

Infrared sensitivity of screening and damping in a quark-gluon plasma

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All the next-to-leading order contributions to the quasiparticle dispersion laws of a quark-gluon plasma, which, because of infrared singularities, are sensitive to the magnetic-mass scale, are calculated using Braaten-Pisarski resummation. These relative-order- $g\ln(g)$ corrections are shown here to contribute generally to the dynamical screening of gluonic fields with frequencies below the plasma frequency as well as to the damping of propagating gluonic and fermionic quasiparticles. In the limit of a vanishing wave vector the infrared singularities disappear, but in a way that raises the possibility for formally higher orders of the Braaten-Pisarski scheme to contribute equally at next-to-leading order when the wave vector is of the order of or less than the magnetic-mass scale. This is argued to be a problem in particular for the fermionic damping rate.

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I. INTRODUCTION

The leading-order results for the dispersion laws of quasiparticles in a quark-gluon plasma [1] are well known and are readily obtained from the high-temperature limit of one-loop Green's functions or from solving the collisionless Boltzmann equation [2]. However, anything beyond the leading terms becomes accessible only through a resummation of the conventional perturbation series.

A systematically improved perturbation theory, which turns out to involve single powers of the QCD coupling constant g rather than the usual g^2 , has been established some years ago by Braaten and Pisarski [3]. It is based on a resummation of all the leading-order self-energies and vertex functions furnished by the so-called hard thermal loops [3, 4]. In a first application this led to the solution of the long-standing problem [5] of how to calculate the damping constant of the lowest QCD plasmon excitation with a vanishing wave vector in a gauge-invariant way [6]. In a later work it was found that with a nonzero wave vector the damping of gluonic as well as fermionic excitations is infrared divergent with a logarithmic singularity which can be cut off by a finite screening mass for static magnetic fields. This gives rise to a contribution proportional to $\ln(m_{\text{el}}/m_{\text{magn}}) \sim \ln(1/g)$ whose coefficient is calculable perturbatively [7–11]. Similar singularities have also been found recently in the next-to-leading-order calculation of the non-abelian Debye screening mass [12, 13] from the pole of the static gluon propagator (at imag-

inary wave vectors) as well as in the perturbative evaluation of the correlator of Polyakov loops [13, 14].

Such a sensitivity to the magnetic mass scale comes as a surprise since by superficial infrared power counting [15] one would not expect it already at (resummed) one-loop order. It is in fact due to the necessity to evaluate the loop diagrams at the position of the pole of the leading-order propagator, which leads to “mass-shell” singularities in the presence of the massless magnetostatic mode. Indeed, these singularities appear also in the case of QED, where no magnetic screening mass can be generated. Presumably the finite width γ of the full propagators is also able to provide the required infrared cutoff [8], which would again lead to $\ln(m_{\text{el}}/\gamma) \sim \ln(1/g)$, or $\ln(1/e)$ in the case of QED.

In this work we shall present a unified treatment of all these next-to-leading-order corrections to gluonic and fermionic dispersion laws which are infrared singular due to mass-shell singularities (Sec. II) and therefore sensitive to the “magnetic-mass scale,” by which we mean simply the scale of new physics that acts as an effective infrared cutoff, be it an actual magnetic mass or the shielding of the singularities through a finite width of the full propagators or another mechanism. For moving quasiparticle excitations we reproduce the results of Pisarski [11] who found that the infrared-singular contributions are proportional to the group velocity of the respective modes. For the case of dynamical screening of perturbations with frequencies below the plasma frequency we find direct proportionality to the modulus of the (now imaginary) wave vector (Secs. III and IV). We also take a somewhat closer look (Sec. V) at the limit of a vanishing wave vector where the infrared singularities seem to be absent and we shall argue that for wave vectors

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of the order of or smaller than the magnetic-mass scale these infrared singularities could still leave non-negligible imprints which would render the resummation scheme of Braaten and Pisarski incomplete in this limit.

II. QUASIPARTICLE MASS-SHELL SINGULARITIES

The leading-order finite-temperature corrections to the gluon and fermion self-energies [1] give rise to effective (albeit momentum-dependent) thermal masses of the order gT , where g is the QCD coupling constant and T the temperature. Because of a nontrivial tensor structure, they give rise to different dispersion laws for spatially longitudinal and transverse gluons according to

$$\begin{aligned} Q^2 &= \Pi_\ell(Q_0, q) \\ &= -\frac{Q^2}{q^2} \Pi_{00} = 3m^2 \left(1 - \frac{Q_0^2}{q^2}\right) \\ &\quad \times \left[1 - \frac{Q_0}{2q} \ln \left(\frac{Q_0 + q}{Q_0 - q}\right)\right], \end{aligned} \quad (1)$$

$$\begin{aligned} Q^2 &= \Pi_t(Q_0, q) \\ &= \frac{1}{2} (3m^2 - \Pi_\ell), \end{aligned} \quad (2)$$

where $\Pi_{\mu\nu}$ is the gluon self-energy and $m^2 = (N + N_f/2)(gT/3)^2$ for gauge group $SU(N)$ with N_f flavors. For fermions (whose zero-temperature rest mass we assume to be $\ll gT$), there are also two different modes at finite temperature from the solution of

$$\det(Q - \Sigma) = 0, \quad (3)$$

where Σ is the fermion self-energy. Equation (3) has two solutions for positive q , which are given by

$$Q_0 = \pm q + \Pi_\pm(Q_0, q) \quad (4)$$

$$\text{with } \Pi_\pm = \frac{M^2}{2q} \left[\left(1 \mp \frac{Q_0}{q}\right) \ln \left(\frac{Q_0 + q}{Q_0 - q}\right) \pm 2 \right] \quad (5)$$

and $M^2 = C_F(gT)^2/8$, $C_F = (N^2 - 1)/(2N)$.

