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Small x Contributions to the Structure Function $F_L(x, Q^2)$

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Abstract

The gluon contributions to $F_L(x, Q^2)$ in $\mathcal{O}(\alpha_s)$ are calculated taking into account the transverse momentum of the initial state parton. In comparison with collinear factorization $F_L(x, Q^2)$, is not affected at large x but takes smaller values in the small x range. The onset of the $k_\perp$ effect is a function of Q^2.

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Small x Contributions to the Structure Function $F_L(x,Q^2)$

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1. Introduction

In the small x range a novel behaviour of nucleon structure functions is expected. Among possible dynamical effects are those due to non strong $k_\perp$ ordering [1] and screening [2]. Their description requires a generalization the factorization of the hadronic matrix elements. The $k_\perp$ dependence of the parton distributions can no longer be neglected in the hard scattering cross sections and $K^2 = -k_\perp^2$ dependent parton distributions must be used [3], i.e.

$$|\mathcal{M}|^2 \sim \int^{\mu^2} dK^2 \hat{\sigma}(x,K^2,\mu^2) \otimes \frac{\partial xG(x,K^2)}{\partial K^2} \qquad (1)$$

instead of the collinear relation

$$|\mathcal{M}|^2 \sim \hat{\sigma}(x,K^2=0,\mu^2) \otimes xG(x,\mu^2). \qquad (2)$$

The present paper aims on finding a consistent solution of the above problem without any approximations of the x and $k_\perp$ behaviour of the coefficient functions. In this way former investigations [4,5] are extended. We aim on a *general formulation* of the gluon contribution to structure functions which offers the possibility to unfold the $k_\perp$ dependence of the gluon distribution at small x. Theoretical predictions of its small x behaviour can thus be *directly* compared with the data and test the $k_\perp$ dependence.

2. $k_\perp$ Factorization

The K^2 integral in (1) extends to $K^2 = 0$. However, a perturbative definition of a gluon distribution is only possible at suitably large virtualities. Therefore, we use [6]

$$\begin{aligned} F_L^g(x,Q^2) &= \int_x^1 \frac{dz}{z} f_L^{g,0}(z) \frac{x}{z} G(z,Q_0^2) \\ &+ \int_x^1 \frac{dz}{z} \int_{Q_0^2}^{K^2_{max}} dK^2 f_L^g(z,\frac{K^2}{Q^2}) \frac{x}{z} \frac{\partial G(x/z,K^2)}{\partial K^2} \\ &\times \theta(K^2_{max} - Q_0^2) \end{aligned} \qquad (3)$$

where $K^2_{max} = Q^2(1-z)/z$. We introduced a scale Q_0 for which we demand that $Q_0^2 << Q^2$. (3) is equivalent to (1) up to terms of $\mathcal{O}((Q_0^2/Q^2)^n)$. Note that eq. (3) contains the gluon distribution $G(x,K^2)$ only at virtualities in the perturbative range.

3. $F_L(x,Q^2)$ in $O(\alpha_s)$

For the gluonic contribution to $F_L(x,Q^2)$ eq. (3) the coefficient function takes the following form

$$\begin{aligned} f_L^g(z,\frac{K^2}{Q^2}) &= \frac{2}{\pi}\alpha_s(Q^2) \sum_{q=1}^{N_f/2} \left(e_{q_u}^2 + e_{q_d}^2\right) \\ &\times \left\{ \frac{1}{64z}\left(\frac{Q^2}{K^2}\right)^2 G_{1L}^{(0,4)}(\omega,\beta) \right. \\ &+ z\frac{1}{16}\frac{Q^2}{K^2}\left[G_{2L}^{(2,4)}(\omega,\beta)\right. \\ &+ \left.\left. G_{3L}^{(1,5)}(\omega,\beta)\log\left|\frac{1-\omega}{1+\omega}\right|\right]\right\} \end{aligned} \qquad (4)$$

with $\omega = \sqrt{1-4K^2z/Q^2}$, $\zeta = 4K^2z/Q^2$, $\cos\beta = (1-\zeta/2)/\sqrt{1-z\zeta}$, and $G_{iL}^{(a,b)}(\omega,\beta) = \sum_{k=a}^b g_{ki}^{(L)}(\beta)/\omega^k$. The coefficients g_{ki} are:

