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Phase Structure and Phase Transition of the SU(2) Higgs Model in Three Dimensions

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Abstract

We derive a set of gauge independent gap equations for Higgs boson and vector boson masses for the SU(2) Higgs model in three dimensions. The solutions can be associated with the Higgs phase and the symmetric phase, respectively. In the Higgs phase the calculated masses are in agreement with results from perturbation theory. In the symmetric phase a non-perturbative vector boson mass is generated by the non-abelian gauge interactions, whose value is rather independent of the scalar self-coupling λ. For small values of λ the phase transition is first-order. Its strength decreases with increasing λ, and at a critical value λ_c the first-order transition changes to a crossover. Based on a perturbative matching the three-dimensional theory is related to the four-dimensional theory at high temperatures. The critical Higgs mass m_H^c, corresponding to the critical coupling λ_c, is estimated to be below 100 GeV. The "symmetric phase" of the theory can be interpreted as a Higgs phase whose parameters are determined non-perturbatively. The obtained Higgs boson and vector boson masses are compared with recent results from lattice Monte Carlo simulations.

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1 Introduction

An important aspect of the standard model of strong and electroweak interactions is the prediction of a phase transition at high temperatures, where the electroweak symmetry is restored [1]. This transition is a direct consequence of the Higgs mechanism of electroweak symmetry breaking. It is of great cosmological importance because baryon-number violating processes come into thermal equilibrium as the temperature approaches the critical temperature of the transition [2]. As a consequence, the present value of the baryon asymmetry of the universe has finally been determined at the electroweak transition[1].

The dynamics of the electroweak phase transition has recently been studied in detail by means of perturbation theory [4]-[7] and lattice Monte Carlo simulations [8]-[12]. As a first step towards the treatment of the full standard model, the pure SU(2) Higgs model is usually investigated, neglecting the effects of fermions and the mixing between photon and neutral vector boson. So far we know that the transition is weakly first-order for Higgs masses m_H small compared to the vector boson mass m_W. Already several years ago it has been shown by lattice simulations that the transition is consistent with a crossover for very large Higgs masses [13]. However, for Higgs masses $m_H = \mathcal{O}(m_W)$ the strength, and even the nature of the transition are not yet known. Since the mass of the physical Higgs boson may very well be close to or larger than the W-boson mass, it is crucial to improve our understanding of the transition in this mass range.

At high temperatures the SU(2) Higgs model in four dimensions can be approximated by an effective three-dimensional theory [14]. In fact, the order of the transition and the properties of the symmetric phase are essentially determined by the quanta with Matsubara frequency zero, i.e., by the three-dimensional theory. Also, the size of non-perturbative effects is related to the confinement scale of the three-dimensional gauge theory [15]-[18] and, based on confinement in three dimensions, it has been suggested that the transition between the Higgs phase and the symmetric phase can only be of first-order or a crossover[15]. Hence, one may hope to gain insight into the nature of the phase transition at large Higgs masses by exploring directly the Higgs model in three dimensions. The connection between this theory and the high-temperature expansion of the Higgs model in four dimensions has already been investigated in detail in perturbation theory [19],[20].

In this paper we shall attempt to study some non-perturbative aspects of the Higgs

[1]For a recent review, see [3]

model in three dimensions by means of gap equations. In the Higgs phase as well as in the symmetric phase one expects non-zero masses for both, the vector boson and the Higgs boson. On the contrary, in perturbation theory the vector boson mass vanishes at any finite order in the symmetric phase. This suggests to perform an improved loop expansion using masses which are self-consistently determined. In a similar way, finite-temperature perturbation theory requires a resummation of plasma mass effects.

Our guiding principle in the derivation of gap equations for Higgs boson and vector boson masses will be the preservation of gauge invariance. As we shall see, this requires a vertex resummation in addition to the mass resummation in a unique way. The resulting gap equations have solutions which can be associated with the Higgs phase and the symmetric phase, respectively. Using perturbative matching equations, which relate the three-dimensional Higgs model and the four-dimensional Higgs model in the high temperature limit, we can then study the implications of our results for the electroweak transition.

The paper is organized as follows. In sect. 2 some general aspects of the SU(2) Higgs model are discussed. In sect. 3 gauge independent gap equations are derived, whose solutions are described in sect. 4. Sect. 5 deals with implications for the electroweak transition, and our results are summarized in sect. 6.

2 General properties of the Higgs model

The SU(2) Higgs model in three dimensions is defined by the action

$$S = \int d^3x \; \mathrm{Tr}\left[\frac{1}{2}W_{\mu\nu}W_{\mu\nu} + (D_\mu\Phi)^\dagger D_\mu\Phi + \mu^2\Phi^\dagger\Phi + 2\lambda(\Phi^\dagger\Phi)^2\right] , \qquad (1)$$

with

$$\Phi = \frac{1}{2}(\sigma + i\vec{\pi}\cdot\vec{\tau}) , \quad D_\mu\Phi = (\partial_\mu - igW_\mu)\Phi , \quad W_\mu = \frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu \; . \qquad (2)$$

Here $\vec{W}_\mu$ is the vector field, σ is the Higgs field, $\vec{\pi}$ is the Goldstone boson field and $\vec{\tau}$ is the triplet of Pauli matrices. The gauge coupling g and the scalar coupling λ have mass dimension 1/2 and 1, respectively. For perturbative calculations gauge fixing and ghost terms have to be added.

The model is known to have only one phase[2]. However, depending on the values of the parameters the physical properties of the model may be rather different. For some range of parameters it is meaningful to distinguish a "Higgs phase" from a "symmetric

[2] For a general discussion and references, see [21]

phase". Varying μ^2/g^4 one then expects a phase transition which, at least for small values of λ/g^2, should be of first order.

The physical content of the theory is contained in the properties of correlation functions of gauge invariant operators. Consider the composite vector and scalar fields

$$\tilde{W}^a_\mu(x) = \mathrm{Tr}\left[\Phi^\dagger(x) D_\mu \Phi \frac{\tau^a}{2}(x)\right] \; , \; \tilde{\sigma}(x) = \mathrm{Tr}\left[\Phi^\dagger(x)\Phi(x)\right] \; . \tag{3}$$

The expectation value $< \tilde{\sigma} >$ plays the role of an "order parameter", which can distinguish between the two phases. A transition from the symmetric phase to the Higgs phase is characterized by an increase of $< \tilde{\sigma} >$.

Other important parameters are the Higgs boson and vector boson masses in both phases. They determine the exponential falloff of the corresponding two-point functions at large separation $|x-y|$,

$$\begin{aligned} < \tilde{\sigma}(x)\tilde{\sigma}(y) > &\sim e^{-M|x-y|} \; , \\ < \tilde{W}(x)\tilde{W}(y) > &\sim e^{-m|x-y|} \; . \end{aligned} \tag{4}$$

In the Higgs phase, after fixing a gauge, these 2-point functions can be evaluated in perturbation theory. One shifts the scalar field Φ around a gauge dependent vacuum expectation value, and the masses m and M are then given by the exponential falloff of the gauge dependent 2-point functions of the fields $\sigma'(x)$ and $W^a_\mu(x)$. In momentum-space these masses are given by poles of the corresponding propagators, which are gauge independent, contrary to "masses" defined at zero momentum [22].

One expects that in the Higgs phase, for $\mu^2/g^4 < 0$ and $\lambda/g^2 < 1$, the masses M and m can be accurately calculated using ordinary perturbation theory in g and λ. On the other hand, these masses cannot be evaluated perturbatively in the symmetric phase, in particular near $\mu^2 = 0$. At a first-order transition one expects a jump in both masses. The masses in the symmetric phase may be $\mathcal{O}(g^2)$, as in the pure gauge theory in three dimensions [23]. However, they may also be smaller, especially near a critical point, where the properties of the Higgs model can be expected to be rather different from the pure gauge theory.

In principle, the Higgs mass and the vector boson mass can be directly measured in both phases by means of lattice Monte Carlo simulations. In practice, this becomes difficult if a large ratio of masses occurs which could be the case near a critical point. In any case, it appears desirable to gain some insight into the structure of the symmetric phase also by means of analytical methods. In the past, gap equations have often been a useful tool to estimate a non-perturbative mass gap. In the following we shall apply this approach to the Higgs model in three dimensions. This theory may be easier to solve than the four-dimensional theory since the couplings have positive mass dimension. Furthermore, it is conceivable that the Higgs model has a simpler structure than the pure gauge theory since Higgs phase and symmetric phase are analytically connected.

3 The gap equations

Our starting point is perturbation theory in the Higgs phase. Hence, we shift the Higgs field σ around its vacuum expectation value v, $\sigma = v+\sigma'$, and supplement the lagrangian (1) by gauge fixing and ghost terms,

$$L_{GF} = \frac{1}{2\xi}(G^a)^2 \ , \qquad L_{FP} = -c^{*a}M^{ab}c^b \ . \tag{5}$$

Here M^{ab} is the variation of G^a under a gauge transformation, $\delta G^a = M^{ab}\Lambda^b$, and c^{*a}, c^a are the ghost fields. We will perform our calculations in R_ξ-gauge, which corresponds to the choice

$$G^a = \partial_\mu W_\mu^a + \xi\frac{g}{2}v\pi^a \ . \tag{6}$$

The complete lagrangian then reads explicitly,

$$\begin{aligned}
L \ = \ & \frac{1}{4}\vec{W}_{\mu\nu}\vec{W}_{\mu\nu} + \frac{1}{2\xi}(\partial_\mu\vec{W}_\mu)^2 + \frac{g^2}{8}v^2\vec{W}_\mu^2 \\
& +\frac{1}{2}(\partial_\mu\sigma')^2 + \lambda v^2\sigma'^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \xi\frac{g^2}{8}v^2\vec{\pi}^2 \\
& +\frac{g^2}{4}v\sigma'\vec{W}_\mu^2 + \frac{g}{2}\vec{W}_\mu\cdot(\vec{\pi}\partial_\mu\sigma' - \sigma'\partial_\mu\vec{\pi}) + \frac{g}{2}(\vec{W}_\mu\times\vec{\pi})\cdot\partial_\mu\vec{\pi} \\
& +\frac{g^2}{8}\vec{W}_\mu^2(\sigma'^2+\vec{\pi}^2) + \lambda v\sigma'(\sigma'^2+\vec{\pi}^2) + \frac{\lambda}{4}(\sigma'^2+\vec{\pi}^2)^2 \\
& +\partial_\mu\vec{c^*}\partial_\mu\vec{c} + \xi\frac{g^2}{4}v^2\vec{c^*}\vec{c} \\
& +g\partial_\mu\vec{c^*}\cdot(\vec{W}_\mu\times\vec{c}) + \xi\frac{g^2}{4}v\sigma'\vec{c^*}\vec{c} + \xi\frac{g^2}{4}v\vec{c^*}\cdot(\vec{\pi}\times\vec{c}) + \frac{1}{2}\mu^2v^2 + \frac{1}{4}\lambda v^4 \\
& +\frac{1}{2}(\mu^2+\lambda v^2)(\sigma'^2+\vec{\pi}^2) + v(\mu^2+\lambda v^2)\sigma' \ .
\end{aligned} \tag{7}$$

The last two terms arise from the scalar part of the lagrangian (1) after the shift in the Higgs field σ. For $\mu^2 < 0$, they vanish if one expands around the classical minimum $v^2 = -\mu^2/\lambda$. In general, however, these terms have to be kept.

