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T. Helbig

Deutsches Elektronen-Synchrotron DESY, Hamburg

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Thomas Helbig
Deutsches Elektronen-Synchrotron DESY, Hamburg

Abstract

The post-Newtonian metric of the general scalar-tensor theory with a massive scalar field is calculated. The result is used to recalculate several relativistic effects in the solar system. Experimental bounds for the massless scalar-tensor theory that have been obtained from these effects are used to get new bounds for the massive case. Furthermore we give a counter-example to the conjecture that all theories with only one gravitational field obey the strong equivalence principle.

1 Introduction

In recent years the interest in scalar-tensor theories of gravitation has increased, since very weakly coupled scalar fields play a crucial role in models for the damping of the cosmological constant, in the low-energy limit of string theories and also in inflationary scenarios. Hence the question of experimental tests for these theories arises. The strongest bounds for the well known Jordan-Brans-Dicke (JBD) theory [1,2] are obtained from experiments in the solar system and from binary pulsars. As the scalar field of the JBD theory is massless there is no difference to general relativity in the Newtonian approximation. All effects, which can be used to distinguish the JBD theory from general relativity, are therefore relativistic. The framework to calculate these effects and to compare the experimental results with the theoretical predictions is the parametrized post-Newtonian (PPN) formalism (see [3]). The experimental results are usually expressed as bounds on the PPN parameters.

The scalar field in the JBD theory is massless. This is needed in order to determine the gravitational constant dynamically by incorporating Mach's principle [2]. From a field-theoretical point of view it is to be expected that the scalar part of the gravitational field is massive and leads to a force with a finite range because a mass term violates no symmetry. The PPN formalism describes only forces of infinite range, therefore the bounds to the PPN parameters do not apply in the massive case. In this work an extended PPN formalism is developed which can be used to include forces of finite range. The experimental results can be reexpressed as bounds to the parameters of this formalism. The previous bounds for massive scalar fields are based on measurements of the Newtonian inverse square law [4]. The calculation of the relativistic effects in the extended PPN formalism gives additional bounds and improves the known ones.

The extended PPN formalism is developed in section 3, after a discussion of the general scalar-tensor theory with a massive scalar field in section 2. In section 4 the Nordtvedt effect, the perihelion shift and the time-delay of light are discussed in the extended PPN formalism and the known experimental bounds for the PPN parameters are used to determine bounds for the parameters of the extended formalism which depend upon the range l of the scalar field. The planned gyroscope experiment is also discussed. In section 5 a result of section 4 is used to give a counter-example to the conjecture [3] that all theories which contain only the metric as the gravitational field obey the strong principle of equivalence. This shows that the strong principle of equivalence distinguishes general relativity from most other theories of gravitation.

In this paper the units $c = \langle\phi\rangle = \hbar = 1$ are used, where $\langle\phi\rangle$ is the value of the scalar field ϕ far from all matter. If the scalar field is massive, it makes no sense to choose the units by fixing the Newtonian gravitational constant, because it is not constant. The conventions are those of [5], with the exception that greek letters are used for four-dimensional indices and latin letters $(i, j, k, ...)$ for three-dimensional indices.

2 The General Scalar-Tensor Theory

The general scalar-tensor theory of gravitation is described by the lagrangian

$$\mathcal{L} = \sqrt{-g}\frac{1}{16\pi}\left[-f(\phi)R + \frac{\omega(\phi)}{\phi}(\partial\phi)^2 - M(\phi)\right] + \mathcal{L}_M(g(\phi)g_{\mu\nu}), \tag{2.1}$$

with arbitrary functions $f(\phi)$, $g(\phi)$, $\omega(\phi)$ and $M(\phi)$ of the scalar field ϕ. The matter lagrangian $\mathcal{L}_M$ depends only on the product of the metric and a function of the scalar field, to ensure that the principle of equivalence is satisfied (in the Newtonian approximation and for weak gravitational fields). By means of a conformal transformation

$$g_{\mu\nu} \longrightarrow \frac{g_{\mu\nu}}{g(\phi)} \tag{2.2}$$

the scalar field can be removed from the matter lagrangian. After a redefinition of the scalar field to eliminate the function $f(\phi)$ the lagrangian reads

$$\mathcal{L} = \sqrt{-g}\frac{1}{16\pi}\left[-\phi R + \frac{\omega(\phi)}{\phi}(\partial\phi)^2 - M(\phi)\right] + \mathcal{L}_M(g_{\mu\nu}), \tag{2.3}$$

where the functions $\omega(\phi)$ and $M(\phi)$ have been redefined. Note that this simplification of (2.1) is not possible in all cases. An exception is, for example, $f(\phi) = g(\phi)$, but in this case the PPN parameters are the same as in general relativity, because the field equations have the solution $\phi = const$, even in the presence of matter. The lagrangian (2.3) corresponds to the Bergmann-Wagoner theory [6] with an additional potential term $M(\phi)$. The JBD theory is the special case with $\omega(\phi) = const$ and $M(\phi) = 0$.

The lagrangian (2.3) yields the field equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{\phi}\left[8\pi T_{\mu\nu} + \frac{\omega}{\phi}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\frac{\omega}{\phi}(\partial\phi)^2 g_{\mu\nu} + \frac{M}{2}g_{\mu\nu} + \phi_{;\mu\nu} - g_{\mu\nu}\Box\phi\right], \tag{2.4}$$

$$\Box\phi + \frac{\phi M' - 2M}{2\omega + 3} = \frac{1}{2\omega + 3}\left(8\pi T - \omega'(\partial\phi)^2\right). \tag{2.5}$$

In the JBD theory the value $\langle\phi\rangle$ of the scalar field far from all matter has to be obtained from cosmological solutions of the field equations. This solves the puzzle of the weakness of the gravitational force [1,2], because the gravitational "constant" G depends on $\langle\phi\rangle$. But if the function $M(\phi)$ is not zero, this leads to a potential for the scalar field so that its vacuum value $\langle\phi\rangle$ is fixed. The gravitational constant is now contained in $M(\phi)$ and the scalar field gets a mass m. In general this is expected, since there is no known symmetry, which forbids a mass term in the lagrangian (2.3). Although the gravitational part of the JBD theory is invariant under global Weyl

transformations, the matter part does not posses this symmetry. Therefore the JBD theory does not really solve the problem of the weakness of the gravitational force, because it raises the new question why the scalar field should have no mass.

