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# MIRROR FERMIONS IN CHIRAL GAUGE THEORIES

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Mirror fermions appear naturally in lattice formulations of the standard model. The phenomenological limits on their existence and discovery limits at future colliders are discussed. After an introduction of lattice actions for chiral Yukawa-models, a recent numerical simulation is presented. In particular, the emerging phase structure and features of the allowed region in renormalized couplings are discussed.

## 1. INTRODUCTION

In the presently known energy range the spectrum of the elementary particles is *"chiral"* in the sense that no explicit fermion mass terms are allowed by the symmetry. Fermion masses, as well as all other masses, are entirely generated by spontaneous symmetry breaking, due to the nonzero vacuum expectation value $v_R \simeq 250$ GeV of the Higgs-boson field. One of the consequences is the V-A structure of weak interactions, resulting in the breaking of space-reflection (parity) symmetry.

A natural question is whether "chirality" and the accompanying parity breaking is perhaps only a low-energy phenomenon, and at high energy the space-reflection symmetry is restored by the existence of opposite chirality *"mirror fermions"* [1]. If the presently known (almost complete) three fermion families were duplicated at the electroweak energy scale, in the range 100-1000 GeV, by three mirror fermion families with opposite chiralities and hence V+A couplings to the weak gauge vector bosons [2], then the whole fermion spectrum would be "vectorlike". This would very much simplify the nonperturbative lattice formulation of the Standard Model [3,4]. Of course, since no effects of the mirror fermions are experimentally observed up to now, first one has to ask whether the limits implied by the presently known experimental data allow their existence at all.

## 2. MIRROR FERMION PHENOMENOLOGY

The mirror partners of fermions have the same $SU(3)\otimes SU(2)_L\otimes U(1)_Y$ quantum numbers but opposite chiralities. For instance, the right-handed chiral components of mirror leptons form a doublet with $Y=-1$ with respect to $SU(2)_L\otimes U(1)_Y$. Such particles appear in several extensions of the minimal Standard Model, for instance, in grand unified theories with large groups as O(16) or SU(15) [5]. In general mirror fermion models there might be some other quantum numbers which are different for fermions and mirror fermions, and the set of representations containing the mirror fermions may also be different. The lattice formulation of the Standard Model suggests a simple doubling of the fermion spectrum [6], resulting in three mirror pairs of fermion families [2].



### 2.1. *Present limits*

The direct pair production of mirror fermions is not observed at LEP. This puts a lower limit on their masses of about 45 GeV. Heavier mirror fermions could be produced via their mixing to ordinary fermions. This implies some constraints on the mixing angles versus the masses.

In order to discuss the mixing schemes, let us first consider the simplest case of a single fermion ($\psi$) mirror fermion ($\chi$) pair. The mass matrix on the $(\overline{\psi}_R, \overline{\psi}_L, \overline{\chi}_R, \overline{\chi}_L) \otimes (\psi_L, \psi_R, \chi_L, \chi_R)$ basis is

$$M = \begin{pmatrix} \mu_\psi & 0 & \mu_R & 0 \\ 0 & \mu_\psi & 0 & \mu_L \\ \mu_L & 0 & \mu_\chi & 0 \\ 0 & \mu_R & 0 & \mu_\chi \end{pmatrix} . \qquad (1)$$

Here $\mu_{(L,R)}$ are the fermion mirror fermion mixing mass parameters, and the diagonal elements are produced by spontaneous symmetry breaking:

$$\mu_\psi = G_{R\psi} v_R \;, \qquad \mu_\chi = G_{R\chi} v_R \;, \qquad (2)$$

with the renormalized Yukawa-couplings $G_{R\psi}$, $G_{R\chi}$.

For $\mu_R \neq \mu_L$ the mass matrix $M$ in (1) is not symmetric, hence one has to diagonalize $M^T M$ by $O^T_{(LR)} M^T M O_{(LR)}$, and $M M^T$ by $O^T_{(RL)} M M^T O_{(RL)}$, where

$$O_{(LR)} = \begin{pmatrix} \cos\alpha_L & 0 & \sin\alpha_L & 0 \\ 0 & \cos\alpha_R & 0 & \sin\alpha_R \\ -\sin\alpha_L & 0 & \cos\alpha_L & 0 \\ 0 & -\sin\alpha_R & 0 & \cos\alpha_R \end{pmatrix} ,$$

$$O_{(RL)} = \begin{pmatrix} \cos\alpha_R & 0 & \sin\alpha_R & 0 \\ 0 & \cos\alpha_L & 0 & \sin\alpha_L \\ -\sin\alpha_R & 0 & \cos\alpha_R & 0 \\ 0 & -\sin\alpha_L & 0 & \cos\alpha_L \end{pmatrix} . \qquad (3)$$

The rotation angles of the left-handed, respectively, right-handed components satisfy

$$\tan(2\alpha_L) = \frac{2(\mu_\chi \mu_L + \mu_\psi \mu_R)}{\mu_\chi^2 + \mu_R^2 - \mu_\psi^2 - \mu_L^2} ,$$

$$\tan(2\alpha_R) = \frac{2(\mu_\chi \mu_R + \mu_\psi \mu_L)}{\mu_\chi^2 + \mu_L^2 - \mu_\psi^2 - \mu_R^2} , \qquad (4)$$

and the two (positive) mass-squared eigenvalues are given by

$$\mu_{1,2}^2 = \tfrac{1}{2}\Big\{\mu_\chi^2 + \mu_\psi^2 + \mu_L^2 + \mu_R^2 \mp \Big[(\mu_\chi^2 - \mu_\psi^2)^2 + (\mu_L^2 - \mu_R^2)^2 + 2(\mu_\chi^2 + \mu_\psi^2)(\mu_L^2 + \mu_R^2) + 8\mu_\chi\mu_\psi\mu_L\mu_R\Big]^{\frac{1}{2}}\Big\} . \qquad (5)$$