From eq. (7) one reads off the propagators for vector boson, Goldstone boson, ghost and Higgs boson, respectively,

$$\begin{aligned}
D^{ab}_{\mu\nu}(p) &= \delta_{ab}\left[D_T(p)\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)+D_L(p)\frac{p_\mu p_\nu}{p^2}\right] ,\\
D_T(p) &= \frac{1}{p^2+m_0^2}\ ,\ D_L(p)=\frac{\xi}{p^2+\xi m_0^2}\ ,\\
\Delta^{ab}_\pi(p) &= \Delta^{ab}_c(p)=\frac{\delta_{ab}}{p^2+\xi m_0^2}\ ,\\
\Delta_\sigma(p) &= \frac{1}{p^2+M_0^2}\ ,
\end{aligned} \tag{8}$$

with the tree level masses

$$m_0^2=\frac{g^2}{4}v^2\ ,\ M_0^2=\mu^2+3\lambda v^2. \tag{9}$$

In perturbation theory the vacuum expectation value vanishes in the symmetric phase, $v=0$. This implies that the vector boson mass vanishes at tree level, $m_0=0$. It is generally expected that in the symmetric phase a non-zero vector boson mass $\mathcal{O}(g^2)$ is generated non-perturbatively. Since the loop expansion in three dimensions generates a series in powers of g^2/m_0, ordinary perturbation theory with a vanishing vector boson mass m_0 appears to be seriously deficient in the symmetric phase.

A non-vanishing vector boson mass can be obtained from a coupled set of gap equations for Higgs boson and vector boson masses as follows. The tree level masses m_0^2 and M_0^2 are expressed as

$$m_0^2=m^2-\delta m^2\ ,\ M_0^2=M^2-\delta M^2\ , \tag{10}$$

where m and M enter the propagators of the loop expansion, and δm^2 and δM^2 are treated perturbatively as counter terms. In R_ξ-gauge the tree-level ghost and Goldstone boson masses are given by $\sqrt{\xi}m_0$, where m_0 is the tree-level vector boson mass. Correspondingly, we define $\sqrt{\xi}m$ as resummed ghost and Goldstone boson mass. One then obtains the coupled set of gap equations for Higgs boson and vector boson masses,

$$\begin{aligned}
\delta m^2+\Pi_T(p^2=-m^2,m,M,\xi)=0\ ,\\
\delta M^2+\Sigma(p^2=-M^2,m,M,\xi)=0\ ,
\end{aligned} \tag{11}$$

where $\Pi_T(p^2)$ is the transverse part of the vacuum polarization tensor,

$$\Pi^{ab}_{\mu\nu}(p)=\delta_{ab}\left[\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\Pi_T(p^2)+\frac{p_\mu p_\nu}{p^2}\Pi_L(p^2)\right]\ . \tag{12}$$

In the gap equations the self-energy corrections are evaluated on the mass shell. This yields the physical screening lengths (cf. eq. (4)), and therefore a gauge independent result [22]. The one-loop self-energy contributions $\Pi^{ab}_{\mu\nu}(p)$ and $\Sigma(p)$ are given by the graphs shown in figs. (1) and (2), respectively.

To obtain gauge independent masses from gap equations is a non-trivial task. In fact, the "magnetic mass" [6],[24], which has been derived from gap equations in the high-temperature expansion, is gauge dependent. Our calculation shows that the mass resummation has to be supplemented by a vertex resummation. This is not unexpected since various 2-point, 3-point and 4-point couplings of the lagrangian (7) are related by gauge invariance.

Let us consider the necessary vertex resummations in detail. First, the vertices appearing in the graphs fig. (1a)-(1d) only involve the gauge coupling g. The gauge dependent terms of these graphs cancel among themselves. In order to obtain a gauge independent result for the contributions (1e)-(1l) all cubic vertices linear in σ' and π^a are rewritten as

$$\frac{g^2 v}{2} = gm - \delta V^g_{\phi\phi\phi}\ ,\ \phi = W,\ c,\ \pi^a,\ \sigma\ , \tag{13}$$

and the terms cubic in σ' and π^a are resummed as

$$\lambda v = \frac{gM^2}{4m} - \delta V^\lambda_{\phi\phi\phi}\ ,\ \phi = \sigma',\ \pi^a\ . \tag{14}$$

The explicit calculation shows that these resummations are necessary and sufficient to obtain a gauge independent result for $\Pi_T(p)$. A gauge independent result for $\Sigma(p)$ is only obtained if also the scalar self-coupling is resummed,

$$\lambda = \frac{g^2 M^2}{8m^2} - \delta V^\lambda_{\phi\phi\phi\phi}\ ,\ \phi = \sigma'\ ,\ \pi^a\ . \tag{15}$$

Combining equations (10) and (13)-(15), the lagrangian (7) takes the form,

$$\begin{aligned}
L &= L_R + L_1 + L_0\ ,\\
L_R &= \frac{1}{4}\vec{W}_{\mu\nu}\vec{W}_{\mu\nu} + \frac{1}{2\xi}(\partial_\mu\vec{W}_\mu)^2 + \frac{1}{2}m^2\vec{W}_\mu^2\\
&\quad + \frac{1}{2}(\partial_\mu\sigma')^2 + \frac{1}{2}M^2\sigma'^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{\xi}{2}m^2\vec{\pi}^2\\
&\quad + \frac{g}{2}m\sigma'\vec{W}_\mu^2 + \frac{g}{2}\vec{W}_\mu\cdot(\vec{\pi}\partial_\mu\sigma' - \sigma'\partial_\mu\vec{\pi}) + \frac{g}{2}(\vec{W}_\mu\times\vec{\pi})\cdot\partial_\mu\vec{\pi}\\
&\quad + \frac{g^2}{8}\vec{W}_\mu^2(\sigma'^2+\vec{\pi}^2) + \frac{g}{4}\frac{M^2}{m}\sigma'(\sigma'^2+\vec{\pi}^2) + \frac{g^2}{32}\frac{M^2}{m^2}(\sigma'^2+\vec{\pi}^2)^2\\
&\quad + \partial_\mu\vec{c}^*\partial_\mu\vec{c} + \xi m^2\vec{c}^*\vec{c}\\
&\quad + g\partial_\mu\vec{c}^*\cdot(\vec{W}_\mu\times\vec{c}) + \xi\frac{g}{2}m\sigma'\vec{c}^*\vec{c} + \xi\frac{g}{2}m\vec{c}^*\cdot(\vec{\pi}\times\vec{c})\ ,
\end{aligned}$$

$$
\begin{aligned}
L_1 &= -\delta m^2 \left(\frac{1}{2}\vec{W}_\mu^2 + \frac{\xi}{2}\vec{\pi}^2 + \xi \vec{c}^* \vec{c}\right) - \frac{1}{2}\delta M^2 \sigma'^2 + \frac{1}{2}(\mu^2 + \lambda v^2)\vec{\pi}^2 \\
&\quad + v(\mu^2 + \lambda v^2)\sigma' - \delta L_{\phi\phi\phi} - \delta L_{\phi\phi\phi\phi} \ , \\
L_0 &= \frac{1}{2}\mu^2 v^2 + \frac{1}{4}\lambda v^4 \ .
\end{aligned} \tag{16}
$$

Here $\delta L_{\phi\phi\phi}$ and $\delta L_{\phi\phi\phi\phi}$ denote the difference between tree level and resummed cubic and quartic vertices. In the resummed perturbation theory only vertices from L_R contribute at one-loop order. In higher orders also the vertices from L_1 have to be taken into account, like counter terms in ordinary perturbation theory.