Now the potential term in (2.5) can be expanded in the deviation $\bar{\phi} = \phi - 1$ from the background field $\langle\phi\rangle = 1$,

$$\frac{\phi M' - 2M}{2\omega + 3} = m^2\bar{\phi} - m^2 k\bar{\phi}^2 + \ldots, \tag{2.6}$$

where the parameter k is defined through (2.6). There is no constant term on the right hand side of (2.6). In the following the vanishing of the cosmological constant is assumed.

The mass of the scalar field is of course completely unknown. The natural scale of the gravitational part of the lagrangian (2.3) that appears in the potential $M(\phi)$ is given by the Planck mass, but it may be that the scalar field has something to do with the mass scales of strong or electroweak interactions. This would lead to a range l of the order of kilometers [7] or centimeters [8], respectively. Another scale that appears in the potential $M(\phi)$ is that of the cosmological constant, which is very tiny or zero. Therefore it is also possible that the range l of the scalar field is of astronomical size or even infinite.

3 The Post-Newtonian Metric

The parametrized post-Newtonian formalism (see [3]) cannot be used to describe theories with massive fields. This can be seen from the fact that the potential

$$U_l = \sum_a \frac{m_a}{|\vec{r} - \vec{r}_a|} e^{-\frac{|\vec{r} - \vec{r}_a|}{l}}, \tag{3.1}$$

with finite range $l = \frac{1}{m}$ can not be expanded into a power series of $\frac{1}{r}$ (Here m_a $(a = 1, 2, \ldots)$ are the rest masses of point particles). A description with the usual PPN metric is therefore impossible. Of course, the post-Newtonian approximation method can be used to derive an extended PPN metric, which allows a comparison of (2.3) with experiments. As in the massless case the metric has to be expanded in powers of the point particle velocities, generically denoted by v, namely

$$\begin{aligned} h_{00} &\quad \text{to order} \quad v^4, \\ h_{0i} &\quad \text{to order} \quad v^3, \\ h_{ij} &\quad \text{to order} \quad v^2, \end{aligned} \tag{3.2}$$

where $h_{\mu\nu}$ is the deviation of the metric from flat space-time $\eta_{\mu\nu}$. If the metric in the post- Newtonian approximation (3.2) is inserted into the lagrangian $\mathcal{L}_M$ of point particles the resulting equations of motion are correct to order v^4. The scalar field $\bar{\phi}$ does not appear in the matter lagrangian and is only needed to calculate $h_{\mu\nu}$.

In the Newtonian approximation only h_{00} is needed. The field equations (2.4) and (2.5) are solved by

$$\begin{aligned} h_{00} &= -2U - 2\alpha U_l + O(v^4), \\ h_{0i} &= O(v^3), \\ h_{ij} &= O(v^2), \\ \bar{\phi} &= 2\alpha U_l + O(v^4), \end{aligned} \tag{3.3}$$

where U is the Newtonian potential

$$U = \sum_a \frac{m_a}{|\vec{r} - \vec{r_a}|}, \tag{3.4}$$

and α is the coupling constant of the scalar field

$$\alpha = \left. \frac{1}{3 + 2\omega} \right|_{\bar{\phi}=0}. \tag{3.5}$$

The potentials U and U_l are of order v^2 or less, as can be seen from the virial theorem

$$U + \alpha U_l = O(v^2). \tag{3.6}$$

Before the next order can be calculated the post-Newtonian potentials have to be defined.

For the massless theory only four potentials with infinite range are needed

$$\begin{aligned} \phi_1 &= \sum_a \frac{m_a {v_a}^2}{|\vec{r} - \vec{r_a}|}, \\ \phi_2 &= \sum_a \frac{m_a U(\vec{r_a})}{|\vec{r} - \vec{r_a}|}, \\ \vec{V} &= \sum_a \frac{m_a \vec{v_a}}{|\vec{r} - \vec{r_a}|}, \\ \vec{W} &= \sum_a \frac{m_a \vec{n_a}(\vec{v_a}\vec{n_a})}{|\vec{r} - \vec{r_a}|}, \end{aligned} \tag{3.7}$$

with

$$\vec{n_a} = \frac{\vec{r} - \vec{r_a}}{|\vec{r} - \vec{r_a}|}. \tag{3.8}$$

The choice of the potentials fixes the gauge and fulfills the requirements of non-relativistic mechanics, for example the potentials depend in a rotational invariant

way on the coordinates $\vec{r_a}$ and velocities $\vec{v_a}$ of the particles, but not on the accelerations (for details see [3]). If the scalar field is massive, more potentials are needed. The calculation is given in appendix A. Here we present only the results. The finite range form of the potential ϕ_1 is

$$\phi_{1l} = \sum_a \frac{m_a {v_a}^2}{|\vec{r} - \vec{r_a}|} e^{-\frac{|\vec{r}-\vec{r_a}|}{l}}. \tag{3.9}$$

For the potential ϕ_2 there are three possibilities to insert a finite range

$$\begin{aligned} {\phi_2}^l &= \sum_a \frac{m_a U_l(\vec{r_a})}{|\vec{r} - \vec{r_a}|}, \\ \phi_{2l} &= \sum_a \frac{m_a U(\vec{r_a})}{|\vec{r} - \vec{r_a}|} e^{-\frac{|\vec{r}-\vec{r_a}|}{l}}, \\ {\phi_{2l}}^l &= \sum_a \frac{m_a U_l(\vec{r_a})}{|\vec{r} - \vec{r_a}|} e^{-\frac{|\vec{r}-\vec{r_a}|}{l}}. \end{aligned} \tag{3.10}$$

The gauge freedom in h_{0i} has to be used to avoid acceleration dependent terms in h_{00}. It turns out that the following finite range form of $\vec{V}$ and $\vec{W}$ obeys this requirement (see appendix A)

$$\begin{aligned} \vec{V_l} &= \sum_a \frac{m_a \vec{v_a}}{|\vec{r} - \vec{r_a}|} e^{-\frac{|\vec{r}-\vec{r_a}|}{l}}, \\ \vec{W_l} &= \sum_a m_a \vec{n_a}(\vec{v_a}\vec{n_a}) \left(\frac{1}{l} + \frac{1}{|\vec{r} - \vec{r_a}|} \right) e^{-\frac{|\vec{r}-\vec{r_a}|}{l}}. \end{aligned} \tag{3.11}$$

Furthermore there are three potentials which vanish if the range l is infinite

$$\begin{aligned} S^l &= -\frac{1}{4\pi l^2} \int dV' \frac{U_l^2(\vec{r}')}{|\vec{r} - \vec{r}'|}, \\ S_l &= -\frac{1}{4\pi l^2} \int dV' \frac{U_l(\vec{r}')U(\vec{r}')}{|\vec{r} - \vec{r}'|} e^{-\frac{|\vec{r}-\vec{r}'|}{l}}, \\ S_l^l &= -\frac{1}{4\pi l^2} \int dV' \frac{U_l^2(\vec{r}')}{|\vec{r} - \vec{r}'|} e^{-\frac{|\vec{r}-\vec{r}'|}{l}}. \end{aligned} \tag{3.12}$$

Note that not all new potentials have a finite range (for example ${\phi_2}^l$). The reason is a coupling of the long-range (spin 2) part of the gravitational field to the energy-momentum tensor of the short-range scalar part.

