for B_s^0 agrees with this value.

Using the average $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)$ from Eq. (7.2), we recalculate f_s in [2]

$$f_s = (21.8 \pm 0.2 \pm 2.0)\%. \quad (7.5)$$

To improve the accuracy of f_s , the relation

$$f_s + f_{B\bar{B}X} + f_{\mathcal{B}} = 1 \quad (7.6)$$

is used, where $f_{B\bar{B}X} = (75.1 \pm 4.0)\%$ [18] is the production rate of $B\bar{B}X$ events at the $\Upsilon(5S)$ and $f_{\mathcal{B}}$ is the production rate of $b\bar{b}$ events without open-bottom mesons in final states; the contribution of known channels is $f_{\mathcal{B}}^{known} = (4.9 \pm 0.6)\%$ [18]. We fit to three measurements, f_s , $f_{B\bar{B}X}$ and $f_{\mathcal{B}}$, applying (7.6) as a constraint. We obtain

$$\begin{aligned} f_s &= (21.4^{+1.5}_{-1.7})\%; \\ f_{B\bar{B}X} &= (73.8^{+1.5}_{-2.9})\%; \\ f_{\mathcal{B}} &= (4.8^{+3.6}_{-0.5})\%. \end{aligned} \quad (7.7)$$

This result for f_s supersedes the previous value of the B_s^0 production rate $f_s = (22.0_{-2.1}^{+2.0})\%$ [2].

8 Conclusions

To conclude, we have measured the inclusive branching fractions for B_s^0 decays into D mesons, using full hadronic reconstruction of one B_s^0 in $e^+e^- \rightarrow B_s^* \bar{B}_s^*$. We find

$$\begin{aligned}\mathcal{B}(B_s^0 \rightarrow D_s^\pm X) &= (68.6 \pm 7.2 \pm 4.0)\%, \\ \mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X) &= (21.5 \pm 6.1 \pm 1.8)\%, \\ \mathcal{B}(B_s^0 \rightarrow D^\pm X) &= (12.6 \pm 4.6 \pm 1.3)\%.\end{aligned}\tag{8.1}$$

We improve the accuracy of $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)$ by averaging with the result of previous measurement [3] and obtain

$$\mathcal{B}(B_s^0 \rightarrow D_s^\pm X) = (63.4 \pm 4.5 \pm 2.2)\%.\tag{8.2}$$

Multiplying this value by the ratio $\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X)/\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)$ measured in ref. [2] and averaging the result obtained for $\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X)$ with the direct measurement presented in Eq. (8.1), we find

$$\mathcal{B}(B_s^0 \rightarrow D^0/\bar{D}^0 X) = (23.9 \pm 4.1 \pm 1.8)\%.\tag{8.3}$$

Using the average value of $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)$, we update the production fractions

$$\begin{aligned}f_s &= (21.4_{-1.7}^{+1.5})\%; \\ f_{B\bar{B}X} &= (73.8_{-2.9}^{+1.5})\%; \\ f_{\mathcal{B}} &= (4.8_{-0.5}^{+3.6})\%.\end{aligned}\tag{8.4}$$

These results supersede those reported in refs. [2, 18].

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Appendix A Channels used to reconstruct B and D mesons in the FEI

Table 9. Decay channels of B_s^0 , B^+ and B^0 mesons used in the FEI.

$B_s^0 \rightarrow$	$B^+ \rightarrow$	$B^0 \rightarrow$
$D_s^- \pi^+$	$\bar{D}^0 \pi^+$	$D^- \pi^+$
$D_s^- \pi^+ \pi^0$	$\bar{D}^0 \pi^+ \pi^0$	$D^- \pi^+ \pi^0$
$D_s^- \pi^+ \pi^+ \pi^-$	$\bar{D}^0 \pi^+ \pi^+ \pi^-$	$D^- \pi^+ \pi^+ \pi^-$
$D_s^{*-} \pi^+$	$\bar{D}^{*0} \pi^+$	$D^{*-} \pi^+$
$D_s^{*-} \pi^0 \pi^+$	$\bar{D}^{*0} \pi^+ \pi^0$	$D^{*-} \pi^+ \pi^0$
$D_s^{*-} \pi^+ \pi^+ \pi^-$	$\bar{D}^{*0} \pi^+ \pi^+ \pi^-$	$D^{*-} \pi^+ \pi^+ \pi^-$
$D_s^- D_s^+$	$D_s^+ \bar{D}^0$	$D_s^+ D^-$
$D_s^{*-} D_s^+$	$D_s^{*+} \bar{D}^0$	$D_s^{*+} D^-$
$D_s^- D_s^{*+}$	$D_s^+ \bar{D}^{*0}$	$D_s^+ D^{*-}$
$D_s^{*-} D_s^{*+}$	$D_s^{*+} \bar{D}^{*0}$	$D_s^{*+} D^{*-}$
$J/\psi K^+ K^-$	$J/\psi K^+$	$J/\psi K_S^0$
$J/\psi K^+ K^- \pi^0$	$J/\psi K_S^0 \pi^+$	$J/\psi K^+ \pi^-$
	$J/\psi K^+ \pi^+ \pi^-$	
$\bar{D}^0 K^- \pi^+$	$D^- \pi^+ \pi^+$	$D^{*-} K^+ K^- \pi^+$
$\bar{D}^{*0} K^- \pi^+$	$D^{*-} \pi^+ \pi^+$	
$D_s^- K^+$		

Table 10. Decay channels of D^0 , D^+ and D_s^+ mesons used in the FEI.

