

Phase structure and chiral limit of compact lattice QED with Wilson fermions ^{*} [†]

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We study the phase structure and chiral limit of $4d$ compact lattice QED with Wilson fermions (both dynamical and quenched). We use the standard Wilson action (WA) and also the modified action (MA) with some lattice artifacts suppressed. We show that lattice artifacts influence the distributions of eigenvalues λ_i of the fermionic matrix especially for small values of λ_i . Our main conclusion is that the chiral limit of compact QED can be efficiently located using different techniques.

1. Introduction

The lattice formulation of QED is not unique. One has to decide on a physical ground which version of QED is realised in nature if different lattice versions of QED do not belong to the same universality class. The *old* physics, i.e., known from experiment, has to be reproduced.

When we consider QED as arising from a subgroup of some non-abelian (e.g., grand unified) gauge theory we are necessarily dealing with the compact version. Our choice in this work is a *compact* formulation of QED.

In the theory with Wilson's fermions [1] chiral symmetry is broken explicitly, and, presumably, can be only restored by fine-tuning the parameters in the continuum limit if we are dealing with a meaningful lattice discretization. What one can expect at nonzero spacing is that at some $\kappa_c \equiv \kappa_c(\beta)$ a so-called *partial* symmetry restoration takes place [2,3] when the Wilson mass term and ordinary mass term cancel at zero momentum in certain vertex functions. If so one can approach continuum limit and chiral symmetry

restoration along the line $\kappa_c(\beta)$. Another question is if in the continuum limit of our theory the chiral symmetry is realised explicitly or is spontaneously broken.

2. Actions and order parameters

The modified lattice action $S_{MA}(U, \bar{\psi}, \psi)$ for $4d$ $U(1)$ gauge theory (QED) is

$$S_{MA} = \beta \cdot S_G(U) + S_F(U, \bar{\psi}, \psi) + \delta S_G(U). \quad (1)$$

In eq.(1) $S_G(U)$ is the standard plaquette (Wilson) action for the pure gauge $U(1)$ theory, and the additional term δS_G suppresses lattice artifacts (i.e., monopoles and negative plaquettes). The fermionic part of the action $S_F(U, \bar{\psi}, \psi)$ is

$$S_F = \sum_{f=1}^{N_f} \sum_{x,y} \sum_{s,s'=1}^4 \bar{\psi}_x^{f,s} \mathcal{M}_{xy}^{ss'} \psi_y^{f,s'} \equiv \bar{\psi} \mathcal{M} \psi, \quad (2)$$

$$\mathcal{M} \equiv \hat{1} + \kappa \cdot \tilde{\mathcal{M}}(U),$$

where \mathcal{M} is Wilson's fermionic matrix, N_f is the number of flavours and κ is the hopping parameter. The first two terms in r.h.s of eq.(1) make up the standard Wilson action S_{WA} .

In our work we used both S_{WA} and S_{MA} with $N_f = 2$ for dynamical fermions. Apart from $\langle \bar{\psi} \psi \rangle$ and $\langle \bar{\psi} \gamma_5 \psi \rangle$ we calculated the *pion norm* $\langle \Pi \rangle$ [4]

$$\Pi(U) = \frac{1}{L^4} \cdot \sum_x \text{Tr} \left(\mathcal{M}_{0x}^{-1} \gamma_5 \mathcal{M}_{x0}^{-1} \gamma_5 \right), \quad (3)$$

^{*}TALK GIVEN AT THE LATTICE '93 INTERNATIONAL SYMPOSIUM LATTICE FIELD THEORY, DALLAS, USA, OCTOBER 12-16, 1993

[†]Work supported by the Deutsche Forschungsgemeinschaft under research grant Mu 932/1-1

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where L is the lattice size. Its advantage is that it appears to be a very sensitive quantity in the 'critical' region. Introducing eigenvectors $g_n(s, x)$ of $\gamma_5 \mathcal{M}$ with eigenvalues μ_n : $\gamma_5 \mathcal{M} g_n = \mu_n \cdot g_n$, one can easily obtain a spectral representation

$$\Pi = \frac{1}{L^4} \sum_n \frac{1}{\mu_n^2} \sum_s |g_n(s, 0)|^2. \quad (4)$$

Following the common practice we identify here the chiral transition with the appearance of zero or near to zero eigenvalues of the fermionic matrix \mathcal{M} . Evidently, an eigenstate of \mathcal{M} with eigenvalue zero is also an eigenstate of $\gamma_5 \mathcal{M}$. So, the presence of configurations which belong to zero eigenvalues of \mathcal{M} gives rise to poles in Π .

3. Conjugate gradient (cg-) method

To locate $\kappa_c(\beta)$ one can use the convergence rate of the cg-method. This is the iterative method of solving a system of linear equations $D \cdot X = \varphi$, where D is a hermitian $n \times n$ matrix and φ is an input vector ($D = \mathcal{M}^\dagger \mathcal{M}$ in our case). The convergence of the cg-method should be controlled by the condition number $\xi \equiv \lambda_{max}/\lambda_{min}$, where λ_{max} and λ_{min} are maximal and minimal eigenvalues of D . Close to κ_c the minimal eigenvalue of $\mathcal{M}^\dagger \mathcal{M}$ is small and is supposed to be $\sim (1 - \kappa/\kappa_c)^2$.