The field equations are solved to post-Newtonian order by the metric (see appendix A):

$$\begin{aligned} h_{00} = & -2U - 2\alpha U_l + 2U^2 + 4\alpha U U_l + (2\alpha^2 + 2\bar{\beta})U_l^2 - 3\phi_1 + \alpha\phi_{1l} + 2\phi_2 + \\ & + 2\alpha\phi_{2l} + 2\alpha{\phi_2}^l + (2\alpha^2 + 4\alpha^3\omega_0'){\phi_{2l}}^l - 2\alpha S^l - 4\alpha S_l + \end{aligned}$$

$$
\begin{aligned}
& + (6\alpha^2 + 4\alpha^2 k - 2\bar\beta) S_l^{\,l}, \\
h_{0i} &= 4V_i + \frac{1}{2}(W_i - V_i) + \frac{\alpha}{2}(W_i^{\,l} - V_i^{\,l}), \\
h_{ij} &= (-2U + 2\alpha U_l)\delta_{ij},
\end{aligned} \tag{3.13}
$$

where the parameter $\bar\beta$ is defined by

$$
\bar\beta = \alpha^3 \frac{\partial\omega}{\partial\phi}\bigg|_{\bar\phi=0}. \tag{3.14}
$$

The insertion of the metric (3.13) into the matter lagrangian leads to the equations of motion for the point particles to order v^4. As for classical electrodynamics [5] and general relativity [9, 5] the lagrangian of the N-body problem in the post-Newtonian approximation can be obtained. The result is given in appendix C.

If the scalar field is massless, (3.13) reduces to the PPN metric of the Bergmann-Wagoner theory [10]

$$
\begin{aligned}
h_{00} &= -2G_{eff}U + 2\beta {G_{eff}}^2 U^2 - (2\gamma+1)G_{eff}\phi_1 + (4\beta - 2){G_{eff}}^2\phi_2, \\
h_{0i} &= (2\gamma+2)G_{eff}V_i + \frac{1}{2}G_{eff}(W_i - V_i), \\
h_{ij} &= -2\gamma G_{eff} U \delta_{ij}.
\end{aligned} \tag{3.15}
$$

Here the PPN parameters are given by

$$
\begin{aligned}
\gamma = \frac{1-\alpha}{1+\alpha} &= \frac{1+\omega}{2+\omega}\bigg|_{\bar\phi=0}, \\
\beta = 1 + \frac{\bar\beta}{(1+\alpha)^2} &= 1 + \frac{\omega(\phi)'}{G_{eff}(3+2\omega)^2(4+2\omega)}\bigg|_{\bar\phi=0},
\end{aligned} \tag{3.16}
$$

where the effective (Newtonian) gravitational constant $G_{eff} = 1 + \alpha$ has been used. The parameters γ, β and G_{eff} loose there meaning if the range l is finite. This can be seen by expressing the metric (3.13) in terms of these parameters. (Therefore it makes no sense to choose units by $G_{eff} = 1$.)

4 Relativistic Effects of Massive Scalar Fields

The metric (3.13) can now be used to calculate relativistic effects, which have been tested by experiments in the solar system (see for example [3]). If the function $g(\phi)$ in (2.1) differs for different kinds of particles, the principle of equivalence is violated. Therefore it was assumed in section 2 that all kinds of particles couple in the same way to the scalar field. There is no symmetry that can be used to justify this assumption. Furthermore the appearance of the Nordtvedt effect in scalar-tensor theories shows that the effective theory always contains terms, that violate the principle of

equivalence. If such a violation is present in (2.1) the parameters α and $\bar\beta$ depend on the particle species. In the following we assume the principle of equivalence for the constituents of macroscopic bodies.

4.1 The Nordtvedt Effect

In scalar-tensor theories a body with a significant amount of gravitational self-energy does not move on a geodesic of the background metric. This is known as the Nordtvedt effect [11]. In the following an idea of Dicke [12] is used to calculate this effect for the case of a massive scalar field. The method originally used by Nordtvedt gives the same result if it is applied to the metric (3.13).

The self-energy E_{in} of a macroscopic body depends on the effective gravitational constant G_{eff} and therefore on the average value ϕ_0 of the scalar field. If the body moves in a slowly varying external scalar field ϕ_{ex} then ϕ_0 can be defined as the value $\langle\phi_{ex}\rangle$ of the external scalar field averaged over the body. The external field is the field which remains if the body where absent. Therefore E_{in} will be a function of the coordinates. These results in an additional force $M\vec{g_\phi}$

$$
M\vec{g_\phi} = -\frac{\partial E_{in}(\phi_0)}{\partial\phi_0}\vec\nabla\phi_{ex}(\vec r). \tag{4.1}
$$

The convention $\phi_0 = 1$ can not be used here, because ϕ_0 is varying. It is assumed, that ϕ_{ex} is nearly constant all over the body so that ϕ_0 is well defined. This means that the radius r_e of the body has to be much less than l, because the external scalar field changes on the scale given by l. Therefore the self- energy can be calculated with $l = \infty$. The energy E_{in} is only needed in the Newtonian approximation

$$
E_{in} = -\frac{1}{2\phi_0}\sum_{a\neq b}\frac{m_a m_b}{r_{ab}}\left(1 + \frac{1}{3+2\omega(\phi_0)}\right), \tag{4.2}
$$

$$
\phi_{ex} = 2\alpha {U_l}^{ex}. \tag{4.3}
$$

This leads to

$$
\vec{g_\phi} = -(2\alpha + 2\alpha^2 + 4\bar\beta)\vec\nabla {U_l}^{ex}\frac{1}{2M}\sum_{a\neq b}\frac{m_a m_b}{r_{ab}}, \tag{4.4}
$$

where the convention $\phi_0 = 1$ can now be used. This effect has been searched for in the earth-moon-sun system, where the external field is the one of the sun. Lunar laser ranging data lead to the bound [13]

$$
\eta = -\frac{1}{{G_{eff}}^2}(2\alpha + 2\alpha^2 + 4\bar\beta) = 0{,}001 \pm 0{,}015. \tag{4.5}
$$

