

# **Scaling Violations of the Proton Structure Function $F_2$ at Small $x$**

H1 Collaboration

## **Abstract**

An analysis is presented of scaling violations of the proton structure function  $F_2(x, Q^2)$  measured with the H1 detector at HERA in the range of Bjorken  $x$  values between  $x = 3 \cdot 10^{-4}$  and  $10^{-2}$  for four-momentum transfers  $Q^2$  larger than  $8.7 \text{ GeV}^2$ . The structure function  $F_2(x, Q^2)$  is observed to rise linearly with  $\ln Q^2$ . Under the assumption that the observed scaling violations at small  $x \leq 0.01$  are described correctly by perturbative QCD, an estimate is obtained of the gluon distribution function  $G(x, Q_o^2)$  at  $Q_o^2 = 20 \text{ GeV}^2$ .

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<sup>b</sup> *Supported by the UK Science and Engineering Research Council*

<sup>c</sup> *Supported by IISN-IHKW, NATO CRG-890478*

<sup>d</sup> *Supported by the Polish State Committee for Scientific Research, grant No. 204209101*

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# 1 Introduction

The observation of scaling of the proton structure function  $F_2(x, Q^2)$  at SLAC demonstrated the existence of point-like constituents of the proton [1]. Subsequent fixed target lepton-nucleon scattering experiments have established the existence of violations of the scaling behaviour which has been one of the most dramatic successes of perturbative Quantum Chromodynamics. The fixed target experiments, though precise, are limited in their range both of four-momentum transfers  $Q^2 (\leq 300 \text{ GeV}^2)$  and of Bjorken  $x (\geq 0.01)$ . At the electron-proton collider HERA the c.m.s. energy  $\sqrt{s}$  is  $296 \text{ GeV}$ , which permits for the first time measurements of deep inelastic scattering in the new kinematic region of very small  $x$ .

Recently we have reported our first measurement of the structure function  $F_2$  [2] in the  $x$  range below  $10^{-2}$  using a data sample from an integrated luminosity of  $22.5 \text{ nb}^{-1}$ . The data were taken at HERA in the autumn 1992 with beam energies of  $E_e = 26.7 \text{ GeV}$  electrons and  $E_p = 820 \text{ GeV}$  protons. The analysis [2] focused on the  $x$  dependence in the small  $x$  region. A strong rise of  $F_2$  for decreasing  $x$  was found in the deep inelastic region  $Q^2 \geq 5 \text{ GeV}^2$  as anticipated in early QCD analyses [3]. Traditionally, theoretical attention has concentrated on the  $Q^2$  dependence of  $F_2$ , predicted by the DGLAP evolution equations [4], and on precision tests of QCD at higher  $x$  [5]. Here we present a re-analysis of our structure function data in the new region now focussing on the  $Q^2$  dependence of  $F_2$ .

The analysis follows largely the lines of our publication reporting on the  $x$  dependence of  $F_2$ . It uses the same data and the same methods to determine the kinematics from both the scattered electron and the final state hadrons. It is therefore subject to the same, sometimes highly correlated, systematic errors. By working directly with our data sample to derive the  $Q^2$  dependence of  $F_2$  in the form of a measurement of  $\partial F_2 / \partial \ln Q^2$ , all errors are propagated correctly through both to the  $F_2$  data points and to the derivative  $\partial F_2 / \partial \ln Q^2$ .

Assuming that perturbative QCD is applicable in the new kinematic region, in the framework of the DGLAP equation the slopes  $\partial F_2 / \partial \ln Q^2$  depend on the momentum weighted gluon distribution  $G(x, Q^2)$ . The  $Q^2$  dependence of  $F_2$  can therefore be used to determine the distribution of the gluons in the proton despite the fact that the latter cannot be probed directly in deep inelastic scattering. At small  $x$  the gluon term in the evolution equation is expected to dominate leading to the approximate relation (for 4 flavours):

$$\frac{\partial F_2}{\partial \ln Q^2} \simeq \frac{5\alpha_s(Q^2)}{9\pi} \int_x^1 (w^2 + (1-w)^2) \cdot G(x/w, Q^2) \, dw \quad (1)$$

An approximate way of extraction the gluon distribution from this relation has been proposed recently [6]. The method is adequate given the size of the experimental errors of our present measurement of  $F_2$ . It is used here to give a first estimate of  $G(x, Q^2)$  at low  $x$ . A preliminary result was presented in [7]. More precise determinations of  $G$  at HERA will be based on measurements of the longitudinal structure function,  $J/\psi$  production and a full analysis of future high statistics  $F_2$  data.

