Mass Spectrum and Bounds on the Couplings in Yukawa Models With Mirror-Fermions

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The $SU(2)_L \otimes SU(2)_R$ symmetric Yukawa model with mirror-fermions in the limit where the mirror-fermion is decoupled is studied both analytically and numerically. The bare scalar self-coupling λ is fixed at zero and infinity. The phase structure is explored and the relevant phase transition is found to be consistent with a second order one. The fermionic mass spectrum close to that transition is discussed and a first non-perturbative estimate of the influence of fermions on the upper and lower bounds on the renormalized scalar self-coupling is given. Numerical results are confronted with perturbative predictions.

1. INTRODUCTION

This contribution represents the talks "Phase Structure of a Chiral SU(2) Yukawa Model", "Mass Spectrum and Bounds on the Couplings in Yukawa Models with Mirror-Fermions" delivered at the international conference on lattice field theory "Lattice 92", Amsterdam, 15–19 September 1992, by Lee Lin and Hartmut Wittig, respectively.

The minimal Standard Model (SM) has been very successful. So far all the experimental data are in good agreement with its perturbative predictions, but there are still too many free parameters (18, if all neutrinos are massless) in the SM, hence physicists believe that it is at most a good low energy effective theory, and new physics which cannot be described by the SM will have noticeable effects at a higher energy scale. Here,

we concentrate on two free parameters in the SM: the top quark mass and the mass of the Higgs particle which is the remnant scalar particle after spontaneous symmetry breaking. We would like to investigate whether these two parameters are completely free, or whether there are some limitations on their values. Hopefully, one can also get some hints about the new physics by studying this Higgs-fermion sector in the SM.

Since the presently quoted upper bound on the top quark mass of 200 GeV is a one-loop perturbative result, nonperturbative (i.e. lattice) studies of this issue can be very helpful. In order to deal with the problem of fermion doublers in lattice formulations, we take the mirror-fermion approach [1].

Our calculations were done in the SU(2) version of the Yukawa model with explicit mirror pairs of fermion doublet fields. The lattice action is a sum of the $O(4) \cong SU(2)_L \otimes SU(2)_R)$ symmetric pure

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scalar part S_{φ} and the fermionic part S_{Ψ} :

$$S = S_{\varphi} + S_{\Psi} . \tag{1}$$

 φ_x is the $2\otimes 2$ matrix scalar field, and $\Psi_x\equiv (\psi_x,\chi_x)$ stands for the mirror pair of fermion doublet fields where ψ is the fermion doublet and χ the mirror-fermion doublet. In the usual normalization conventions for numerical simulations we have

$$S_{\varphi} = \sum_{x} \left\{ \frac{1}{2} \operatorname{Tr} \left(\varphi_{x}^{+} \varphi_{x} \right) + \lambda \left[\frac{1}{2} \operatorname{Tr} \left(\varphi_{x}^{+} \varphi_{x} \right) - 1 \right]^{2} \right.$$

$$- \kappa \sum_{\mu = \pm 1}^{\pm 4} \operatorname{Tr} \left(\varphi_{x+\hat{\mu}}^{+} \varphi_{x} \right) \right\}, \qquad (2)$$

$$S_{\Psi} = \sum_{x} \left\{ \mu_{\psi\chi} \left[\left(\overline{\chi}_{x} \psi_{x} \right) + \left(\overline{\psi}_{x} \chi_{x} \right) \right] \right.$$

$$- K \sum_{\mu = \pm 1}^{\pm 4} \left[\left(\overline{\psi}_{x+\hat{\mu}} \gamma_{\mu} \psi_{x} \right) + \left(\overline{\chi}_{x+\hat{\mu}} \gamma_{\mu} \chi_{x} \right) \right.$$

$$+ r \left[\left(\overline{\chi}_{x+\hat{\mu}} \psi_{x} \right) - \left(\overline{\chi}_{x} \psi_{x} \right) + \left(\overline{\psi}_{x+\hat{\mu}} \chi_{x} \right) - \left(\overline{\psi}_{x} \chi_{x} \right) \right] \right]$$

$$+ G_{\psi} \left[\left(\overline{\psi}_{Rx} \varphi_{x}^{+} \psi_{Lx} \right) + \left(\overline{\psi}_{Lx} \varphi_{x} \psi_{Rx} \right) \right]$$

$$+ G_{\chi} \left[\left(\overline{\chi}_{Rx} \varphi_{x} \chi_{Lx} \right) + \left(\overline{\chi}_{Lx} \varphi_{x}^{+} \chi_{Rx} \right) \right] \right\}. \qquad (3)$$