Of course this bound applies only to the case $l = \infty$. But the parameter l appears in (4.4) only through the potential ${U_l}^{ex}$. So a new bound can be obtained from (4.5)

$$
\eta\frac{|\vec\nabla {U_l}^{ex}|}{|\vec\nabla U^{ex}|} = \eta\left(1 + \frac{r_{AU}}{l}\right)e^{-\frac{r_{AU}}{l}} = 0{,}001 \pm 0{,}015, \tag{4.6}
$$

where r_{AU} is the distance between the earth and the sun. This gives the range in which α and $\bar{\beta}$ can vary as a function of the range l. If the post- Newtonian expansion is assumed to converge, $\bar{\beta}$ has to be of order α^2. In fig. 1 the parameter $\bar{\beta}$ is set to zero which corresponds to $\omega = const.$ It should be noted that the bound (4.5) was calculated with the assumption that the effective gravitational constant G_{eff} is really constant. A corresponding correction to (4.6) can be neglected, because of the strong bounds on a variation of G_{eff}. This applies also to the other experiments under consideration.

4.2 The Perihelion Shift

Another strong bound stems from the perihelion shift of mercury. The Hamilton-Jacobi equation for a particle with mass m,

$$g^{\mu\nu}\partial_\mu S \partial_\nu S = m^2, \tag{4.7}$$

is solved by

$$S = -(E+m)t + L\varphi + \int dr \sqrt{2m\,(E - V(r)) - \frac{L^2}{r^2}}. \tag{4.8}$$

The use of the metric (3.13) leads to the potential

$$\begin{aligned} V &= -\frac{E^2}{2m} - 4EU - m\Big(U + \alpha U_l + 3U^2 + 2\alpha U U_l - (\alpha^2 + \bar{\beta}) {U_l}^2 + \\ &\quad + \alpha S^l + 2\alpha S_l + (\bar{\beta} - 2\alpha^2 k - 3\alpha^2) {S_l}^l\Big), \end{aligned} \tag{4.9}$$

from which the perihelion shift can be calculated (see [5])

$$\delta\varphi = \frac{2\pi b m}{L^2} = \frac{2\pi b}{a_p G(a_p) M}. \tag{4.10}$$

Here a_p is the semi-major axis of the orbit, M is the mass of the sun, $G(a_p)$ is the variable gravitational constant

$$G(a_p) = 1 + \alpha\left(1 + \frac{a_p}{l}\right) e^{-\frac{a_p}{l}}, \tag{4.11}$$

and the eccentricity of the orbit is neglected. A straightforward calculation yields for the parameter b:

$$\begin{aligned} b &= -\frac{1}{2m}\partial^2_{r^{-1}} V(a_p) \\ &\approx 3M^2 + \left(\frac{1}{2}\alpha M \frac{{a_p}^3}{l^2} + 2\alpha M^2\right) e^{-\frac{a_p}{l}} - \left(\alpha^2 M^2 + \bar{\beta} M^2\right) e^{-\frac{2a_p}{l}}. \end{aligned} \tag{4.12}$$

For the range $l \approx a_p$ the Newtonian term is the major term. Therefore relativistic terms are neglected if they vanish for $l \ll a_p$ and also for $l \gg a_p$. (The S-potentials are

given in appendix B). The result (4.10) can now be compared with the experimental limits to the difference $\delta\varphi_a$ between $\delta\varphi$ and the value predicted by general relativity [4]

$$\begin{aligned} \text{Mercury:}\ \delta\varphi_a &= (-80 \pm 210)\cdot 10^{-11}, \\ \text{Mars:}\ \delta\varphi_a &= (-130 \pm 180)\cdot 10^{-11}, \end{aligned} \tag{4.13}$$

which gives bounds to α and $\bar{\beta}$ in dependency on l. The bounds to α are shown in fig. 1 ($\bar{\beta} = 0$).

4.3 The Time-Delay of Light

The electromagnetic field equations are invariant under a conformal redefinition of the metric. So the scalar field has no effect on the motion of light. This makes it possible to measure the "bare" gravitational constant $\langle\phi\rangle^{-1}$ as if the scalar field $\bar{\phi}$ were absent. By comparison with the Newtonian constant $G_{eff} = 1 + \alpha$ it is possible to constrain α.

If we define the Newtonian constant as the "constant" in the inverse square law then for finite range of the scalar field the Newtonian constant is a function of the distance r_0

$$G(r_0) = 1 + \alpha\left(1 + \frac{r_0}{l}\right) e^{-\frac{r_0}{l}}. \tag{4.14}$$

The determination of the time delay of electromagnetic waves is therefore a measurement of $G(\infty)$. This is an ideal supplement to other limits [4] on the variation of the gravitational constant. The measurement of the time delay of light in the solar system [14] gives for the PPN parameter γ

$$\gamma + 1 = 2 \pm 0,002, \tag{4.15}$$

where an infinite-range force was assumed. If the range is finite, equation (4.15) has to be interpreted as

$$(\gamma + 1) G_{eff} = (2 \pm 0,002) G(r_0). \tag{4.16}$$

With the help of (3.16) this takes the form

$$\frac{1}{G(r_0)} = (1 \pm 0,001), \tag{4.17}$$

where the value of $G(r_0)$ comes from other measurements in the solar system (in fig. 1 $r_0 \approx 1AU$ is assumed). With the help of (4.14) this gives a bound on α as a function of l, which is shown in fig. 1.

The bending of light by the sun gives other bounds for γ. These bounds are weaker than (4.15), but the determination of $G(r_0)$ is the same. Although the light comes nearer to the sun there is no new information in these bounds, because the dependence on l is only through $G(r_0)$.