## 2 Data Analysis

The proton structure function analysis of the deep inelastic scattering data taken with the H1 detector [8] has been described in detail in [2]. Here we summarize the most important steps in this analysis and specify the modifications necessary for our study of the  $Q^2$  dependence.

The inclusive scattering cross section is determined by measuring the four-momentum transfer squared  $Q^2$  and the scaling variable  $x = Q^2/sy$ . The momentum transfer is calculated from

the energy  $E'_e$  and the polar angle  $\theta_e$  with respect to the proton beam axis of the scattered electron according to  $Q_e^2 = 4E_e E'_e \cos^2(\theta_e/2)$ . The  $Q^2$  resolution is better than 10% in the region of experimental acceptance. To achieve the largest possible kinematic coverage we determine the scaling variable  $y$  using both the scattered electron and the hadronic final state. From the electron information the  $y$  variable is given as  $y_e = 1 - E'_e/E_e \cdot \sin^2(\theta_e/2)$ . The  $y_e$  resolution is about 10% at  $y \sim 0.5$  but deteriorates like  $1/y$  at lower  $y$ . The use of  $y_e$  is therefore limited to the region  $y_e > 0.05$ . From the hadronic information  $y$  is determined using the relation  $y_h = \sum_{\text{hadrons}} (E_h - p_{z,h})/2E_e$  [9], where  $E_h$  is the energy of a hadron and  $p_{z,h}$  is its longitudinal momentum component. For  $y \geq 0.03$  the  $y_h$  resolution is better than 20%. Above  $y = 0.4$ , however, a sizeable fraction of hadrons is poorly measured and the  $y_h$  measurement deteriorates. In the intermediate  $y$  region we have two reliable determinations of the structure function with different systematics and radiative corrections.

The scattered electron energy,  $E'_e$ , is measured in the backward electromagnetic calorimeter (BEMC). The energy scale of this detector is known to 2% from a comparison of  $E'_e$  with the energy derived from the angles of the electron and the hadronic system. The polar angle  $\theta_e$  is calculated from the event vertex and a space point in the backward proportional chamber (BPC) with an accuracy of better than 5 *mrad*. For the determination of  $y_h$  a combination of energies in the liquid argon and BEMC calorimeters and of track momenta in the central tracker is used. The electromagnetic and the hadronic energy scales in the liquid argon calorimeter are presently known to an accuracy of 3% and 7%, respectively. The detectors used for the luminosity measurement are described in [8]. The error in the determination of the luminosity is 7%.

The deep inelastic events are selected in the region  $8.7 \text{ GeV}^2 \leq Q^2 \leq 80 \text{ GeV}^2$ ,  $3 \cdot 10^{-4} \leq x \leq 10^{-2}$  and  $160^\circ \leq \theta_e \leq 172.5^\circ$  where  $\theta_e$  is determined with respect to the proton beam direction. Since this analysis is optimized to measure the  $Q^2$  dependence in bins of  $x$ , these kinematic cuts differ slightly from those in our previous publication.

For the selection of deep inelastic events, the trigger requires a local energy deposit, or cluster, of more than 4 *GeV* in the BEMC. This cluster must not be vetoed by the time of flight system to improve the discrimination against proton induced background. The following cuts are imposed on the data sample to suppress efficiently beam induced and photoproduction background:

- a BEMC cluster of more than 10.6 *GeV* energy must exist within a radius of 5 *cm* from a reconstructed BPC point;
- the lateral cluster size has to be smaller than 5 *cm*;
- an event vertex must be reconstructed within  $\pm 50$  *cm* from the nominal interaction point;
- more than 25% of the tracks have to point to the event vertex.

The efficiencies of these cuts are discussed in [2]. The data presented here consist of 947 events with less than 2% residual background from beam gas and wall interactions. The surviving photoproduction background is estimated from detailed simulation studies to be low, with a maximum of 6% in one bin at high  $y$ . It is subtracted statistically from the data sample.

The detector response is simulated in detail in order to calculate the acceptance, smearing and radiative corrections to the deep inelastic cross section. Distributions of the basic kinematic quantities  $E'_e$ ,  $\theta_e$  and  $y_h$  are described well by the Monte Carlo calculations, see [2]. For the coarser binning in  $x$  and finer binning in  $Q^2$  of this analysis the smearing corrections are small. All corrections are performed using the event generator HERAKLES [10] interfaced to the Monte Carlo program LEPTO [11]. As a reference parton distribution we use the  $D^-$  parametrization [12] which agrees well with the measured  $F_2(x, Q^2)$  [2]. The simulation of the detector is described in [8].

