Interplay of intrinsic motion of partons and soft gluon emissions in Drell-Yan production studied with PYTHIA

I. Bubanja[*1,2], H. Jung [†][3,4], N. Raičević[‡][1], and S. Taheri Monfared[§][3]

[1]Faculty of Science and Mathematics, University of Montenegro, Podgorica, Montenegro
[2]Interuniversity Institute for High Energies (IIHE), Université libre de Bruxelles, Belgium
[3]Deutsches Elektronen-Synchrotron DESY, Germany
[4]II. Institut für Theoretische Physik, Universität Hamburg, Hamburg, Germany

[*]itana.bubanja@cern.ch
[†]hannes.jung@desy.de
[‡]natasar@ucg.ac.me
[§]sara.taheri.monfared@desy.de

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Abstract

Understanding the intrinsic transverse momentum (intrinsic-k_T) of partons within colliding hadrons, typically modeled with a Gaussian distribution characterized by a specific width (the intrinsic-$k_\mathrm{T}$width), has been an extremely challenging issue. This difficulty arises because event generators like PYTHIA require an intrinsic-k_T width that unexpectedly varies with collision energy, reaching unphysical values at high energies.

This paper investigates the underlying physics behind this energy dependence in PYTHIA, revealing that it arises from an interplay between two non-perturbative processes: the internal transverse motion of partons and non-perturbative soft gluon emissions. These contributions are most constrained in the production of Drell-Yan pairs with very low transverse momentum, where soft gluon effects become increasingly prominent with rising collision energy—contrary to initial expectations.

Through a detailed analysis of the non-perturbative Sudakov form factor and its influence on intrinsic-k_T width, we clarify the observed energy scaling behavior in PYTHIA, providing insight into a longstanding issue in parton shower modeling.

1   Introduction

A parton in the initial state within a hadron has longitudinal momentum in the direction of the hadron to which it belongs, but it also possesses a certain momentum perpendicular to the hadrons flight direction due to its internal motion (intrinsic-k_T) inside the hadron. Studies of intrinsic-k_T distributions have been recently published by several groups and collaborations [1–3]. While the Parton Branching (PB) method [4, 5], which uses Transverse Momentum Dependent (TMD) parton distributions, can describe the Drell-Yan (DY) pair cross section [6,7] as a function of the transverse momentum, $p_\mathrm{T}(\ell\ell)$, an intrinsic-k_T distribution that remains nearly independent of $\sqrt{s}$ (as shown in [1]) has been observed. However, in parton-shower-based Monte Carlo event generators (MC) like PYTHIA [8, 9] and HERWIG [10], the width of the intrinsic-k_T distribution depends on the center-of-mass energy, $\sqrt{s}$, and increases with it [11,12]. In Ref. [2] it was shown that by excluding the non-perturbative region, near the soft-gluon resolution boundary in the PB method approach, a dependence of the width of the intrinsic-k_T distribution on $\sqrt{s}$ is observed and has similar dependence as in parton shower MCs.

The main aim of this study is to understand the physical reason for the $\sqrt{s}$-dependent width of the intrinsic-k_T distribution in PYTHIA by confirming that it is related to the exclusion of soft and non-perturbative real parton emissions as was indicated in the PB study. To verify this, it is necessary that the region which is not strongly influenced by such non-perturbative contributions is well constrained, so that by varying the width of the intrinsic-k_T distribution, it stays stable and provides a good description of data.

This paper is organized as follows: In the first part of Section 2 we discuss recent studies of the internal transverse momentum of the partons, which serve as a guide for following investigations. Additionally, we highlight the importance of the intrinsic-k_T on the transverse momentum of the DY pairs. In the second part of Section 2 we discuss how the shape of the predicted DY cross section as a function of $p_\mathrm{T}(\ell\ell)$ changes with the choice of the selected

PYTHIA tunes, focusing on the transition region between non-perturbative and perturbative regions, which is less influenced by the choice of the intrinsic-k_T width. Section 3 describes how fits to DY data are performed and how the width of the intrinsic-k_T distribution is determined. In section 4, we discuss in detail the influence of soft parton emissions on the obtained values of the width of the intrinsic-k_T distribution, which can be interpreted as the interplay between the internal transverse motion and the soft gluon emissions at very low DY transverse momenta.

2 Low transverse momentum distribution of Drell-Yan pairs

While the longitudinal momentum of partons inside the colliding hadrons is linked to the energy of the hadron, the internal motion of the partons inside the hadron results in a small transverse momentum (intrinsic-k_T), which is believed to be connected to the hadron, but independent of the hadron's longitudinal momentum.

This intrinsic-k_T is introduced into parton evolution as a non-perturbative parameter and is, for simplicity, treated as a Gaussian distribution with a width of σ, $e^{-k_T^2/\sigma^2}$, which is multiplied by the parton density function at the starting scale [1].

2.1 The impact of the intrinsic-k_T on the transverse momentum of DY pairs

In Ref. [1], the width of the intrinsic-k_T distribution, σ, was determined using the PB method [6,7] and it was shown, that σ has little to no dependence on the center-of-mass energy, $\sqrt{s}$. In the PB method, by construction, all the soft gluon emissions were taken into account since the minimal transverse momentum, q_0, of a parton emitted at any branching is close to zero. This condition is essential for obtaining collinear parton densities, which are consistent with collinear factorization.

However, this result is different from the results obtained from parton shower Monte Carlo event generators [11,12] where a center-of-mass energy dependence of the width of the intrinsic transverse momentum distribution was observed. In Ref. [2], using the PB method, this behavior was traced back to the lower limit of the transverse momentum, q_0, allowed in parton showers: by increasing q_0 to typical values of 1-2 GeV used in parton shower event generators, a $\sqrt{s}$ dependence of σ was observed. Obviously, setting $q_0 \sim$ 1-2 GeV excludes soft and non-perturbative gluon emissions, which are essential for inclusive parton distribution functions, as shown in detail in Ref. [1,13].