4.4 The Gyroscope Experiment

The angular momentum $\vec{S}$ of a satellite orbiting around the earth undergoes a precession, because of the gravitational field of the earth [15]. With the metric (3.13) the frequency of precession $\vec{\Omega}$ is given by

$$\begin{aligned}\vec{\Omega} &= -(\vec{v_0} \times \vec{\nabla})\left(\frac{h_{ii}}{6} + \frac{h_{00}}{4}\right) + \frac{1}{2}\vec{\nabla} \times \vec{h_0} \\ &= \frac{\alpha}{2}(\vec{v_0} \times \vec{e_r})\left(\frac{M}{r^2} + \frac{M}{rl}\right)e^{-\frac{r}{l}} - \frac{3}{2}(\vec{v_0} \times \vec{e_r})\frac{M}{r^2} + \frac{1}{r^3}\left(3\vec{e_r}(\vec{e_r}\vec{L}) - \vec{L}\right), \quad (4.18)\end{aligned}$$

where $\vec{v_0}$ is the speed of the satellite, $\vec{h_0}$ means $h_{0i}\vec{e_i}$, $\vec{L}$ is the angular momentum and M the mass of the earth. The gyroscope experiment, which is in the planning stage, will measure (4.18) and will give limits to α and to the variation of the gravitational constant close to the earth. (A measurement of the last term determines $MG(\infty)$.) The gyroscope experiment is therefore interesting for forces with a range much below one astronomical unit, which can not be measured in the solar system.

5 Theories With Only One Gravitational Field

General relativity complies with the strong principle of equivalence, which means that there is no Nordtvedt effect. The question arises, whether this is a general feature of all theories where the metric is the only gravitational field [3]. The following counter-example, which is based on a work by Whitt [16], disproves this conjecture. The Lagrangian

$$\mathcal{L} = \frac{1}{16\pi}\sqrt{-g}\left[-\phi R - \frac{1}{4a}(\phi - 1)^2\right] + \mathcal{L}_M \quad (5.1)$$

is a special example of (2.3), with

$$\begin{aligned}\omega &= 0, \\ M &= \frac{1}{4a}(\phi - 1)^2, \\ \alpha &= \frac{1}{3}, \\ m_\phi &= \sqrt{6a}^{-1}. \quad (5.2)\end{aligned}$$

One equation of motion is

$$\phi = 1 - 2aR. \quad (5.3)$$

Although the lagrangian (5.1) contains no kinetic term for the scalar field, the physical degrees of freedom are the same as for the general theory, i.e., one has a dilaton in addition to the graviton. It is now possible to eliminate the scalar field from the equations of motion. The new field equations have the lagrangian

$$\mathcal{L} = \frac{1}{16\pi}\sqrt{-g}\left(-R + aR^2\right) + \mathcal{L}_M. \quad (5.4)$$

This can be seen by inserting (5.3) into (5.1). The matter lagrangian $\mathcal{L}_M$ remains unchanged. The lagrangian (5.4) contains only the metric, but it is a scalar-tensor theory with a massive scalar field. Because of this the theory (5.4) has a Nordtvedt effect, as is shown in section 4.1. (Apart from this (5.4) makes scalar-tensor theories even more interesting.)

Therefore the conjecture [3], that the strong principle of equivalence distinguishes general relativity from theories with additional gravitational fields (like scalar-tensor theories), can be extended to the conjecture that the strong principle of equivalence implies general relativity. There are only two known exceptions. One is the Einstein-Fokker theory, which is a pure scalar theory of gravitation and is therefore ruled out by the bending of light by the sun. The other theory is Barker's scalar- tensor theory (see [3] for details) which fulfills the strong principle of equivalence only accidentally in the post-Newtonian approximation.

6 Conclusions

We have developed an extended PPN formalism, which describes a massive scalar-tensor theory in the post- Newtonian approximation. The insertion of a finite range for the scalar field allows a clear separation of the scalar from the tensor contribution to the metric and shows the complicated nonlinear couplings between both fields. As in the massless theory the construction of a Lagrangian for point particles is possible. In the limit of vanishing range l the post-Newtonian metric approaches that of general relativity after a proper definition of particle masses. Therefore, as is to be expected, all experimental bounds vanish if the range l is to short.

Every experiment, that gives bounds for the PPN parameters γ and β in the case of massless fields, can also be used to obtain bounds for the parameters α, $\bar{\beta}$ and l in the extended PPN formalism for a massive scalar field. This has been done for three important experiments in the solar system. The results are shown in fig. 1 for the case of the massive JBD theory ($\bar{\beta} = 0$) together with the results from measurements of Newton's constant at different distance scales [4]. The new parameter $\bar{\beta}$ is also constrained by the discussed experiments. The relativistic bounds for the parameters vanish if l is below the typical length-scale of the solar system. The nonrelativistic bounds are maximal on this scale because they are only possible through the finite value of l. All bounds vanish very fast if l is much below the typical length of the corresponding experiment. Note that there are laboratory based experiments which give bounds at a much smaller range l. Further applications of the formalism are possible. Examples are the planned gyroscope experiment and binary pulsars, which are interesting because of the shorter lenght-scales involved.

Furthermore an example was given which shows that theories with higher derivatives of the metric in general do not fulfill the strong principle of equivalence. Therefore this principle can be used to distinguish general relativity from higher-order theories. This is especially interesting because such terms appear as counter-terms if general relativity is quantized.

I would like to thank W. Buchmüller for suggesting this topic and for many helpful discussions, D. Walliser for reading the manuscript and Prof. H. A. Kastrup for raising the question of the strong equivalence principle in theories with dilatons.

A Derivation of the Post-Newtonian Metric

To calculate the metric (3.13) equation (2.4) and equation (2.5) are expanded in powers of $h_{\mu\nu}$ and $\bar{\phi}$. In the Newtonian approximation only h_{00} to order v^2 is needed. The field equation in this approximation are:

$$R_{00} = \tfrac{1}{2}\nabla^2 h_{00} = 4\pi\varrho - \tfrac{1}{2}\nabla^2\bar{\phi}, \tag{A.1}$$
$$\nabla^2\bar{\phi} - m^2\bar{\phi} = -8\pi\alpha\varrho \tag{A.2}$$

where the abbreviation

$$\varrho = \sum_a m_a \delta(\vec{r} - \vec{r_a}) \tag{A.3}$$

is used. The solution to these equations is:

$$\bar{\phi} = 2\alpha U_l, \tag{A.4}$$
$$h_{00} = -2U - 2\alpha U_l. \tag{A.5}$$