In Fig. 1, the dispersion laws $Q_0 = \omega(q)$ of the above modes are shown as curves in the ω^2, q^2 plane. For real q , $q^2 > 0$, they correspond to propagating quasiparticles; for $q^2 < 0$, they describe the screening of fields oscillating below the plasma frequency, where $|q|$ is the inverse screening length.

The physical significance of the various modes have been discussed in full length by Weldon [1], to which we refer for further detail. Let us just mention here that the spatially longitudinal gluonic excitation (which is often referred to as *the plasmon*) and the fermionic one corresponding to the lower sign in Eq. (4) (sometimes called the *plasmino*) have no counterpart at zero temperature. They can only be understood as collective phenomena.

If one attempts to go beyond the leading-order results just presented, one has to face the problem that the conventional, bare perturbation theory becomes insufficient whenever the loop momenta probe the new ("soft") scale $\sim m, M$. This is true in particular for the next-to-leading-order terms in Eqs. (1),(2),(4). An improved perturbation theory, due to Braaten and Pisarski [3], requires a resummation of both the leading-order contributions to the various self-energies and also to the vertices, collectively termed "hard thermal loops." However, in a perturbative treatment of the solutions to Eqs. (1),(2),(4) one has to evaluate the correction terms to the self-energies at the location of the poles of the leading-order propagators, which in the presence of massless modes generally will give rise to "mass-shell" singularities. Indeed, while all the propagating physical modes have nonvanishing effective masses, the thermal mass of the transverse gluon propagator vanishes in the static limit, corresponding to the absence of screening of static chromomagnetic fields up to distances $\sim 1/(gT)$.

Consider the following integral which appears in the resummed one-loop contribution to the self-energy of mode i , where in the loop integral the propagator for mode i is coupled to the one of transverse gluons:

$$\begin{aligned} \mathcal{S}_i(Q_0, q) &= \sum_{P_0=2\pi i n T} \int \frac{d^3 p}{(2\pi)^3} \Delta_t(P) \Delta_i(Q - P) \Big|_{\Delta_i^{-1}(Q)=0} \\ &\equiv \sum_{P_0=2\pi i n T} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{P^2 - \Pi_t(P)} \frac{1}{(Q - P)^2 - \Pi_i(Q - P)} \Big|_{Q^2 = \Pi_i(Q)}. \end{aligned} \quad (6)$$

For gluons, i either means t or ℓ ; for fermions the second propagator in (6) has the form

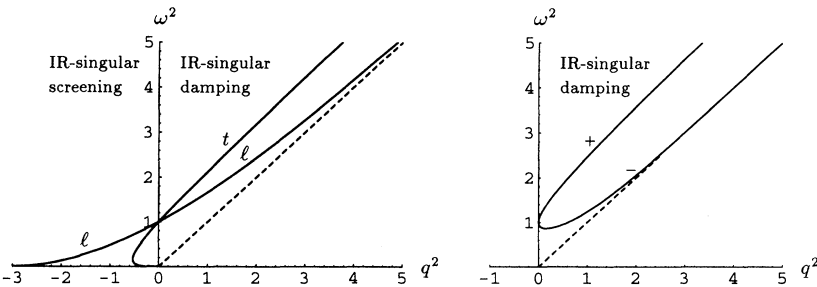


FIG. 1. The leading-order gluonic and fermionic dispersion curves and their infrared singularities at subleading order. For the transverse (t) and longitudinal (ℓ) dispersion curves of the gluonic modes ω^2 and q^2 are given in units of m^2 ; for the quark (+) and plasmino (−) modes the unit is M^2 .

$$\frac{1}{[K_0 - k - \Pi_+(K)][K_0 + k - \Pi_-(K)]} \equiv \Delta_f(K), \quad (7)$$

with $K \equiv Q - P$.

The $n = 0$ contribution in the above sum comes with a vanishing $\Pi_t(P_0 = 0, p) = 0$, and for small p the second propagator is $\sim 1/(\mathbf{p} \cdot \mathbf{q})$ on the mass shell of mode i , so this contribution involves a logarithmic singularity on mass shell. Concentrating on the infrared-singular part, we therefore have

$$\begin{aligned} \mathcal{S}_i(Q_0, q) &\simeq T \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p^2} \frac{1}{p^2 - 2\mathbf{p} \cdot \mathbf{q} + \Pi_i(Q - P) - \Pi_i(Q) - i\epsilon\sigma} \Big|_{P_0=0} \\ &\simeq T [1 + \partial_{q^2} \Pi_i(Q_0, q)]^{-1} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p^2} \frac{1}{p^2 - 2\mathbf{p} \cdot \mathbf{q} - i\epsilon\sigma'}, \end{aligned} \quad (8)$$

where here and in the following \simeq indicates that we have dropped regular terms. In the last line of Eq. (8) we have used that the difference of the two Π_i when taken at $P_0 = 0$ is a function of Q and $p^2 - 2\mathbf{p} \cdot \mathbf{q}$. Only the first term of its Taylor series with respect to the latter variable is relevant for the infrared singular contribution; the others lead to regular integrals of the form $\int d^3 p p^{-2} (p^2 - 2\mathbf{p} \cdot \mathbf{q})^n$, $n \geq 0$. In Eq. (8) σ is a sign fixed in accordance to retarded boundary conditions by $Q_0 \rightarrow \omega + i\epsilon$:

$$\sigma = \text{sgn}(\omega [1 - \partial_{\omega^2} \Pi_i(\omega, q)]) \quad (9)$$

which leads to

$$\sigma' = \text{sgn}(\omega [1 - \partial_{\omega^2} \Pi_i(\omega, q)] / [1 + \partial_{q^2} \Pi_i(\omega, q)]) = \text{sgn}(v_i), \quad (10)$$

with v_i the group velocity $d\omega/dq$ of mode i . Note that $-\omega^2 \partial_{\omega^2} = q^2 \partial_{q^2}$ since $\Pi_i(\omega, q)$ depends only on ω/q .