We observed that at large enough number of iteration steps N_{cg} ($N_{cg} > N_0$) the average residue $\langle R \rangle \equiv \langle R \rangle(N_{cg})$ behaves as

$$\langle R \rangle = C \cdot \exp(-\alpha \cdot N_{cg}), \quad \alpha = \ln \frac{\sqrt{\xi} + 1}{\sqrt{\xi} - 1} \quad (5)$$

independently of the distribution of eigenvalues λ_i provided n is large enough. To check it we generated D with different (uniform, gaussian, double-peaked) distributions of eigenvalues and given λ_{min} and λ_{max} . The components of the initial vector X_0 and of φ were chosen every time randomly with gaussian distributions.

It follows from the above that, for the inversion of $\mathcal{M}^\dagger \mathcal{M}$, the $\langle N_{cg}^{-1} \rangle$ required for convergence to some small but fixed R will behave as $\langle N_{cg}^{-1} \rangle \sim 1 - \kappa/\kappa_c$. In Fig.1 we show the dependence of $\langle N_{cg}^{-1} \rangle$ on κ at $\beta = 0.8$ for quenched MA (qMA) for different L .

Figure 1. $\langle N_{cg}^{-1} \rangle(\kappa)$ for qMA for different L .

At $\kappa \sim \kappa_c$ the data fit nicely to straight lines giving reasonable estimation of κ_c . The volume dependence becomes rather weak for larger L . A similar picture was obtained for the case of dynamical fermions.

4. Phase diagram and chiral limit

For WA with dynamical fermions (dWA) at $\beta < \beta_0 \sim 1.0$ thermal cycles with respect to κ or β have a typical hysteresis behaviour for $\bar{\psi}\psi$ and plaquette \square . Time histories (TH's) of $\bar{\psi}\psi$ and plaquette \square for different starts show the existence of metastable states. So, we conclude that for dWA there is a 1st order phase transition (PT) line from $(\beta; \kappa) \simeq (1.; 0.)$ to $(\beta; \kappa) = (0.; 0.25)$ which is in agreement with [5].

After the suppression of lattice artifacts (i.e., for dMA) this line disappears (see also [6]).

At $\beta > \beta_0$ and $\kappa < \kappa_c(\beta)$ the system with dWA is in the Coulomb phase. The photon correlator $\Gamma(\tau)$ is well consistent with that corresponding to a zero photon mass. For $\beta > \beta_0$ and $\kappa > \kappa_c(\beta)$ the correlator $\Gamma(\tau)$ shows a tachyonic-type behaviour ($m_\gamma^2 < 0$). Thus, we can conclude that there is a higher- κ phase (or phases) differing from the Coulomb phase.

For quenched WA (qWA) at $\beta < \beta_0$ TH's of Π (as well as of $\bar{\psi}\psi$ and $\bar{\psi}\gamma_5\psi$) show very sharp peaks at $\kappa \sim \kappa_c(\beta)$ which means the appearance

of small eigenvalues λ_i . It is worth noting that for WA at $\beta < \beta_0$ those peaks do *not* disappear at $\kappa > \kappa_c$ but instead become even more strong.

For WA at $\beta > \beta_0$ and for MA at any (positive) β the dependence of TH's on κ changes drastically. We don't find peaks of comparable amplitude ($\sim 10^4$) but, nevertheless, in some 'critical' region $\kappa \sim \kappa_c$ TH's of Π become much more rough than at smaller or *larger* values of κ . As far as for every configuration Π is the arithmetic average of $4L^4$ terms corresponding to $4L^4$ different eigenvalues one can conclude that rather small λ_i appeared.

The (renormalized) variance of the pion norm $\sigma^2(\Pi) \equiv L^4 \cdot \text{Var}(\Pi)$ appears to be a suitable 'order parameter'.

Figure 2. $\sigma^2(\Pi)$ for qMA at $\beta = 0.8$.

Fig.2 shows $\sigma^2(\Pi)$ for qMA at $\beta = 0.8$. There is a clear signal at some κ_c . With increasing L this peak becomes even more sharp.

For dynamical fermions the signal becomes less sharp (at least at small L) because the fermionic determinant suppresses small eigenvalues. However also in this case $\sigma^2(\Pi)$ develops a pronounced maximum thus allowing to locate κ_c .

In Fig.3 we show the phase diagrams for both dWA and dMA. At the moment we have no clear interpretation of the phase in the upper left corner in Fig.3a.

Figure 3. Phase diagrams for WA(a) and MA(b).

5. Conclusions

For the standard compact Wilson action we observe a presumably 1st order PT which disappears after suppressing lattice artifacts.

These lattice artifacts influence strongly the distribution of eigenvalues λ_i of the fermionic matrix \mathcal{M} . This influence is especially pronounced for the near-to-zero values of λ_i .

After suppression of artifacts a chiral transition (i.e., appearance of near-to-zero eigenvalues of \mathcal{M}) is left on a 'horizontal' line $\kappa = \kappa_c(\beta)$.

As a preliminary conclusion, we have *no* sign for a qualitative change of the behaviour along the line $\kappa = \kappa_c(\beta)$.

REFERENCES

1. K. Wilson, Phys. Rev. **D10** (1974) 2445; in New phenomena in subnuclear physics, ed. A. Zichichi (Plenum, New York, 1977)
2. N. Kawamoto, Nucl. Phys. **B190** (1981) 617.
3. N. Kawamoto and J. Smit, Nucl. Phys. **B192** (1981) 100.
4. K. Bitar, A. D. Kennedy and P. Rossy, Phys. Lett. **B234** (1990) 333.
5. C. Hege and A. Nakamura, Nucl. Phys. **B** (Proc. Suppl.) 9 (1989) 114.
6. V.G. Bornyakov, V.K. Mitrjushkin and M. Müller-Preussker, Nucl. Phys. **B** (Proc. Suppl.) 30 (1993) 587

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