### 3 Scaling Violations of $F_2(x, Q^2)$

Fig. 1 presents the structure function  $F_2(x, Q^2)$  as function of  $\ln Q^2$  for four values of  $x$ . The circles show  $F_2$  values based on the electron variables  $Q_e^2$  and  $x_e = Q_e^2/sy_e$  and the triangles show  $F_2$  values calculated with the mixed set  $Q_e^2$  and  $x_m = Q_e^2/sy_h$ . The agreement between both structure functions is good. Note that the lowest  $x$  bin is dominated by events measured at large  $y$  and low jet energies where the mixed analysis is less accurate than the electron analysis. Within the statistical accuracy the  $\ln Q^2$  dependence of the  $F_2$  is linear. The systematic uncertainties in the slopes  $\partial F_2/\partial \ln Q^2$  are smallest using the electron variables for  $x \leq 0.001$  and the mixed analysis at larger  $x$ . The NMC collaboration [13] has measured  $F_2$  at  $x = 0.008$  and  $Q^2 = 3.5 \text{ GeV}^2$ . The extrapolation of our data down to this  $Q^2$  value agrees well with the NMC measurement.

The systematic errors are divided into those which are correlated and others which are point to point dependent. The former are determined by recalculating the structure function and redetermining its  $\ln Q^2$  derivative. The differences of the central values are taken as the error on  $\partial F_2/\partial \ln Q^2$ . The major contributions are due to the  $5 \text{ mrad}$  uncertainty in  $\theta$  for the correlated errors and to the 7% error in vertex efficiency [2].

| $x$     | $Q^2/\text{GeV}^2$ | $\partial F_2/\partial \ln Q^2$ | stat.error | syst.error |
|---------|--------------------|---------------------------------|------------|------------|
| 0.00042 | 13.5               | 0.93                            | 0.46       | 0.43       |
| 0.00098 | 20.6               | 0.33                            | 0.20       | 0.08       |
| 0.0027  | 24.1               | 0.14                            | 0.12       | 0.06       |
| 0.0068  | 26.8               | 0.13                            | 0.11       | 0.07       |

Table 1: Derivative  $\partial F_2/\partial \ln Q^2$  of  $F_2$  with statistical and systematic errors. The  $Q^2$  values are the average values in a given  $x$  bin.

Table 1 presents the measured derivatives with statistical and systematic errors. The gross features of the observed scaling violations in the new small  $x$  domain, namely an increasing derivative  $\partial F_2/\partial \ln Q^2$  with decreasing  $x$ , follow the well established trend of measurements at larger  $x$ . Exact scaling would require all derivatives to be zero. The mean value of the four measured slopes is different from zero at the 95% confidence level taking into account the systematic error correlations. As discussed in our previous publication [2] the data sample comprises 6% of the events which have a large rapidity gap between the secondary hadrons and the proton beam direction. The normalized raw  $Q^2$  distribution of those events is not significantly different from the full sample distribution.

According to [6] the measured derivatives can be utilized to extract a leading-order estimate of the gluon distribution at small  $x$  based on the relation:

$$\frac{\partial F_2(x, Q^2)}{\partial \ln Q^2} \simeq \frac{10\alpha_s(Q^2)}{27\pi} \cdot G(2x, Q^2) \quad (2)$$

Using parton distributions from the QCD analyses at higher  $x$  we found that this relation holds to within 20% in the  $x$  range of this analysis. Since  $F_2$  is observed to depend linearly on  $\ln Q^2$  we choose to determine the gluon distribution at  $Q_o^2 = 20 \text{ GeV}^2$ . For the calculation of  $G(x, Q_o^2)$  we assume  $\alpha_s = 0.24$  corresponding to four quark flavours and  $\Lambda_{QCD} = 200 \text{ MeV}$  in leading-order approximation.