In the recent paper of the CMS Collaboration [3], the $\sqrt{s}$-dependence of the intrinsic-k_T width was confirmed for the event generators PYTHIA (for tunes CP3, CP4 and CP5 [14]) and HERWIG (for tunes CH1 and CH2 [15]).

In the following we show that in parton shower event generators the $\sqrt{s}$-dependence of the intrinsic-k_T width, σ, is coming from the exclusion of soft real gluon emission by a detailed study of the low transverse momentum distributions of the DY pairs. Next-to-leading order (NLO) calculations of DY production are performed using MCatNLO [16], supplemented with parton showers simulated by PYTHIA8. The NNPDF3.1 collinear parton

densities [17] are employed for the hard process calculation. Predictions are compared with experimental data obtained at different collision energies, aiming to determine the best value for the width of the intrinsic-k_T distribution and to investigate the origin of its center-of-mass energy dependence.

The parameter that controls soft gluon emissions in PYTHIA is the initial-state-radiation (ISR) cut-off scale, SPACESHOWER:pT0Ref which has default value of 2 GeV. We study the energy dependence of the width of the intrinsic-k_T distribution for the three values of the ISR cutoff scale: SPACESHOWER:pT0Ref=2, 1 and 0.5 GeV (in the following abbreviated as pT0Ref). The intrinsic-k_T width in the Gaussian distribution, σ, corresponds to the parameter denoted as BeamRemnants:primodialkThard in PYTHIA.

Figure 1 illustrates the impact of intrinsic-k_T on the transverse momentum distribution of DY pairs, $p_T(\ell\ell)$, in the Z-peak region obtained from proton-proton collisions at 13 TeV using the default settings from PYTHIA for the two values of ISR cutoff parameter: the default one, pT0Ref=2 GeV, and pt0Ref=0.5 GeV (all other parameters are set to the Monash 2013 tune [18]).

As shown in the figure, the impact of intrinsic-k_T is greatest at the lowest DY pair transverse momenta. The figure also shows that by reducing the ISR cutoff parameter, the sensitivity on intrinsic-k_T decreases. Furthermore, it indicates that the effect of the intrinsic-k_T distribution on $p_T(\ell\ell)$ above 3-4 GeV is minimal, suggesting that this range could be used for tuning other parameters—in particular, the PDF set, the α_s value and ordering in PDFs as well as in showers.
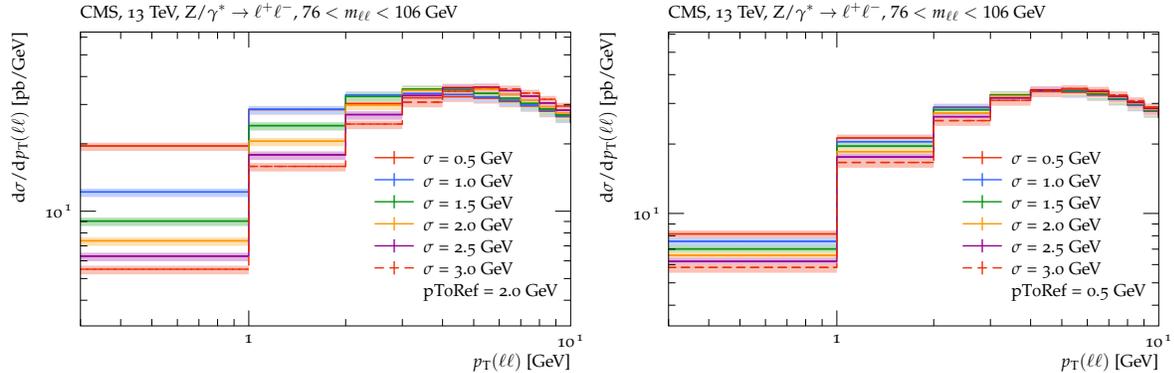

Figure 1: The DY cross section as a function of $p_T(\ell\ell)$ in the Z-peak region obtained by the default settings of PYTHIA for proton-proton interactions at $\sqrt{s} = 13$ GeV for the two ISR cutoff parameter values, pT0Ref=2 GeV(left) and pT0Ref=0.5 GeV(right), with different σ values of $0.5, 1.0, 1.5, 2.0, 2.5$ and 3.0 GeV. The binning follows the measurement in Ref. [19].

In the following, we illustrate how the most recent DY cross section measurements as a function of $p_T(\ell\ell)$, obtained at high center-of-mass energy from LHC, $\sqrt{s} = 13$ TeV [19], is described by PYTHIA using various parameter settings and tunes.

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2.2 Optimal settings in PYTHIA

The default setting in PYTHIA is based on the Monash 2013 tune [18], which is used to model the initial- and final-state radiation (ISR and FSR), multiparton interactions (MPI), and hadronization. The NNPDF2.3 LO [20] parton density, with a strong coupling value of $\alpha_s(m_Z) = 0.130$, is used. However, a different strong coupling of $\alpha_s(m_Z) = 0.1365$ is used for ISR and FSR. The default value of the ISR cutoff parameter is `pT0Ref=2.0` GeV, and the width of the intrinsic-k_T distribution is set to $\sigma = 1.8$ GeV.
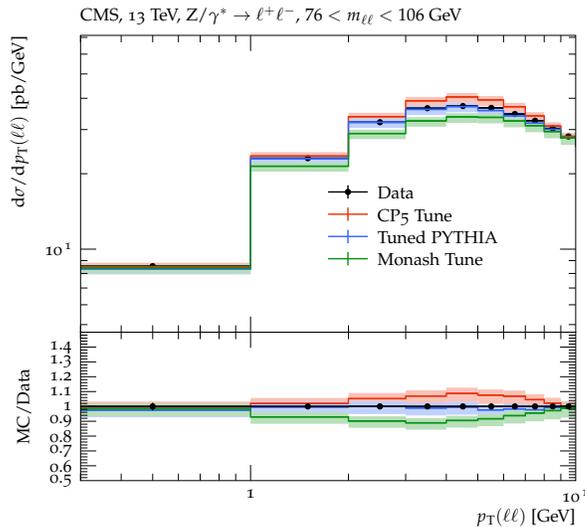

Figure 2: Comparison of the LHC DY cross-section measurement as a function of $p_T(\ell\ell)$ in the Z-peak region from proton-proton collisions at $\sqrt{s} = 13$ GeV GeV [19] with PYTHIA using the Monash and CP5 tunes as well as a *intrinsick_T-optimized* configuration. The value `pT0Ref=0.5` GeV has been used and the width of the intrinsic-k_T distribution has been adjusted to provide the best description of the measurement.