Starting from the lagrangian L_R, it is straightforward to evaluate the one-loop self-energy contributions for vector boson and Higgs boson. The corresponding graphs are shown in figs. (1) and (2). Vertices with full bubbles denote resummed vertices, and lines with full bubbles represent resummed propagators, which are obtained from eq. (8) by replacing the tree level masses by the full masses. For the transverse part of the vacuum polarization tensor we then obtain the result

$$
\begin{aligned}
\Pi_T(p^2) &= g^2 \Bigg[\frac{m}{gM^2} v(\mu^2 + \lambda v^2) + \left(\frac{p^4}{4m^4} - \frac{p^2}{m^2} - \frac{15}{8} + \frac{3m^2}{M^2} - \frac{m^2}{8p^2} + \frac{M^2}{8p^2} \right. \\
&\quad \left. - \frac{1}{4m^4 p^2}(p^2 + m^2)^2 (p^2 + (\xi - 1)m^2) \right) A_0(m^2) + \left(\frac{5}{8} - \frac{M^2}{8p^2} + \frac{m^2}{8p^2} \right) A_0(M^2) \\
&\quad + \left(\frac{3}{4m^4}(m^4 - p^4) + \frac{1}{4m^4 p^2}(p^2 + m^2)^2 (3p^2 + (\xi - 1)m^2) \right) A_0(\xi m^2) \\
&\quad - \left(\frac{p^6}{8m^4} - \frac{p^4}{m^2} - 5p^2 + 4m^2 \right) B_0(p^2, m^2, m^2) \\
&\quad + \left(\frac{m^2}{2} - \frac{1}{8p^2}(p^2 + M^2 - m^2)^2 \right) B_0(p^2, m^2, M^2) \\
&\quad + \frac{1}{8m^4}(m^4 - p^4) \left(4\xi m^2 + p^2 \right) B_0(p^2, \xi m^2, \xi m^2) \\
&\quad + \frac{1}{4m^4 p^2}(p^2 + m^2)^2 \left((p^2 + (\xi - 1)m^2)^2 - 4m^2 p^2 \right) B_0(p^2, m^2, \xi m^2) \Bigg] \ .
\end{aligned} \tag{17}
$$

Similarly, we find for the Higgs boson self-energy

$$
\begin{aligned}
\Sigma(p^2) &= g^2 \Bigg[\frac{3}{2gm} v(\mu^2 + \lambda v^2) + \frac{3}{4m^2}(4m^2 - p^2) A_0(m^2) + \frac{3M^2}{4m^2} A_0(M^2) \\
&\quad + \frac{3}{4m^2}(M^2 + p^2) A_0(\xi m^2) + \frac{3}{8m^2}(8m^4 + 4m^2 p^2 + p^4) B_0(p^2, m^2, m^2) \\
&\quad + \frac{9M^4}{8m^2} B_0(p^2, M^2, M^2) + \frac{3}{8m^2}(M^4 - p^4) B_0(p^2, \xi m^2, \xi m^2) \Bigg] \ .
\end{aligned} \tag{18}
$$

Here A_0 and B_0 are the three-dimensional integrals

$$\begin{aligned}
A_0(m^2) &= \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2+m^2} \\
B_0(p^2, m_1^2, m_2^2) &= \int \frac{d^3k}{(2\pi)^3} \frac{1}{(k^2+m_1^2)((k+p)^2+m_2^2)} \ .
\end{aligned} \tag{19}$$

The integral A_0 is linearly divergent. The divergence can be cancelled by a counter term generated by an additive renormalization of the mass parameter μ^2 in the lagrangian (7). We will remove the divergent part of the integral by dimensional regularization.

From eqs. (17) and (18) one reads off that the resummed one-loop self-energy contributions are gauge independent on the mass shell. As described above this has been achieved by supplementing the mass resummations by vertex resummations. Why did this work? One easily verifies that the lagrangian L_R can essentially be obtained from the gauge invariant lagrangian

$$L = \mathrm{Tr}\left[\frac{1}{2}W_{\mu\nu}W_{\mu\nu} + (D_\mu\Phi)^\dagger D_\mu\Phi - \frac{1}{2}M^2\Phi^\dagger\Phi + \frac{g^2}{4}\frac{M^2}{m^2}(\Phi^\dagger\Phi)^2\right] \ , \tag{20}$$

by shifting the Higgs field $\sigma = \mathrm{Tr}[\Phi]$ around the "classical" minimum,

$$\sigma = \sigma' + \frac{2m}{g} \ , \tag{21}$$

and by adding the corresponding gauge fixing and ghost lagrangians defined by (cf. (5))

$$G^a = \partial_\mu W_\mu^a + \xi m \pi^a \ . \tag{22}$$

The resulting lagrangian differs from L_R in eq. (16) only by a constant. Hence, the lagrangian L_R is invariant under BRS transformations and we expect a gauge independent result for the position of the pole of a propagator.

The functions A_0 and B_0 are easily evaluated, and from eqs. (11), (17) and (18) one obtains the gap equations,

$$\begin{aligned}
m^2 &= m_0^2 - \frac{gz}{M}v(\mu^2+\lambda v^2) + mg^2\bar{f}(z) \ , \\
M^2 &= M_0^2 - \frac{3g}{2m}v(\mu^2+\lambda v^2) + Mg^2\bar{F}(z) \ ,
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
\bar{f}(z) &= \frac{1}{\pi}\left[\frac{63}{64}\ln 3 - \frac{1}{8} + \frac{1}{32z^3} - \frac{1}{32z^2} + \frac{1}{8z} \right. \\
&\quad \left. + \frac{3}{4}z^2 - \left(\frac{1}{64z^4} - \frac{1}{16z^2} + \frac{1}{8}\right)\ln(1+2z)\right] \ ,
\end{aligned} \tag{24}$$

$$\begin{aligned}
\bar{F}(z) &= \frac{1}{\pi}\left[\frac{3}{64}(4-3\ln 3)\frac{1}{z^2} + \frac{3}{16z} + \frac{3}{4}z \right. \\
&\quad \left. - \left(\frac{3}{8}z^2 - \frac{3}{16} + \frac{3}{64z^2}\right)\ln\frac{2z+1}{2z-1}\right] \ ,
\end{aligned} \tag{25}$$

with $z = m/M$. Note, that the equation for M becomes complex for $M > 2m$, i.e., $z < 1/2$, since in this case the Higgs boson can decay into two vector bosons. We will therefore restrict our discussion to the mass range $M < 2m$. In order to find solutions of the gap equations we have to specify the value of the vacuum expectation value v. This will be discussed in the following section.

In our derivation of the gauge independent gap equations the scalar degrees of freedom have played a crucial role. In fact, it does not seem possible to derive a gauge independent gap equation for the pure SU(2) gauge theory. The physical reason for this appears rather obvious. In the Higgs model the Goldstone bosons can screen the colour of the gauge bosons, which is not the case in the pure gauge theory. Hence, the ground state and the spectrum of excitations can be very different for the two theories.

4 Phase structure in three dimensions

In the previous section we have performed a calculation in the "Higgs phase", i.e., we have shifted the Higgs field around an unspecified vacuum expectation value v. The value of v can be self-consistently determined from the requirement that the vacuum expectation value of the shifted field is zero,

$$< \sigma' > = 0 . \tag{26}$$

This equation simply means that the sum of all tadpole contributions to the self-energies, which are shown in figs. (1i) - (1m) and (2i) - (2m), vanishes. From eq. (26) one obtains in resummed perturbation theory at one-loop order,

$$\begin{aligned} v(\mu^2 + \lambda v^2) &= -\frac{3}{4} g m \left(4 A_0(m^2) + \frac{M^2}{m^2} A_0(\xi m^2) + \frac{M^2}{m^2} A_0(M^2) \right) \\ &= \frac{3}{16\pi} g \left(4m^2 + \sqrt{\xi} M^2 + \frac{M^3}{m} \right) . \end{aligned} \tag{27}$$

The vacuum expectation value v of the Higgs field is not a physical observable and expected to be gauge dependent, as in ordinary perturbation theory. Numerically, the gauge dependence becomes important for large Higgs masses, $M > m$. On the other hand, the masses obtained from the gap equations (23) are physical observables and must therefore be gauge independent. The weak gauge dependence induced by the gauge dependence of v has to be cancelled by higher order contributions. In the following we shall work in Landau gauge, $\xi = 0$.

For any solution v of eq. (27), only the irreducible parts of the self-energies contribute in the gap equations (23). From eqs. (17) and (18) one easily finds ($z = m/M$),

$$
\begin{aligned}
m^2 &= \frac{g^2}{4}v^2 + mg^2 f(z) \ , && (28)\\
M^2 &= \mu^2 + 3\lambda v^2 + Mg^2 F(z)\,, && (29)
\end{aligned}
$$

where the functions $f(z)$ and $F(z)$ are given by

$$
\begin{aligned}
f(z) &= \frac{1}{\pi}\left[\frac{63}{64}\ln 3 - \frac{1}{8} + \frac{1}{32z^3} - \frac{1}{32z^2} - \frac{1}{16z} - \frac{3\sqrt{\xi}}{16}\right.\\
&\qquad \left. - \left(\frac{1}{64z^4} - \frac{1}{16z^2} + \frac{1}{8}\right)\ln(1+2z)\right] \ , && (30)\\
F(z) &= \frac{1}{\pi}\left[-\left(\frac{3}{32} + \frac{9}{64}\ln 3\right)\frac{1}{z^2} + \frac{3}{16}\left(1 - \frac{3}{2}\sqrt{\xi}\right)\frac{1}{z}\right.\\
&\qquad \left. -\frac{3}{8}z - \left(\frac{3}{8}z^2 - \frac{3}{16} + \frac{3}{64z^2}\right)\ln\frac{2z+1}{2z-1}\right] \ . && (31)
\end{aligned}
$$

The solutions of the gap equations depend crucially on the properties of these functions. They are plotted in figs. (3) and (4). $F(z)$ is always negative, whereas $f(z)$ has a zero at a large value z_0. Their asymptotic behaviour for large z reads

$$
f(z) \sim -\frac{1}{8\pi}\ln z \ , \quad F(z) \sim -\frac{3}{4\pi}z \ . \qquad (32)
$$

Also important is the behaviour of $f(z)$ in the vicinity of its zero which is given by

$$
f(z) = -\frac{1}{8\pi}\frac{z-z_0}{z_0} + \mathcal{O}\left(\left(\frac{z-z_0}{z_0}\right)^2\right) \ . \qquad (33)
$$

From equations (23) and (27) we can also obtain the one-loop results of ordinary perturbation theory for vacuum expectation value and masses. In this case, the masses $m_0 = gv/2$ and $M_0 = \sqrt{2\lambda}v$, with the fixed ratio $z = \sqrt{g^2/8\lambda}$, appear in the one-loop expressions, and the vacuum expectation value v is determined from the one-loop effective potential. One then finds (cf. eqs. (24), (25)),

$$
\begin{aligned}
v(\mu^2 + \lambda v^2) &= \frac{v^2}{4\pi}\left(\frac{3}{4}g^3 + 3\sqrt{2}\lambda^{3/2} + \frac{3}{2}\sqrt{\xi}\lambda g\right) \ , && (34)\\
m^2 &= -\frac{g^2}{4\lambda}\mu^2 + \frac{g^3}{2}v\bar{f}\left(\sqrt{\frac{g^2}{8\lambda}}\right) \ , && (35)\\
M^2 &= -2\mu^2 + g^2\sqrt{2\lambda}v\bar{F}\left(\sqrt{\frac{g^2}{8\lambda}}\right) \ . && (36)
\end{aligned}
$$

From these equations v, m and M are easily obtained as functions of g, λ and μ^2.