In order to give meaning to the singular expression (8), we introduce an infrared cutoff $\lambda \ll gT$ for p :

$$\mathcal{S}_i(\omega, q; \lambda) = \frac{T}{4\pi^2} [1 + \partial_{q^2} \Pi_i(\omega, q)]^{-1} \int_{\lambda}^{\infty} \frac{dp}{p} \frac{1}{2q} \ln \left(\frac{p + 2q - i\epsilon\sigma'}{p - 2q - i\epsilon\sigma'} \right), \quad (11)$$

which has a regular real part but a singular imaginary part, where λ cannot be removed:

$$\mathcal{S}_i(\omega, q; \lambda) = i\sigma' \frac{T}{8\pi q} [1 + \partial_{q^2} \Pi_i]^{-1} \ln \left(\frac{q}{\lambda} \right) + O(\lambda^0). \quad (12)$$

For frequencies below the plasma frequency m , where the gluonic dispersion laws describe screening corresponding to poles at purely imaginary values of $q = \pm i|q|$, we have, instead,

$$\begin{aligned} \mathcal{S}_i(\omega, q = \pm i|q|; \lambda) &= \frac{T}{4\pi^2} [1 + \partial_{q^2} \Pi_i]^{-1} \int_{\lambda}^{\infty} \frac{dp}{p} \frac{1}{|q|} \arctan \left(\frac{2|q|}{p} \right) \\ &= \frac{T}{8\pi|q|} [1 + \partial_{q^2} \Pi_i]^{-1} \ln \left(\frac{|q|}{\lambda} \right) + O(\lambda^0), \end{aligned} \quad (13)$$

which now has a singular *real* part.

The results (12), (13) are only valid when $|q| \gg \lambda$. In the limit $|q| \rightarrow 0$, \mathcal{S} is in fact linearly divergent rather than only logarithmically, which makes \mathcal{S} proportional to $1/\lambda$:

$$\mathcal{S}_i(\omega, q; \lambda) \rightarrow \frac{T}{4\pi^2 \lambda} [1 + \partial_{q^2} \Pi_i]^{-1} + O\left(\frac{Tq}{\lambda^2}\right) \quad \text{when } |q| \ll \lambda. \quad (14)$$

In the applications below it will turn out that all the infrared singular contributions to the next-to-leading-order self-energies involve rational functions of the momenta times the paradigmatic expression \mathcal{S} such that the limit $q \rightarrow 0$ is infrared finite and, apparently, the linear divergence in (14) is defused. However, we shall find reason to reassess the case $q \rightarrow 0$ more cautiously later, so let us for the time being restrict our attention to the case of a nonzero q .

With nonzero $q \sim gT$, the infrared singularities are only logarithmic and it suffices to assume that an actual infrared cutoff will be generated at some scale $\mu \sim g^x T$, $x > 1$ to make sense of the calculations. The technical cutoff λ can then be identified with μ as far as the logarithmic term is concerned, which turns into $(x-1)\ln(1/g)$. We expect $\mu \sim g^2 T$ and shall refer to it loosely as the magnetic-mass scale, but actually we mean simply the mass scale where new (infrared) physics eventually removes the mass-shell singularities of Eq. (8). This may be through the nonperturbative generation of a screening mass for static chromomagnetic fields or something else, e.g., the finite width of the true quasiparticle excitations [8], which, as we shall see, is again of the order of $g^2 T$ [up to logarithms $\ln(g)$].

At any rate, this assumption fixes the term involving $\ln(1/g)$, while the finer details of the physics pertaining to the supersoft scales, which are outside the scope of this paper, will be relevant only for the subleading contributions.

III. GLUON SELF-ENERGY

In resummed perturbation theory, the one-loop correction to the gluon self-energy is usually a rather complicated object, since also the vertices have to be dressed. For a nonzero external wave vector the resulting algebra is considerable. Some of its properties have recently been studied by two of the present authors [16]. Fortunately, when concentrating on the infrared-singular parts, this algebra can be greatly simplified and we shall in the following describe only the shortcut to obtaining them. In an appendix we display the next-to-leading-order self-energy in full detail, from which the following results can be straightforwardly reproduced, albeit in a less transparent manner.

First notice that only the diagrams containing two propagators are able to produce mass-shell singularities, of which one has vanishing four-momentum. The most singular terms are those that have no further loop momenta in the numerators, so all the momentum algebra factorizes into a prefactor to singular integrals of the form (6). In particular, only the part of the complicated dressed vertices contributes where one leg has vanishing momentum. Thanks to the gauge invariance of the hard-thermal-loop effective action [3], these vertices are determined by differential Ward identities,

$$*\Gamma_{\mu\alpha\sigma}(-Q, Q, 0) = \partial_{Q^\sigma} * \Pi_{\mu\alpha}(Q) , \quad (15)$$

where the vertex $*\Gamma$ is the sum of the bare and the hard-thermal-loop vertex and $*\Pi$ the sum of the bare kinetic term and the hard-thermal-loop self-energy :

$$\begin{aligned} * \Pi^{\mu\nu} &= Q^\mu Q^\nu - Q^2 g^{\mu\nu} + \Pi^{\mu\nu} \\ &= (\Pi_t - Q^2) A^{\mu\nu} + (\Pi_\ell - Q^2) B^{\mu\nu} . \end{aligned} \quad (16)$$

Here A is the projector on spatially transverse vectors [see (17) below] and $B^{\mu\nu} = g^{\mu\nu} - Q^\mu Q^\nu / Q^2 - A^{\mu\nu}$.