The CP5 tune [14] is the main tune for the underlying event (UE) in the CMS collaboration for RUN-2 data and is used for studying energy scaling behaviour in [3]. It uses a strong coupling value of $\alpha_s(m_Z) = 0.118$ and the NLO strong coupling evolution. The settings use NLO or NNLO PDFs for ISR, FSR and MPI. The ISR cutoff parameter is set to `pT0Ref=2.0` GeV, and the intrinsic-k_T width is set to $\sigma = 1.8$ GeV.

In Fig. 2, the predictions using the Monash 2013 and CP5 tunes are compared to the measurement [19], with `pT0Ref=0.5` GeV (the minimal value accessible in PYTHIA) and adjusting the width of the intrinsic-k_T distribution to best describe the cross section measurements at lowest measured $p_T(\ell\ell)$. One can observe, that using the CP5 tune leads to an overestimation of the cross section in the region of 2-5 GeV while using the Monash tune results in an underestimation of the cross section in this region. * By changing the value of

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*"CMS Tune MonashStar", alias CUETP8M1-NNPDF2.3LO [21], an underlying-event tune based on the

$\alpha_s(m_Z) = 0.130$ in the Monash tune, an excellent description is achieved. In the following, we take this value and label it as the *intrinsic*k_T*-optimized* tune, where the intrinsic-k_T width is adjusted depending on $\sqrt{s}$.

The *intrinsic*k_T*-optimized* tune prediction provides a very good description even at very low DY pair invariant mass when the intrinsic-k_T width is adjusted. Figure 3 shows a comparison between the prediction and measurements of the DY pair cross section as a function of $p_T(\ell\ell)$ obtained from $\sqrt{s} = 200$ GeV [22] and $\sqrt{s} = 38.8$ GeV [23] for the ranges $7 < m(\ell\ell) < 8$ GeV and $4.8 < m(\ell\ell) < 8.2$ GeV, respectively. The figure shows a very good agreement between the data and the prediction with a reasonable values of χ^2/NDF as indicated in the figure. The adjusted σ values, obtained through the procedure described in the next section, are also provided.
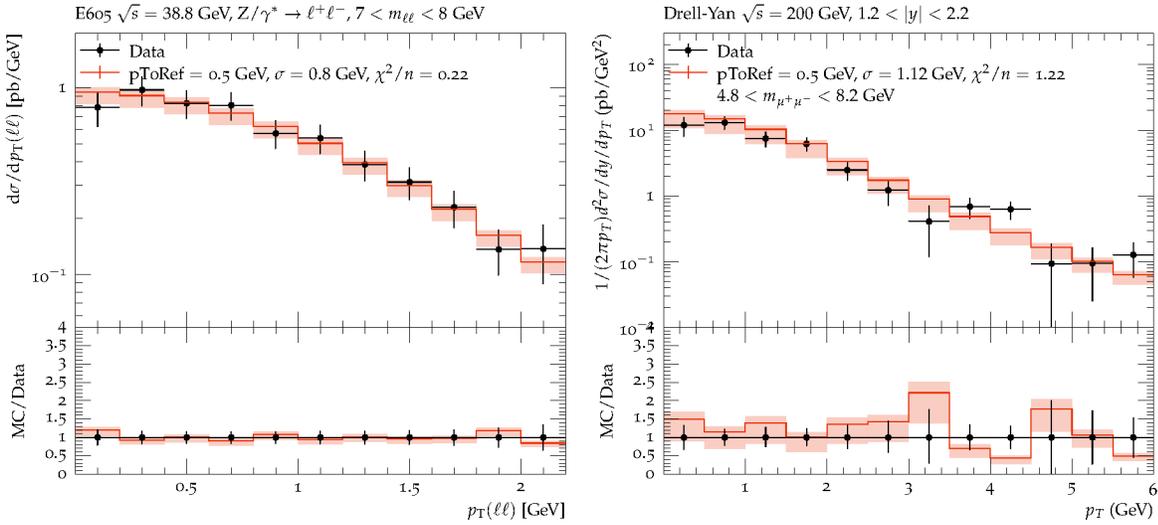

Figure 3: Comparison of the DY cross section measurement as a function of $p_T(ll)$ in the low invariant mass region at $\sqrt{s} = 200$ GeV [22] (left) and at $\sqrt{s} = 38.8$ GeV [23] (right) with *intrinsic*k_T*-optimized* tune, using `pT0Ref = 0.5` GeV. The adjusted values of σ are displayed.

3   Determination of the intrinsic-k_T width

As demonstrated in the previous section, low-energy and high-energy measurements of DY cross sections are well described using the *intrinsic*k_T*-optimized* tune with an adjusted value of σ. In this section, we outline the procedure for determining the optimal value of σ. The width of the intrinsic-k_T distribution is determined by fitting the predicted cross section to

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Monash 2013 tune gives similar results as the default tune in PYTHIA. Similar results as in CP5 are obtained using CP3 and CP4 tunes [14].