The coupled system of equations (27) - (29) for vacuum expectation value and masses can be solved numerically. For a given value of λ/g^2 one can determine v/g, m/g^2 and M/g^2 as functions of μ^2/g^4. Let us first choose a small value, $\lambda/g^2 = 1/128$. In the Higgs phase, this correponds to $m_H \sim m_W/4$. The result for v/g is shown in fig. (5). For $\mu^2/g^4 < 0$, v/g is large, as expected for the Higgs phase. For large positive values of μ^2/g^4, which correspond to the symmetric phase of the theory, v/g is small but non-zero. Furthermore, v/g is rather independent of μ^2/g^4. Of particular interest is the region of small positive μ^2/g^4. Here one obtains two solutions, i.e., the theory has one metastable state. The Higgs phase is generated by quantum corrections, as in the Coleman-Weinberg mechanism of radiative symmetry breaking [25]. The vacuum expectation value in the Higgs phase is not continuously connected to the one in the symmetric phase. Hence, the phase transition is first-order. In fig. (5) the solutions of the gap equations are also compared with the results from ordinary perturbation theory. In the Higgs phase the perturbative value of v/g is slightly smaller than the one obtained from the gap equations. The main difference concerns the symmetric phase. Here, in perturbation theory $v = 0$, whereas the gap equations yield $0 < v/g < 1$.

Each solution $v/g(\mu^2/g^4)$ is connected with Higgs boson and vector boson masses $M/g^2(\mu^2/g^4)$ and $m/g^2(\mu^2/g^4)$. In fig. (6) the results are shown and compared with perturbation theory. The masses in the Higgs phase are in accord with the results of perturbation theory. In the symmetric phase, the vector boson mass is rather small and independent of μ^2/g^4. Its value is of the same order of magnitude as the "magnetic mass" which has previously been obtained for the symmetric phase of the SU(2) Higgs model at high temperatures in Landau gauge [6],[24]. Rather intriguing is the behaviour of vector boson and Higgs boson masses in the symmetric phase in the metastability domain. As μ^2/g^4 approaches zero, both, m/g^2 and M/g^2 tend to zero with a fixed ratio $M/m = 1/z_0 \ll 1$, a behaviour very different from that obtained in perturbation theory.

In the limit $\mu^2/g^4 \to 0$, the solution of equations (27)-(29) takes a simple form,

$$v \sim \frac{4\pi}{3}\frac{\mu^2}{g^3} \quad , \tag{37}$$

$$\frac{z - z_0}{z_0} \sim -8\pi^2 \frac{\mu^2}{g^4} \quad , \tag{38}$$

$$M \sim \frac{4\pi}{3z_0}\frac{\mu^2}{g^2} \quad , \tag{39}$$

where z_0 is the zero of the function $f(z)$. As in the ordinary Higgs phase, vector boson and Higgs boson masses are proportional to the vacuum expectation value v,

$$m \sim v \ , \quad M \sim 2\sqrt{2\lambda_R} v \ , \tag{40}$$

where $\lambda_R = g^2/8z_0^2$ is the resummed scalar coupling. Hence, near the point $\mu^2 = 0$ the smallness of the Higgs mass reflects the smallness of the resummed scalar coupling. Note, that the suppression of the non-perturbative contribution to the vector boson mass and the smallness of the effective scalar coupling for small μ^2/g^4 are also characteristic features of the ϵ-expansion applied to the electroweak phase transition [26].

For sufficiently small values of λ/g^2 the phase transition is first-order. However, larger values of λ are of particular importance because of the present lower experimental bound for the Higgs boson mass. Let us first consider the point $\mu^2 = 0$ where the solution, which corresponds to the symmetric phase, vanishes. For large values of λ/g^2, the solution of eqs. (27) - (29) takes a form similar to the one obtained for small μ^2/g^4. In both cases the ratio m/M is close to the zero of $f(z)$. One finds,

$$v \sim \frac{1}{12\pi}\frac{g^3}{\lambda} \ , \tag{41}$$

$$\frac{z - z_0}{z_0} \sim \frac{5}{18}\frac{g^2}{\lambda} \ , \tag{42}$$

$$M \sim \frac{1}{36\pi z_0}\frac{g^4}{\lambda} \ . \tag{43}$$

Hence, at $\mu^2 = 0$ one has $v > 0$ for arbitrarily large values of λ/g^2. This means that for a small range of μ^2/g^4 near $\mu^2 = 0$ there exist at least two solutions of the gap equations, one of which represents a metastable state.

What is the nature of the transition for large values of λ/g^2? In fig. (7) the solution $v/g(\mu^2/g^4)$ of the gap equations is shown for $\lambda/g^2 = 1/8$. In the Higgs phase, this corresponds to $m_H \sim m_W$. Compared to fig. (5), where $\lambda/g^2 = 1/128$, a dramatic change has taken place. The Higgs phase and the symmetric phase are now continuously connected. Hence, the first-order transition has changed to a crossover! The phase transition changes its character at a critical coupling λ_c. At this value of λ the two solutions of the gap equations merge at a critical value μ_c^2. Numerically, we find $\lambda_c/g^2 \approx 0.053$ and $\mu_c^2/g^4 \approx 0.054$. The nature of the transition at this endpoint of the first-order transition line will be studied in more detail elsewhere.

An important quantity is the vector boson mass in the symmetric phase. A comparison of figs. (6) and (8) shows that its value is essentially independent of λ. In both cases the vector boson mass agrees within 10% with the mass obtained from the nonlinear

σ-model, where only the graphs fig. (1a)-(1d) contribute, which yield [27]

$$\begin{aligned} m_{SM} &= C g^2 \ , \\ C &= \frac{3}{64\pi}(21 \ln 3 - 4) = 0.28 \ . \end{aligned} \tag{44}$$

This is in accord with the intuitive picture that for large μ^2/g^4 the Higgs degree of freedom should be unimportant for the vector boson mass. As μ^2/g^4 is increased, the ratio of the masses, $z = m/M$, crosses the value 1/2 at some point μ_e^2/g^4, and real solutions to the gap equations cease to exist. The position of this point, $\mu_e^2/g^4 \approx 0.8$, is essentially independent of λ.

The Higgs boson and vector boson masses, which we have determined in this section, are solutions of one-loop gap equations. What can be said about the size of corrections in higher orders of the loop expansion? In particular one might worry about the well known infrared problem in the symmetric phase, where the effective gauge coupling and also the scalar coupling can become very large [15]-[18]. The expansion parameter ρ_V for vector loops is (cf. [5], [17], [28]),

$$\rho_V = \frac{1}{6\pi}\frac{g^2}{m} \ . \tag{45}$$

In the Higgs phase where, for $\mu^2 \approx 0$, $v \approx 3g^3/(16\pi\lambda)$ (cf. (34)), one then has

$$\rho_V^{Higgs} \approx \frac{16}{9}\frac{\lambda}{g^2} \ . \tag{46}$$

For $m_H \sim m_W$, i.e. $\lambda/g^2 \sim 1/8$, this yields $\rho_V^{Higgs} \approx 0.22$. On the other hand, in the symmetric phase one has

$$\rho_V^{sym} = \frac{1}{6\pi C} \approx 0.19 \ , \tag{47}$$

which corresponds to the expansion parameter in the Higgs phase for a Higgs mass slightly below the vector boson mass.

Alternatively, one may estimate the effect of higher order corrections by means of the running coupling in $4-\epsilon$ dimensions at $\epsilon = 1$,

$$g^2(\mu) = \frac{g^2}{1 + \beta_0 g^2/\mu} \ , \tag{48}$$

where $\beta_0 = -43/(48\pi^2)$ for the SU(2) gauge theory with one doublet of Higgs fields. At the scale of the vector boson mass in the symmetric phase one finds $g^2(m_{SM}) \approx 1.48 g^2$, i.e., the running coupling is still rather far away from the infrared singularity. The Higgs boson mass is much smaller than the vector boson mass. However, since the scalar coupling in the symmetric phase is also suppressed like M^2/m^2, it is conceivable that higher order corrections involving the Higgs field are small.

We conclude that the solution of the one-loop gap equations discussed in this section may indeed provide a suitable starting point for a systematic improved loop expansion in the Higgs phase as well as in the symmetric phase. Most difficult is the region near the phase transition point, especially for large λ/g^2. Here, Higgs boson and vector boson masses in the symmetric phase are small, and the one-loop results are therefore not reliable. This also applies to the region near $\mu^2 = 0$ in the symmetric phase. In general, a strong gauge dependence of a one-loop result can be used as an indication for the importance of higher order corrections.

5 The electroweak transition

At high temperatures the SU(2) Higgs model in four dimensions can be described by an effective three-dimensional theory. The connection between the two theories has been discussed in detail in perturbation theory [19],[20]. For sufficiently large values of λ, the SU(2) Higgs model in three dimensions is expected to be the appropriate effective theory. Perturbation theory at one-loop order yields the following relations between the parameters (cf., e.g. [17]),

$$\begin{aligned} g^2 &= \bar{g}^2(T)T\ ,\ \lambda = \left(\bar{\lambda}(T) - \frac{3}{128\pi}\sqrt{\frac{6}{5}}\bar{g}^3(T) + \mathcal{O}(\bar{g}^4,\bar{\lambda}^2)\right)T\ , \\ \mu^2 &= \left(\frac{3}{16}\bar{g}^2(T) + \frac{1}{2}\bar{\lambda}(T) - \frac{3}{16\pi}\sqrt{\frac{5}{6}}\bar{g}^3(T) + \mathcal{O}(\bar{g}^4,\bar{\lambda}^2)\right)(T^2 - T_b^2)\ . \end{aligned} \tag{49}$$

Here $\bar{g}$ and $\bar{\lambda}$ are the dimensionless couplings in four dimensions, T is the temperature and T_b is the "barrier temperature". These matching equations can be used to interpret the results of the previous section in terms of the high-temperature theory and to compare them with recent results obtained by means of lattice Monte Carlo simulations [9],[11]. Note, however, that for a precise quantitative comparison between the three-dimensional theory and the four-dimensional theory at high temperatures the perturbative matching equations (49) are not sufficient [29].