The mass matrix itself is diagonalized by

$$O^T_{(RL)} M O_{(LR)} = O^T_{(LR)} M^T O_{(RL)} = \begin{pmatrix} \mu_1 & 0 & 0 & 0 \\ 0 & \mu_1 & 0 & 0 \\ 0 & 0 & \mu_2 & 0 \\ 0 & 0 & 0 & \mu_2 \end{pmatrix} . \qquad (6)$$

This shows that for $\mu_\psi, \mu_L, \mu_R \ll \mu_\chi$ there is a light state with mass $\mu_1 = O(\mu_\psi, \mu_L, \mu_R)$ and a heavy state with mass $\mu_2 = O(\mu_\chi)$. In general, both the light and heavy states are mixtures of the original fermion and mirror fermion. According to (4), for $\mu_L \neq \mu_R$ the fermion-mirror-fermion mixing angle in the left-handed sector is different from the one in the right-handed sector.

In case of three mirror pairs of fermion families the diagonalization of the mass matrix is in principle similar but, of course, more complicated. A particular class of mixing schemes will be discussed in the next subsection. The mirror fermions can be produced through their mixing to ordinary fermions. The upper limits on the absolute value of mixing angles depend on the mixing scheme.

Indirect limits on the existence of heavy mirror fermions can also be deduced from the absence of observed effects in 1-loop radiative corrections [7,8], because of the non-decoupling of heavy fermions. The question of non-decoupling in higher loop orders is, however, open. In fact, one of the goals of nonperturbative lattice studies is to investigate this in the nonperturbative regime of couplings.

### *2.2. Mixing schemes*

The strongest constraints on mixing angles between ordinary fermions and mirror fermions arise from the conservation of $e$-, $\mu$- and $\tau$- lepton numbers and from the absence of flavour changing neutral currents [5]. In a particular scheme these constraints can be avoided [2]. In this *"monogamous mixing"* scheme the structure of the mass matrix is such that there is a one-to-one correspondence between fermions and mirror fermions, due to the fact that the family structure of the mass matrix for mirror fermions is closely related to the one for ordinary fermions.

Let us denote doublet indices by $A = 1, 2$, colour indices by $c = 1, 2, 3$ in such a way that the leptons belong to the fourth value of colour $c = 4$, and family indices by $K = 1, 2, 3$. In general the entries of the mass matrix for three mirror pairs of fermion families are diagonal in isospin and colour, hence they have the form

$$
\begin{aligned}
\mu_{(\psi,\chi);A_2c_2K_2,A_1c_1K_1} &= \delta_{A_2A_1}\delta_{c_2c_1}\mu^{(A_1c_1)}_{(\psi,\chi);K_2K_1} , \\
\mu_{L;A_2c_2K_2,A_1c_1K_1} &= \delta_{A_2A_1}\delta_{c_2c_1}\mu^{(c_1)}_{L;K_2K_1} , \\
\mu_{R;A_2c_2K_2,A_1c_1K_1} &= \delta_{A_2A_1}\delta_{c_2c_1}\mu^{(A_1c_1)}_{R;K_2K_1} . \qquad (7)
\end{aligned}
$$

The diagonalization of the mass matrix can be achieved for given indices $A$ and $c$ by two $6 \otimes 6$ unitary matrices $F_L^{(Ac)}$ and $F_R^{(Ac)}$ acting, respectively, on the L-handed and R-handed subspaces:

$$
\begin{aligned}
& F_L^{(Ac)\dagger}(M^\dagger M)_L F_L^{(Ac)} , \\
& F_R^{(Ac)\dagger}(M^\dagger M)_R F_R^{(Ac)} . \qquad (8)
\end{aligned}
$$

The main assumption of the "monogamous" mixing scheme is that in the family space $\mu_\psi, \mu_\chi, \mu_L, \mu_R$ are hermitian and simultaneously diagonalizable, that is

$$
F_L^{(Ac)} = F_R^{(Ac)} = \begin{pmatrix} F^{(Ac)} & 0 \\ 0 & F^{(Ac)} \end{pmatrix} , \qquad (9)
$$

where the block matrix is in $(\psi, \chi)$-space. The Kobayashi-Maskawa matrix of quarks is given by

$$
C \equiv F^{(2c)\dagger} F^{(1c)} , \qquad (10)
$$

independently for $c = 1, 2, 3$. The corresponding matrix with $c = 4$ and $A = 1 \leftrightarrow 2$ describes the mixing of neutrinos, if the Dirac-mass of the neutrinos is nonzero. (Majorana masses of the neutrinos are not considered here, but in principle, they can also be introduced.)

A simple example for the "monogamous" mixing is the following:

$$
\begin{aligned}
\mu^{(Ac)}_{\chi;K_2K_1} &= \lambda_\chi^{(Ac)}\mu^{(Ac)}_{\psi;K_2K_1} + \delta_{K_2K_1}\Delta^{(Ac)} , \\
\mu^{(c)}_{L;K_2K_1} &= \delta_{K_2K_1}\delta_L^{(c)} , \\
\mu^{(Ac)}_{R;K_2K_1} &= \lambda_R^{(Ac)}\mu^{(Ac)}_{\psi;K_2K_1} + \delta_{K_2K_1}\delta_R^{(Ac)} , \qquad (11)
\end{aligned}
$$

where $\lambda_\chi^{(Ac)}$, $\Delta^{(Ac)}$, $\delta_L^{(c)}$, $\lambda_R^{(Ac)}$, $\delta_R^{(Ac)}$ do not depend on the family index.