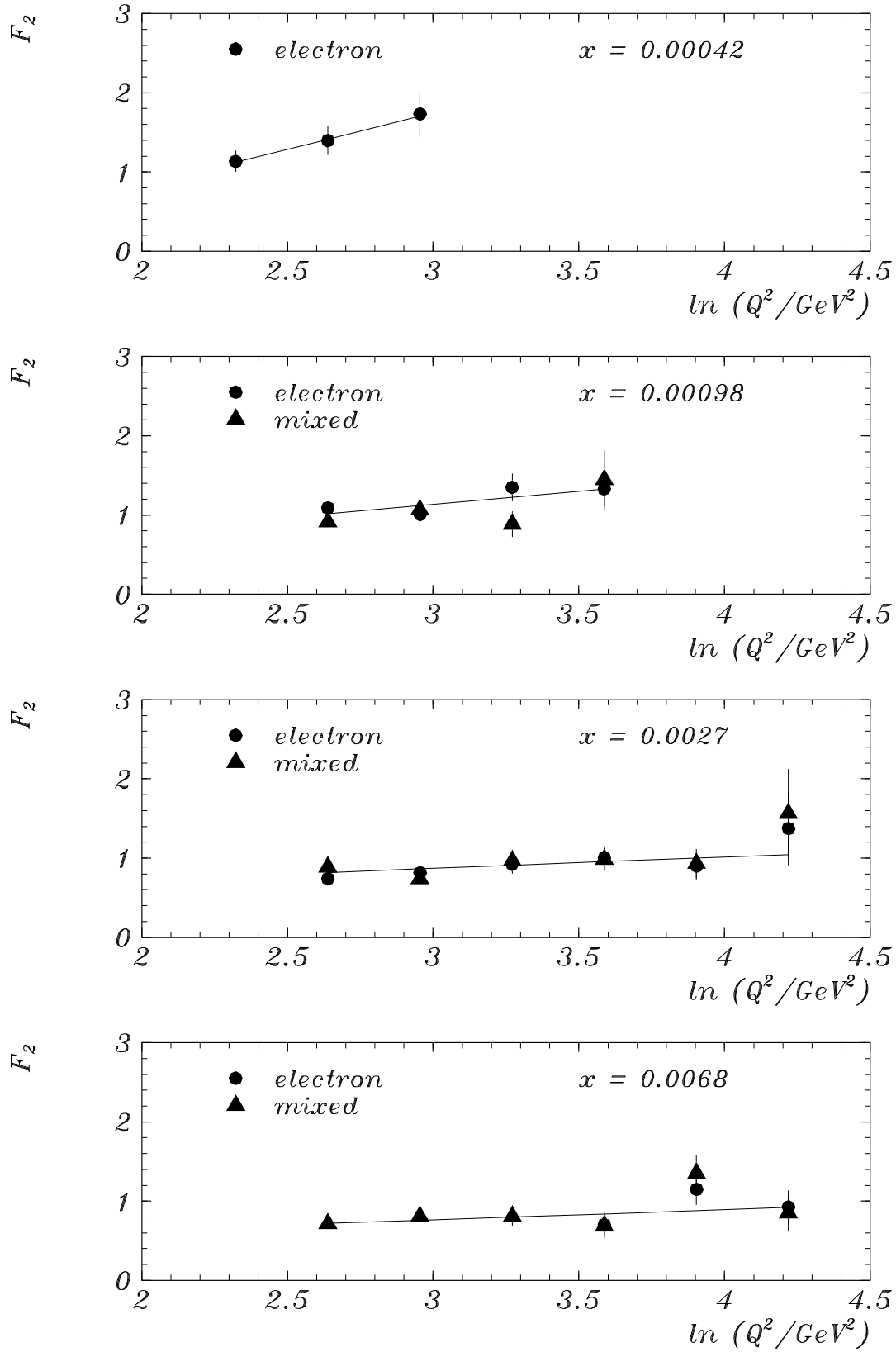


Figure 1: The measured structure function  $F_2(x, Q^2)$  for different values of  $x$  with statistical errors only. All points are subject to a global normalization uncertainty of 8% [2]. The lines are straight line fits to the electron data for  $x < 0.001$  and to the mixed data at higher  $x$ .