the measured DY cross section as a function of $p_T(\ell\ell)$. The prediction is obtained using MCatNLO supplemented with parton showers from *intrinsick$_T$-optimized* tune. The generated events are passed through the Rivet package [24] for a detailed comparison with the measurement. The predictions are based on a width of the intrinsic-k_T distribution, which is adjusted to minimize χ^2:

$$\chi^2(\texttt{primodialkThard}) = \sum_{i=1}^{n} \frac{\left(p_i(\texttt{primodialkThard}) - m_i\right)^2}{s_i^2} \ , \tag{1}$$

where m_i and p_i are measured and predicted cross sections for the i-th bin, n is the number of bins from the $p_T(\ell\ell)$ distribution used to evaluate the χ^2. The value of `BeamRemnants:primodialkThard` (in the figures and formulas denoted as `primodialkThard` for brevity) for which the χ^2 distribution, fitted with a cubic spline function interpolated through the points, has a minimum is taken to be the width, σ, of the intrinsic-k_T contribution. The experimental uncertainties are treated as uncorrelated (since most published experimental data do not provide an uncertainty breakdown) and are added in quadrature to the uncertainty from the model prediction to obtain the total uncertainty s_i for bin i.
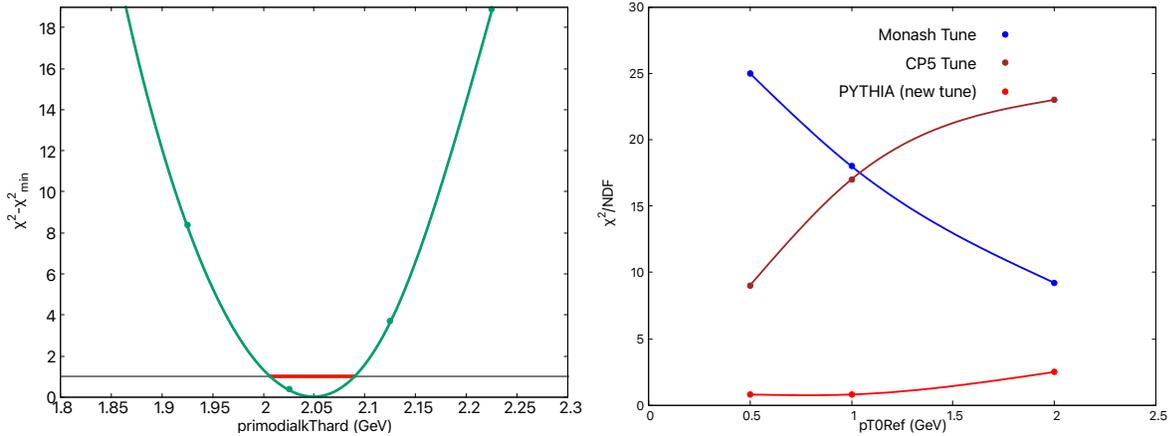

Figure 4: $\chi^2(\texttt{primodialkThard}) - \chi^2_{\min}$ distribution obtained from the comparison of CMS measurements at $\sqrt{s} = 13$ TeV [19] with the *intrinsick$_T$-optimized* tune prediction for `pT0Ref` = 2 GeV(left). The green line represents the result of the fit using a cubic spline function interpolated through the points. The position of the minimum corresponds to the tuned parameter σ, and the red line at $\chi^2 - \chi^2_{\min}$ indicates its uncertainty. The right plot shows the minimum χ^2/NDF obtained in the Z-peak region as a function of `pT0Ref` for different tunes.

The uncertainty of the tuned value of σ is estimated as the range of `primodialkThard` for which $\chi^2 - \chi^2_{\min} = 1$, as illustrated by the red line in the left part of Fig. 4. The right part of figure 4 shows the obtained values of χ^2/NDF, calculated from predictions obtained using

the Monash, CP5 and *intrinsick_T-optimized* tunes. The number of degrees of freedom (NDF) is equal to the number of bins used in the χ^2 calculation. The upper value of the $p_T(\ell\ell)$ used for the χ^2 calculation was set to 8 GeV when applicable.

Figure 4 shows that χ^2 is significantly improved using *intrinsick_T-optimized* tune compared to the other tunes. We are now in a position to fit σ as a function of `pT0Ref`.
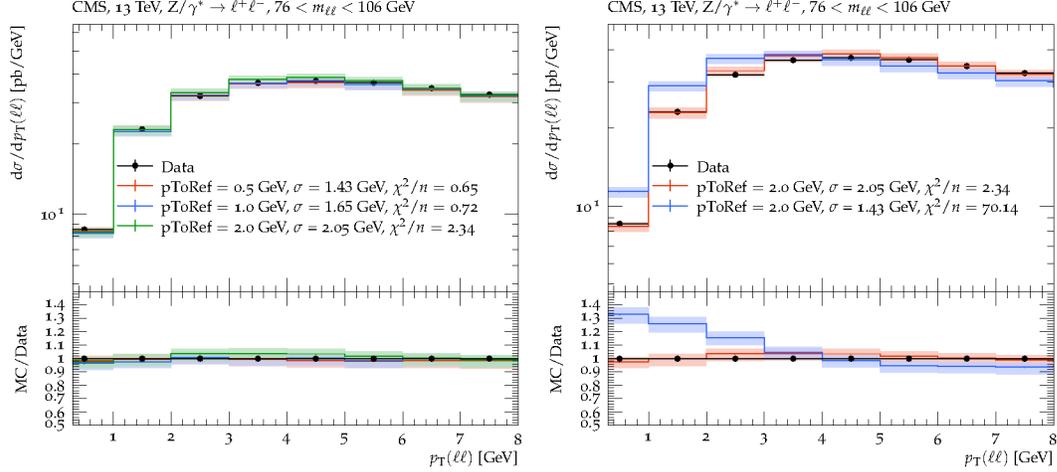

Figure 5: The cross section as a function of $p_T(\ell\ell)$ in the Z-peak region, obtained from proton-proton collisions at $\sqrt{s} = 13$ TeV [19], compared with *intrinsick_T-optimized* tune. (Left): Predictions using different `pT0Ref` values, with the intrinsic-k_T width adjusted accordingly. (Right): Prediction using `pT0Ref` = 2.0 GeV with both the optimal and "wrong" intrinsic-k_T width (the "wrong" width is the one that was obtained as an optimal one for `pT0Ref` = 0.5 GeV). The intrinsic-k_T width σ as well as the obtained χ^2 are given in the figures.