The general case can be parametrized by the eigenvalues $\mu_\psi^{(AcK)}$, $\mu_\chi^{(AcK)}$, $\mu_R^{(AcK)}$, $\mu_L^{(cK)}$ and matrices $F^{(Ac)}$:

$$
\begin{aligned}
\mu^{(Ac)}_{(\psi,\chi,R);K_2K_1} &= \sum_K F^{(Ac)}_{K_2K}\mu^{(AcK)}_{(\psi,\chi,R)}F^{(Ac)\dagger}_{KK_1} , \\
\mu^{(c)}_{L;K_2K_1} &= \sum_K F^{(Ac)}_{K_2K}\mu^{(cK)}_{L}F^{(Ac)\dagger}_{KK_1} . \qquad (12)
\end{aligned}
$$



Here in the second line the left hand side has to be independent of the value of $A = 1, 2$.

The full diagonalization of the mass matrix on the $(\psi_L, \psi_R, \chi_L, \chi_R)$ basis of all three family pairs is achieved by the $96 \otimes 96$ matrix

$$
\mathcal{O}^{(LR)}_{A'c'K',AcK} = \delta_{A'A}\delta_{c'c}F^{(Ac)}_{K'K}
\cdot \begin{pmatrix}
\cos\alpha_L^{(AcK)} & 0 & \sin\alpha_L^{(AcK)} & 0 \\
0 & \cos\alpha_R^{(AcK)} & 0 & \sin\alpha_R^{(AcK)} \\
-\sin\alpha_L^{(AcK)} & 0 & \cos\alpha_L^{(AcK)} & 0 \\
0 & -\sin\alpha_R^{(AcK)} & 0 & \cos\alpha_R^{(AcK)}
\end{pmatrix} . \qquad (13)
$$

$M^\dagger M$ is diagonalized by

$$
\mathcal{O}^{(LR)\dagger} M^\dagger M \mathcal{O}^{(LR)} , \qquad (14)
$$

and $MM^\dagger$ by

$$
\mathcal{O}^{(RL)\dagger} M M^\dagger \mathcal{O}^{(RL)} , \qquad (15)
$$

where $\mathcal{O}^{(RL)}$ is obtained from $\mathcal{O}^{(LR)}$ by $\alpha_L \leftrightarrow \alpha_R$.

In case of $\mu_R = \mu_L$, which happens for instance in (11) if $\lambda_R = 0$ and $\delta_R = \delta_L$, the left-handed and right-handed mixing angles are the same:

$$
\alpha^{(AcK)} \equiv \alpha_L^{(AcK)} = \alpha_R^{(AcK)} . \qquad (16)
$$

In Ref. [2] only this special case was considered. The importance of the left-right-asymmetric mixing was pointed out in Ref. [9], where the constraints arising from the measured values of anomalous magnetic moments were determined. It turned out that for the electron and muon the upper limit is

$$
|\alpha_L\alpha_R| \leq 0.0004 , \qquad (17)
$$

which is much stronger than the limits obtained from all other data [10]:

$$
\alpha_L^2,\ \alpha_R^2 \leq 0.02 . \qquad (18)
$$

In case of the L-R asymmetric mixing the constraint (17) can be satisfied, if either the left- or right-handed mixing exactly vanishes (or is very small): $\alpha_L = 0$ or $\alpha_R = 0$.

### *2.3. Future colliders*

The hypothetical mirror fermions can be discovered at the next generation of high energy colliders. At HERA the first family mirror fermions can be produced via mixing to ordinary fermions up to masses of about 200 GeV, if the mixing angles are close to their present upper limits [11,12]. At SSC and LHC mirror lepton pair production can be observed up to masses of a few hundred GeV [13]. This has the advantage of being essentially independent of the small mixing. At a high energy $e^+e^-$ collider, e. g. LEP-200 or NLC, mirror fermions can be pair produced and easily identified up to roughly half of the total energy, and also produced via mixing almost up to the total energy [14].

## 3. LATTICE SIMULATION OF YUKAWA MODELS

### *3.1. Lattice actions*

The lattice formulation of the electroweak Standard Model is difficult because of the doubler fermions [6]. In fact, at present no completely satisfactory formulation is known [4]: if one insists on explicit chiral gauge invariance, then mirror fermion fields have to be introduced [3], otherwise one has to fix the gauge as in the "Rome-approach" [15].

The situation is different if the $SU(3)_{colour} \otimes U(1)_{hypercharge}$ gauge couplings are neglected. In this case, as a consequence of the pseudo-reality of SU(2) representations, mirror fermions can be transformed to normal fermions by charge conjugation. This allows the gauge invariant lattice formulation of $SU(2)_L$ symmetric models with an even number of fermion doublets. For simplicity, let us also neglect here the $SU(2)_L$ gauge interaction, and consider a chiral Yukawa-model of two fermion doublets. A global $SU(2)_L \otimes U(1)$ symmetric chiral Yukawa-model can be formulated by the lattice action

$$S = S_{scalar} + S_{fermion} , \tag{19}$$

where the pure scalar part in terms of the $2 \otimes 2$ matrix Higgs-boson field $\varphi$ is

$$S_{scalar} = \frac{1}{4} \sum_x \Big\{ m_0^2 \mathrm{Tr}\,(\varphi_x^\dagger \varphi_x) + \lambda \left[\mathrm{Tr}\,(\varphi_x^\dagger \varphi_x)\right]^2 + \sum_\mu [\mathrm{Tr}\,(\varphi_x^\dagger \varphi_x) - \mathrm{Tr}\,(\varphi_{x+\hat\mu}^\dagger \varphi_x)] \Big\} , \tag{20}$$