Here K is the fermion hopping parameter, r the Wilson-parameter, which will always be fixed to r=1, the indices L,R denote the chiral components of fermion fields, and Tr acts on the SU(2) indices only. In this normalization the fermion mirror-fermion mixing mass is $\mu_{\psi\chi}=1-8rK$. The lattice spacing a is set to unity.

At $G_{\chi}=0$, the action has the Golterman-Petcher shift-symmetry such that all higher vertex functions containing the χ -field vanish identically [2,3]. In this limit, the χ - χ and χ - ψ components of the two-point fermion vertex function (the inverse fermion propagator) $\tilde{\Gamma}_{\Psi}(p)$ are equal to the corresponding components of the free inverse propagator [4]. By setting K=1/8, one can easily show that in the broken phase, there is no mixing between the fermion and mirror-fermion and the mirror-fermion completely decouples like a right-handed neutrino. This $G_{\chi}=0$, K=

1/8 combination is sometimes called the mirrorfermion decoupling limit. This leaves us only one mass parameter κ to tune when we try to approach the continuum limit and hence saves a lot of CPU time. Numerical simulations can also be guided by analytic results derived in this limit such that fluctuations can hopefully be reduced. The Hybrid Monte Carlo algorithm [5] is used in our simulations. We therefore need to double the flavour of the fermion spectrum to guarantee a positive definite fermion matrix determinant.

One of the main goals in the analysis of the model is the determination of the range of renormalized quartic couplings g_R that can be realized for a certain value of the cut-off as a function of the renormalized Yukawa coupling $G_{R\psi}$. This so-called "Allowed Region" is bounded by the (upper) triviality bound on g_R (saturated at $\lambda = \infty$) and the (lower) vacuum stability bound, which is defined at $\lambda = 0$. Hence for the bare quartic coupling λ the values $\lambda = \infty$, 0 are crucial in the numerical simulations.

Reflection positivity, which is required for any Euclidean quantum field theory respecting unitarity, can be proven to hold in a wide range of the parameter space of our model [6]. In particular, reflexion positivity only holds for $\kappa \geq 0$. Although the criterion of reflexion positivity is only a sufficient but not a necessary condition for the reconstruction of the theory in Minkowski space, we would like to stay where unitarity is guaranteed in order to be on the safe side. So we always study the "Allowed Region" for the renormalized couplings in the subspace where κ is non-negative.

The β -functions have been calculated up to one loop on the lattice and two loops in the continuum [3]. There are basically two possibilities for the cut-off dependence of the "Allowed Region": either this region shrinks to the origin as the cut-off grows or it expands and eventually fills the whole space of renormalized couplings at infinite cut-off. The first possibility is due to the fact that there is only one fixed point at zero couplings and that fixed point is infrared stable. One-loop β -functions are behaving qualitatively like this. In the second scenario, there is an ultraviolet stable fixed point at nonzero couplings which changes the dependence of the "Allowed".

Region" on the cut-off qualitatively. Assuming that the β -functions are behaving qualitatively like at one-loop level, then our model is trivial in the continuum limit, and the upper bound on the renormalized scalar self-coupling g_R will be given by setting $\lambda = \infty$ while the lower bound comes from the $\lambda \to 0$ limit [6].

2. PHASE STRUCTURE

In order to know where and how the cut-off can be removed, we need to explore the phase structure. If there is a first order phase transition somewhere in the bare parameter space, we would like to know how it is going to affect the "Allowed Region".