In ref. [11] a first-order phase transition was found for $m_H = 49$ GeV, $m_W = 80$ GeV and $\bar{g}^2 = 0.576$, which implies for the scalar coupling $\bar{\lambda} = 0.027$. The critical temperature was measured to be $T_c = 93.7$ GeV, and Higgs boson and vector boson masses were determined in the symmetric phase and the Higgs phase. In order to compare these masses with the solutions of the gap equations, one has to determine the critical mass parameter μ_c^2 where the energy densities of the two solutions in the metastability region are equal. In the loop expansion, the energy density is calculated as function of couplings, masses and vacuum expectation value. It is not clear that a good approximation is obtained at one-loop order if the on-shell masses are used. This problem requires further study. However, even without knowing μ_c^2, useful bounds on the masses in the Higgs phase and the symmetric phase can be obtained by considering the edge of the metastability range.

According to eqs. (49) the parameters of the lattice simulations correspond to $\lambda/g^2 = 0.0406$. The corresponding vector boson and Higgs boson masses are shown in fig. (9) as functions of μ^2/g^4. The values at the upper end of the metastability range yield lower and upper bounds for the masses in the Higgs phase and the symmetric phase, respectively. The dimensionless quantities $m/(g^2T_c)$ and M/m obtained from gap equations and lattice simulations are given in table 1.

In the Higgs phase the two approaches yield consistent results. With respect to the symmetric phase, however, there is a severe discrepancy. The lattice simulations indicate a spectrum of states, such that the smallest mass in the symmetric phase is larger than the mass in the Higgs phase – a result which may seem surprising. The opposite is true for the solution of the gap equations. Here the masses in the symmetric phase are much smaller than the ones in the Higgs phase. Since the masses are rather small, the one-loop result may not be reliable quantitatively. However, the qualitative feature that the masses in the symmetric phase are smaller than the ones in the Higgs phase is in accord with the mass predictions deep in the symmetric phase where the loop expansion parameter is small (cf. (47)). It is conceivable that the small masses, predicted by the gap equations, could not yet be identified by the lattice simulations. Simulations on larger lattices should be able to resolve this puzzle.

	Higgs phase		symmetric phase	
	$m/(g^2T_c)$	M/m	$m/(g^2T_c)$	M/m
gap equations	> 0.59	> 0.25	< 0.27	< 0.10
lattice simulations	0.76 - 0.93	0.35	1.2 - 1.7	0.2 - 0.3

Table 1: *Comparison of vector boson and Higgs boson masses obtained from gap equations and lattice Monte Carlo simulations. The lattice data are from ref. [11].*

As discussed in the previous section, the vector boson mass in the symmetric phase is given to good accuracy by the contributions shown in fig. (1a) - (1d). This suggests that in the symmetric phase the dynamics of the vector bosons is rather independent of the Higgs field, and described by the effective lagrangian

$$\begin{aligned} L_{eff} &= \frac{1}{4}\vec{W}_{\mu\nu}\vec{W}_{\mu\nu} + \frac{1}{2\xi}(\partial_\mu \vec{W}_\mu)^2 + \frac{1}{2}m^2\vec{W}_\mu^2 \\ &\quad + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{\xi}{2}m^2\vec{\pi}^2 + \partial_\mu \vec{c}^* \partial_\mu \vec{c} + \xi m^2 \vec{c}^* \vec{c} \\ &\quad + \frac{g}{2}(\vec{W}_\mu \times \vec{\pi}) \cdot \partial_\mu \vec{\pi} + g\partial_\mu \vec{c}^* \cdot (\vec{W}_\mu \times \vec{c}) + \xi \frac{g}{2} m \vec{c}^* \cdot (\vec{\pi} \times \vec{c}). \end{aligned} \tag{50}$$

This lagrangian can be obtained from the gauged non-linear σ-model by applying the resummation procedure described in sect. 3 and by neglecting terms $\mathcal{O}(\pi^3, \pi^4, \pi^3 W, \ldots)$ [27]. Since the lagrangian (50) yields the correct vector boson mass in the symmetric phase, it may also be an appropriate starting point for evaluating the rate of baryon- and lepton-number changing processes. Hence, our results support previous work on $B + L$-violation in the symmetric phase [30].

In the previous section we found that the first-order transition changes to a crossover at a critical value of the scalar coupling, $\lambda_c/g^2 \approx 0.053$. For the quoted parameters from ref. [11] this correponds to a critical Higgs mass $m_H^c \sim 55$ GeV. This value strongly depends on the size of the vector boson mass in the symmetric phase. If, for instance, two-loop corrections would reduce the one-loop value, $m_{SM} = 0.28 g^2 T$, by 50% to $\bar{m}_{SM} = 0.14 g^2 T$, the critical Higgs mass would increase to $\bar{m}_H^c \sim 100$ GeV. Hence, from our approach we can only conclude that the change from a first-order transition to a crossover should take place for Higgs masses below 100 GeV. This is compatible with results from lattice calculations for the SU(2) Higgs model in four dimensions at high temperature [8], as well as for the reduced three-dimensional theory [9], where evidence for a first-order transition at $m_H \sim m_W$ is claimed.

Many features of the electroweak transition, as described in this section, are similar to effects of a "magnetic mass", which have been discussed previously [6],[24]. In fact, in ref. [6] a change from a first-order to a second-order transition was predicted at $\tilde{m}_H^c = 85/\sqrt{\gamma}$ GeV for a magnetic mass $m_{magn} = \gamma g^2 T/(3\pi)$. For $m_{magn} = 0.28 g^2 T$ the corresponding critical Higgs mass is $\tilde{m}_H^c = 52$ GeV. The weakness of the electroweak phase transition for Higgs masses $m_H = \mathcal{O}(m_W)$ again raises the question of how the transition actually takes place and how important subcritical bubbles [31] are as compared to the nucleation and growth of critical bubbles.

6 Summary and conclusions

In the previous sections we have derived gauge independent gap equations for vector boson and Higgs boson masses, and we have found different solutions of these equations whose properties depend on the dimensionless parameters of the theory, λ/g^2 and μ^2/g^4. The presence of a Higgs doublet of scalar fields turned out to be crucial to obtain gauge independent equations. For the pure SU(2) gauge theory no gauge independent equation could be found.

Two kinds of solutions of the gap equations were found, with large and small vacuum expectation values v/g of the Higgs field, which could be associated with the Higgs phase and the symmetric phase of the theory, respectively. In the Higgs phase the obtained masses for vector boson and Higgs boson are in agreement with ordinary perturbation theory. In the symmetric phase, a non-perturbative vector boson mass is generated whose value is rather independent of μ^2/g^4 and λ/g^2. Except for a small range of μ^2/g^4 close to the phase transition point, Higgs boson and vector boson masses are large enough to yield a sufficiently small loop expansion parameter. Hence, we expect that the properties of the symmetric phase obtained at one-loop order will not be qualitatively changed by higher order corrections. The obtained masses in the symmetric phase appear to be at variance with recent measurements of lattice simulations, a puzzle which can be resolved by considering larger lattices.

An intriguing aspect of the picture emerging from the gap equations is the difference between the scalar couplings in the symmetric phase and the Higgs phase, respectively. The resummed scalar coupling λ_R in the symmetric phase near $\mu^2 = 0$ is very small, which is related to the large ratio of vector boson and Higgs boson masses. A similar behaviour of the scalar coupling is suggested by the renormalization group equations in $4 - \epsilon$ dimensions.

The nature of the transition between Higgs phase and symmetric phase depends on the value of the scalar coupling λ. Below a critical coupling λ_c we find a first-order transition, which changes to a crossover at $\lambda = \lambda_c$. The precise value of λ_c is strongly correlated with the size of the vector boson mass in the symmetric phase. Based on the perturbative matching between the three-dimensional theory and the high-temperature expansion of the four-dimensional theory the critical Higgs mass m_H^c, corresponding to the critical coupling λ_c, is estimated to be smaller than 100 GeV.

The guiding principle in the design of a resummation procedure for masses and vertices leading to the gap equations has been the preservation of gauge invariance.

The result has a very simple interpretation. The lagrangian with resummed masses and couplings is nothing but the ordinary Higgs model with modified parameters, shifted around the corresponding "classical" minimum. Hence, the "symmetric" phase is again a Higgs phase, just with different parameters. These parameters are determined self-consistently at one-loop order, like the vacuum expectation value of the Higgs field in the case of Coleman-Weinberg type symmetry breaking by radiative corrections. Hence, no "symmetry restoration" takes place at high temperatures, the vacuum expectation value of the Higgs field is always non-zero. This result is in accord with the known property of the Higgs model to have only one phase.

The solutions of the gap equations provide a clear physical picture of the symmetric phase and the transition between Higgs phase and symmetric phase. More work is needed to prove the stability of the one-loop results with respect to higher order corrections and to understand the connection between the small masses obtained from the gap equations and the mass spectrum measured with lattice simulations. The results of these further investigations will be of crucial importance for achieving a full understanding of the electroweak phase transition and its implications for the cosmological baryon asymmetry.

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Figure captions

Fig.1 One-loop contributions to the vector boson propagator.

Fig.2 One-loop contributions to the Higgs boson propagator.

Fig.3 The function $f(z)$ entering the gap equation for the vector boson mass, $\xi = 0$.

Fig.4 The function $F(z)$ entering the gap equation for the Higgs boson mass, $\xi = 0$.