and the fermionic part with two doublet fields $\psi_{1,2}$ is

$$\begin{aligned} S_{fermion} = \sum_x \Big\{ & \frac{\mu_0}{2} \big[(\psi_{2x}^T \epsilon C \psi_{1x}) - (\psi_{1x}^T \epsilon C \psi_{2x}) \\ & + (\overline{\psi}_{2x} \epsilon C \overline{\psi}_{1x}^T) - (\overline{\psi}_{1x} \epsilon C \overline{\psi}_{2x}^T)\big] \\ & - \tfrac{1}{2} \sum_\mu \big[(\overline{\psi}_{1x+\hat\mu} \gamma_\mu \psi_{1x}) + (\overline{\psi}_{2x+\hat\mu} \gamma_\mu \psi_{2x}) \\ & - \frac{r}{2} \big((\psi_{2x}^T \epsilon C \psi_{1x}) - (\psi_{2x+\hat\mu}^T \epsilon C \psi_{1x}) \\ & - (\psi_{1x}^T \epsilon C \psi_{2x}) + (\psi_{1x+\hat\mu}^T \epsilon C \psi_{2x}) \\ & + (\overline{\psi}_{2x} \epsilon C \overline{\psi}_{1x}^T) - (\overline{\psi}_{2x+\hat\mu} \epsilon C \overline{\psi}_{1x}^T) \\ & - (\overline{\psi}_{1x} \epsilon C \overline{\psi}_{2x}^T) + (\overline{\psi}_{1x+\hat\mu} \epsilon C \overline{\psi}_{2x}^T)\big)\big] \\ & + (\overline{\psi}_{1Rx} G_1 \varphi_x^+ \psi_{1Lx}) + (\overline{\psi}_{1Lx} \varphi_x G_1 \psi_{1Rx}) \\ & + (\overline{\psi}_{2Rx} G_2 \varphi_x^+ \psi_{2Lx}) + (\overline{\psi}_{2Lx} \varphi_x G_2 \psi_{2Rx}) \Big\} . \end{aligned} \tag{21}$$

Here the summations $\sum_\mu$ always go over eight directions of the neighbouring sites, $C$ is the Dirac matrix for charge conjugation and $\epsilon = i\tau_2$ is the antisymmetric unit matrix in isospin space. In the scalar part of the action $m_0^2$ is the bare mass squared and $\lambda$ the bare quartic coupling. In the fermionic part $\mu_0$ is an off-diagonal Majorana mass term, $r$ is the Wilson parameter for removing lattice fermion doublers, which is usually chosen to be $r = 1$, and $G_{(1,2)}$ are diagonal $2 \otimes 2$ matrices in isospin space for the bare Yukawa-couplings:

$$G_{(1,2)} \equiv \begin{pmatrix} G_{(1,2)u} & 0 \\ 0 & G_{(1,2)d} \end{pmatrix} . \tag{22}$$

The global $SU(2)_L$ symmetry is acting on the fields as

$$\begin{aligned} \varphi'_x &= U_L^{-1} \varphi_x , \\ \psi'_{(1,2)Lx} &= U_L^{-1} \psi_{(1,2)Lx} , \\ \overline{\psi}'_{(1,2)Lx} &= \overline{\psi}_{(1,2)Lx} U_L . \end{aligned} \tag{23}$$

The right-handed components of the fermion fields $\psi_{(1,2)Rx}$ and $\overline{\psi}_{(1,2)Rx}$ are, of course, invariant.

The global U(1) symmetry corresponds to the conservation of the difference of the fermion number of $\psi_1$ minus the fermion number of $\psi_2$. (This means that if it is identified by the hypercharge $U(1)_Y$, then $\psi_1$ and $\psi_2$ have opposite hypercharges.)

An equivalent form of the fermionic part of the above action is obtained, if one introduces the mirror fermion fields by charge conjugation:

$$\chi_x \equiv \epsilon^{-1} C \overline{\psi}_{2x}^T , \quad \overline{\chi}_x \equiv \psi_{2x}^T \epsilon C . \tag{24}$$

Since under charge conjugation the left- and right-handed components are interchanged, and $\epsilon^{-1} U_L^T \epsilon = U_L^{-1}$, under $SU(2)_L$ transformations we have

$$\chi'_{Rx} = U_L^{-1} \chi_{Rx} , \quad \overline{\chi}'_{Rx} = \overline{\chi}_{Rx} U_L , \tag{25}$$



and now the left-handed components $\chi_{Lx}$ and $\overline{\chi}_{Lx}$ are invariant. Omitting the index on $\psi_1$, one gets the action in terms of the mirror pair of fermion fields [3]

$$\begin{aligned} S_{fermion} = \sum_x \Big\{ & \mu_0 \left[(\overline{\chi}_x \psi_x) + (\overline{\psi}_x \chi_x)\right] \\ & - \tfrac{1}{2} \sum_\mu \big[(\overline{\psi}_{x+\hat\mu} \gamma_\mu \psi_x) + (\overline{\chi}_{x+\hat\mu} \gamma_\mu \chi_x) \\ & - r \big((\overline{\chi}_x \psi_x) - (\overline{\chi}_{x+\hat\mu} \psi_x) \\ & + (\overline{\psi}_x \chi_x) - (\overline{\psi}_{x+\hat\mu} \chi_x)\big)\big] \\ & + (\overline{\psi}_{Rx} G_\psi \varphi_x^\dagger \psi_{Lx}) + (\overline{\psi}_{Lx} \varphi_x G_\psi \psi_{Rx}) \\ & + (\overline{\chi}_{Lx} G_\chi \varphi_x^\dagger \chi_{Rx}) + (\overline{\chi}_{Rx} \varphi_x G_\chi \chi_{Lx}) \Big\} . \end{aligned} \tag{26}$$

The Yukawa-coupling of the fermion doublet is denoted here by $G_\psi = G_1$, and the Yukawa-coupling of the mirror fermion doublet is $G_\chi = \epsilon^{-1} G_2 \epsilon$. This means that in $G_\chi$ the isospin components are interchanged. The mass term proportional to $\mu_0$ and the off-diagonal Wilson term multiplied by $r$ look in the second form (26) not Majorana-like but Dirac-like.