We basically use the magnetization $\langle |\phi| \rangle$ and staggered magnetization $\langle |\hat{\phi}| \rangle$ as the order parameters to distinguish different phases where $\langle \ \rangle$ is the statistical average, and

$$|\phi| \equiv \sqrt{\phi_a^2} , |\hat{\phi}| \equiv \sqrt{\hat{\phi}_a^2} ,$$

$$\phi_a \equiv \frac{1}{L^3 \cdot T} \sum_x \phi_a(x) ,$$

$$\hat{\phi}_a \equiv \frac{1}{L^3 \cdot T} \sum_x (-1)^x \phi_a(x) , a = 1, \dots, 4$$

where L and T are lattice sizes along the spatial and time directions. The symmetry broken (FM) phase has nonzero $\langle |\phi| \rangle$ and vanishing $\langle |\hat{\phi}| \rangle$, the symmetric (PM) phase has zero $\langle |\phi| \rangle$ and $\langle |\hat{\phi}| \rangle$ while $\langle |\phi| \rangle = 0$, $\langle |\hat{\phi}| \rangle \neq 0$ in the antiferromagnetic (AFM) phase. In the PM phase, the mirror-fermion and fermion are degenerate and hence it is not physical. The physically relevant phase is the FM phase where the splitting between fermion and mirror-fermion masses is possible due to spontaneous symmetry breaking and by tuning the bare couplings appropriately. The AFM phase is like the FM phase except that the staggered scalar field $\hat{\phi}$ is now playing the role of ϕ .

The phase structure can be investigated analytically using various expansions in several limits [7]. We find that at zero or infinite Yukawa couplings, or at $K = \infty$, the system goes to a purely scalar four-component ϕ^4 theory plus free

fermions. Therefore, the system exists in the FM, PM and AFM phases with the transitions between them being second order and Gaussian. At K=0, which is the limit where the bare fermion mass is infinite, a first order phase transition line was found in the U(1) version at $G_{\psi} \cdot G_{\chi} > 0$ and finite λ [8]. This first order transition is due to the singularity of a log-term in the effective action. It becomes weaker and weaker and eventually vanishes at $\lambda = \infty$. In the SU(2) version, everything is qualitatively the same. Therefore, this first order transition is also present when $G_{\psi} \cdot G_{\chi} > 0$, $\lambda = \text{finite.}$ We did not further investigate this issue. At $\lambda = \infty$, when Yukawa couplings are weak or strong, one can do expansions up to the nextleading order in |G| or 1/|G| plus the small-K expansion and find out that κ_c , the value where the transition between the FM and PM phases happens, will go down as |G| or 1/|G| increase. Furthermore the transition remains a second order one on which the renormalized scalar mass vanishes.

We also need to look for the fermionic critical plane on which the renormalized fermionic mixing mass μ_R vanishes. When $G_{\chi} = 0$ at any λ , this happens at K = 1/8 and any G_{ψ} value. When both G's are nonzero, we can use one-loop bare perturbation theory to estimate its position at weak K and G's.

The correct continuum limit should be taken by approaching the critical line where both scalar and fermion masses are zero from within the FM phase while at the same time the mixing mass is fixed at zero and all mass ratios are kept constant.

The phase structure at intermediate values of the Yukawa couplings must be explored numerically. That was always done on the $4^3 \cdot 8$ lattice. The strategy is to fix λ at infinity or zero (actually 10^{-6}), which are the relevant values to studying upper and lower limits on g_R respectively. For the remaining parameters, since we chose to decouple the mirror-fermion like a right-handed neutrino in the SU(2) model, we always set $G_{\chi} = 0, K = 1/8$, and then studied the phase diagram in the (κ, G_{ψ}) plane.

At $\lambda = \infty$, the phase structure was explored numerically up to G_{ψ} around 2.0. The result was shown in Fig.1 in ref. [4]. Everything ap-

peared to be qualitatively the same as the phase structure of the U(1) model at infinite λ . At $G_{\psi} \geq 1.5, \ \kappa = -0.5, \ \text{a new phase with nonva-}$ nishing magnetization and staggered magnetization was found. We call it the ferri-magnetic (FI) phase. The phase transition between the FM and PM phases was found to be consistent with a second order one because of the smooth behaviour of $\langle |\phi| \rangle$ across the transition. This was also true for other transitions in the phase diagram. Based on our analytic analyses, we know that as we move to larger and larger G_{ψ} values, eventually the FI phase will disappear and the system again exists only in the FM, PM and AFM phases. We therefore did not spend CPU time to investigate further. (This has been numerically confirmed in our $U(1) \mod [9]$.