$$g_{01}^{(L)}(\beta) = \frac{5}{2} + 3\cos^2\beta - \frac{3}{2}\cos^4\beta$$

$$\begin{aligned}
g_{11}^{(L)}(\beta) &= 4\cos\beta - 12\cos^3\beta \\
g_{21}^{(L)}(\beta) &= 3 - 18\cos^2\beta + 15\cos^4\beta \\
g_{31}^{(L)}(\beta) &= -12\cos\beta + 20\cos^3\beta \\
g_{41}^{(L)}(\beta) &= -\frac{3}{2} + 15\cos^2\beta - \frac{35}{2}\cos^4\beta \\
g_{22}^{(L)}(\beta) &= 4 - 12\cos^2\beta \\
g_{23}^{(L)}(\beta) &= -24\cos\beta + 40\cos^3\beta \\
g_{24}^{(L)}(\beta) &= -3 + 30\cos^2\beta - 35\cos^4\beta \\
g_{31}^{(L)}(\beta) &= \frac{3}{4} + \frac{1}{2}\cos^2\beta + \frac{3}{4}\cos^4\beta \\
g_{32}^{(L)}(\beta) &= 2\cos\beta - 6\cos^3\beta \\
g_{33}^{(L)}(\beta) &= \frac{7}{2} - 15\cos^2\beta + \frac{15}{2}\cos^4\beta \\
g_{34}^{(L)}(\beta) &= -18\cos\beta + 30\cos^3\beta \\
g_{35}^{(L)}(\beta) &= -\frac{9}{4} + \frac{45}{2}\cos^2\beta - \frac{105}{4}\cos^4\beta \qquad (5)
\end{aligned}$$

In the limit $K^2 \to 0$ one obtains the well–known result [9]

$$f_L^{g,0}(z) = \frac{2}{\pi}\alpha_s(Q^2)\sum_{q=1}^{N_f/2}\left(e_{q_u}^2 + e_{q_d}^2\right) z^2(1-z). \qquad (6)$$

4. Numerical Results

Figure 1 shows the logarithmic derivative of the gluon distribution $dxG(x,Q^2)/d\log Q^2$ for different sets of parton parametrizations in the $\overline{\mathrm{MS}}$ scheme. The most recent results, CTEQ2M and MRSA, were determined using the data measured at HERA, and do practically coincide, while earlier ones show some variation at small x. We will refer to the CTEQ2 parametrization [8] as an input in the following.

In figure 2 the gluonic contributions to $F_L(x,Q^2)$ using either eq. (6) or (3) are compared. At large x coinciding results are obtained, but at small x the collinear approach yields larger values for F_L. Setting $\mu^2 = Q^2(1-z)/z$, the kinematical upper limit of the K^2 integral (3), instead of $\mu^2 = Q^2$, in (6) leads to a lowering of $F_L(x,Q^2)$ already. Figure 2 shows that with rising Q^2, the effect due to finite $k_\perp$ emerges at smaller values of x. As expected, the onset of small x effects is Q^2 dependent.

The separation scale Q_0^2 required in (3) affects $F_L(x,Q^2)$ very weakly as long as $Q^2 \gg Q_0^2$, which we assume. This is illustrated in figure 3. The effect of this choice of scale is comparable to that of Q'^2_0, the starting point of QCD evolution.

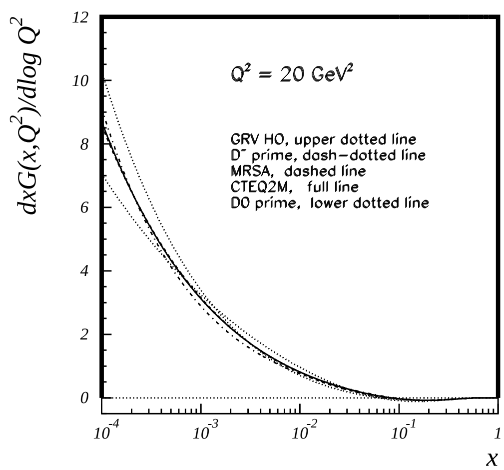

Fig. 1 Logarithmic slope of the gluon momentum distribution vs x for different parton parametrizations [7].

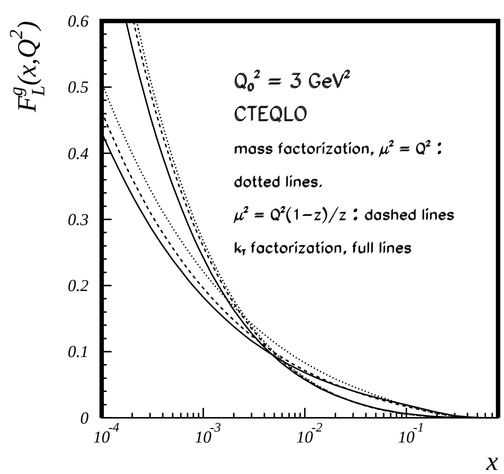

Fig. 2 Comparison of the gluonic contributions to $F_L(x,Q^2)$ in the collinear case and $k_\perp$ factorization. The steeper lines are at $Q^2 = 10^4\,\mathrm{GeV}^2$, the others at $Q^2 = 20\,\mathrm{GeV}^2$.