Note that if the doublets are degenerate, that is the Yukawa-couplings are proportional to the unit matrix in isospin space, then the $SU(2)_L \otimes U(1)$ symmetry is enlarged to $SU(2)_L \otimes SU(2)_R$ defined by

$$\begin{aligned} \varphi'_x &= U_L^{-1} \varphi_x U_R , \\ \psi'_{(L,R)x} &= U_{(L,R)}^{-1} \psi_{(L,R)x} , \\ \overline{\psi}'_{(L,R)x} &= \overline{\psi}_{(L,R)x} U_{(L,R)} , \\ \chi'_{(R,L)x} &= U_{(L,R)}^{-1} \chi_{(R,L)x} , \\ \overline{\chi}'_{(R,L)x} &= \overline{\chi}_{(R,L)x} U_{(L,R)} . \end{aligned} \tag{27}$$

### 3.2. Chirality and decoupling

An important property of the lattice action in the previous subsection is the possibility of decoupling half of the fermions from the interacting sector [16]. Let us formulate this in the mirror fermion langauge corresponding to (26). For vanishing Yukawa-coupling of the mirror fermion doublet $G_\chi = 0$ and fermion mirror fermion mixing mass $\mu_0 = 0$ the action is invariant with respect to the Golterman-Petcher shift symmetry

$$\chi_x \to \chi_x + \epsilon , \quad \overline{\chi}_x \to \overline{\chi}_x + \overline{\epsilon} . \tag{28}$$

This implies [17,18] that all higher vertex functions containing the $\chi$-field identically vanish, and the $\chi$-$\chi$ and $\chi$-$\psi$ components of the inverse propagator are equal to the corresponding components of the free inverse propagator:

$$\tilde{\Gamma}_{\psi\chi} = \mu_0 + \frac{r}{2} \hat{p}^2 , \quad \tilde{\Gamma}_{\chi\chi} = i\gamma \cdot \bar{p} , \tag{29}$$

where, as usual, $\bar{p}_\mu \equiv \sin p_\mu$ and $\hat{p}_\mu \equiv 2 \sin \frac{1}{2} p_\mu$.

The consequence of (29) is that the fermion mirror fermion mixing mass $\mu_0$ is not renormalized by the Yukawa-interaction of the $\psi$-field. This is very useful in numerical simulations, because the corresponding bare parameter (usually the fermionic hopping parameter $K \equiv (2\mu_0 + 8r)^{-1}$) is fixed, and the number of bare parameters to be tuned is less.

There is also another possible interpretation of the fermion decoupling. Namely, interchanging the rôles of $\psi$ and $\chi$, in the case of $G_\psi = \mu_0 = 0$ the ordinary fermions are decoupled. This decoupling scenario is in fact a rather good approximation to the situation in phenomenological models with mirror fermions discussed in the previous section. This is due to the fact that all known physical fermions have very small Yukawa-couplings. The only fermion states with strong Yukawa-coupling would be the members of the mirror families, if they would exist. In fact, the smallness of the known fermion masses on the electroweak scale could then be explained by the approximate validity of the Golterman-Petcher shift symmetry. *Low energy chirality would be the consequence of the approximate decoupling of* *light fermions.* In this sense the mirror fermion model is natural, because the smallness of some of its parameters is connected to an approximate symmetry [19].

Let us shortly discuss the form of the broken Golterman-Petcher identities. They are broken in general by the small Yukawa-coupling $G_\chi$, by small mixing mass $\mu_0$, and by the small $SU(3) \otimes SU(2) \otimes U(1)$ gauge couplings. For definiteness, let us consider here the case of $G_\chi \simeq 0$. Consider the generating function of the connected Green's functions $W[\eta, \overline{\eta}, \zeta, \overline{\zeta}, j]$, where the external sources $\eta, \zeta, j$ belong, respectively, to $\psi, \chi, \varphi$. The identities obtained by shifting the $\chi$- and $\overline{\chi}$-fields are

$$\overline{\zeta}_x + \tfrac{1}{2}\sum_{\mu=1}^{4}(\Delta^f_\mu + \Delta^b_\mu)\frac{\partial W}{\partial \zeta_x}\gamma_\mu$$
$$+\frac{r}{2}\sum_{\mu=1}^{4}\Delta^f_\mu\Delta^b_\mu\frac{\partial W}{\partial \eta_x} = \mu_0\frac{\partial W}{\partial \eta_x}$$
$$+G_\chi\left(\frac{\partial^2 W}{\partial j_{Rx}\partial\zeta_x} + \frac{\partial W}{\partial j_{Rx}}\frac{\partial W}{\partial \zeta_x}\right)\Gamma^\dagger_R ,$$
$$-\zeta_x - \tfrac{1}{2}\sum_{\mu=1}^{4}\gamma_\mu(\Delta^f_\mu + \Delta^b_\mu)\frac{\partial W}{\partial \overline{\zeta}_x}$$
$$+\frac{r}{2}\sum_{\mu=1}^{4}\Delta^f_\mu\Delta^b_\mu\frac{\partial W}{\partial \overline{\eta}_x} = \mu_0\frac{\partial W}{\partial \overline{\eta}_x}$$
$$+ G_\chi\left(\frac{\partial^2 W}{\partial j_{Rx}\partial\overline{\zeta}_x} + \frac{\partial W}{\partial j_{Rx}}\frac{\partial W}{\partial \overline{\zeta}_x}\right)\Gamma^\dagger_R . \qquad (30)$$