The phase structure at very small λ and large negative values of κ was actually not thoroughly explored yet. In the U(1) model we only made sure that the FM-PM transition was consistent with a second order one and then went on to study the lower bound on g_R [6]. We now spent more computer time to look into the phase structure of the SU(2) model at $\lambda = 10^{-6}$. The transitions from the FM to PM and from the PM to AFM phases were found to be consistent with a second order one at weak G_{ψ} . Again the magnetization behaves smoothly across the PM to FM transition. Plotting histograms of the magnetization we found that there is no evidence at all for a twostate signal. The two transition lines bend down as G_{ψ} increases, and come quite close to each other at $G_{\psi} = 0.75$, $\kappa = -0.17$. At an even more negative κ -value: -0.19 and at $G_{\psi} = 0.6, 0.8$, we observed that the system seemed to "tunnel" from the FI phase to the AFM phase. At this stage, we cannot decide whether it is a real tunneling or whether the system has not equilibrated yet. Even if we find a tunneling event (or a hysteresis loop), it still might be due to a very long auto-correlation length in the vicinity of a critical point and therefore is still not the final word. We think it is better to measure the shape of the effective potential to see if a double-well structure developes. If there is a first order phase transition, we suspect that it might be the continuation of the first order transition we discovered at K=0. On the other hand, a leading order large-N calculation of the effective potential of our SU(2) model shows that the usual PM, FM, AFM and FI phases exist at small λ , and that all transitions are second order due to the absence of a quartic term in the effective potential in leading order. This issue will be further investigated in the future.

Most importantly, the physically relevant FM-PM phase transition at $\lambda = 10^{-6}$ is consistent with a second order one, and we can define the continuum limit by approaching it from the FM phase. Therefore, studies of the lower limit on g_R should not be affected by a possibly existing first order transition.

3. MASSES AND COUPLINGS

We now describe the Monte Carlo simulations of the model in the broken (FM) phase. Besides the massive component σ_x of the scalar field φ_x three massless Goldstone bosons π_{jx} , j=1,2,3 appear which cause the strong finite-size effects encountered in the simulations.

As noted before, the model was simulated using the Hybrid Monte Carlo algorithm. For the fermions, periodic spatial boundary conditions but antiperiodic boundary conditions in the time direction were chosen. This fixes the minimum lattice momentum for fermionic quantities at $p_{\min} = (0, 0, 0, \pi/T)$ where T is the time extent of the lattice. Exploiting the shift symmetry, the decoupling of the mirror-fermion was ensured by setting $G_{\chi} = 0$. In order to exclude mixing between ψ - and χ -states we chose a slightly different value than K = 1/8 which was originally suggested. Namely, by setting $\mu_{p_{\min}} = 0$, where $\mu_p = (\mu_{\psi\chi} + Kr\hat{p}^2)/2K$, the fermionic hopping parameter K turns out slightly greater than 1/8, but will eventually approach this value once the time extent of the lattice goes to infinity. This particular choice corresponds to exact decoupling in the continuum limit whilst ensuring a smooth behaviour of the propagator on a finite lattice near K = 1/8 (see also ref. [4]).

With the parameters G_{χ} and K fixed by the decoupling condition our further strategy was as follows. For $\lambda = \infty$, G_{ψ} was varied from 0.3, 0.6

to 1.0. At each value of G_{ψ} the scalar hopping parameter κ was tuned in order to achieve $m_{R\sigma} \leq 1$. Choosing lattice sizes of $L^3 \cdot T$ of $4^3 \cdot 8$, $6^3 \cdot 12$ and $8^3 \cdot 16$ it was hoped that on the largest lattice $m_{R\sigma}$ could be brought down to about $m_{R\sigma} \simeq 0.5$ in lattice units.

For $\lambda=10^{-6}$, G_{ψ} was fixed at 0.3 and again κ was tuned. A second set of data, however, was generated by setting $\kappa=0$ and tuning G_{ψ} in order to determine the maximum value of the renormalized Yukawa coupling $G_{R\psi}$ for non-negative values of κ , i.e. in the region where reflexion positivity can still be proven.

