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THE REACTION $e^+e^- \to t\bar{t}g$ AT NEXT-TO-LEADING ORDER IN α_s*

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The production of top quark pairs together with a hard gluon in e^+e^- annihilation is studied including next-to-leading order corrections in the strong coupling.

1 Motivation

The production of top quark pairs in association with (at least) one additional parton X carrying a hard momentum k_X,

$$e^+(p_+) + e^-(p_-) \to t(k_t) + \bar{t}(k_{\bar{t}}) + X(k_X), \tag{1}$$

is interesting for several reasons: A comparison of the cross section for (1) (which will be properly defined below) to the inclusive $t\bar{t}$ production cross section allows a measurement of the strong coupling constant α_s. A deviation of this measurement from the determination of α_s using event samples containing light quark jets would indicate a violation of the "flavour independence" of the strong interactions — i.e., would point towards new physics phenomena connected with the top quark. Specific examples of non-standard interactions that could be probed by reaction (1) are possible anomalous couplings of the top quark to photons, Z-bosons and gluons. It has been shown [1] that a large anomalous chromomagnetic $t\bar{t}g$ coupling would modify the gluon energy spectrum in $e^+e^- \to t\bar{t}g$. Furthermore, symmetry tests can be performed utilizing the richer kinematic structure of the final state in (1). These contain tests of the CP symmetry [2] and the search for final state rescattering ('T'-violating) effects using triple momentum correlations.[3] Both the search for heavy quark anomalous couplings [4] and the symmetry tests [5] have been shown to be experimentally feasible in the case of $b\bar{b}g$ production at the Z resonance, and it will be interesting to perform similar studies with top quarks. In order to unravel possible deviations from QCD expectations, it is mandatory to analyse reaction (1) at next-to-leading order in α_s.

2 Leading order analysis

At order α_s, the additional parton X in (1) can only be a gluon. The cross section for $e^+e^- \to t\bar{t}g$ develops a soft singularity as the gluon energy goes to zero. An infrared finite cross section can be defined by demanding $x_g \equiv 2E_g/\sqrt{s} > x_{\text{cut}}$, where E_g is the gluon energy in the c.m. system, $\sqrt{s}$ is the c.m. energy, and x_{cut} is some preset number. Since x_g is no useful variable for final states with four or more partons (which are relevant at higher orders in α_s), we use instead the scaled

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$t\bar{t}$ invariant mass square $x_{t\bar{t}} = (k_t + k_{\bar{t}})^2/s$, i.e. replace the condition $x_g > x_{\rm cut}$ by the (at LO equivalent) condition:[a]

$$1 - x_{t\bar{t}} > x_{\rm cut}. \tag{2}$$

We define $r(x_{\rm cut})$ as the fraction of $t\bar{t}X$ events for which $1 - x_{t\bar{t}} > x_{\rm cut}$ with respect to all $t\bar{t}$ events,

$$r(x_{\rm cut}) = \frac{\sigma\left(e^+e^- \to t\bar{t}X;\ 1 - x_{t\bar{t}} > x_{\rm cut}\right)}{\sigma_{\rm tot}(e^+e^- \to t\bar{t})} \equiv \frac{\sigma_3(x_{\rm cut})}{\sigma_{\rm tot}}. \tag{3}$$

At leading order, a compact analytic result can be derived for $r(x_{\rm cut})$.[6] A remark is in order concerning the experimental distinction of $t\bar{t}$ events with a gluon radiated off the t or $\bar{t}$ from events in which the gluon is radiated off the b or $\bar{b}$ produced in the decays of the top quark pairs. It has been shown [7,8] that the following two constraints efficiently select events where the gluon is produced in association with the top quark pair:

$$\begin{aligned} E_g &> \frac{\sqrt{s}}{2} x_{\rm cut} \gg \Gamma_t, \\ (m_t - 2\Gamma_t)^2 &\leq (k_{W^\pm} + k_{b(\bar{b})})^2 \leq (m_t + 2\Gamma_t)^2. \end{aligned} \tag{4}$$

By requiring that the invariant mass of the Wb system lies in the vicinity of the top quark mass, the probability that a highly energetic gluon jet ($E_g \gg \Gamma_t \approx 1.4$ GeV) is emitted from the b or $\bar{b}$ is very small.