To obtain h_{ij} to the order v^2 the tensor R_{ij} is expanded and inserted into (2.4):

$$\begin{aligned} R_{ij} &= -\frac{1}{2}\partial_i\partial_j h_{00} + \frac{1}{2}\partial_i\partial_j h_{kk} - \frac{1}{2}\partial_k\partial_i h_{jk} - \frac{1}{2}\partial_k\partial_j h_{ik} + \frac{1}{2}\nabla^2 h_{ij} \\ &= 8\pi\left(T_{ij} + \frac{1}{2}\delta_{ij}T\right) + \frac{1}{2}\delta_{ij}\nabla^2\bar{\phi} + \partial_i\partial_j\bar{\phi}. \end{aligned} \tag{A.6}$$

The energy-momentum tensor $T_{\mu\nu}$ ($T = T_\mu{}^\mu$) gives in the needed approximation:

$$T = \varrho, \quad T_{ij} = 0. \tag{A.7}$$

Furthermore h_{00} and $\bar{\phi}$ from (A.1) and (A.2) can be inserted. One possible solution is:

$$h_{ij} = \delta_{ij}(-2U + 2\alpha U_l). \tag{A.8}$$

The equation for h_{0i} to order v^3 is given by:

$$\begin{aligned} R_{0i} &= \frac{1}{2}\nabla^2 h_{0i} - \frac{1}{2}\partial_i\partial_j h_{0j} + \frac{1}{2}\partial_i\dot{h}_{jj} - \frac{1}{2}\partial_j\dot{h}_{ij} \\ &= 8\pi T_{0i} + \partial_i\dot{\bar{\phi}}. \end{aligned} \tag{A.9}$$

This equation is not unique because of the gauge freedom. One possible solution is:

$$h_{0i} = 4V_i. \tag{A.10}$$

In analogy to the PPN-metric a different solution is used:

$$h_{0i} = 4V_i + \frac{1}{2}(W_i - V_i). \tag{A.11}$$

For the calculation of the fourth order terms the matter energy-momentum tensor

$$T_{\mu\nu} = \sum_a \frac{m_a}{\sqrt{-g}}\frac{dx^\mu}{ds}\frac{dx^\nu}{dt}\delta(\vec{r} - \vec{r_a}) \tag{A.12}$$

is expanded up to the order v^2:

$$T_{00} = \varrho\left(1 + \frac{v^2}{2} - 5U + \alpha U_l\right), \tag{A.13}$$
$$T = \varrho\left(1 - \frac{v^2}{2} - 3U + 3\alpha U_l\right), \tag{A.14}$$
$$T_{00} - \frac{1}{2}(1 + h_{00})T = \frac{\varrho}{2}\left(1 + \frac{3}{2}v^2 - 5U + \alpha U_l\right). \tag{A.15}$$

For the calculation of h_{00} to order v^4 the field $\bar{\phi}$ is needed to the same order. This complication is caused by the mass term in (2.3).

After the insertion of the second order terms, the expansion of equation (2.5) to order v^4 results in:

$$\begin{aligned} (-\nabla^2 + m^2)\bar{\phi} = {} & 8\pi\alpha\varrho - 4\alpha(U - \alpha U_l)\nabla^2 U_l + \alpha\vec{\nabla}(4U - 8\alpha U_l)\cdot\vec{\nabla}U_l - 2\alpha\ddot{U}_l - \\ & - 4\alpha\vec{\nabla}(U - \alpha U_l)\cdot\vec{\nabla}U_l + m^2 k 4\alpha^2 U_l^2 + \\ & + 8\pi\alpha\varrho\left(-\frac{v^2}{2} - 3U + 3\alpha U_l\right) - 32\pi\alpha^3\omega_0' U_l\varrho + \\ & + 4\omega_0'\alpha^3(\vec{\nabla}U_l)^2. \end{aligned} \tag{A.16}$$

Before this equation can be solved an auxiliary potential χ_l has to be defined:

$$(\nabla^2 - m^2)\chi_l = \ddot{U}_l. \tag{A.17}$$

With the help of χ_l and the PPN-potentials we are now able to solve (A.16):

$$\begin{aligned} \bar{\phi} = {} & 2\alpha U_l + (2\alpha^2 - 2\alpha^3\omega_0')U_l^2 - \alpha\phi_{1l} - 2\alpha\phi_{2l} - (2\alpha^2 + 4\alpha^3\omega_0')\phi_{2l}^l + 4\alpha S_l + \\ & + (6\alpha^2 - 2\bar{\beta} + 4\alpha^2 k)S_l^l + 2\alpha\chi_l. \end{aligned} \tag{A.18}$$

The next step is the expansion of (2.4) to order v^4:

$$
\begin{aligned}
R_{00} &= \frac{1}{2}\nabla^2 h_{00} + \frac{1}{2}\ddot{h}_{ii} - \partial_i \dot{h}_{0i} + \frac{1}{2}\partial_j h_{00}\cdot\partial_i h_{ij} - \frac{1}{4}\vec{\nabla} h_{00}\cdot\vec{\nabla} h_{ii} - \frac{1}{4}(\vec{\nabla} h_{00})^2 + \\
&\quad + \frac{1}{2} h_{ij}\partial_i\partial_j h_{00} \\
&= 8\pi\left(T_{00} - \frac{1}{2}(1 + h_{00})T\right) - 8\pi\bar{\phi}\left(T_{00} - \frac{1}{2}T\right) + \ddot{\bar{\phi}} - \frac{1}{2}\vec{\nabla} h_{00}\vec{\nabla}\bar{\phi} + \\
&\quad + 4\pi(1 - \bar{\phi} + h_{00})\alpha T - \bar{\phi}\omega_0' 8\pi\alpha^2 T + \frac{1}{2}\omega_0'\alpha(\partial_i\bar{\phi})^2 - \frac{1}{4}\frac{m^2}{\alpha}\bar{\phi}^2 - \\
&\quad - \frac{1}{2}m^2\bar{\phi}(1 + h_{00} - \bar{\phi}) + \frac{1}{2}m^2 k\bar{\phi}^2. \qquad (A.19)
\end{aligned}
$$

After the insertion of T and T_{00} and of the already calculated fields, this equation can be solved:

$$
\begin{aligned}
h_{00} &= -2U - 2\alpha U_l + 2U^2 + 4\alpha U\, U_l + (2\alpha^2 + 2\alpha^3\omega_0')U_l^2 - 3\phi_1 + \alpha\phi_{1l} + 2\phi_2 + \\
&\quad + 2\alpha\phi_{2l} + 2\alpha\phi_2^l + (2\alpha^2 + 2\alpha^3\omega_0')\phi_{2l}^l - 2\alpha S^l - 4\alpha S_l - \\
&\quad - (6\alpha^2 + 4\alpha^2 k - 2\beta)S_l^l - 2\alpha\chi_l. \qquad (A.20)
\end{aligned}
$$

Now the potential χ_l, which was needed because of the gauge in (A.11), has to be removed.