Here $\Delta^f_\mu$ and $\Delta^b_\mu$ denote, respectively, forward and backward lattice derivatives, and real scalar field components $\phi_R$ ($R = 0, 1, 2, 3$) are introduced by

$$\tfrac{1}{2}\left[\varphi_x(1+\gamma_5) + \varphi^\dagger_x(1-\gamma_5)\right]$$
$$= \phi_{0x} + i\gamma_5\tau_r\phi_{rx} \equiv \Gamma_R\phi_{Rx} . \qquad (31)$$

Let us define the composite fermion field $\Psi_x$ by

$$\Gamma_R\phi_{Rx}\chi_x = \varphi_x\chi_{Lx} + \varphi^\dagger_x\chi_{Rx} \equiv \Psi_x . \qquad (32)$$

The mixed $\psi$-$\Psi$ two point function is

$$\langle\psi_y\Psi_x\rangle \equiv \frac{1}{\Omega}\sum_k e^{ik\cdot(y-x)}\tilde{\Delta}^{\psi\Psi}_k . \qquad (33)$$

Taking derivatives at zero sources, with the notation $\langle\phi_{0x}\rangle \equiv v$ for the vacuum expectation value one obtains, for instance,

$$0 = (\mu_0 + \frac{r}{2}\hat{k}^2)\tilde{\Delta}^{\psi\psi}_k + \tilde{\Delta}^{\psi\chi}_k(i\gamma\cdot\bar{k} + G_\chi v) + G_\chi\tilde{\Delta}^{\psi\Psi}_k ,$$
$$1 = (\mu_0 + \frac{r}{2}\hat{k}^2)\tilde{\Delta}^{\chi\psi}_k$$
$$+ \tilde{\Delta}^{\chi\chi}_k(i\gamma\cdot\bar{k} + G_\chi v) + G_\chi\tilde{\Delta}^{\chi\Psi}_k . \qquad (34)$$

For $G_\chi = 0$ this is equivalent to (29). The case of small gauge couplings can be treated similarly.

### *3.3. A simple $SU(2)_L \otimes SU(2)_R$ model*

The lattice actions of Yukawa-models in the form (21) or (26) can be used for numerical simulation studies of chiral Yukawa-models. In order to apply Monte Carlo simulation methods one needs, however, a fermion determinant which is positive. For instance, in the Hybrid Monte Carlo algorithm [20] one has to duplicate the number of fermionic degrees of freedom. Let us denote the fermion matrix corresponding to (26) by $Q$, then the replica fermions have $Q^\dagger$, and the fermion determinant is $\det(QQ^\dagger)$, which is positive. Due to the adjoint, for the replica fermions $\psi_x$ describes a mirror fermion doublet and $\chi_x$ an ordinary fermion doublet. By charge conjugation as in (24) one can transform the $\psi$-field of replica fermions to an ordinary doublet, and the $\chi$-field of replica fermions to a mirror doublet. In this way one can consider two doublets described by the $\psi$-fields and two mirror doublets described by the $\chi$-fields. For simplicity, let us consider only degenerate doublets, that



is, let the Yukawa-couplings $G_{(\psi,\chi)}$ be proportional to the unit matrix in isospin space. In this case the Hybrid Monte Carlo simulation describes two equal mass degenerate doublets plus two equal mass degenerate mirror doublets with exact global $SU(2)_L \otimes SU(2)_R$ symmetry. In the phase with spontaneously broken symmetry the mass of the doublets is proportional to $G_\psi$, and the mass of the mirror doublets to $G_\chi$.

As it was noted in the previous subsection, the limit $G_\chi = \mu_0 = 0$ is particularly interesting, because it has less bare parameters to tune. In this case the $\chi$-fields are exactly decoupled, and one is left in Hybrid Monte Carlo with two degenerate fermion doublets described by the $\psi$-fields. This is the simplest model of heavy fermion doublets one can simulate by present day fermionic simulation techniques [21]. Besides the two bare parameters of the pure scalar sector $(m_0^2, \lambda)$ there is only one additional bare parameter $(G_\psi)$ in the fermionic part of the action:

$$S_{fermion} = \sum_x \left\{ -\tfrac{1}{2}\sum_\mu \left[(\overline{\psi}_{x+\hat{\mu}}\gamma_\mu\psi_x)\right.\right.$$
$$+(\overline{\chi}_{x+\hat{\mu}}\gamma_\mu\chi_x)$$
$$\left. - \left((\overline{\chi}_x\psi_x) - (\overline{\chi}_{x+\hat{\mu}}\psi_x) + (\overline{\psi}_x\chi_x) - (\overline{\psi}_{x+\hat{\mu}}\chi_x)\right)\right]$$
$$\left. +G_\psi\left[(\overline{\psi}_{Rx}\varphi^+_x\psi_{Lx}) + (\overline{\psi}_{Lx}\varphi_x\psi_{Rx})\right]\right\} . \qquad (35)$$

To have the smallest possible number of parameters is very important, in order to keep parameter tuning as easy as possible.