$D^0 \rightarrow$	$D^+ \rightarrow$	$D_s^+ \rightarrow$
$K^- \pi^+$	$K^- \pi^+ \pi^+$	$K^+ K^- \pi^+$
$K^- \pi^+ \pi^0$	$K^- \pi^+ \pi^+ \pi^0$	$K^+ K_S^0$
$K^- \pi^+ \pi^+ \pi^-$	$K_S^0 \pi^+$	$K^+ K^- \pi^+ \pi^0$
$K_S^0 \pi^+ \pi^-$	$K_S^0 \pi^+ \pi^0$	$K^+ K_S^0 \pi^+ \pi^-$
$K_S^0 \pi^+ \pi^- \pi^0$	$K_S^0 \pi^+ \pi^+ \pi^-$	$K^- K_S^0 \pi^+ \pi^+$
$K^+ K^-$	$K^+ K^- \pi^+$	$K^+ K^- \pi^+ \pi^+ \pi^-$
$K^+ K^- K_S^0$		$K^+ \pi^+ \pi^-$
		$\pi^+ \pi^+ \pi^-$
		$K^+ K_S^0 \pi^0$
		$K_S^0 K_S^0 \pi^+$
		$\eta \pi^+$
		$\eta' \pi^+$
		$\eta \pi^+ \pi^0$
		$\eta' \pi^+ \pi^0$

Appendix B Determination of weights for simulated events

We introduce weights for simulated events to take into account the difference between relative yields of B_s^0 channels in simulation and data. To determine the weights, we use the $M(B_s)$ distributions in the $\Upsilon(5S)$ sample. We select B_s^0 candidates using requirements on their momenta measured in the center-of-mass frame, $p^*(B_s)$, and the signal probability, \mathcal{P}_{B_s} , which are close to the optimal requirements for measuring $\mathcal{B}(B_s^0 \rightarrow D_s^\pm X)$. We perform a simultaneous fit to the $M(B_s)$ distributions in data and simulation for each B_s^0 decay channel. The signal in simulation is described by a sum of three Gaussians; the signal in data is described by the same Gaussians with additional free parameters representing overall normalization, a shift in the means, and a broadening of the Gaussian widths to adjust for differences between data and simulation. The background in simulation and in data is described by a second order polynomial.

Appendix C Broken signal and peaking background calibration

We calibrate the simulation of the broken-signal components due to the loss of a photon from the $D_s^* \rightarrow D_s \gamma$ decay and misidentification of a kaon as a pion, as well as the B^0 peaking background. All these peaking structures are situated in the left $M(B_s)$ sideband.

C.1 Loss of a photon from D_s^*

The ratio of the number of broken-signal events due to misreconstructed $D_s^* \rightarrow D_s \gamma$ to the number of events in the B_s^0 peak is determined from simulation to be 9–11%. This ratio is multiplied by the calibration factor to adjust for differences between data and simulation in these effects. This calibration factor is obtained using the $M(B^0)$ distribution of B^0 candidates reconstructed in part of the $\Upsilon(4S)$ data with an integrated luminosity of 571 fb^{-1} . The B^0 decay channels used in the FEI are listed in table 9 in Appendix A. We use the mass distribution of the candidates reconstructed in $B^0 \rightarrow D_s^+ D^-$ channel. Introducing the peaking background component and fitting to the mass distribution in data, we find a calibration factor of 0.75 ± 0.25 . The result of this fit is shown in figure 8. The calibrated relative normalization of this broken-signal component, which depends on the \mathcal{P}_{B_s} requirement, is shown in table 11.

Table 11. The broken-signal (BS) yields relative to the B_s^0 signal yield and the yields of B^0 peaking background for various requirements on the B_s signal probability \mathcal{P}_{B_s} .

Decay	\mathcal{P}_{B_s} cut	D_s^* BS, %	misID BS, %	Number of B^0
$B_s^0 \rightarrow D_s^\pm X$	> 0.0012	8.4 ± 2.8	0.89 ± 0.02	131 ± 9
$B_s^0 \rightarrow D^0/\bar{D}^0 X$	> 0.0050	7.5 ± 2.5	0.85 ± 0.02	97 ± 7
$B_s^0 \rightarrow D^\pm X$	> 0.0200	6.7 ± 2.2	0.79 ± 0.01	57 ± 5