4 The impact of non-perturbative soft gluons and the Sudakov form factor

In a parton shower or parton branching language, the Sudakov form factor $\Delta(\mu^2, \mu_0^2)$ is central to the splitting process, as it gives the probability of no radiation between the scales μ^2 and μ_0^2. This form factor can be split at a scale z_{dyn} into a perturbative ($0 < z < z_{\text{dyn}}$) and

non-perturbative ($z_{\mathrm{dyn}} < z < z_{\mathrm{M}}$) part [13, 25]:

$$
\begin{aligned}
\Delta(\mu^2, \mu_0^2) &= \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mathbf{q}'^2}{\mathbf{q}'^2} \int_0^{z_{\mathrm{M}}} dz \, z \, P_{ba}^{(R)}(\alpha_s, z)\right) \\
&= \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mathbf{q}'^2}{\mathbf{q}'^2} \int_0^{z_{\mathrm{dyn}}} dz \, z \, P_{ba}^{(R)}(\alpha_s, z)\right) \\
&\quad \times \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mathbf{q}'^2}{\mathbf{q}'^2} \int_{z_{\mathrm{dyn}}}^{z_{\mathrm{M}}} dz \, z \, P_{ba}^{(R)}(\alpha_s, z)\right) \\
&= \Delta^{(\mathrm{P})}\left(\mu^2, \mu_0^2, q_0^2\right) \cdot \Delta^{(\mathrm{NP})}\left(\mu^2, \mu_0^2, q_0^2\right) \ .
\end{aligned}
\tag{2}
$$

The splitting functions $P_{ba}^{(R)}(\alpha_s, z)$ for the splitting of parton a into parton b, with the splitting variable z being the ratio of longitudinal momenta of the involved partons, are explicitly given in e.g. Ref. [4]. The parameter z_{M} is introduced for numerical stability with $z_{\mathrm{M}} = 1 - \epsilon$ with $\epsilon \to 0$.

The scale z_{dyn} separates resolvable perturbative gluons from soft, non-perturbative gluons. In the PB-approach, this scale is related to the minimal transverse momentum q_0 via $z_{\mathrm{dyn}} = 1 - q_0/q'$, with q' being the evolution scale. In the parton shower approach of PYTHIA this scale depends implicitly on pT0Ref, though a closed analytic expression cannot be given since several kinematic constraints also play a role [9].

In Ref. [1], it was shown that $\Delta_a^{(\mathrm{NP})}$ plays an important role in inclusive and TMD distributions, and in Ref. [13], it was pointed out that neglecting $\Delta_a^{(\mathrm{NP})}$ can significantly affect predictions.

Here, we investigate the effect of pT0Ref and its role in determining the width of the intrinsic-k_{T} distribution. When soft, non-perturbative gluons are not simulated due to a "large" value of pT0Ref, the intrinsic-k_{T} distribution must compensate for the missing soft gluons, leading to a larger width of the intrinsic-k_{T} distribution. Conversely, if soft gluons are simulated in the parton shower, the intrinsic-k_{T} distribution requires a smaller width.

The Sudakov form factors (both the perturbative and non-perturbative ones) depend on the resolution scale z_{dyn} and the hard scale μ through the integrals in eq.(2). In the following subsections, we investigate the effect of z_{dyn} and μ on the value of σ for the intrinsic-k_{T} distribution, also aiming to study the contribution and distribution of soft gluons in the production of DY pairs in hadron-hadron collisions.

4.1 Dependence on z_{dyn}

In PYTHIA, the z-integral (eq.(2)) is limited by z_{dyn}, which depends on the parameter pT0Ref. To emphasize the effect of the cutoff pT0Ref on the width of the intrinsic-k_{T} distribution, a comparison of the cross section measurement as a function of $p_{\mathrm{T}}(\ell\ell)$ at $\sqrt{s} = 13$ TeV [19], with the prediction obtained from PYTHIA is presented in Fig. 5. In Fig. 5(left), we show predictions with three different values of pT0Ref, using adjusted σ for each pT0Ref. All predic-

tions describe the measurement reasonably well, with acceptable χ^2 values. In Fig. 5(right), we show predictions with `pT0Ref` = 2.0 GeV, using both the optimal intrinsic-$k_{\rm T}$ width and a "wrong" value (the intrinsic-$k_{\rm T}$ width that is optimal for `pT0Ref` = 0.5 GeV). One can clearly see that the value of σ obtained for the prediction with `pT0Ref` = 0.5 GeV fails to describe the measurements when used with `pT0Ref` = 2.0 GeV since a significantly larger fraction of soft gluon contribution is removed when when the ISR cutoff parameter is increased.

This figure also illustrates the strong influence of the soft gluon contribution with transverse momentum of the order of 1 GeV at low DY pair transverse momentum.

To illustrate the effect of the cutoff `pT0Ref` on the width of the intrinsic-$k_{\rm T}$ distribution, we investigate several data sets for DY production cross section as a function of $p_{\rm T}(\ell\ell)$, obtained at different center-of-mass energies: LHC data [19], Tevatron data [26], and data from lower energies [22,23]. The analysed data sets are shown in Table 1.

Given name	Number of bins	$E_{\rm CM}$ [GeV]	Ref.
CMS	8	13000	[19]
CDF	20	1960	[26]
PHENIX	12	200	[22]
E605	11	38.8	[23]

Table 1: List of measurements, collision energies, and the number of bins in $p_{\rm T}(\ell\ell)$ used to determine the width of the intrinsic-$k_{\rm T}$ distribution.

Figure 6 shows the width σ obtained from χ^2 minimization as a function of the cut-off parameter, `pT0Ref` , at different values of $\sqrt{s}$. One can observe that for the same value of `pT0Ref`, the width of the intrinsic-$k_{\rm T}$ distribution increases significantly with $\sqrt{s}$. At higher energies, the contribution of soft, non-perturbative gluons becomes more important compared to lower $\sqrt{s}$. This observation can be understood by the increased number of branchings (emission vertices during the parton shower) at higher energies: not only does the number of perturbative emissions increase, but the number of soft, non-perturbative emissions also increases. In the examined region 0.5 <`pT0Ref`< 2.0 GeV, the value of σ increases nearly linearly with `pT0Ref`, as indicated by the linear fits applied to the points for visual guidance. From the measurements at lower collision energies, 38.8 GeV [23] and 200 GeV [22], it is clear that the contribution from soft gluons with low transverse momentum (around 0.5 GeV) dominates, as the value of σ shows only small variations with respect to the ISR cut-off parameter. At $\sqrt{s} \sim$ TeV, the rate of increase of σ with `pT0Ref` is similar for $\sqrt{s} = 13$ TeV and $\sqrt{s} = 1.96$ TeV.