The full next-to-leading-order gluon self-energy is in general gauge-parameter dependent and nontransverse, but one can show on an algebraic level that the gauge parameter drops out from $\delta\Pi_{t,\ell}$ on the respective mass shells and also that $\delta\Pi_{\mu\nu}$ is transverse (in the four-dimensional sense) on the longitudinal mass shell [16]. This on-shell gauge independence allows us to further simplify the algebra by using Feynman gauge, where indeed only the integral (6) arises, and from on-shell transversality we have

$$\delta\Pi_t(Q) = \frac{1}{2} A^{\mu\nu} \delta\Pi_{\mu\nu} = -\frac{1}{2} \left(\delta_{mn} - \frac{q_m q_n}{q^2} \right) \delta\Pi_{mn} , \quad (17)$$

$$\delta\Pi_\ell(Q) = -\frac{Q^2}{q^2} \delta\Pi_{00} . \quad (18)$$

The whole algebra is now a matter of a few lines only:

$$\begin{aligned} \delta\Pi_t(Q) &\simeq -\frac{g^2 N}{2} \left(\delta_{mn} - \frac{q_m q_n}{q^2} \right) * \Gamma_{mai}(-Q, Q, 0) \\ &\quad \times * \Gamma_{ani}(-Q, Q, 0) \mathcal{S}_t(Q) \\ &= -g^2 N (\partial_{q_i} \Delta_t^{-1})^2 \mathcal{S}_t(Q) \\ &= -g^2 N 4q^2 [1 + \partial_{q^2} \Pi_t(\omega, q)]^2 \mathcal{S}_t(Q) \end{aligned} \quad (19)$$

and

$$\begin{aligned} \delta\Pi_\ell(Q) &\simeq -g^2 N \left(\frac{Q^2}{q^2} \right)^2 * \Gamma_{00i}^2(-Q, Q, 0) \mathcal{S}_\ell(Q) \\ &= -g^2 N (\partial_{q_i} \Delta_\ell^{-1})^2 \mathcal{S}_\ell(Q) \\ &= -g^2 N 4q^2 [1 + \partial_{q^2} \Pi_\ell(\omega, q)]^2 \mathcal{S}_\ell(Q) . \end{aligned} \quad (20)$$

This exhibits the prominent role played by the factors \mathcal{S}_i , $i = t, \ell$, studied in the previous section. As they have no singular real parts at positive q^2 and no singular imaginary part at $q^2 < 0$, (19) and (20) tell us that there is *no* danger from the infrared in the corresponding parts of the polarization functions at next-to-leading order. For the singular parts, on the other hand, \mathcal{S} contains in one simple expression the essentials on both next-to-leading-order damping and screening for $q^2 > 0$ and $q^2 < 0$, respectively (see Fig. 1).

Let us first consider the consequences for propagating quasiparticle excitations which have $\omega > m$ and $q \sim m \gg \mu$. Inserting the on-shell values of the derivatives of the various Π_i we obtain

$$\begin{aligned} \delta\Pi_t &\simeq -ig^2 NT \frac{q}{2\pi} [1 + \partial_{q^2} \Pi_t(\omega, q)] \ln \left(\frac{1}{g} \right) \\ &= -ig^2 NT \frac{3}{4\pi} \left(\frac{m^2 \omega^2}{Q^2} - Q^2 \right) \frac{1}{q} \ln \left(\frac{1}{g} \right) , \end{aligned} \quad (21)$$

$$\begin{aligned} \delta\Pi_\ell(Q) &\simeq -ig^2 NT \frac{q}{2\pi} [1 + \partial_{q^2} \Pi_\ell(\omega, q)] \ln \left(\frac{1}{g} \right) \\ &= -ig^2 NT \frac{3}{4\pi} (m^2 - Q^2) \frac{1}{q} \ln \left(\frac{1}{g} \right) , \end{aligned} \quad (22)$$

where in evaluating \mathcal{S} we have assumed $m/\mu \sim 1/g$ and used that $\sigma' = +1$ for both modes.

In order to derive corrections to the dispersion laws, one has to take into account that the leading-order $\Pi_{t,\ell}$ also varies when $\omega_{t,\ell}(q)$ changes. Expanding the condition $\omega_{t,\ell}^2(q) = q^2 + \Pi_{t,\ell} + \delta\Pi_{t,\ell}$ around the leading-order result $\omega_0(q)^2$, we have

$$\begin{aligned} \delta\omega_{t,\ell}^2 &\equiv (\omega^2 - \omega_0^2)_{t,\ell} = \delta\Pi_{t,\ell}(\omega_0, q) + \delta\omega_{t,\ell}^2 \partial_{\omega_0^2} \Pi_{t,\ell} \\ &\quad \times (\omega_0, q) + O(\delta^2) \\ &= \frac{\delta\Pi_{t,\ell}(\omega_0, q)}{1 - \partial_{\omega_0^2} \Pi_{t,\ell}(\omega_0, q)} + O(\delta^2) . \end{aligned} \quad (23)$$

Interpreting the imaginary correction $\delta\omega^2 = -2i\omega_0\gamma$ as a damping term γ , we obtain (dropping the index 0 on ω in the final results, since the difference is of higher order now)

$$\begin{aligned}\gamma_{t,\ell} &= \frac{g^2 NT}{4\pi} \frac{q}{\omega} \frac{[1 + \partial_{q^2} \Pi_{t,\ell}(\omega, q)]}{[1 - \partial_{\omega^2} \Pi_{t,\ell}(\omega, q)]} \ln\left(\frac{1}{g}\right) \\ &= \frac{g^2 NT}{4\pi} v_{t,\ell} \ln\left(\frac{1}{g}\right),\end{aligned}\quad (24)$$

with

$$v_\ell = \frac{\omega}{q} \frac{3(m^2 - Q^2)}{3m^2 - Q^2} \quad \text{and} \quad v_t = \frac{\omega}{q} \frac{3(m^2 \omega^2 - Q^4)}{3m^2 \omega^2 - Q^4}, \quad (25)$$

both of which are zero at $q = 0$ and approach 1 with increasing q . This exactly reproduces and confirms the results of Ref. [11].