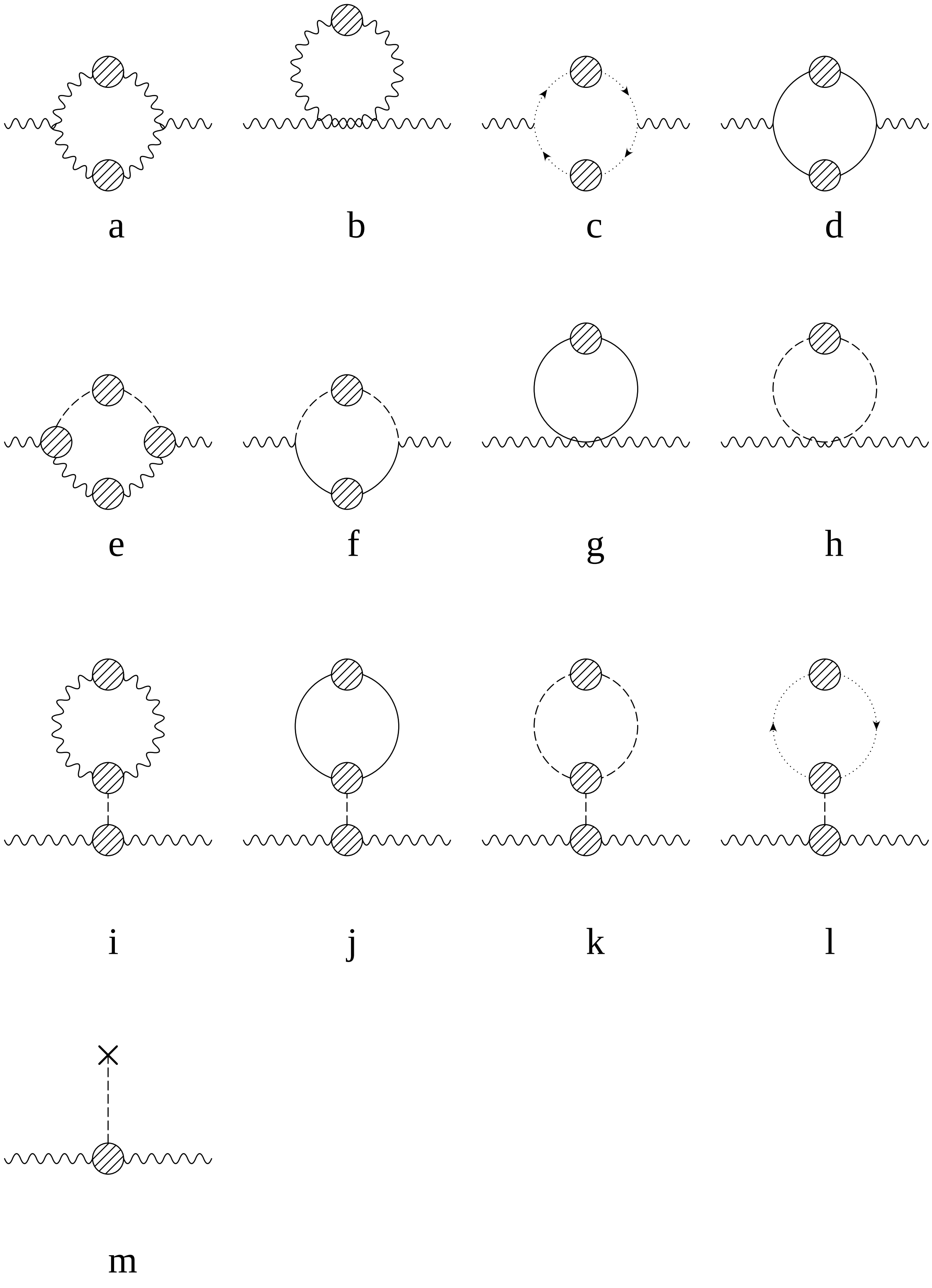
Fig.5 The vacuum expectation value v/g as function of the mass parameter μ^2/g^4. Full line: solution of gap equations, dash-dotted line: perturbation theory. $\lambda/g^2 = 1/128$.

Fig.6 Vector boson and Higgs boson masses for $\lambda/g^2 = 1/128$. Gap equations: m (full line), M (dashed line); perturbation theory: m (dash-dotted line), M (dotted line).

Fig.7 The vacuum expectation value v/g as function of the mass parameter μ^2/g^4. Full line: solution of gap equations, dash-dotted line: perturbation theory. $\lambda/g^2 = 1/8$.

Fig.8 Vector boson and Higgs boson masses for $\lambda/g^2 = 1/8$. Gap equations: m (full line), M (dashed line); perturbation theory: m (dash-dotted line), M (dotted line).

Fig.9 Vector boson mass m (full line) and Higgs boson mass M (dashed line) for $\lambda/g^2 = 0.0406$.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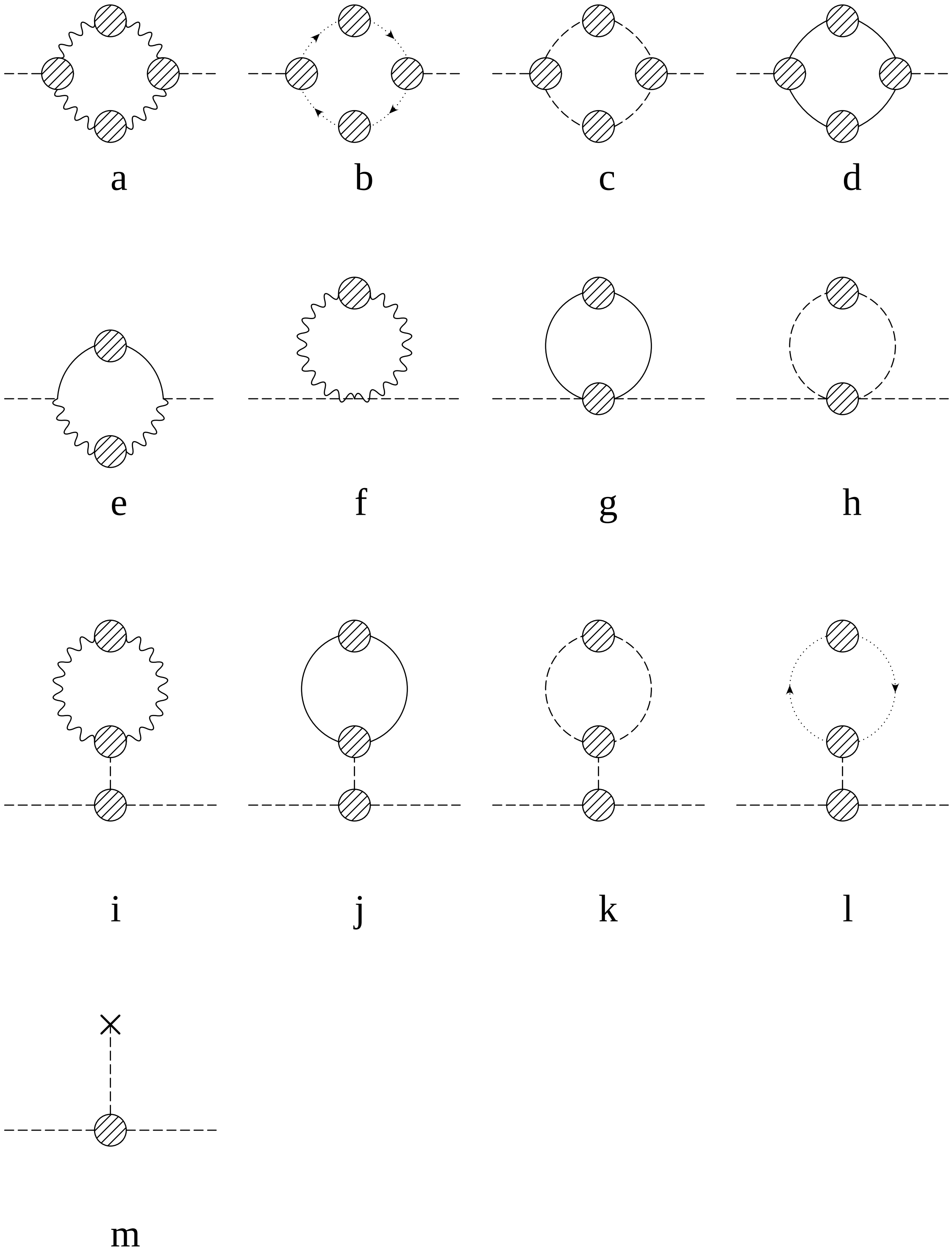
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Fig.3