4.2 Dependence on μ

From the study above, we confirm that the value of the width σ reflects the effect of two processes: the internal parton motion and soft gluon emission. Therefore, by studying the intrinsic-$k_{\rm T}$ width, which quantifies the interplay between these two processes, we can investigate soft gluon emission and estimate its contribution.

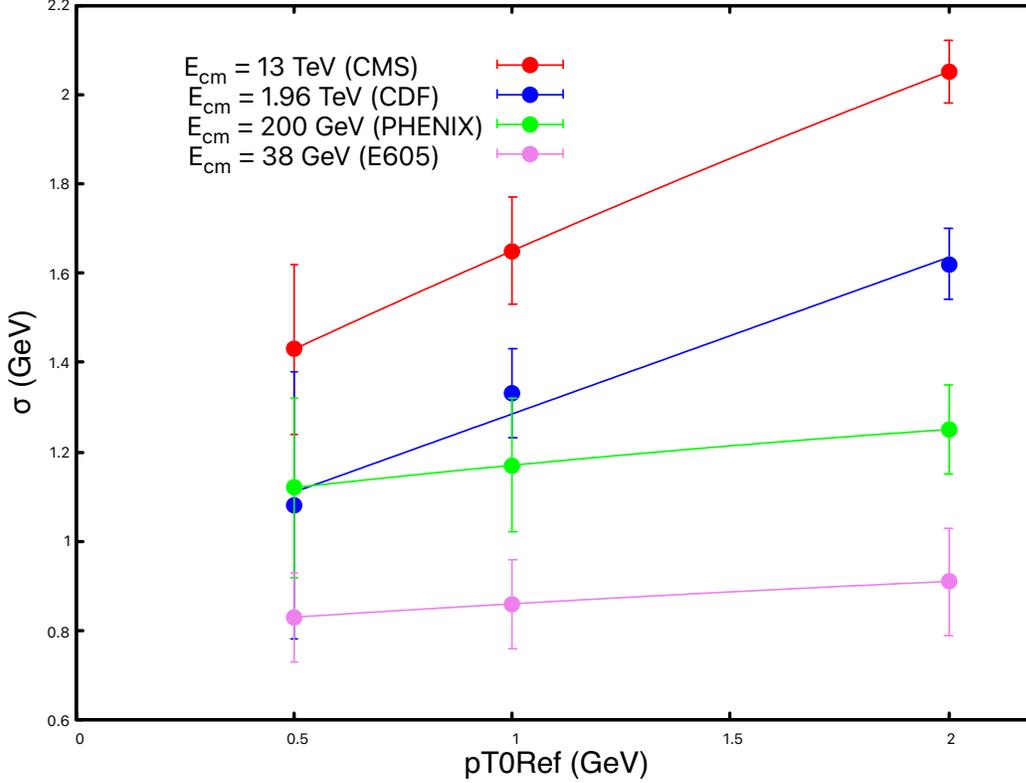Figure 6: Dependence of the intrinsic-k_T width, σ, on the ISR cut-off parameter `pT0Ref` used in *intrinsick_T-optimized* tune, obtained from comparison with the measurements at different collision center of mass energies listed in Table 1. Linear fits are also shown for each collision energy.

To confirm the impact of the μ-dependence of the soft gluon contribution (non-perturbative Sudakov form factor), we now study the width of the intrinsic-k_T distribution at low $p_T(\ell\ell)$ as a function of m_{DY}, which is directly related to the scale value μ. A pure intrinsic-k_T distribution should not vary with the DY-mass m_{DY}, and any variation observed indicates the influence of the soft contribution. Figure 7 shows σ as a function of m_{DY} at $\sqrt{s} = 13$ TeV [19] (left plot) and 38.8GeV [23] (right plot). The measurements at $\sqrt{s} = 13$ TeV do not show a mass dependence of σ, while the measurements at the lower energy of 38.8 GeV show a weak m_{DY}-dependence.

The non-observation of a m_{DY}-dependence of σ suggests that the relative contribution of soft gluon emission, which interplays with the internal transverse motion, is similar across all measured m_{DY} bins at low $p_T(\ell\ell)$.

The measured [19] ratio of the cross section as a function of $p_T(\ell\ell)$ to the one in the Z-peak region in several m_{DY} bins from 50 to 1000 GeV is shown in figure 8 and compared with
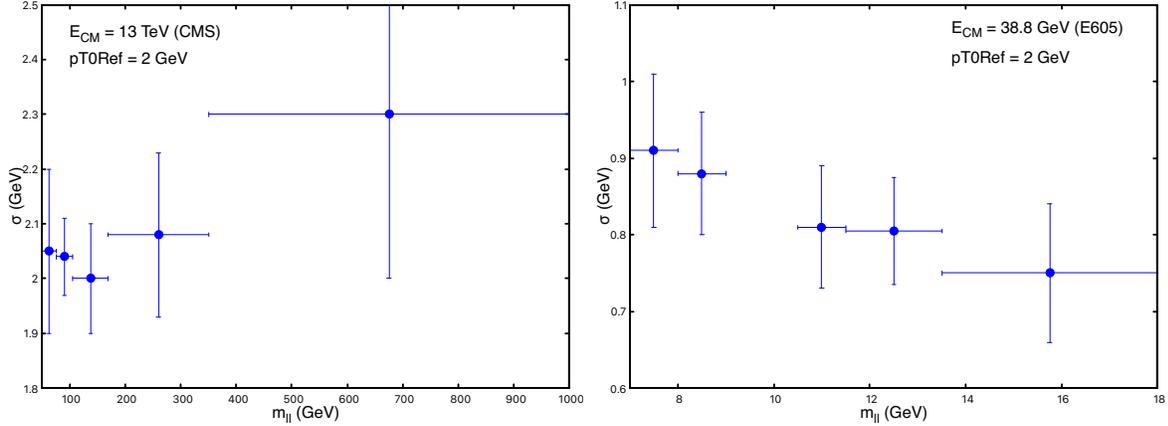
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Figure 7: Intrinsic-k_T width, σ, as a function of the DY pair invariant mass, obtained from the comparison of data at $\sqrt{s} = 13$ TeV [19] (left) and at $\sqrt{s} = 38.8$ GeV [23] (right) with PYTHIA with pT0Ref = 2.0 GeV.