The above calculations allow us equally to derive the correction to the part of the dispersion laws which describe screening of external perturbations with frequencies below the plasma frequency, $\omega < m$ (see Fig. 1). There the gluon propagator exhibits poles for imaginary values of q , which gives the effective screening length for a given frequency ω . In the static limit this reduces to the electric (Debye) mass and the magnetic mass (which is zero at leading order) for mode ℓ and t , respectively. In the general case, both screening lengths are finite (which is sometimes referred to as “dynamical screening”). Here it is more natural to keep ω fixed and to determine $\delta q(\omega)$ according to

$$\delta(-q^2)_{t,\ell} = \frac{\delta \Pi_{t,\ell}(\omega, q)}{1 + \partial_{q^2} \Pi_{t,\ell}(\omega, q)} + O(\delta^2), \quad (26)$$

with $q(\omega)$ as given by the leading-order results.

Together with the results for $\mathcal{S}_{t,\ell}$ for imaginary q , this gives, for $|q| \sim m \gg \mu$,

$$\delta|q|_{t,\ell} = \frac{g^2 NT}{4\pi} \ln\left(\frac{1}{g}\right), \quad (27)$$

so the correction is now a simple constant. In the static limit $|q|_\ell = \sqrt{3} m = m_{e1}$ and (27) agrees with the next-to-leading-order result for the electric (Debye) screening mass of Refs. [12–14].

In the previous section we have seen that the logarithmic singularity of

$$g^2 \mathcal{S} \sim (g^2 T/q) \ln(q/\mu) \sim g \ln(1/g) \quad \text{for } |q| \sim m$$

turns into a linear one for $|q| \rightarrow 0$:

$$g^2 \mathcal{S} \sim gm/\mu \quad \text{for } |q| \lesssim \mu,$$

which for $\mu \sim g^2 T$ is of order 1. The final results for the gluon self-energy (21),(22) come with a prefactor that vanishes like q^2 , so that both the damping coefficient and the correction to dynamical screening, as given above, do not receive contributions from the singular integral \mathcal{S} . This could change, however, when higher-order corrections are taken into account, and we shall resume this point after extending the above calculations to the fermionic spectrum.

IV. QUARK SELF-ENERGY

The calculation of the infrared-singular parts of the one-loop resummed quark self-energy can be simplified in full analogy to the gluonic case of the previous section. Again the result factorizes into an expression involving only the external momentum and the singular integral \mathcal{S}_f introduced in Sec. II. The dressed quark-quark-gluon vertex is needed with vanishing gluon momentum only and can again be derived from the self-energy through a differential Ward identity:

$$*\Gamma_\mu(-Q, Q, 0) = \partial_{Q^\mu} (\not{Q} - \Sigma(Q)), \quad (28)$$

so that

$$\begin{aligned}\delta \Sigma &= -g^2 C_F [\partial_{q_m} (\not{Q} - \Sigma(Q))] [\not{Q} - \Sigma(Q)] \\ &\quad \times [\partial_{q_m} (\not{Q} - \Sigma(Q))] \mathcal{S}_f.\end{aligned}\quad (29)$$

The corrections to the two branches of the fermionic dispersion laws (see Fig. 1) are determined by

$$\delta \Pi_\pm = \frac{1}{4} \text{tr} [(\gamma^0 \mp \not{q}/q) \delta \Sigma]. \quad (30)$$

Introducing

$$D_\pm = \omega \mp q - \Pi_\pm \quad (31)$$

we obtain

$$\delta \Pi_\pm = - \left\{ D_\mp (\partial_q D_\pm)^2 + \frac{1}{2} D_\pm (D_+ - D_-)^2 \right\} \mathcal{S}_f, \quad (32)$$

where only the first term in the curly brackets survives on the respective mass shell $D_\pm = 0$.

In the fermionic case, the propagator has poles only for real values of q , corresponding to the fact that fermionic quantum numbers cannot get screened. For $q \sim M \gg \mu \sim g^2 T$ we have

$$\begin{aligned}\delta \Pi_\pm &= -i \frac{g^2 C_F T}{4\pi} |\partial_q D_\pm| \ln\left(\frac{1}{g}\right) \\ &= -i \frac{g^2 C_F T}{4\pi} \left| 1 \mp \frac{2\omega(Q^2 - M^2)}{qQ^2} \right| \ln\left(\frac{1}{g}\right),\end{aligned}\quad (33)$$

where in evaluating \mathcal{S}_f we have used

$$\partial_{q^2} \Delta_f^{-1} = \frac{1}{2q} \partial_q (D_+ D_-) \quad (34)$$

and $\text{sgn}(v_\pm) = -\text{sgn}(\partial_q D_\pm)$. Determining the correction term to the dispersion law $\omega_\pm(q) = \omega_{0\pm}(q) + \delta\omega_\pm(q)$ requires again to take into account the induced variation of the leading-order self-energy through $\delta\omega_\pm$ according to

$$\delta\omega_\pm = \frac{\delta \Pi_\pm(\omega_0, q)}{1 - \partial_{\omega_0} \Pi_\pm(\omega_0, q)} = \frac{\delta \Pi_\pm(\omega_0, q)}{\partial_{\omega_0} D_\pm(\omega_0, q)}. \quad (35)$$