Equation (A.17) can be solved with:

$$\chi_l = \frac{\varrho}{2}\partial_t^2 f_l, \qquad (A.21)$$

where f_l is defined by

$$(\nabla^2 - m^2)f_l = \frac{2}{r}e^{-\frac{r}{l}}. \qquad (A.22)$$

We get:

$$f_l = \left[c_1\frac{l^2}{r} - l\right]e^{-\frac{r}{l}} + c_2\frac{l^2}{r}e^{\frac{r}{l}}. \qquad (A.23)$$

After the insertion of h_{00} into the matter lagrangian all time derivatives in χ_l can be ignored:

$$
\begin{aligned}
\chi_l &= \sum_a \frac{m_a}{2}\partial_t^2 f_l(\vec{r} - \vec{r_a}) \\
&= -\sum_a \frac{m_a}{2}\partial_t\vec{v_a}\vec{\nabla} f_l \\
&= -\sum_a \frac{m_a}{2}d_t\vec{v_a}\vec{\nabla} f_l + \sum_a \frac{m_a}{2}\partial_t'\vec{v_a}\vec{\nabla} f_l \\
&\doteq \frac{1}{2}\sum_a m_a(\vec{v}\vec{\nabla})(\vec{v_a}\vec{\nabla})f_l \\
&= \frac{1}{2}\sum_a m_a\left[(\vec{v}\vec{n})(\vec{v_a}\vec{n})\partial_r^2 - \frac{(\vec{v}\vec{n})(\vec{v_a}\vec{n})}{r}\partial_r + \frac{\vec{v}\vec{v_a}}{r}\partial_r\right]f_l, \qquad (A.24)
\end{aligned}
$$

where the derivative ∂_t acts on $\vec{r_a}$ and ∂_t' acts on $\vec{r}$. If a finite limit for $r \to \infty$ and the usual limit for $l \to \infty$ is required, we get $c_1 = c_2 = 0$:

$$\chi_l = \frac{1}{2}\sum_a m_a\left[\frac{\vec{v}\vec{v_a} - (\vec{v}\vec{n})(\vec{v_a}\vec{n})}{r}e^{-\frac{r}{l}} - \frac{(\vec{v}\vec{n})(\vec{v_a}\vec{n})}{l}e^{-\frac{r}{l}}\right]. \qquad (A.25)$$

Therefore the potential χ_l can be dropped in h_{00}, if the corresponding terms are added to h_{0i}. The result is (3.13).

B The S-Potentials

The S-potentials 3.12 of a single point particle can be calculated. The result is

$$
\begin{aligned}
S_l &= \frac{1}{2}M^2\frac{1}{rl}\left[e^{\frac{r}{l}}\mathrm{Ei}\left(-\frac{2r}{l}\right) - e^{-\frac{r}{l}}\left(C + \ln\left(\frac{2r}{l}\right)\right)\right], \\
S^l &= \frac{1}{2}M^2\left[\frac{e^{-\frac{2r}{l}}}{rl} - \frac{1}{rl} + \frac{2}{l^2}\mathrm{Ei}\left(-\frac{2r}{l}\right)\right], \\
S_l^l &= \frac{1}{2}M^2\frac{1}{rl}\left[e^{\frac{r}{l}}\mathrm{Ei}\left(-\frac{4r}{l}\right) - e^{-\frac{r}{l}}\mathrm{Ei}\left(-\frac{2r}{l}\right) - e^{-\frac{r}{l}}\ln 2\right], \qquad (B.1)
\end{aligned}
$$

where C is Euler's constant, $\mathrm{Ei}(x)$ is the exponential integral function

$$\mathrm{Ei}(x) = \int_{-\infty}^{x} dt\frac{e^t}{t}, \qquad (B.2)$$

and r is the distance from the body. These potentials contain self-energy terms, which contribute to the Newtonian order (see appendix C).

C The Post-Newtonian Lagrangian

The lagrangian which gives the post-Newtonian equations of motion of the N-body problem takes the form

$$
\begin{aligned}
L &= \sum_a m_a\left(\frac{v_a{}^2}{2} + \frac{v_a{}^4}{8}\right) + \sum_{a\neq b} m_a m_b\left(\frac{1}{2}\frac{1}{r_{ab}} + \frac{\alpha}{2}\frac{1}{r_{ab}}e^{-\frac{r_{ab}}{l}}\right) + \\
&\quad + \sum_{a\neq b} m_a m_b v_a{}^2\left(\frac{3}{2}\frac{1}{r_{ab}} - \frac{\alpha}{2}\frac{1}{r_{ab}}e^{-\frac{r_{ab}}{l}}\right) + \sum_{a\neq b} m_a m_b\left[-\frac{7}{4}\frac{\vec{v_a}\vec{v_b}}{r_{ab}} - \frac{1}{4}\frac{(\vec{v_a}\vec{n})(\vec{v_b}\vec{n})}{r_{ab}} + \right. \\
&\quad \left. + \frac{\alpha}{4}\frac{\vec{v_a}\vec{v_b}}{r_{ab}}e^{-\frac{r_{ab}}{l}} - \frac{\alpha}{4}\frac{(\vec{v_a}\vec{n})(\vec{v_b}\vec{n})}{r_{ab}}e^{-\frac{r_{ab}}{l}} - \frac{\alpha}{4}\frac{(\vec{v_a}\vec{n})(\vec{v_b}\vec{n})}{l}e^{-\frac{r_{ab}}{l}}\right] + \\
&\quad + \sum_{a\neq b, a\neq c} m_a m_b m_c\left[-\frac{1}{2}\frac{1}{r_{ab}r_{ac}} - \frac{\alpha}{r_{ab}r_{ac}}e^{-\frac{r_{ab}}{l}} - \left(\frac{\alpha^2}{2} + \alpha^3\omega_0'\right)\frac{1}{r_{ab}r_{ac}}e^{-\frac{r_{ab}}{l} - \frac{r_{ac}}{l}}\right] + \\
&\quad + {\sum_{a,b,c}}' \frac{m_a m_b m_c}{4\pi l^2}\left[-\alpha\int dV\frac{e^{\frac{|\vec{r_a} - \vec{r}|}{l}}}{|\vec{r_a} - \vec{r}|}\frac{e^{\frac{|\vec{r_b} - \vec{r}|}{l}}}{|\vec{r_b} - \vec{r}|}\frac{1}{|\vec{r_c} - \vec{r}|} + \right.
\end{aligned}
$$