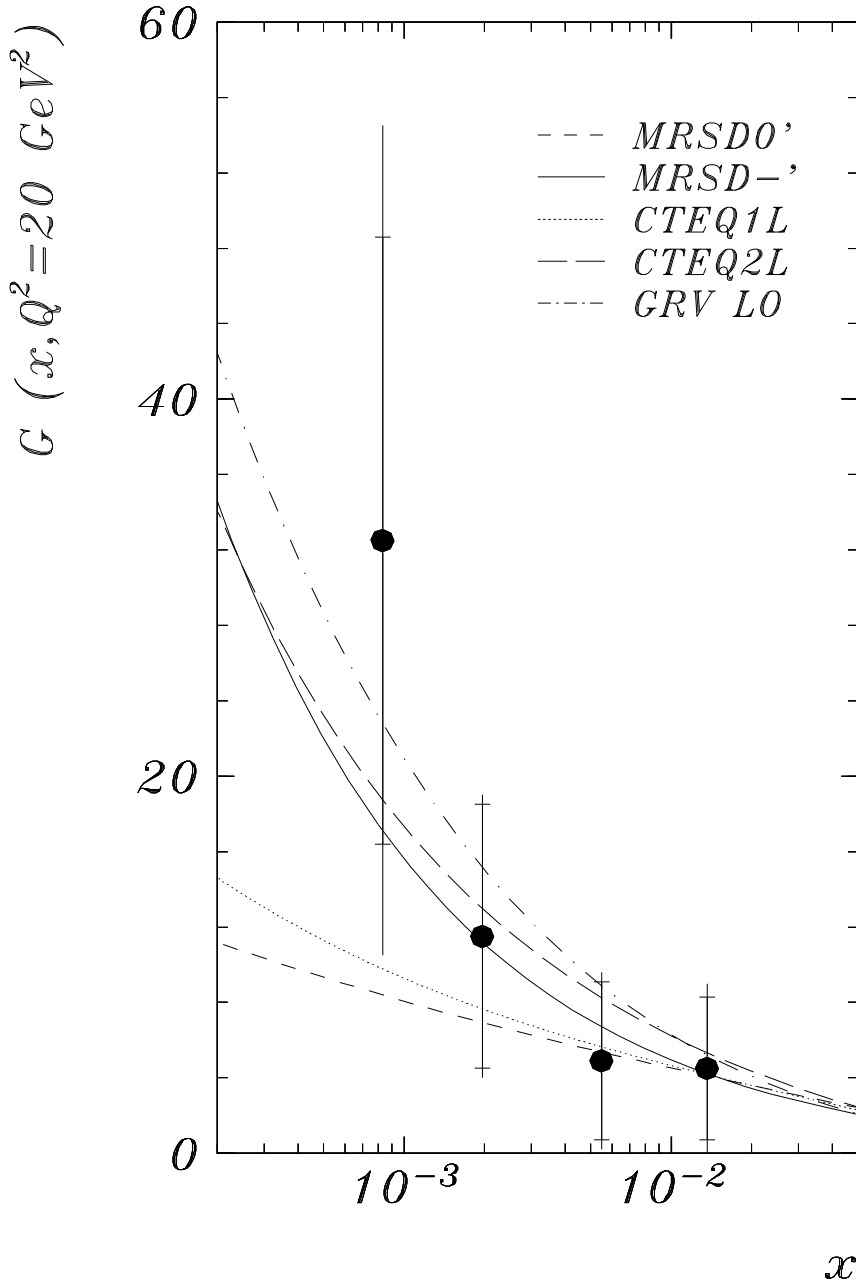


Figure 2: The gluon distribution function  $G(x, Q^2)$  at  $Q^2 = 20 \text{ GeV}^2$ . The inner errors are the statistical errors, the full bars represent the systematic and the statistical errors added in quadrature. The curves represent different parametrizations of  $G$  [14, 15, 16]. The CTEQ and GRV curves are leading-order parametrizations, while the MRS parametrizations are given in the DIS renormalization scheme. CTEQ2 denotes a global analysis including the recent HERA structure function data.

The resulting estimate of the gluon distribution is shown in fig. 2. Also shown are the expectations from various analyses [14, 15, 16] based on deep inelastic, Drell-Yan and direct photon production data. The experimental  $G(x, Q_o^2)$  is seen to rise with decreasing  $x$ . If the  $x$  dependence of the gluon distribution is parametrized as  $\alpha x^\beta$  we obtain  $\beta = -0.81 \pm 0.42(\text{stat}) \pm 0.32(\text{syst})_{-0}^{+0.12}(\text{theory})$  at  $Q_o^2 = 20 \text{ GeV}^2$  where the theoretical error represents the approximations inherent in eq.(2). This is consistent with the expectation, based on the Lipatov evolution, that  $\beta = -12\alpha_s \ln 2 / \pi \simeq -0.64$  [17], albeit with large experimental uncertainty. The neglect

of the sea quark distributions is estimated to reduce  $G$  by about 10% independently of the structure function parametrizations in the covered  $x$  range.