Since $\delta \Pi_\pm$ is purely imaginary, this yields a fermionic damping coefficient

$$\begin{aligned}
\gamma_{\pm} &= \frac{g^2 C_F T}{4\pi} \frac{|\partial_q D_{\pm}|}{\partial_{\omega} D_{\pm}(\omega, q)} \ln\left(\frac{1}{g}\right) \\
&= \frac{g^2 C_F T}{4\pi} |v_{\pm}| \ln\left(\frac{1}{g}\right) \\
&= \frac{g^2 C_F T}{4\pi} \left| \frac{\omega(Q^2 - M^2) \mp qQ^2}{qM^2} \right| \ln\left(\frac{1}{g}\right) \quad (36)
\end{aligned}$$

in accordance with the results of Ref. [11].

The group velocity v_{\pm} equals $\pm 1/3$ in the limit $q \rightarrow 0$, monotonically increasing towards $+1$ for large q (with a zero for the plasmino mode at $q \approx 0.41M$). For $q \rightarrow 0$, (36) is no longer valid but becomes proportional to q/μ in place of $\ln(1/g)$, so that the infrared-singular contribution disappears. This agrees with the calculations performed in Ref. [17] at strictly $q = 0$, where a finite result without $\ln(1/g)$ was obtained.

V. HIGHER-ORDER CONTRIBUTIONS

For $q \neq 0$, we had to invoke higher-order corrections for providing a physical infrared cutoff for the mass-shell singularities in the next-to-leading-order corrections to the various on-shell self-energy components. Since the singularities were only logarithmic, the assumption that such an infrared cutoff will indeed be put into effect by the higher-order contributions at a certain scale μ suffices to fix the dominant term $\propto g \ln(1/g)$. Clearly, the other terms of $O(g)$ depend on the details of the new infrared physics at the scale μ and thus are beyond the perturbative scheme of Braaten and Pisarski, which is based on a resummation of the leading-order terms pertaining to the scale $m \sim gT$. Only the real-part corrections to the dispersion laws at $O(g)$ are infrared safe and calculable this way [18, 16].

In the long-wavelength limit, $q \rightarrow 0$, this sensitivity to scales $\ll m$ disappeared because the potentially singular terms in the self-energy corrections were of the form

$$\delta\Pi_{t,\ell}(\omega, q) \simeq q^2 f_{t,\ell}(\omega/q) g^2 \mathcal{S}_{t,\ell}, \quad (37)$$

$$\delta\Pi_{\pm}(\omega, q) \simeq q f_{\pm}(\omega/q) g^2 \mathcal{S}_f. \quad (38)$$

In Sec. II we have observed that \mathcal{S} becomes *linearly* singular for $q \rightarrow 0$, and assuming that higher-order corrections render \mathcal{S} finite so that $\mathcal{S} \sim T/\mu$, one can indeed drop the above contributions at $q = 0$.

However, this may cease to be justified if there are higher-order corrections to the prefactor of \mathcal{S} that do not vanish when $q \rightarrow 0$. In the resummed perturbation theory, one collects all the contributions at the soft scale gT , consistently disregarding potential terms in the effective action that are proportional to μ/m and therefore suppressed by extra powers of g . So higher-order corrections to the dressed vertices and propagators could in principle change the prefactor in (37) like

$$q^2 \rightarrow q^2 + c_1 g m q + c_2 g m \omega + c_3 g m^2 + \dots \quad (39)$$

and in (38) like

$$q \rightarrow q + d_1 g m + \dots, \quad (40)$$

with dimensionless functions $c_i(\omega, q)$ and $d_i(\omega, q)$.

If μ which cuts off the linear singularity of \mathcal{S} for $q \lesssim \mu$ was much smaller than gm , then there would even be the possibility that the *leading-order* results might get modified by the higher-loop orders, but only for $q \lesssim \mu$. We shall exclude this rather improbable eventuality by assuming that $\mu \sim gm \sim g^2 T$.

Let us first consider the effect of nonzero c 's on the result for dynamical screening, Eq. (26), for $q \lesssim \mu$. The correction to the inverse screening length $|q|$ becomes

$$\begin{aligned}
\delta|q|_{t,\ell} &\sim \frac{gmq}{\mu} + c_1 \frac{g^2 m^2}{\mu} + c_2 \frac{g^2 m^2 \omega}{q\mu} + c_3 \frac{g^2 m^3}{q\mu} + \dots \\
&\sim gq + c_1 gm + c_2 \frac{gm\omega}{q} + c_3 \frac{gm^2}{q} + \dots \quad (41)
\end{aligned}$$

Obviously, $c_2(\omega, q)$ and $c_3(\omega, q)$ should vanish for $q \rightarrow 0$ in order that the kinematical situation for $q \rightarrow 0$ does not become singular, which we shall take for granted.

In the static limit, the transverse branch of the dispersion laws has $|q| = 0$ at leading order, i.e., a vanishing magnetic mass. A nonvanishing $\delta|q|_t$ for $\omega \rightarrow 0$ would be interpreted as the generation of a magnetic mass m_{magn} . The result obtained within the Braaten-Pisarski scheme vanishes in this limit, which is in agreement with the null result of Ref. [12]. But a nonzero c_1 would render $m_{\text{magn}} \sim c_1 gm$, which is consistent with our assumption that the cutoff μ is of the order of the magnetic mass (although not necessarily identical with it). The linear mass-shell singularity of \mathcal{S} for $q \lesssim \mu$ could thus play a prominent role for the generation of a magnetic screening mass through higher-order corrections.