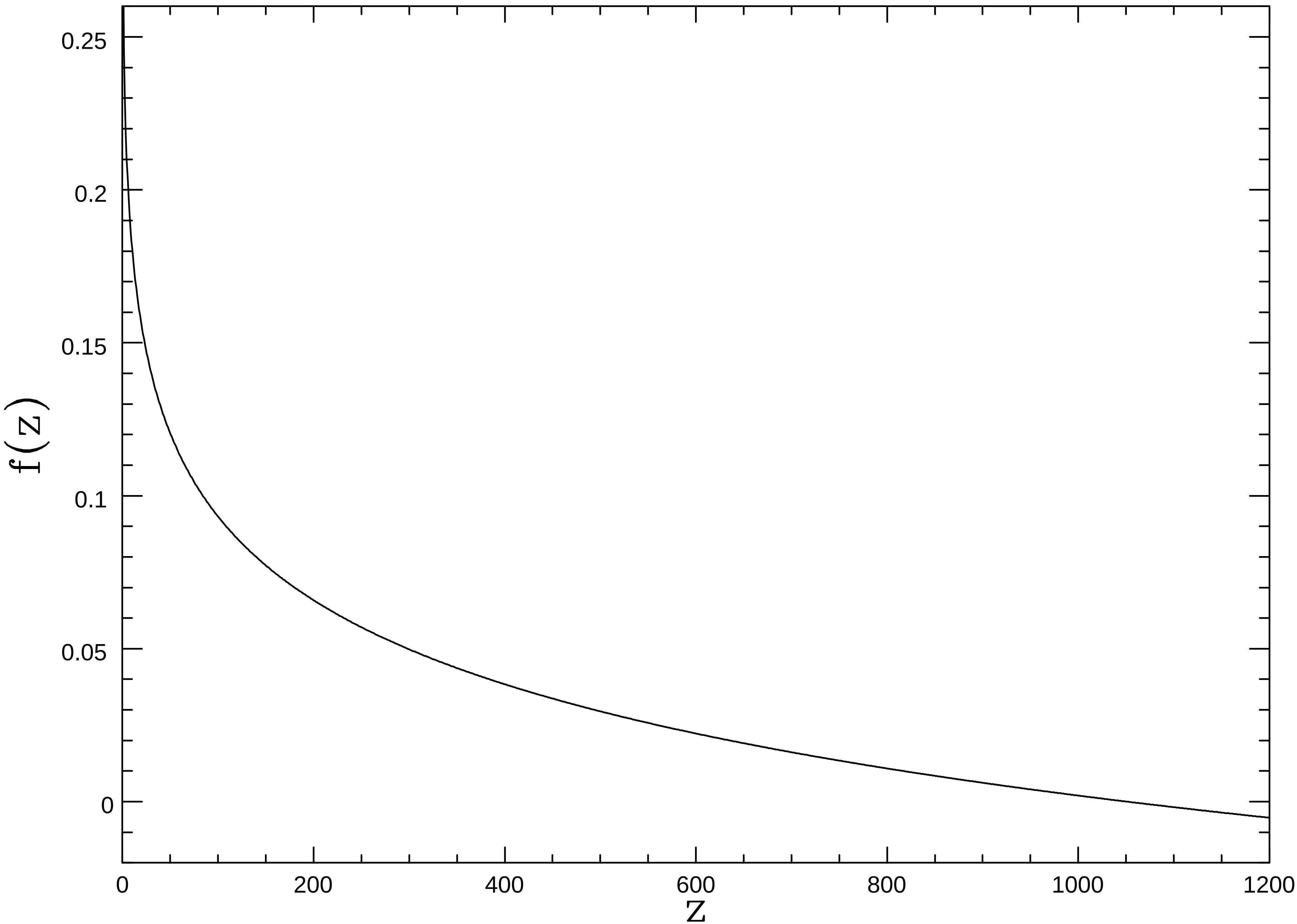

Fig.4

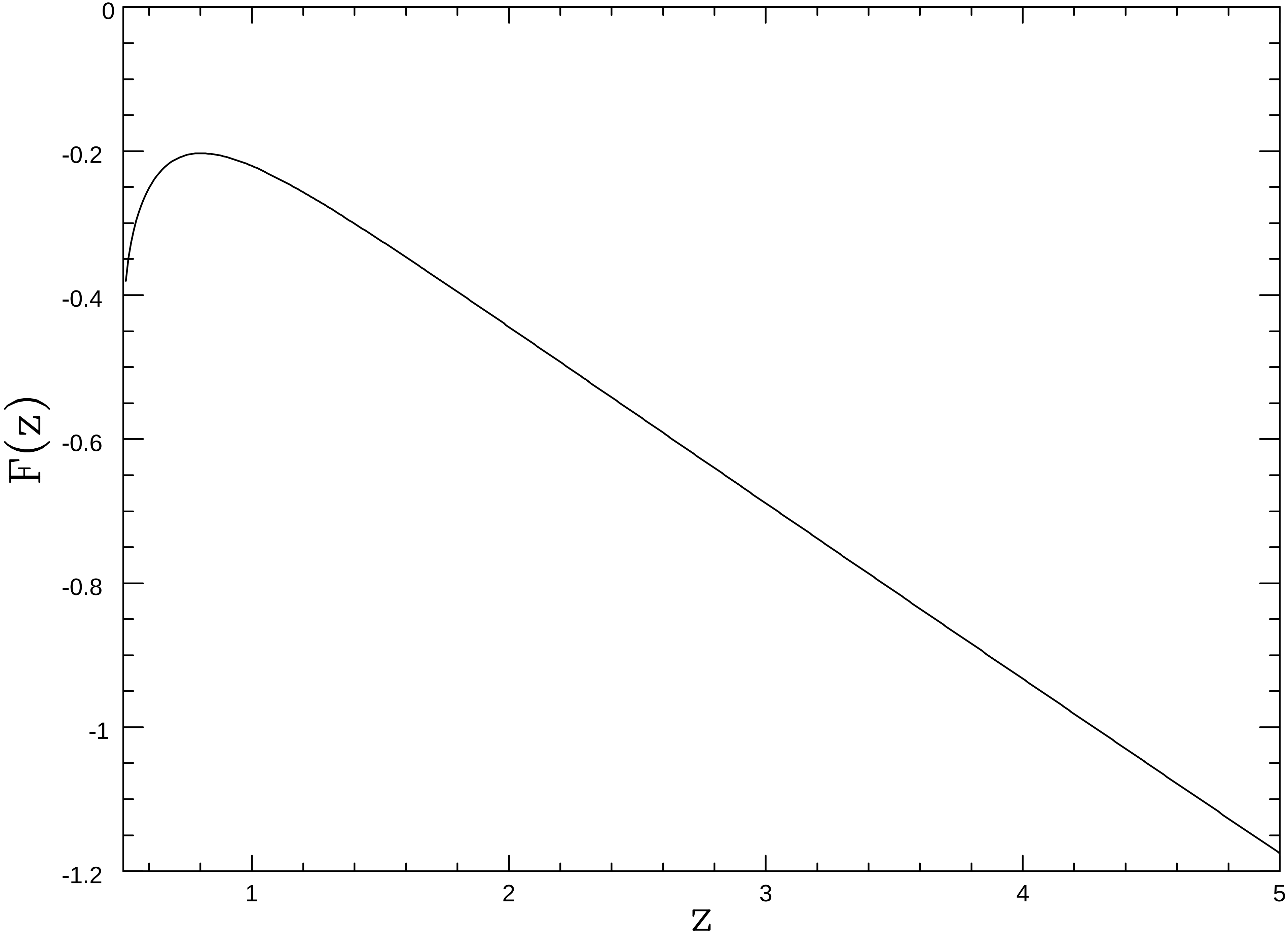

Fig.5

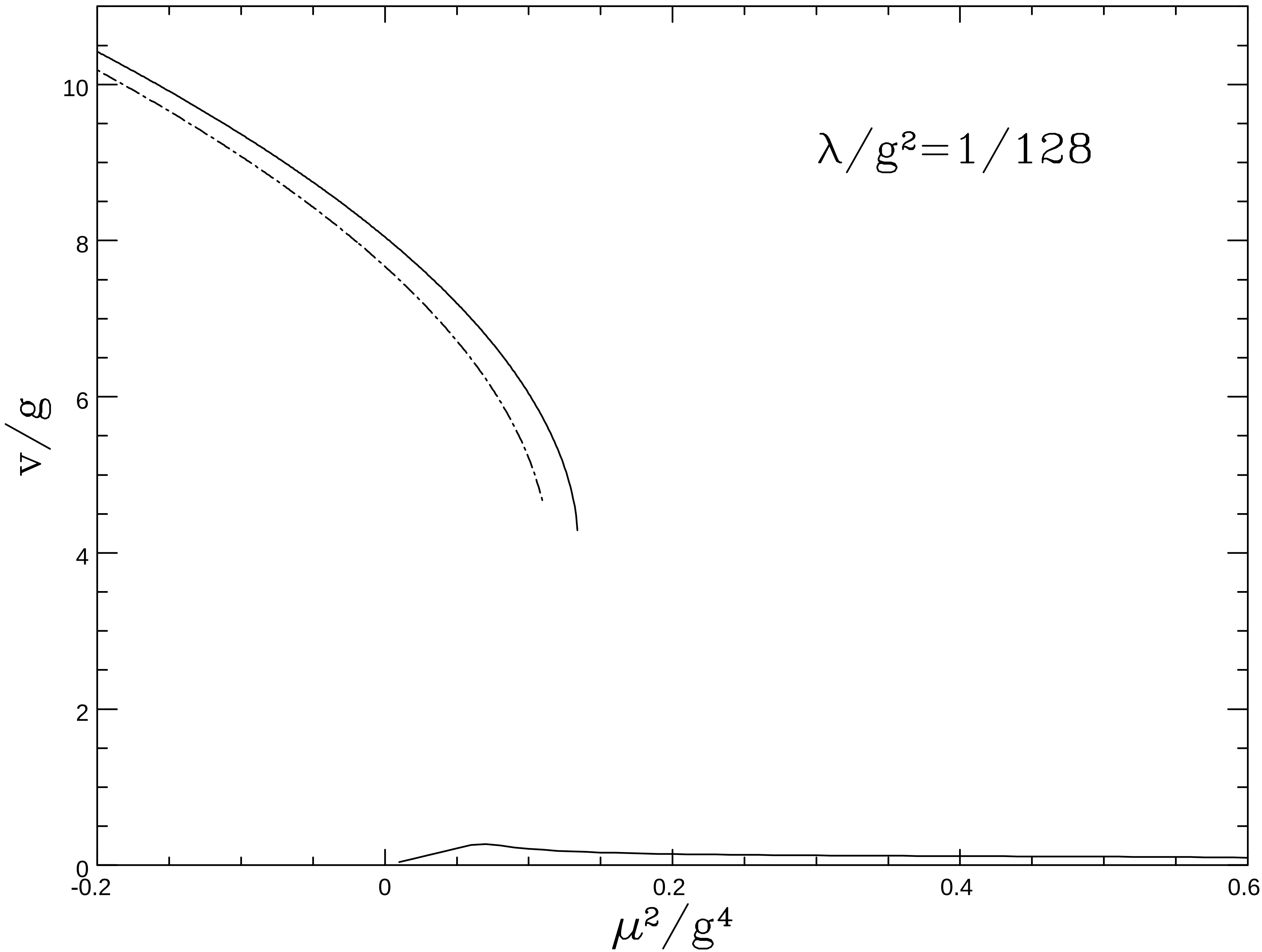

Fig.6