3 Results at next-to-leading order

To evaluate the fraction $r(x_{\rm cut})$ at order α_s^2 we need two ingredients: First, the total cross section $\sigma_{\rm tot}(e^+e^- \to t\bar{t})$ to order α_s, which is well-known.[9] Second, we need $\sigma_3(x_{\rm cut})$ at order α_s^2, i.e. both virtual and real corrections to the reaction $e^+e^- \to t\bar{t}g$. For this we apply the results and techniques developed for the analogous case of three-jet production at NLO involving massive b quarks.[10] The real corrections consist of the processes $e^+e^- \to t\bar{t}gg$, $e^+e^- \to t\bar{t}q\bar{q}$ ($q = u, d, s, c, b$). If $\sqrt{s} > 4m_t$, the production of *two* $t\bar{t}$ pairs becomes possible. However, these rather spectacular events are extremely rare for the c.m. energies considered below and contributions from the process $e^+e^- \to t\bar{t}t\bar{t}$ can therefore be neglected. We renormalize the coupling in the modified minimal subtraction ($\overline{\rm MS}$) scheme. For the results presented in Figs. 1 and 2, the top quark mass is defined as the perturbative pole mass.

Fig. 1 shows the LO and NLO results for r as a function of $x_{\rm cut}$ at $\sqrt{s} = 0.5$ TeV and $\sqrt{s} = 1$ TeV. The renormalization scale is set to $\mu = \sqrt{s}$, and for the top quark pole mass we use $m_t = 175$ GeV. (A detailed discussion of the scale and mass renormalization scheme dependence is presented elsewhere.[6]) For $\sqrt{s} = 0.5$ TeV, the relative size of the QCD corrections varies between 56% (at $x_{\rm cut} = 0.02$) and 36%

[a] Other, at LO equivalent, choices for the cut condition are of course possible, e.g. $2E_X/\sqrt{s} > x_{\rm cut}$, where $E_X = \sqrt{s} - E_t - E_{\bar{t}}$, and all energies are defined in the c.m. system.

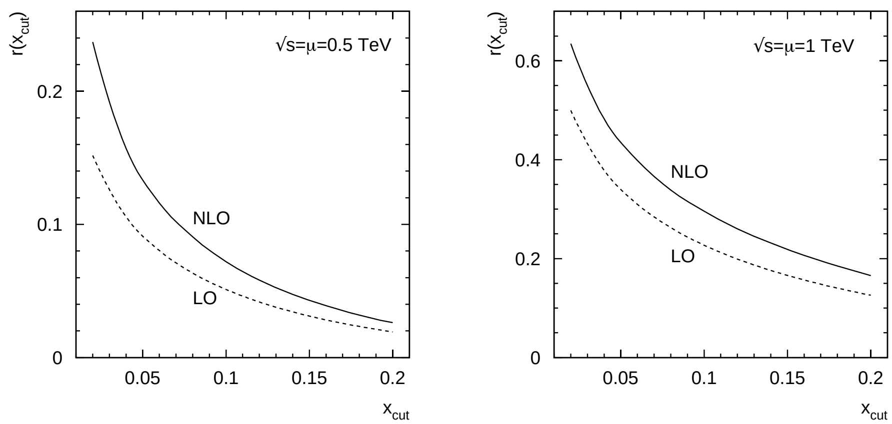

Figure 1: Fraction $r(x_{\rm cut})$ as a function of $x_{\rm cut}$ at $\sqrt{s} = \mu = 0.5$ and 1 TeV.