$$+ \left(\alpha^2 + \frac{2}{3}\alpha^2 k - \frac{1}{3}\alpha^3 \omega_0'\right) \int dV \frac{e^{\frac{|\vec{r_a} - \vec{r}|}{l}}}{|\vec{r_a} - \vec{r}|} \frac{e^{\frac{|\vec{r_b} - \vec{r}|}{l}}}{|\vec{r_b} - \vec{r}|} \frac{e^{\frac{|\vec{r_c} - \vec{r}|}{l}}}{|\vec{r_c} - \vec{r}|} \Bigg], \tag{C.1}$$

where the case $a = b = c$ is excluded in the last sum. The metric (3.13) has been calculated under the assumption that all fields are weak. Hence the lagrangian (C.1) can only be applied to macroscopic bodies, which have a small mass density and a neglectable gravitational self-energy. Therefore the self-interaction terms in (C.1) have to be discarded. For general relativity this can be done by a redefinition of the masses of the particles. This is impossible for a scalar-tensor theory, because the Nordtvedt effect shows that the gravitational self-energy does not simply add to the mass. Furthermore in the massive scalar-tensor theory there are finite contributions to the self-interaction through the S-potentials and an undefined term from the W_l-potential. Therefore (C.1) is only an effective lagrangian for bodies that can be taken as pointlike on the scale of interest. The self-interaction terms have to be calculated from a knowledge of the internal structure of the bodies.

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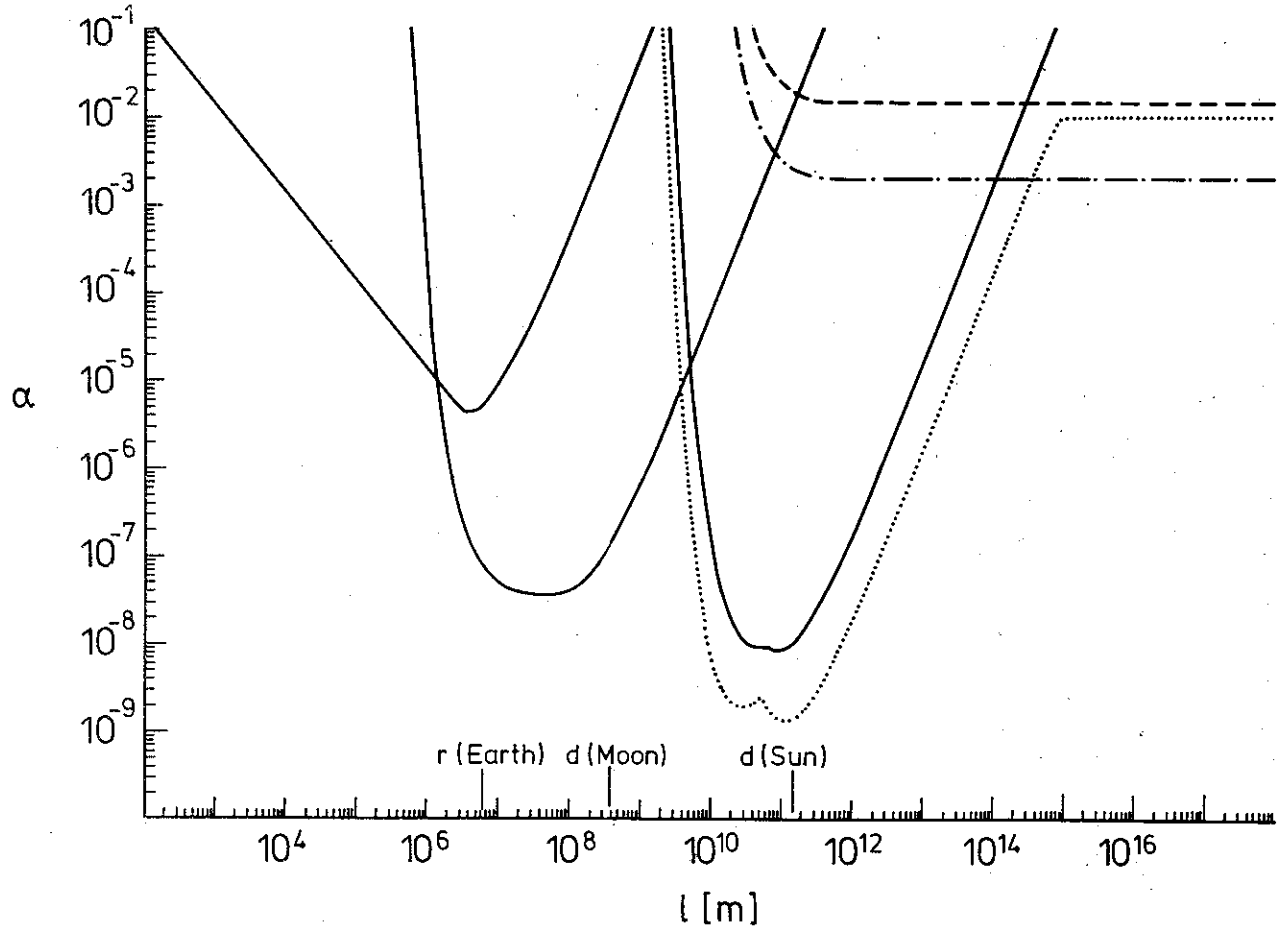
Figure caption:

Fig. 1:

Limits on α as a function of the range l ($\bar{\beta} = 0$). The area above any line is excluded at the 2σ level. The full line corresponds to measurements of Newton's constant from the orbits of Lageos, moon and planets [4]. The dash-dotted line is the bound from the measured time-delay of light (chapter 4.3), the dashed line is caused by the absence of the Nordtvedt effect within the experimental uncertainty (chapter 4.1) and the dotted line corresponds to the measurements of the precession of mercury and mars (chapter 4.2 and [4]). Bounds from laboratory and earth based experiments are not shown.



10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
α
r (Earth)
d (Moon)
d (Sun)
10^{4}
10^{6}
10^{8}
10^{10}
10^{12}
10^{14}
10^{16}
l [m]