predictions from PYTHIA for different values of pT0Ref. This also agrees with the results presented in Ref. [3] using the CP5 tune by the CMS Collaboration. All three predictions describe the measurements reasonably well, indicating that the soft gluon contribution is quite similar in all m_{DY} bins, contrary to the expectation of a scale dependence from eq.(2). A similar result regarding the mass dependence was obtained using the Parton Branching Method [27].

To observe a μ-dependence of the width σ, we study the ratio of the cross section as a function of $p_T(\ell\ell)$ to that in the Z-peak region, using predictions from *intrinsick$_T$-optimized* tune produced in finer binning at smaller m_{DY}.

Figure 9 shows the ratio of the simulated cross section obtained with PYTHIA for two values of the ISR cut-off parameter: pT0Ref = 0.5 GeV and pT0Ref = 2.0 GeV. For high m_{DY} values as measured at the LHC, we do not observe a significant dependence of the width parameter σ, while extending the mass range to smaller values, such a dependence is clearly visible. The difference between the simulations with pT0Ref = 2 GeV and pT0Ref=0.5 GeV increases as m_{DY} decreases. This is consistent with the expectation that most of the contribution from the non-perturbative Sudakov form factor arises at low μ scales.

5   Conclusion

A detailed study was performed to accurately describe the cross section of Drell-Yan pair production as a function of transverse momentum $p_T(\ell\ell)$ using PYTHIA simulations. We focused on the region of the lowest measured transverse momentum, where non-perturbative contributions, internal transverse momentum of partons in colliding hadrons, and soft gluon emissions significantly impact the results.
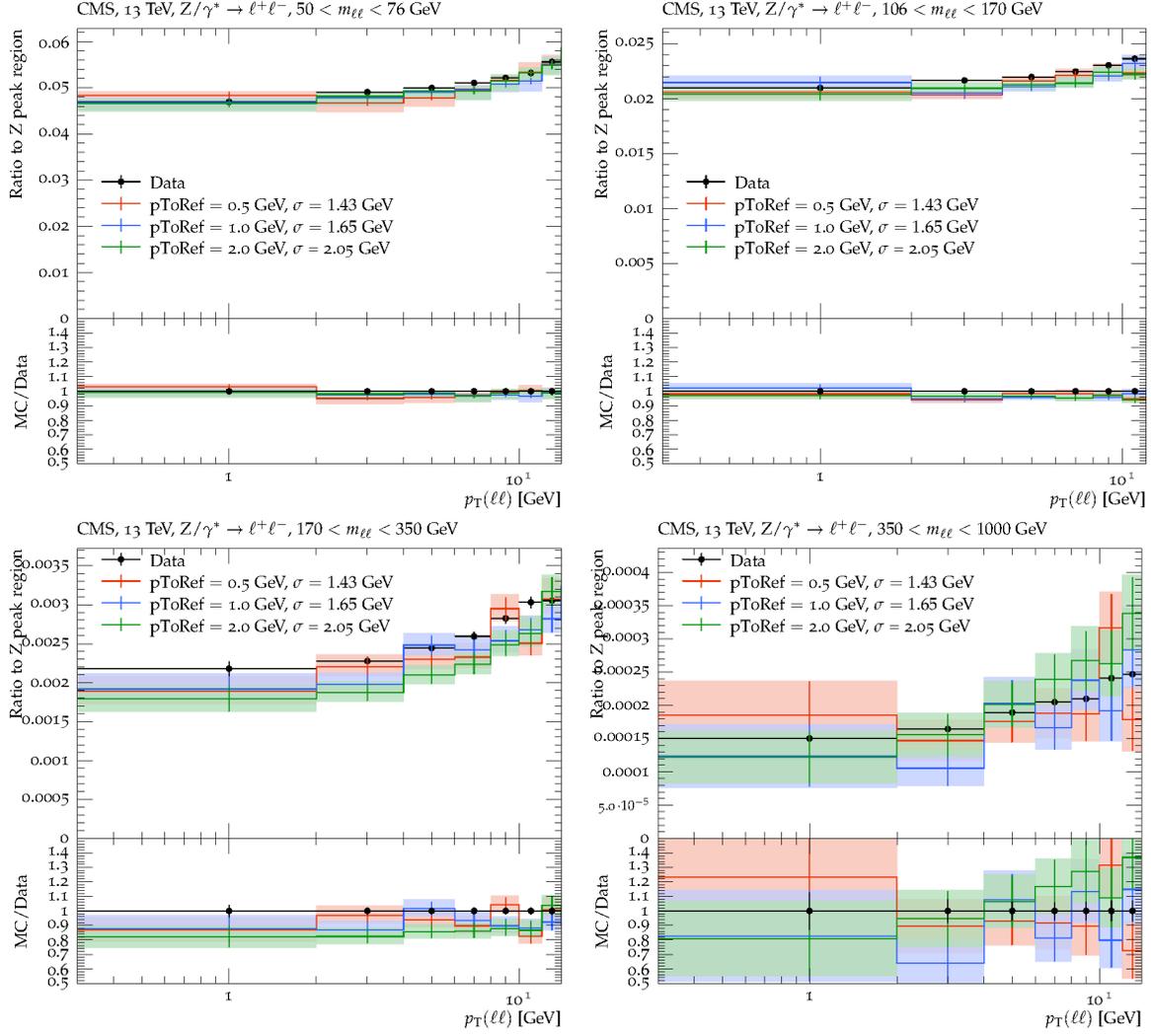
Figure 8: Measurement of the ratio of the cross section as a function of $p_T(\ell\ell)$ to the Z-peak region [19], compared with predictions from *intrinsick_T-optimized* tune with `pT0Ref` = 0.5, 1.0 and 2.0 GeV.