(at $x_{\rm cut} = 0.2$). At $\sqrt{s} = 1$ TeV, the QCD corrections are roughly constant as $x_{\rm cut}$ is varied and of the order of 30%. In Fig. 2 we plot distributions of the cross section $\sigma_3(x_{\rm cut})$ (defined in (3)) w.r.t. to the scaled top quark energy $x = 2E_t/\sqrt{s}$ and the variable $1 - x_{t\bar{t}}$. The distributions are normalized to the total $t\bar{t}$ cross section $\sigma_{\rm tot}$, and we set $\sqrt{s} = \mu = 0.5$ TeV and $x_{\rm cut} = 0.1$.

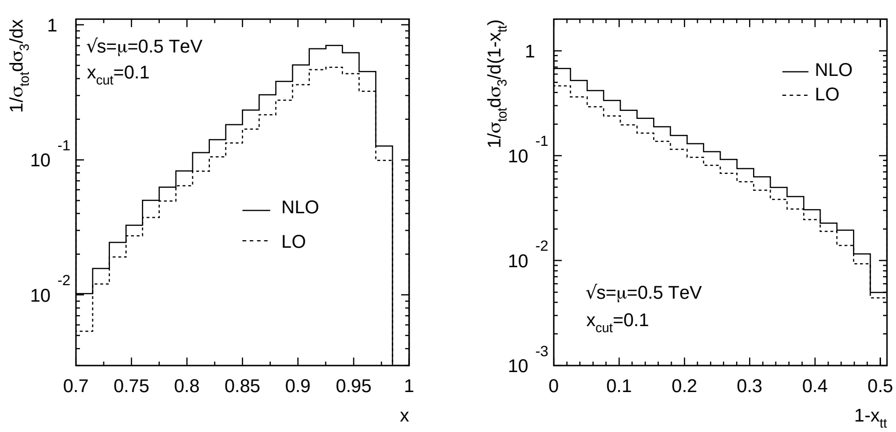

Figure 2: Distributions $1/\sigma_{\rm tot}\, d\sigma_3/dx$ and $1/\sigma_{\rm tot}\, d\sigma_3/d(1 - x_{t\bar{t}})$.

An intriguing question is whether a measurement of the fraction r allows a direct determination of the value of the running ($\overline{\rm MS}$) mass parameter $\overline{m}_t$ of the top quark at high energies. (Analogously, NLO results for three-jet fractions involving b quarks [11,12,10] have been used to extract a value for $\overline{m}_b(\mu = m_Z)$ from the high-statistics LEP [13] and SLD [14] data.) We consider here as a case study the result for

$r(x_{\rm cut}=0.1)$ at $\sqrt{s}=1$ TeV. We first express the fraction r in terms of the running mass $\overline{m}_t$ rather than in terms of the pole mass. This reduces the dependence on the renormalization scale significantly.[6] We then compute $r(x_{\rm cut}=0.1)$ at NLO for values of $\overline{m}_t(\mu=1$ TeV$)$ between, say, 140 and 160 GeV. (The value obtained from the renormalization group evolution is $\overline{m}_t(\mu=1$ TeV$)=148.6$ GeV.) The fraction $r(x_{\rm cut}=0.1)$ decreases by about 4% when the running top quark mass is changed from 140 to 160 GeV. We find that if a measurement of r will be possible with an error of $\pm1\%$, the running top quark mass at $\mu=1$ TeV could be determined up to ±5 GeV. A statistical error of 1% on r is realistic with the envisioned high luminosity of a future linear collider operating at $\sqrt{s}=1$ TeV. The sensitivity of the fraction r on $\overline{m}_t(\mu=1$ TeV$)$ may appear rather poor, but a direct determination of the running top quark mass at such a high scale would provide a nice consistency check of perturbative QCD in the following way: The direct measurement could be compared with the value for $\overline{m}_t(\mu=1$ TeV$)$ which one obtains by a conversion and evolution of the "1S" or "potential subtracted" [15] mass to be extracted from the threshold scan of $\sigma_{\rm tot}$.

In summary, the production of top quark pairs together with one or more additional hard partons at a future e^+e^- linear collider will be an exciting new testing ground for perturbative QCD and possible new interactions of the top quark.

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