## 4 Conclusion

We have measured the  $Q^2$  dependence of the proton structure function  $F_2$  in the Bjorken  $x$  range  $3 \cdot 10^{-4}$  to  $10^{-2}$ . We find that the  $Q^2$  dependence follows the experimentally well established trend of data at larger  $x$ , namely a steeper dependence on  $Q^2$  with decreasing  $x$ . The hypothesis of scale invariance can be rejected at a 95% confidence level. Assuming the validity of the leading log (DGLAP)  $Q^2$  evolution and neglecting the quark contributions in this evolution, we extract a first estimate of the gluon distribution in the proton at small  $x$  which suggests a rising gluon density with decreasing  $x$ . The observed  $x$ -dependence of  $G(x, Q_o^2)$  can be parametrized as  $\alpha x^\beta$  with  $\beta = -0.81 \pm 0.42(stat) \pm 0.32(syst)_{-0}^{+0.12}(theory)$  at  $Q_o^2 = 20 \text{ GeV}^2$ .

## Acknowledgements

We are grateful to the HERA machine group whose outstanding efforts made this experiment possible. We appreciate the immense effort of the engineers and technicians who constructed and maintain the detector. We thank the funding agencies for financial support. We acknowledge the support of the DESY technical staff. We wish to thank the DESY directorate for the support and hospitality extended to the non-DESY members of the collaboration.

## References

- [1] M.Breidenbach et al., Phys.Rev.Lett. **23** (1969) 935.
- [2] I.Abt et al., H1 Collaboration, Nucl.Phys. **B407** (1993) 515.
- [3] A.DeRujula et al., Phys.Rev. **D10** (1974) 1649.
- [4] Yu.L.Dokshitzer, JETP **46** (1977) 641;  
V.N.Gribov and L.N.Lipatov, Sovj.J.Nucl.Phys. **15** (1972) 78;  
G.Altarelli and G.Parisi, Nucl.Phys. **B126** (1977) 298.
- [5] M.Virchaux and A.Milsztajn, Phys.Lett. **274B** (1992) 221.
- [6] K.Prytz, Phys.Lett. **311B** (1993) 286.
- [7] A preliminary determination of  $G$  by the ZEUS collaboration was presented by J.Martin, ZEUS Collaboration, Inv.Talk International Lepton-Photon Symposium, Cornell, USA (1993)
- [8] H1 Collab., I. Abt et al., *The H1 Detector at HERA*, DESY preprint 93-103 (1993).
- [9] A. Blondel and F. Jacquet, Proceedings of the Study of an  $ep$  Facility for Europe, ed. U. Amaldi, DESY 79-48 (1979) 391.
- [10] A. Kwiatkowski, H. Spiesberger, and H.-J. Möhring, Comp. Phys. Commun. **69** (1992) 155 and references therein.

- [11] G. Ingelman, “LEPTO 5.2”, unpublished program manual;  
H.Bengtsson, G.Ingelman, and T.Sjöstrand, Nucl. Phys. **B301** (1988) 554;  
G.A. Schuler and H. Spiesberger, Proceedings of the Workshop “Physics at HERA”, ed.  
W. Buchmüller and G. Ingelman, Hamburg, (1991) 1419.
- [12] A. D. Martin, W. J. Stirling, R. G. Roberts, Phys. Rev. **D47** (1993) 867.
- [13] NMC Collab., P. Amaudruz et al., Phys. Lett. **295B** (1992) 159.
- [14] A. D. Martin, W. J. Stirling, R. G. Roberts, Phys.Lett. **306B** (1993) 145, Erratum **309B**  
(1993) 492.
- [15] M. Glück, E. Reya and A. Vogt, Z. Phys. **C53** (1992) 127;  
M. Glück, E. Reya and A. Vogt, Phys. Lett. **306B** (1993) 391.
- [16] J. Botts et al., Phys. Lett. **304B** (1993) 159;  
J. Botts et al., preprint MSU-HEP 93/18 (1993)
- [17] E.A.Kuraev, L.N.Lipatov and V.S.Fadin, Sov.Phys.JETP **45** (1977) 199;  
Ya.Balitsky and L.N.Lipatov, Sov.J.Nucl.Phys. **28** (1978) 822
- [18] A.Mueller, Nucl.Phys.B(Proc.Suppl.) **18C** (1990) 125.
- [19] A.Mueller, Nucl.Phys.B(Proc.Suppl.) **29A** (1992) 275.