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RESISTIVE WALL IMPEDANCE OF MULTILAYER BEAM PIPES OF GENERAL CROSS SECTION

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Abstract

This paper presents the calculation of the resistive wall coupling impedance of multilayer vacuum chambers with arbitrary cross-sections, which is of particular importance for circular particle accelerators, especially the lepton Future Circular Collider (FCC-ee). The pipes considered in this study are longitudinally uniform but have arbitrary cross-sections. The results obtained from the VACI code are validated by comparing them to those obtained from the well-known ImpedanceWake2D code as well as analytical formulas for round and elliptical pipes. The excellent agreement between the results demonstrates the accuracy and reliability of the VACI suite for calculating the resistive wall coupling impedance of vacuum chambers with arbitrary cross-sections.

INTRUDUCTION

The resistive wall (RW) impedance [1,2] is a major contributor to system impedance and a limiting factor for beam operation properties, particularly in long circular accelerators and colliders. High luminosity of the interaction is directly related to the number of particles in the bunch and the quality of the beam in the interaction region, which is tied to these wakefields. In addition, these impedances or wakefields result in power dissipation on the vacuum chamber walls, which can lead to potential instabilities in the beam.

The RW wakefields can also induce transverse mode coupling instability (TMCI) [3] in the beam. This instability can cause beam loss and limits the beam intensity, so it must be carefully studied and mitigated in the design of particle accelerators.

The RW wakefields are especially significant for the Future Circular e^+e^- Collider (FCC-ee) [4], which has a circumference of approximately ~ 92 km. Due to the high-intensity beam in this collider and its long circumference, the RW impedance is the second major source of power loss for particle beams, and it must be accurately calculated and compensated for by RF power in the accelerator cavities.

In this study, we have introduced the VAcuum Chamber Impedance (VACI) suite, which is a powerful tool for solving Maxwell's equations in general cross-section pipes with arbitrary particle velocities and any number of layers. The code is based on the Finite Element Method (FEM) in the frequency domain, making it highly accurate and efficient. Our benchmarking results show that VACI provides excellent agreement with analytical formulas and other codes like ImpedanceWake2D (IW2D) [5], both in longitudinal and transverse directions. These results demonstrate the reliability and accuracy of VACI in simulating the wakefields and impedances of complex vacuum chamber geometries, which are crucial in designing and optimizing particle accelerators.

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METHODOLOGY

The Maxwell's equation [6] for an ultra-relativistic beam in a long, uniform pipe can be expressed as:

$$\begin{cases} \nabla \times E = -j\omega\mu H \\ \nabla.\epsilon E = \rho \\ \nabla \times H = J + j\omega\epsilon E \\ \nabla.H = 0, \end{cases} \tag{1}$$

where $J = \rho v$ represents the current density and v is the velocity of the particles. Here, we used a method based on the one introduced by Uwe Niedermayer [7] to solve Maxwell's equations. We considered a uniform long pipe ($\partial_t \to j\omega$ and $\partial_z \to -j\omega/v$) with a conductivity of σ and used the Coulomb gauge, which is expressed as:

$$\begin{cases} E = -\nabla\phi - \partial A/\partial t, \\ \nabla.A = 0. \end{cases} \tag{2}$$

By considering the above conditions and Gauss' Law in Eq. (1), the electric field can be divided into two components: $E = E^{sol} + E^{curl}$. However, the main challenge lies in the boundary condition, which is dependent on the frequency. For very high frequencies, a very small mesh size is required to solve Maxwell's equations, resulting in high computational time. To overcome this issue, we used the Leontovich Boundary Condition, also known as Surface Impedance Boundary Condition (SIBC). This boundary condition expresses the effect of the normal conductivity of the surface as a surface impedance, which can be expressed as:

$$n \times (n \times E) = -Z_s n \times H, \tag{3}$$

where $Z_s = (1 + J)/\delta\sigma$ is the surface impedance and δ is the skin depth.