In the Abelian case this possibility does not exist for photonic quasiparticles because the latter do not interact directly with transverse gauge fields. Indeed, there the absence of a magnetic mass can be proved rigorously [19, 20].

Turning now to the propagating gluonic modes, we note that for $q \rightarrow 0$, the singular contributions to the damping constant still vanish like

$$\gamma_{t,\ell} \sim \frac{gq^2}{\mu} + c_1 \frac{g^2 qm}{\mu} + \dots, \quad (42)$$

since we have excluded c_2 and c_3 for $q \rightarrow 0$. Thus there seems to be little danger that the relative-order- g results that have been obtained previously for $q = 0$ [6, 18] could be modified by higher-order corrections. Also for nonzero $q \lesssim \mu$, the contribution from the mass-shell singularities remains below $O(gm)$.

The situation is somewhat different for the fermionic modes, however. There we have

$$\gamma_{\pm} \sim \frac{gqm}{\mu} + d_1 \frac{g^2 m^2}{\mu} + \dots, \quad (43)$$

and already the first term, which arises within the Braaten-Pisarski scheme, is of the order $gm \sim g^2 T$ for $q \sim \mu$. For such momenta γ is obviously not calculable within the one-loop resummed approximation, because through the linear divergence of \mathcal{S} , loop momenta of the order of μ contribute on a par with the ones of order m . For $q \rightarrow 0$, these contributions are suppressed, but

if higher-order terms could produce a nonvanishing d_1 , then also the strict $q = 0$ result for γ at order g^2T would become infested by higher-order contributions.

However, the proportionality of $\delta\Pi_\pm$ to q can be traced back to the Ward identity (28) which is responsible for the simple form of (32): therein both D_+ and D_- vanish like q for $q \rightarrow 0$. In the Abelian case, the tree-level-like Ward identities hold also beyond the level of hard thermal loops, whereas in the non-Abelian case, one can retain simple Ward identities by choosing to work in axial gauge, so it seems plausible that $d_1 \propto q$, which makes the results obtained for strictly $q = 0$ in Ref. [17] stable against higher-order contributions, despite the lurking mass-shell singularities.

VI. CONCLUSION

We have found that the next-to-leading-order corrections to dynamical screening masses for gluonic fields with frequencies below the plasma frequency as well as the damping constants for the propagating quasiparticle modes are strongly sensitive to the magnetic-mass scale except when the wave vector is exactly zero. On the other hand, the next-to-leading-order corrections to the real part of the dispersion laws of the propagating modes have turned out to be infrared safe.

The infrared singular contributions for both screening and damping are determined in essence by one simple expression exhibiting (quasiparticle) mass-shell singularities. For large enough modulus of the wave vector the latter are logarithmic, yielding contributions of relative order $g \ln(1/g)$. The coefficient in front of the logarithm is calculable once the scale of the cutoff brought about by higher-order contributions has been determined. The coefficients under the logarithm $\ln(1/g)$, which are of obvious importance when the coupling constant is not infinitesimally small, are clearly beyond these perturbative considerations.

For a small wave vector, we have found that the mass-shell singularities become even linear, which in principle opens a way for higher-order corrections to contribute on a par with the ones obtained within the resummed perturbation theory of Braaten and Pisarski. We have argued that it is plausible that the case of an exactly vanishing wave vector is stable against such corrections, whereas the particular case of the damping constant of fermionic modes with nonzero wave vector of the order of the magnetic mass scale remains uncalculable.

Linear mass-shell singularities have previously been identified as the root of a potential problem with gauge independence [21] of next-to-leading-order corrections to the dispersion laws. Whereas formally one can prove gauge-fixing independence [22], in covariant gauges the unphysical modes of the gluons behave like zero-mass particles, and lead to linear divergences in the residues of the quasiparticle propagators. Unless they are identified as such by the introduction of a (purely technical) infrared cutoff, they can mimic contributions to the pole position [23].

However, the type of mass-shell singularities that we have discussed in this paper appear directly in the corrections to the pole position. They call for a physical cutoff to be provided by higher-order corrections. Indeed, in the results for dynamical screening of transverse gluonic modes, we have seen that the linear mass-shell singularities themselves could play an important role in a dynamical generation of a magnetic screening mass $\sim g^2T$, which would be the most obvious candidate for such a physical infrared cutoff.

On the other hand, in the Abelian case, a magnetic screening mass cannot be generated. This is no problem for the photonic dispersion laws, which are infrared safe. But mass-shell singularities are there in the next-to-leading-order corrections to the electron propagator, and there has been some controversy [24–26] on whether the finite width of the internal propagators alone can provide the necessary cutoff μ , in particular when the singular contributions are evaluated on the corrected quasiparticle massshells, i.e., including damping. In fact, an analogous problem exists for the screening of static scalar fields in scalar electrodynamics [20], where damping of the internal propagators can hardly be operative as an effective infrared cutoff.

Our approach, however, was a strictly perturbative one. Through Eq. (35) it requires us to evaluate the corrections at the location of the (real) leading-order position, which, at this level, is in accordance with the findings of Ref. [27]. Our results and conclusions should therefore be quite independent of the details of the actual higher-order effects, as long as they are indeed able to cut off the quasiparticle mass-shell singularities.

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APPENDIX

In the following we describe some of the steps which are encountered when deriving the main results of Secs. II and III in a more pedestrian way.