In figure 4, the $\mathcal{O}(\alpha_s)$ result using $k_\perp$ factorization is compared with results of a $\mathcal{O}(\alpha_s^2)$ calculation in the collinear approach [10]. The gluon contribution to F_L in the collinear approach is diminished by about 10% by the $\mathcal{O}(\alpha_s^2)$ term for $x \sim 10^{-4}$ and $Q^2 \sim \mathcal{O}(20\,\mathrm{GeV}^2)$. The $\mathcal{O}(\alpha_s)$ value of F_L using $k_\perp$ factorization is somewhat smaller than the $\mathcal{O}(\alpha_s^2)$ value in the collinear approach. Note that the results are nearly equal in the range $x \sim 10^{-4}$. The quark contribution to F_L in $\mathcal{O}(\alpha_s^2)$ [10] amounts to $\sim 10\%$ at $x \sim 10^{-4}$.

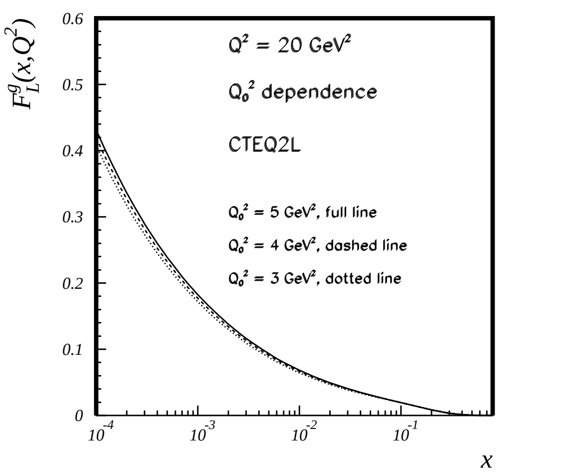

Fig. 3 Dependence of $F_L^g(x,Q^2)$ on the choice of the separation scale Q_0^2.

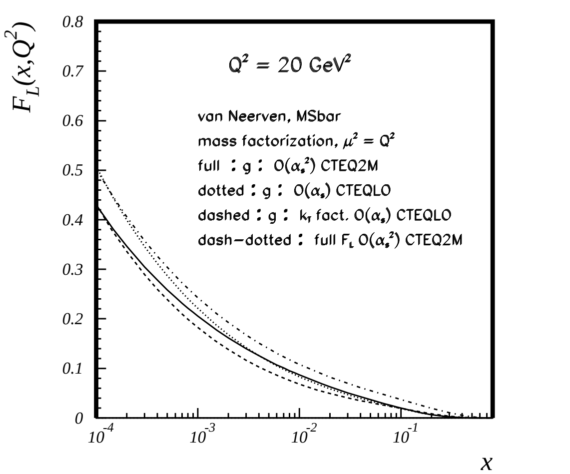

Fig. 4 Comparison of the $\mathcal{O}(\alpha_s^2)$ calculation [10] with the result obtained in the $k_\perp$ factorization scheme.

5. Conclusions

A representation of $k_\perp$ factorization which is consistent with perturbative QCD has been given. The gluon contribution to the structure function $F_L(x,Q^2)$ was calculated using $k_\perp$ factorization *without* using any approximations of the Mellin convolution or the x dependence of the coefficient functions, unlike some earlier investigations. The contributions to the structure functions obtained are *positive* in the whole x range.

The derived coefficient functions approach those found using mass factorization in the limit $K^2 \to 0$. The numerical value obtained in $k_\perp$ factorization for suitably 'large' values of x approach the result which ignores the $k_\perp$ dependence of the coefficient functions. This has been an expectation in the parton model [3]. There is *no fixed onset* (e.g. $x \sim 10^{-2}$ [5]) of the small x effects observed. Deviations from the collinear result become smaller with rising Q^2 at constant x. The effect of the separation scale Q_0 is found to be subleading.

The $k_\perp$ dependence of the coefficient function and gluon distribution results into *smaller values* of F_L in $\mathcal{O}(\alpha_s)$ in the small x range. Quite similar values are obtained for F_L in $\mathcal{O}(\alpha_s^2)$ [10] using mass factorization.

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REFERENCES

1. E. Kuraev et al., Sov. Phys. JETP, **44** (1976) 443; Y. Balitzkii and L. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 882.
2. L. Gribov et al., Nucl. Phys. **B188** (1981) 555.
3. S. Drell and T. Yan, Ann. Phys. **66** (1971) 578.
4. E. Levin and M. Ryskin, Sov. J. Nucl. Phys. **53** (1991) 657.
5. A.J. Askew et al., Phys. Rev. **D47** (1993) 3775.
6. J. Blümlein, J. Phys. **G19** (1993) 1623; J. Blümlein, DESY report to appear.
7. H. Plothow–Besch, CERN Progr. Lib. W5051.
8. J. Botts et al., MSU–HEP 93/24 (1993).
9. A. Zee et al. Phys. Rev. **D10** (1974) 2881.
10. E.B. Zijlstra and W.L. van Neerven, Nucl. Phys. **B383** (1992) 525.