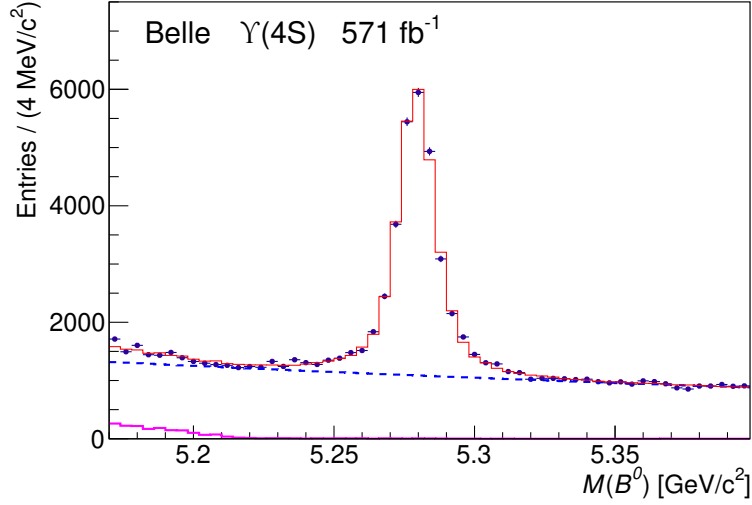


Figure 8. The mass distribution for the selected $B^0 \rightarrow D_s^+ D^-$ candidates. The points with error bars are data, the solid red histogram is the result of the fit, the solid magenta histogram is the broken-signal component caused by the loss of a photon, and the dashed blue histogram is the smooth background component of the fit function.

C.2 Misidentification of a kaon

The normalization of the misidentification component is 0.75–0.85% of the number of events in the B_s^0 peak. The difference between MC and data in kaon misidentification rate is studied using $D^{*+} \rightarrow D^0(K^- \pi^+) \pi^+$ decays [16]. The calibration factor for the broken signal due to kaon misidentification is found to be 1.06 ± 0.02 . The calibrated relative normalization of this broken signal for various \mathcal{P}_{B_s} requirements is shown in table 11.

C.3 Reconstruction of a B^0 meson as B_s^0

To satisfy our momentum selection criteria, $|p_{cm} - p^*(B_s)| < 0.09$, B^0 mesons must be produced in $e^+e^- \rightarrow \Upsilon(4S)(\rightarrow B^0 \bar{B}^0) \gamma_{ISR}$ or $e^+e^- \rightarrow B^* \bar{B}^* \pi$ processes whose cross-section in simulation may differ from that in data. To eliminate this uncertainty, we use part of the $\Upsilon(4S)$ data with an integrated luminosity of 571 fb^{-1} , in which events with the B^0 meson misreconstructed as B_s^0 also take place. The number of these events is obtained fitting to the $M(B_s)$ distribution and found to be 36670 ± 2110 , 27150 ± 1220 , and 16020 ± 650 , depending on the \mathcal{P}_{B_s} requirement. The result of the fit when the $\mathcal{P}_{B_s} > 0.005$ selection requirement is applied is shown in figure 9. The number of signal B^0 reconstructed in the same data sample in channels shown in table 9 is equal to 439530 ± 950 . At the $\Upsilon(5S)$ we reconstruct 1565 ± 130 signal B^0 satisfying the $p^*(B)$ selection criteria. Assuming that the B^0 reconstruction efficiency does not depend on the B^0 momentum, we obtain the expected number of fake B_s^0 candidates at the $\Upsilon(5S)$ resonance given in table 11.

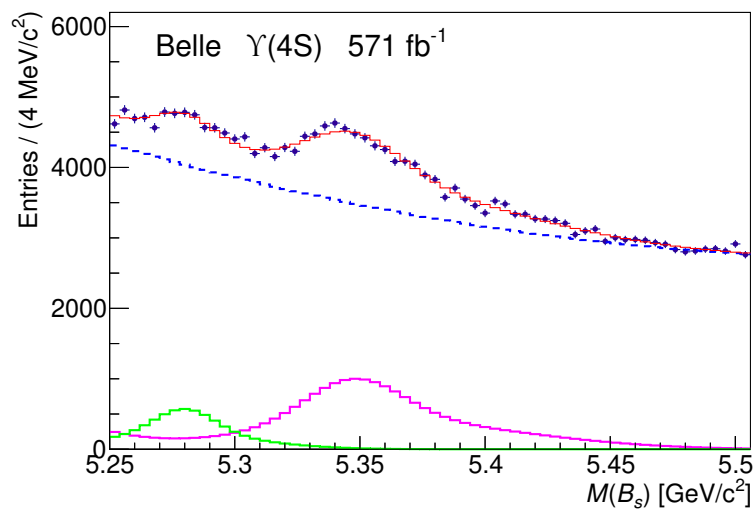


Figure 9. The mass distribution for the selected B_s^0 candidates at the $\Upsilon(4S)$ resonance. The points with error bars are data, the solid red histogram is the result of the fit, the solid green histogram is the component corresponding to correct reconstruction of all B^0 decay products, and the solid magenta histogram corresponds to the B^0 candidates in which daughter pion was misidentified as a kaon. The dashed blue histogram is the smooth background component of the fit function.