The complete next-to-leading-order correction to the longitudinal gluon polarization function $\delta\Pi_\ell$, on the longitudinal mass shell, has recently been written down in full detail in Eq. (4.5) of Ref. [16], albeit still at a purely algebraic level. To cover also the transverse case, we generalize the result obtained in [16] and give an expression for $\delta\Pi^{\mu\nu}$ which is valid when used under the trace *either* with the matrix $B_{\mu\nu}$ introduced in Sec. III and taken at the longitudinal mass shell *or* with $A_{\mu\nu}$ and taken at the transverse mass shell,

$$\delta\Pi^{\mu\nu} = g^2 N \sum (c_0^{\mu\nu} + \Delta_\ell^- \Delta_\ell c_{\ell\ell}^{\mu\nu} + \Delta_\ell^- \Delta_t c_{t\ell}^{\mu\nu} + \Delta_t^- \Delta_t c_{tt}^{\mu\nu}) \quad (\text{A1})$$

with the coefficient matrices $c^{\mu\nu}$:

$$c_0^{\mu\nu} = \Delta_0^- \Delta_0 [2P^2 g^{\mu\nu} - 4P^\mu P^\nu] , \quad (A2)$$

$$c_{\ell\ell}^{\mu\nu} = \frac{P^2 K^2}{2p^2 k^2} * \Gamma^{\mu 00} * \Gamma^{\nu 00} - \frac{P^2}{2p^2} \delta_\ell^- * \Gamma^{\mu\nu 00} , \quad (A3)$$

$$c_{\ell t}^{\mu\nu} = -\frac{P^2 K^2}{p^2 k^2} * \Gamma^{\mu 00} * \Gamma^{\nu 00} - \frac{K^2}{k^2} * \Gamma^{\mu 0\rho} * \Gamma^{\nu 0}_\rho + \frac{P^2}{2p^2} \delta_\ell^- * \Gamma^{\mu\nu 00} , \quad (A4)$$

$$c_{tt}^{\mu\nu} = \frac{P^2 K^2}{2p^2 k^2} * \Gamma^{\mu 00} * \Gamma^{\nu 00} + \frac{K^2}{k^2} * \Gamma^{\mu 0\rho} * \Gamma^{\nu 0}_\rho + \frac{1}{2} * \Gamma^{\mu\rho\lambda} * \Gamma^{\nu}_{\rho\lambda} \\ - 3g^{\mu\nu} \delta_t + \delta_\ell^- \delta_t \frac{P_0 K_0}{P^2 K^2} U^\mu U^\nu + \frac{1}{2p^2 k^2} [R^\mu R_\nu + R_-^\mu R_-^\nu] . \quad (A5)$$

The notation is that of [16]: $K = Q - P$, $\mathbf{k} = \mathbf{q} - \mathbf{p}$, $\Delta_0 = 1/P^2$, $\Delta_i = 1/[P^2 - \Pi_i(P)]$ ($i = \ell, t$), $\delta_i = P^2 - \Pi_i(P) = \Delta_i^{-1}$, $U = (1, \mathbf{0})$. An index minus refers to the shift $P \rightarrow K$ in the corresponding quantity. The four-vector R has no zeroth component: $R^\mu = (\delta_t - \delta_\ell P_0^2/P^2)(P_0 U^\mu - P^\mu)$. The arguments of the four-leg vertices $*\Gamma$ are $Q, -Q, -P, P$; those of all three-leg $*\Gamma$'s are $Q, -K, -P$. Finally, the unadorned sum symbol is shorthand for $\sum_{P_0} \int d^3p/(2\pi)^3$.

The above result (A1) has a convergent sum over P_0 , and its p integration is restricted to soft values automatically due to the subtraction of the hard contribution. The calculation was done in general covariant gauge with gauge parameter α , which dropped out algebraically.

As in Sec. III, we concentrate on contributions involving a transverse propagator:

$$\sum \Delta_t \Delta_i^- c(P_0, \mathbf{p}) . \quad (A6)$$

Using the spectral representation

$$\Delta_t = \int_{-\infty}^{\infty} dx x \frac{\rho_t(x, p)}{P_0^2 - x^2} , \quad (A7)$$

we may write

$$\sum \Delta_t \Delta_i^- c = \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\infty} dx \frac{1}{x} \rho_t(x, p) \sum_{P_0} \frac{x^2}{P_0^2 - x^2} \\ \times \Delta_i(Q_0 - P_0, \mathbf{q} - \mathbf{p}) c(P_0, \mathbf{p}) . \quad (A8)$$

Inspecting now the integration region of small p , one realizes that the weight of the transversal (but not the longitudinal) density (times $1/x$) is concentrated at $x = 0$ [see, e.g., Eq. (B13) of Ref. [18]]:

$$\frac{1}{x} \rho_t(x, p) \rightarrow \frac{1}{p^2} \delta(x) \quad (p^2 \ll m^2) . \quad (A9)$$

Inserting (A9) in (A8) and performing the x integration, one finds that the sum over P_0 reduces to the term $P_0 = 0$ provided $c(P_0, \mathbf{p})$ has no poles at $P_0 = 0$, but this is easily excluded by inspection. Infrared singularities can now occur when Δ_i^- or $c(0, \mathbf{p})$ diverges for $p \rightarrow 0$. By inspection one finds that $c(0, \mathbf{p})$ is regular, so all singularities are due to the mass shell singularities when approaching mass-shell i . The first term of the Taylor series for $c(0, \mathbf{p})$ is responsible for the dominant contribution, which thus involves only the paradigmatic term \mathcal{S}_i studied in Sec. II times functions of the external momentum. This way Eqs. (19) and (20) are readily reproduced.

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