By a further duplication of the fermion fields one can also simulate four degenerate fermion doublets, which correspond to a heavy degenerate family. The $SU(4)_{Pati-Salam}$ symmetry [22] of the four $\psi$-doublets, including $SU(3)_{colour}$ for the quarks, is exact in the continuum limit, but for finite lattice spacings it is broken by the off-diagonal Wilson-terms which mix the $\psi$- and $\chi$-fields. Note, however, that there is an exact $SU(4)$ symmetry also at finite lattice spacings, if one transforms the $\psi$- and $\chi$-fields simultaneously. Of course, the $\chi$'s are mirror fermion fields, which mix with the $\psi$'s through the nonzero $\tilde{\Gamma}_{\psi\chi}$ in (29). This mixing goes to zero only in the continuum limit. The decoupling in the continuum limit is exact in the Yukawa-model, but for nonzero gauge couplings decoupling the $\chi$'s in a gauge invariant way does not work.

Another way of simulating a heavy degenerate fermion family with only two pairs of $(\psi, \chi)$-doublet fields is to choose $G_\chi = \pm G_\psi$ (the opposite sign is preferred by the study of the $K = 0$ limit [23]). Since without $SU(3) \otimes U(1)$ gauge fields the mirror fermion doublets are equivalent to ordinary fermion doublets, this describes the same model in the continuum limit as the one with twice as much fields and decoupling. In this case, however, the fermion hopping parameter $K$ has to be tuned, too, which can be worse than having more field components per lattice sites.

### *3.4. Phase structure*

The first step in a recent numerical simulation of the $SU(2)_L \otimes SU(2)_R$ symmetric Yukawa-model with $N_f = 2$ fermion doublets in the decoupling limit $G_\chi = 0$ [21] was to check the phase structure at infinite bare quartic coupling $\lambda = \infty$. On the basis of experience in several different lattice Yukawa models [24–26], this is expected to possess several phase transitions between the *"ferromagnetic" (FM)*, *"antiferromagnetic" (AFM)*, *"paramagnetic" (PM)* and *"ferrimagnetic" (FI)* phases. The resulting picture in the $(G_\psi, \kappa)$-plane is shown in fig. 1. ($\kappa \equiv (1-2\lambda)/(m_0^2+8)$ is the bare parameter which is usually taken in numerical simulations instead of $m_0^2$.)

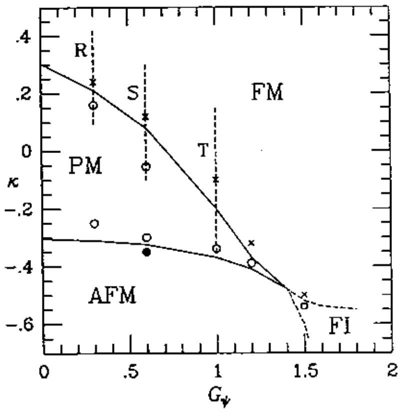


**Fig. 1. Phase structure of the $SU(2)_L \otimes SU(2)_R$ symmetric Yukawa model at $\lambda = \infty$ and $G_\chi = \mu_0 = 0$ in the $(G_\psi, \kappa)$-plane. Open circles denote points in the PM phase, crosses represent points in the FM phase. The points in the AFM and FI phases are denoted by full circles and open squares, respectively. The dashed lines labelled R,S,T each show the range of $\kappa$ used for a systematic scan of renormalized parameters at fixed $G_\psi$. The crosses along those lines denote the kappa values where the minimum scalar mass in the broken phase is encountered. Solid lines connect the critical values for $\kappa$ estimated from the behaviour of the magnetization on $4^3 \cdot 8$. Dashed lines around the FI phase show the expected continuation of the critical lines.**

### *3.5. Allowed region in renormalized couplings*

An important question for numerical simulations is the determination of the nonperturbative cut-off dependent *allowed region* in the space of renormalized quartic and Yukawa-couplings. If the continuum limit of Yukawa-models is trivial, then there are cut-off dependent upper bounds on both the renormalized quartic and Yukawa-couplings, which tend to zero in the continuum limit. In pure $\phi^4$ models the upper bound is qualitatively well described by the 1-loop perturbative $\beta$-function, if the Landau-pole in the renormalization group equations is assumed to occur at the scale of the cut-off. The same might be true for scalar-fermion models with Yukawa-couplings. For instance, in the model with $SU(2)_L \otimes SU(2)_R$ symmetry and $N_f$ degenerate fermion doublets the 1-loop $\beta$-functions for the quartic ($g_R$) and Yukawa- ($G_{R\psi}$) couplings are:

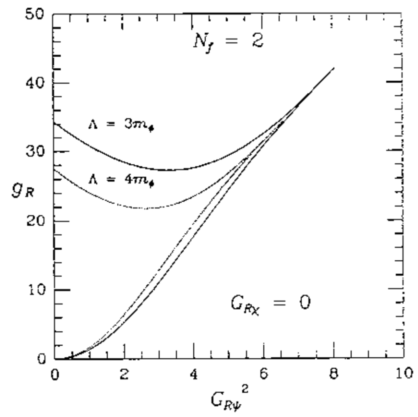


Fig. 2. The cut-off dependent allowed region in the $(G_{R\psi}^2, g_R)$ plane for cut-off values equal to some multiples of the Higgs-boson mass $m_\phi \equiv m_{R\sigma}$. In the Yukawa-model describing two degenerate heavy fermion doublets without gauge couplings the perturbative 1-loop $\beta$-functions are assumed.

