

# Small instanton-induced flavour invariants and the axion potential

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**ABSTRACT:** Small instantons, which increase the axion mass due to a modification of QCD at a UV scale  $\Lambda_{\text{SI}}$ , can also enhance CP-violating operators to shift the axion potential minimum by an amount,  $\theta_{\text{ind}}$  proportional to the flavorful couplings in the SMEFT. Since physical observables must be flavor basis independent, we construct a basis of determinant-like flavor invariants that arise from instanton calculations containing the effects of dimension-six CP-odd operators at the scale  $\Lambda_{\text{CP}}$ . This new basis provides a more reliable estimate of the shift,  $\theta_{\text{ind}}$  induced by the SMEFT operator, that is severely constrained by neutron EDM experiments. In particular, for the case of four-quark, semi-leptonic and gluon dipole operators, these invariants are then used to provide improved limits on the ratio of scales  $\Lambda_{\text{SI}}/\Lambda_{\text{CP}}$  for different flavor scenarios. The CP-odd flavor invariants also provide a classification of the leading effects from Wilson coefficients, and as an example we show that a semi-leptonic four-fermion operator is subdominant compared to the four-quark operators. More generally, the flavor invariants, together with an instanton NDA, can be used to more accurately estimate small instanton effects in the axion potential that arise from any SMEFT operator.

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