The results for the scalar mass show that $m_{R\sigma}$ is decreasing as one approaches the critical line from above and then starts rising again. This rise close to the phase transition is explained as a strong finite-size effect. The minimum of $m_{R\sigma}$ increases with increasing G_{ψ} [4]. For $\lambda = \infty$, $G_{\psi} = 0.6$, it is necessary to have lattices as large as $8^3 \cdot 16$ in order to obtain scalar masses smaller than 1 in lattice units.

At $\lambda=10^{-6}$ the tuning of $m_{R\sigma}$ is easier in the sense that one can achieve smaller values than at $\lambda=\infty$, however the rise of the minimum of $m_{R\sigma}$ as G_{ψ} is increased, remains. All that illustrates that there are strong finite-size effects which get stronger as the bare couplings λ and/or G_{ψ} are increased. Nevertheless, our experience has shown that for $\kappa\geq 0$ a lattice size of $8^3\cdot 16$ is sufficient to have control on the finite volume effects.

For the fermion masses one can make the following statements: firstly, setting $G_{\chi} = 0$ ensured indeed that the mass of the mirror-fermion μ_{R_Y} was always zero within errors. Secondly, the condition $\mu_{p_{\min}} = 0$ guaranteed that the fermionic mixing mass μ_R was zero as well. Fig. 1 shows the fermion mass $\mu_{R\psi}$ (full squares) and the lowest doubler states $\mu_{R\psi}^d$ (open squares) and $\mu_{R\chi}^d$ (open circles) as a function of κ . It is seen that the lowest fermion doublers have masses of 1.5 in lattice units. Furthermore, it is evident from the figure that only at the lowest value of κ there is a clear separation of $\mu_{R\psi}$ and the doubler states. Since this κ -value coincided with the minimum of the scalar mass $(m_{R\sigma} = 0.75(3))$ at $\kappa = 0.09$, $\lambda = 10^{-6}, G_{\psi} = 0.3 \text{ on } 6^3 \cdot 12), \text{ it is precisely}$ this data point which was subsequently included in the plot of the Allowed Region. To summarize, we note that in general $\mu_{R\psi}$ decreases as κ and/or G_{ψ} is decreased, and that the lowest doubler state of the mirror-fermion has a mass of about 1.5, irrespective of the actual choice of the tunable bare parameters.

Using the relation $\mu_{R\psi}=G_{R\psi}v_R$, where v_R is the VEV of the scalar field, it is instructive to plot $G_{R\psi}$ versus G_{ψ} in order to check whether the linear rise of $G_{R\psi}$ reported for the symmetric phase [3] prevails. This is done in Fig. 2. It is seen that there is still a linear behaviour which, however, is much flatter as in the symmetric phase. The data obtained at $\lambda=10^{-6}$ are slightly larger than those at $\lambda=\infty$. The maximum value of $G_{R\psi}$ for $\kappa\geq 0$ was found at $\kappa=0$, $G_{\psi}=0.63$ where $G_{R\psi}^{\max}=3.5\pm 0,4$. This has to be confronted with the tree unitarity bound which yields $G_{R\psi}\simeq 2.5$ for $N_f=2$. Hence the maximum value for $G_{R\psi}$ is not very much above this bound, suggesting that for $\kappa\geq 0$ the coupling $G_{R\psi}$ cannot grow indefinitely large.

Fig. 3 shows our data for the Allowed Region, together with the perturbative expectation coming from the numerical integration of the oneloop β -functions for cut-off's corresponding to $m_{R\sigma} = 0.75$ (dotted curve) and $m_{R\sigma} = 1$ (solid curve), respectively. All data were obtained on lattices of size $6^3 \cdot 12$ and $8^3 \cdot 16$. The point with the largest couplings was on $6^3 \cdot 12$ at $\lambda = 10^{-6}$, $G_{\psi} = 0.63, \ \kappa = 0.$ Despite the large error bars the data are in remarkable agreement with the perturbative estimates even at values of $G_{R\psi}$ around or even above the tree unitarity limit. Therefore, the data support the view that the perturbative behaviour of the bounds on the scalar coupling observed in pure ϕ^4 theory persists when fermions are included.