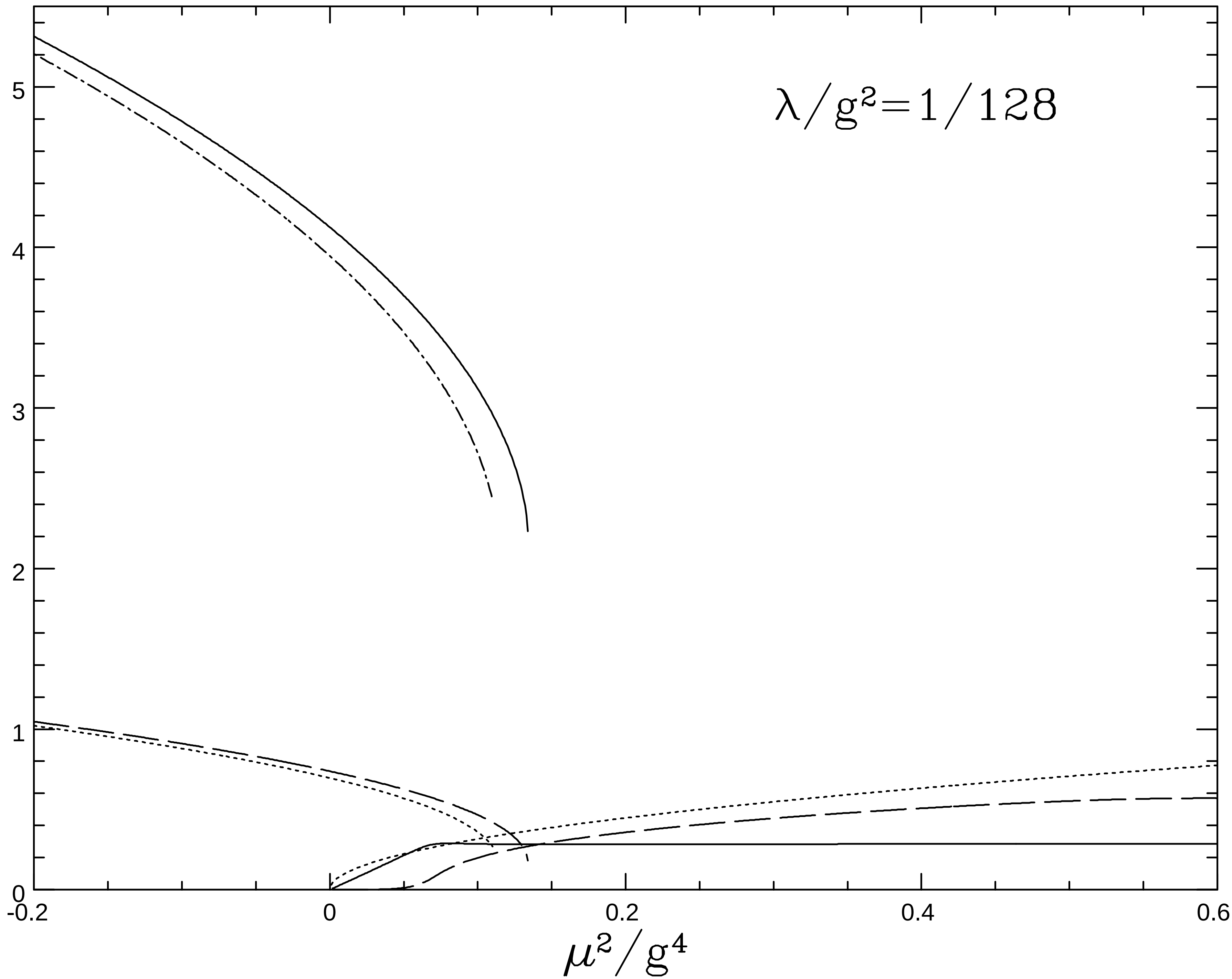

Fig.7

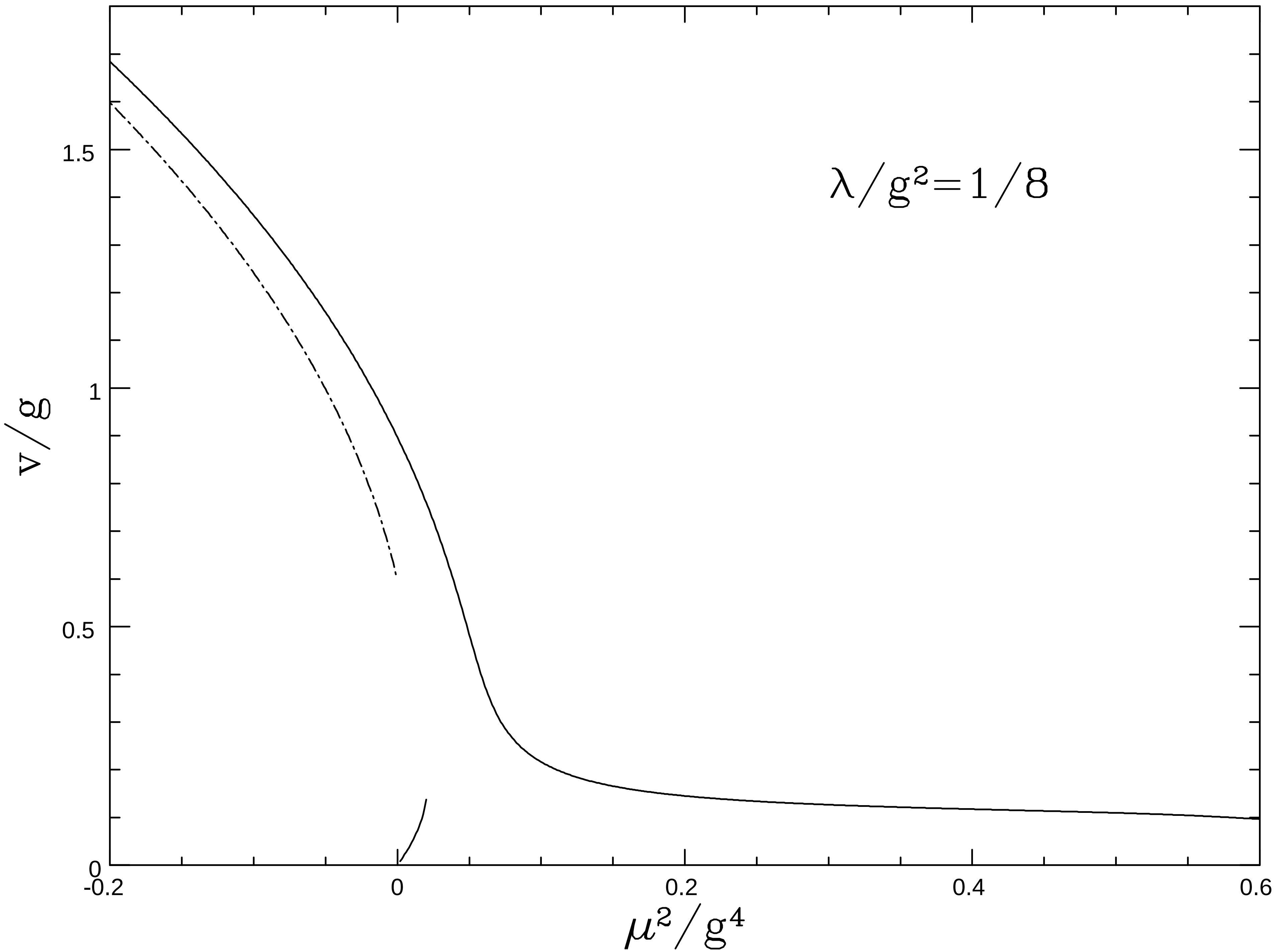

Fig.8

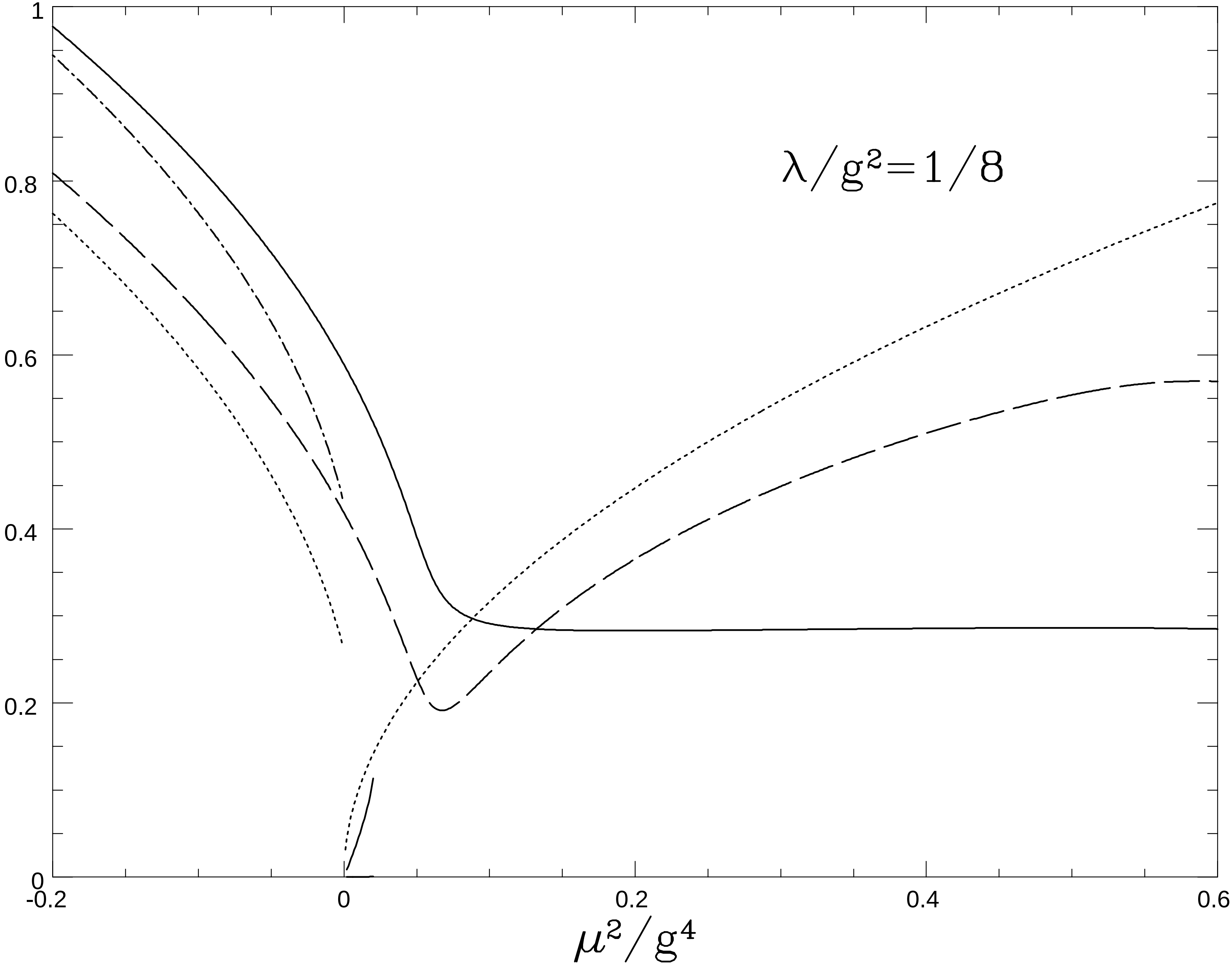

Fig.9