RESULTS AND DISCUSSION

To benchmark our results, we used some well-known cases including round and elliptical pipes. There are many papers and textbooks that have tackled the problem of solving the equation analytically, such as Alex Chao's book, which presents a formula for resistive wall impedance. Another tool we used for benchmarking our results is IW2D, which is a well-known simulation code from CERN. IW2D can obtain impedance and wakefields for multilayer pipes, but

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only for round pipes. With Yokoya's form factors [2] the code can estimate RW impedance for rectangular, parallel, and elliptical pipes based on the analytical results of round pipes with appropriate diameters. For a round copper pipe with 35 mm radius, Fig. 1 shows a greate agreement with analytical formula and IW2D results:

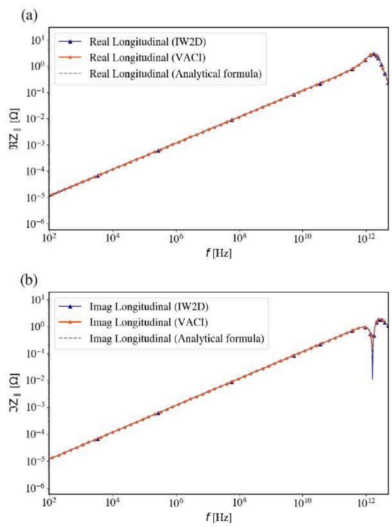

Figure 1: Lonngitudinal Resisitive Wall impedance for a round copper pipe with 35 mm radius. a) is the real part and b) is the imaginary part.

The wakefield can be obtained from impedance by an inverse Fourier Transform as:

$$W(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega e^{j\omega t} f(\omega). \quad (4)$$

To compare the results, we present the wakefield results obtained from Yokoya's code. Figure 2 shows the comparison of the results obtained from the VACI code with those obtained from IW2D and Yokoya's code.

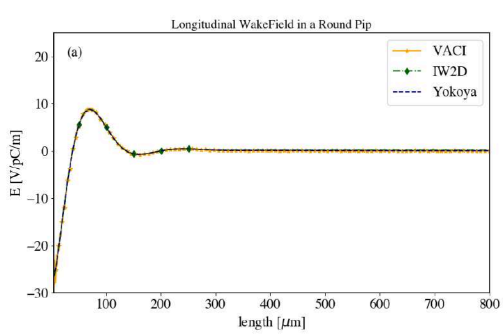

Figure 2: Lonngitudinal Resisitive Wall wakefield for a round copper pipe with 35 mm radius.

Considering a multilayer pipe with a layer of Non-evaporable Getter (NEG) coated inside the copper pipe, we can also obtain the wakefield and impedance for a round pipe. Figure 3 shows the comparison between three codes in the longitudinal direction for different thickness of NEG coating.

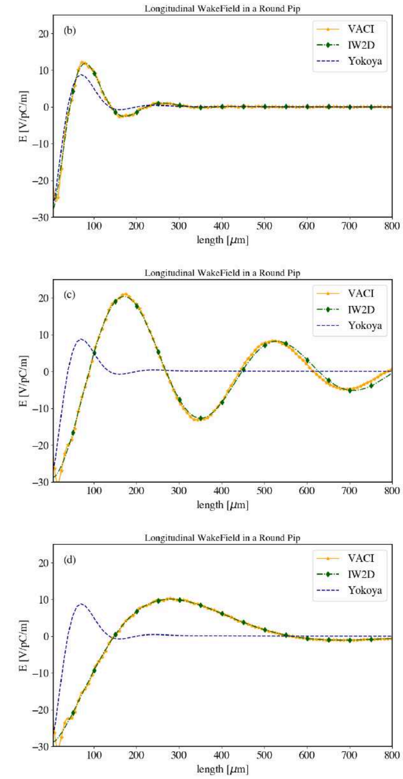

Figure 3: Lonngitudinal Resisitive Wall wakefield for a round copper pipe with 35 mm radius and three thicknesses of NEG coating, a) 10 nm, b) 150 nm, c) 1000 nm.

The same benchmarking results can also be obtained for an elliptical pipe. In this case, we considered an oval-shaped pipe with minor and major semi-axes of 20 mm and 35 mm, respectively, and only a single layer. We compared the longitudinal and transverse wakefield and impedance obtained from VACI code and IW2D. The reason for focusing on a single-layer pipe is to show the accuracy of the code compared to IW2D, which uses Yokoya's form factors. Furthermore, when the pipe becomes different from a fully symmetric round pipe, another impedance also appears that can raise instabilities in multi-bunch operation of the storage ring. We call it Detuning Impedance, which is also known as quadrupole impedance in some literature. Figure 4 shows a comparison between the analytical formula and the results obtained from VACI code and IW2D in the longitudinal direction.