We investigated the width of the intrinsic-k_T distribution and its relationship to soft gluon emissions to clarify the energy dependence in PYTHIA, as noted in several publications. We confirm that the $\sqrt{s}$-dependence of the intrinsic-k_T width is connected to soft gluon emissions, which are controlled by introducing a lower cut on the transverse momentum of emitted partons in initial state radiation.

Our results demonstrate that the intrinsic-k_T width, reflecting the interplay between two non-perturbative processes (internal transverse motion inside the hadrons and soft gluon
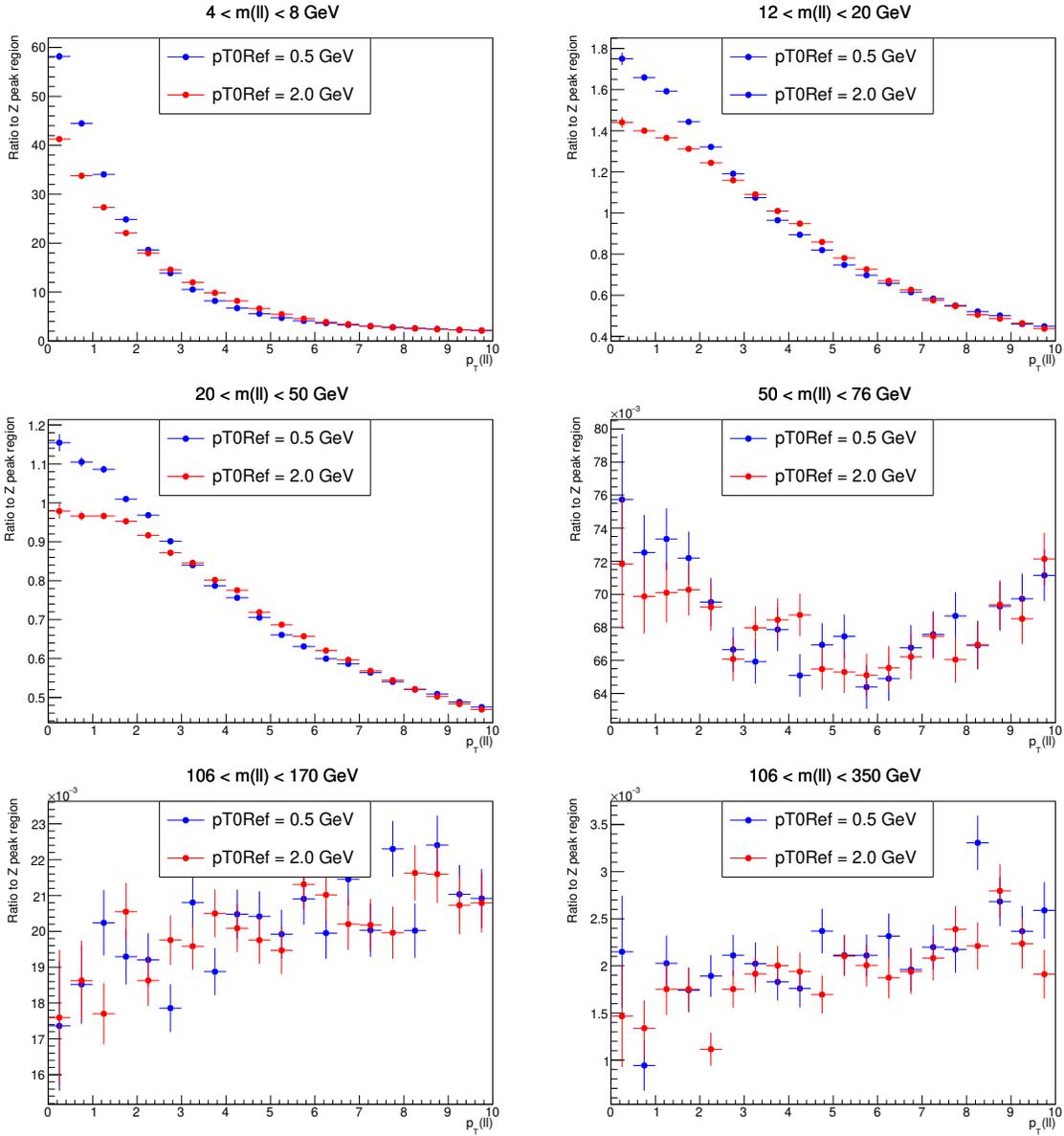
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Figure 9: The ratio of the cross section as a function of $p_T(\ell\ell)$ to Z-peak region from predictions from *intrinsick_T-optimized* tune in different DY pair invariant mass bins, shown for the two values of the ISR cut-off parameter: `pT0Ref = 0.5` GeV and `pT0Ref = 2.0` GeV.

emissions), increases approximately linearly with the ISR cut-off parameter in the range $0.5 < \texttt{pT0Ref} < 2.0$ GeV. This observation provides clear evidence that the $\sqrt{s}$-dependence is related to the no-emission probability, Sudakov form factor, through its dependence on the

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scale z_{dyn}, which in turn is related to the ISR cut-off parameter. The μ-dependent part of the Sudakov form factor was also confirmed by examining the dependence of the width σ on m_{DY}, which is directly linked to the evolution scale μ and is particularly visible at low m_{DY}.

The findings of this study address longstanding questions about the energy dependence of the intrinsic-k_{T} width in standard Monte Carlo event generators, tracing it back to contributions from soft, non-perturbative gluon emissions during parton evolution and the parton shower.

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