Finally, Fig. 4 shows the mass ratio $\mu_{R\psi}/m_{R\sigma}$ as a function of $G_{R\psi}$, together with the perturbative prediction from the integration of the one-loop β -functions for the upper and lower bound, respectively. It is seen that for larger values of $G_{R\psi}$ (where there is only a weak cut-off dependence) the data are clustered around 0.7, which is close to the fixed point for the corresponding ratio of couplings, $\sqrt{3} G_{R\psi}/\sqrt{g_R} = 0.759\ldots$ that is reached as the cut-off scale becomes infinite. The

Figure 1. The fermion mass $\mu_{R\psi}$ and the lowest doubler states plotted versus κ at $\lambda=10^{-6},$ $G_{\psi}=0.3$ on $6^3\dot{1}2$.

Figure 3. The Allowed Region in the $(g_R, G_{R\psi}^2)$ -plane together with the integration of the β -functions for scalar mass $m_{R\sigma} = 1$ (solid line) and $m_{R\sigma} = 0.75$ (dotted line).

Figure 2. The renormalized Yukawa coupling $G_{R\psi}$ versus G_{ψ} for $\lambda = \infty$ (open circles) and $\lambda = 0$ (full squares) on $6^3 \cdot 12$.

Figure 4. The mass ratio $\mu_{R\psi}/m_{R\sigma}$ versus $G_{R\psi}$ in comparison with 1-loop perturbative estimates for $m_{R\sigma}=0.75$ (dotted curve), $m_{R\sigma}=1$ (full curve) and $m_{R\sigma}=1.25$ (dashed curve).

fixed point is indicated by the horizontal line in the figure. For smaller $G_{R\psi}$ the data obtained for $\lambda = \infty$ deviate from the fixed point in a fashion that is well described by the expected cut-off dependence of the upper bound for finite cut-off.

4. CONCLUSIONS

According to our results, the Higgs-Yukawa sector of the Standard Model can indeed be studied non-perturbatively following the mirrorfermion approach. The phase diagram exhibits a rich structure with the familiar PM, FM, AFM and FI phases encountered in other lattice Yukawa models as well. For both $\lambda = \infty$ and $\lambda \simeq$ 0 the physically relevant phase transition from PM to FM is second order and hence permits taking the continuum limit along this transition line. Exploiting the Golterman-Petcher shift symmetry, numerical simulations in the decoupling limit in the broken phase are feasible and greatly facilitated since only κ remains to be tuned once G_{ψ} is fixed. The resulting fermionic mass spectrum consists of a fermion of mass $\mu_{R\psi}$ < 1, whereas μ_{R_X} and the mixing mass μ_R vanish identically. The doublers for both fermion and mirror-fermion receive masses of at least 1.5 in lattice units.

The most severe limitations arise from finitesize effects on the scalar mass $m_{R\sigma}$ in the broken phase. However, for an exploratory study such as this a lattice size of $8^3 \cdot 16$ appears sufficiently large in order to control finite-size effects.

Restricting the physical analysis to data points with $\kappa \geq 0$ (i.e. data which always will permit a particle interpretation in Minkowski spacetime), it is evident that the results for the couplings g_R and $G_{R\psi}$ agree with perturbative estimates. In particular the data for the Allowed Region suggest that the perturbative behaviour of the bounds on g_R encountered in pure ϕ^4 theory prevails once the effects of heavy fermions are taken into account.

During the preparation of this paper, we have accumulated about 3500 trajectories for a run at $\lambda = 10^{-6}$, $G_{\psi} = 0.25$, $G_{\chi} = 0.0$ and $\kappa = 0.099$. It shows that with $m_{R\sigma} = 0.624(56)$, $\mu_{R\psi} = 0.413(12)$, we obtain $g_R = 16 \pm 5$, $G_{R\psi} = 1.56(7)$ and $G_{R\psi}^2 = 2.43(22)$. Readers can see that the

central value is slightly above the dotted oneloop curve we show in Fig. 3. Since the scalar mass in that data point turns out to be higher than $m_{R\sigma} = 0.75$ for the dotted curve, our findings still support the perturbative scenario for the bounds on g_R . Data with better statistics will be published in a forthcoming paper.

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