In the case of transverse impedance, there are two cases of dipolar and Detuning impedances. Figure 5 shows these difference for the same pipe.

The results in the x-direction show good agreement between the analytical formula, VACI code, and IW2D. However, at very high frequencies, there are small differences that are not significant for long electron bunches. These differences can affect the conversion of impedance to wakefield. To ensure a precise comparison between VACI suite and IW2D, we also included the results of Yokoya's code (Fig. 6).

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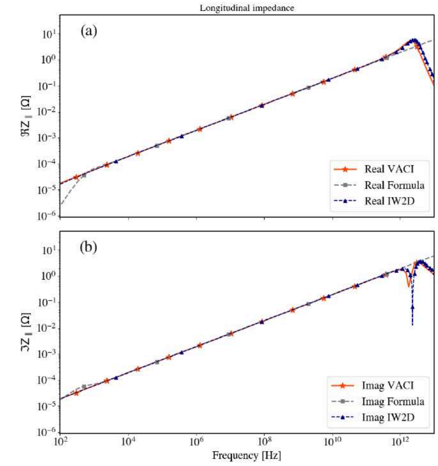

Figure 4: Lonngitudinal Resisitive Wall impedance for an Elliptical copper pipe with minor and major semi-axes of 20 mm and 35 mm, respectively.

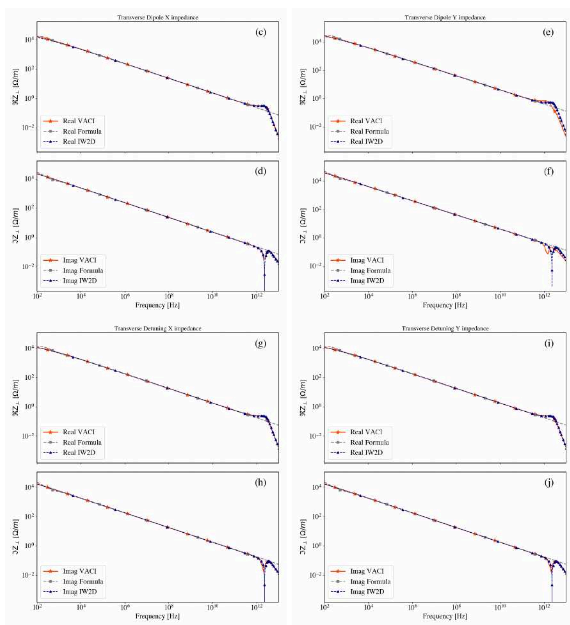

Figure 5: Transverse Resisitive Wall impedance for an Elliptical copper pipe with minor and major semi-axes of 20 mm and 35 mm, respectively.

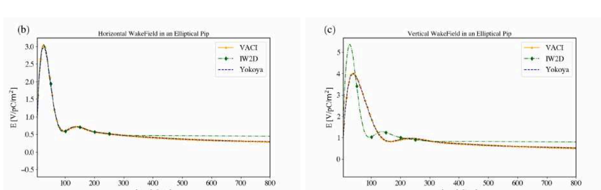

Figure 6: Transverse Resisitive Wall wakefield for an Elliptical copper pipe with minor and major semi-axes of 20 mm and 35 mm, respectively.

It can be seen that the estimation of Yokoya's form factor doesn't work very well for high frequencies, and a precise algorithm like Finite Element Method or Boundary Element Method should be used to solve Maxwell's equations.

CONCLUSIONS

In this study, we have used the VACI code to calculate the wakefield and impedance of round and elliptical pipes with and without a layer of non-evaporable getter (NEG) coating. We compared the results obtained from VACI code with those from IW2D and analytical formulas. Our results show that the VACI code provides accurate and reliable results that are in excellent agreement with IW2D and analytical formulas.

The accuracy of the VACI code is particularly evident in the case of elliptical pipes, where the differences between VACI and IW2D are negligible in the x-direction and only present at very high frequencies. This suggests that VACI can provide reliable predictions of the wakefield and impedance of non-symmetric structures, which are of great importance in multi-bunch operation of storage rings.

In conclusion, our results demonstrate that VACI is a powerful tool for the simulation of wakefield and impedance in accelerator structures. Its accuracy and efficiency make it a suitable alternative to other numerical methods such as IW2D, especially in the case of non-symmetric structures.

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