$$\beta_{g_R} = \frac{1}{16\pi^2}\left(4g_R^2 + 16N_f g_R G_{R\psi}^2 - 96N_f G_{R\psi}^4\right) ,$$

$$\beta_{G_{R\psi}} = \frac{1}{16\pi^2} \cdot 4N_f G_{R\psi}^3 . \qquad (36)$$



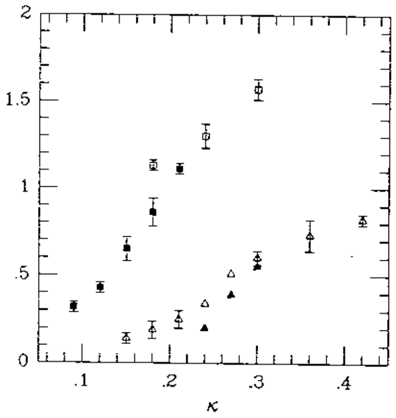


Fig. 3. The fermion mass in lattice units $\mu_{R\psi}$ plotted versus $\kappa$ for $G_\psi = 0.3$ (triangles) and $G_\psi = 0,6$ (squares) on lattices of size $4^3 \cdot 8$ (open symbols) and $6^3 \cdot 12$ (filled-in symbols). Errorbars are omitted when the variation is of the size of the symbols. It is seen that larger bare couplings $G_\psi$ in general yield larger fermion masses.

Since in the region where $G_{R\psi}^2 \gg g_R$ the 1-loop $\beta$-function of the quartic coupling $\beta_{g_R}$ is negative, besides the upper bounds there is also a lower bound on $g_R$ for fixed $G_{R\psi}$, which is called in the literature *vacuum stability bound* [27]. On the lattice, if one assumes the qualitative behaviour of the 1-loop $\beta$-function to be valid also nonperturbatively, the vacuum stability lower bound occurs at zero bare quartic coupling $\lambda = 0$, whereas the upper bound at $\lambda = \infty$ [28–30]. Negative $\lambda$-values are excluded, because there the path integral is divergent. For the 1-loop $\beta$-functions in (36) with $N_f = 2$ the bounds in the plane of $(G_{R\psi}^2, g_R)$ are shown in fig. 2.

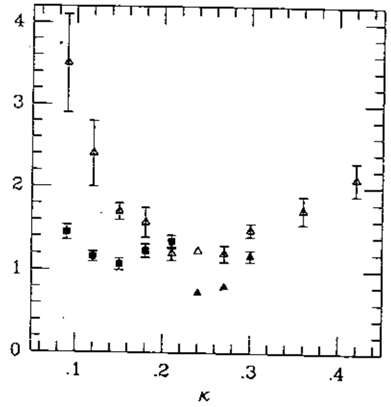


**Fig. 4. The scalar mass in lattice units $m_{R\sigma}$ plotted versus $\kappa$. The explanation of symbols is similar to the previous figure. When approaching the phase transition the scalar masses increase again after going through a minimum.**

These curves have to be confronted with the results of the numerical simulations. It turned out (see fig. 3) that on $4^3 \cdot 8$ and $6^3 \cdot 12$ lattices the fermion mass in lattice units tends to zero, if one is approaching the FM-PM phase transition from above (i. e. from the FM-side). This is as expected in the continuum limit on an infinite lattice.

The behaviour of the Higgs-boson mass $m_{R\sigma}$ on finite lattices is more involved. This is because the decrease of $m_{R\sigma}$ is stopped by a minimum, and instead of a further decrease there is an increase (see fig. 4). This can be understood as a finite size effect. The value at the minimum is smaller on larger lattices. Nevertheless, for increasing bare Yukawa-coupling $G_\psi$ the required lattice size is growing. Reasonably small masses

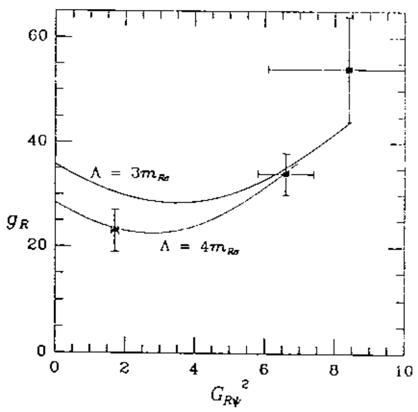


Fig. 5. The obtained numerical estimate of the upper bound on the renormalized quartic coupling $g_R$ (or Higgs-boson mass $m_{R\sigma}$) as a function of the renormalized Yukawa-coupling squared $G^2_{R\psi}$. Two points with the smaller errors are on $6^3 \cdot 12$, the third one on $8^3 \cdot 16$ lattice. The lines are the upper bounds according to 1-loop perturbation theory at a cut-off $\Lambda \simeq 3m_{R\sigma}$ and $4m_{R\sigma}$, respectively.

$m_{R\sigma} \simeq 0.5 - 0.7$ can be achieved at $G_\psi = 0.3$ on $6^3 \cdot 12$, at $G_\psi = 0.6$ on $8^3 \cdot 16$ lattices [21]. At $G_\psi = 1.0$ presumably lattices with spatial extension of at least $16^3$ are necessary. Taking the values of the Higgs-boson mass at $\kappa$-values above the minimum, one obtains for the upper limit on the renormalized quartic coupling the estimates in fig. 5. These agree within errors with the 1-loop estimates, although the values of the renormalized couplings are close to the tree unitarity limits. The continuation of the upper bound on $g_R$ towards larger Yukawa-couplings can only be obtained on larger lattices. Particularly interesting is the behaviour in the vicinity of the FM-FI phase transition near $G_\psi = 1.0 - 1.5$. A further interesting question is the phase structure near $\lambda \simeq 0$, which has an influence on the vacuum stability lower bound.

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