FREQUENCY OF PARAMETRIC X-RAY RADIATION

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The exact solution of the equation for the frequency of parametric X-ray radiation (PXR) of relativistic charged particles moving in a crystal is obtained and compared to the approximate solution. It is found that the exact solution is in good agreement with the approximate one and that the approximate PXR frequency solution is practically correct for a comparison to experimental data.

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1. INTRODUCTION

Parametric Cherenkov radiation emitted by fast charged particles moving in a layered dielectric medium was first predicted in [1]. Later on, the equation for the frequency of parametric X-ray radiation (PXR) emitted by a relativistic charged particle moving in a crystal was derived by Ter-Mikaelian [2]. The approximate solution of the equation was confirmed in a number of experimental researches of PXR properties, see e.g. [3-5]. The influence of the medium where the radiation is emitted, is neglected in the approximate solution, and PXR is considered as emitted in vacuum due to the periodical interaction with crystallographic planes. However, sufficient influence of the medium on the radiation frequency was found at the interaction of particles with macroscopic periodical structures, as transition radiation from a stack of thick foils [5,6], and at undulator radiation from an undulator filled by an amorphous medium [7], and at undulator radiation from a volume reflection undulator [8]. In the present paper, we consider an exact analytical solution of the equation for the frequency of PXR emitted in a crystal and compare it to the approximate solution and experimental data.

2. CALCULATIONS

The equation for the angular frequency ω of PXR, excited by a relativistic charged particle in a crystal, was derived by Ter-Mikaelian [2] and reads

$$\omega = \frac{2\pi}{l} \frac{nV}{1 - \frac{\sqrt{\varepsilon}}{c} V\cos\theta} \qquad (1)$$

with V the particle velocity, l the distance between crystallographic planes along the particle velocity vector $\vec{V}$, c the speed of light, $\theta >> \gamma^{-1}$ the observation angle between particle velocity vector $\vec{V}$ and observation direction, $\gamma >> 1$ the relativistic Lorentz factor of the incident particles, n the harmonic number (in crystals n is associated with the crystal structure factor, therefore PXR does not exists for every n), and

$$\varepsilon = 1 - \left(\frac{\omega_p}{\omega}\right)^2 \qquad (2)$$

the frequency dependent dielectric permittivity of the crystal in the X-ray range for frequencies exceeding the atomic frequencies in the crystal such that

$$\omega >> \omega_p , \qquad (3)$$

ω_p being the crystal plasma frequency. With $\frac{\omega_p}{\omega} << 1$ in Eq. (2), the solution of Eq. (1) often can be used in the approximate form

$$\omega_0 = \frac{2\pi}{l} \frac{nV}{1 - \frac{V}{c}\cos\theta} . \qquad (4)$$

The approximate solution (4) can be considered as "vacuum" solution because the dielectric permittivity of the crystal (2) (where PXR is generated) is not taken into account. Eq. (4) was confirmed experimentally in a number of experimental investigations of PXR properties. In the following, taking Eq. (1) as start point it is indicated how the exact solution differs from the approximate vacuum solution (4).

Eq. (1) together with the dielectric permittivity Eq. (2) results in a quadratic equation that can be written in the form

$$\omega^2 (1 - \frac{V}{c}\cos\theta) - \omega \frac{2\pi V n}{l} + \frac{V}{2c}\omega_p^2 \cos\theta = 0 . \qquad (5)$$

The two solutions of Eq. (5) are

$$\omega_{1,2} = \frac{\omega_0}{2} \pm \sqrt{\left(\frac{\omega_0}{2}\right)^2 - \frac{V\omega_p^2 \cos\theta}{2c\left(1 - \frac{V}{c}\cos\theta\right)}} . \qquad (6)$$

The second term under the root in Eq. (5) is much smaller than the first one because of condition (3). Therefore, one can write two separate solutions. The low-frequency solution ω_1 ("-" sign in Eq. (6)) is

$$\omega_1 = \frac{l\omega_p^2 \cos\theta}{4\pi cn}, \quad (7)$$

and the high-frequency solution ω_2 ("+" sign in Eq. (6)) is

$$\omega_2 = \frac{2\pi}{l}\frac{nV}{1-\frac{V}{c}\cos\theta} - \frac{l\omega_p^2\cos\theta}{4\pi cn} = \omega_0 - \omega_1 . \quad (8)$$

The low-frequency solution (7) does not correspond to PXR because it does not satisfy the condition $\omega >> \omega_p$, see Eq. (3). The high-frequency solution ω_2 (8) differs from the vacuum solution (4) by the value of the low-frequency solution ω_1. The difference leads to an increase in the PXR frequency (8) in the PXR reflection emitted in the backward hemisphere at $\pi \ge \theta > \frac{\pi}{2}$, and to a decrease in the PXR frequency for radiation emitted in the forward hemisphere at $\frac{\pi}{2} > \theta >> \gamma^{-1}$. The difference is absent if PXR is emitted at a right angle with respect to the particle velocity vector, i.e. $\theta = \frac{\pi}{2}$. The value of the difference ω_1 ($\hbar\omega_1$ is in the eV range in typical cases) between the exact (Eq. (8)) and the approximate vacuum solution (Eq. (4)) is much smaller than the PXR quanta energy (typically $\hbar\omega_0$ is in the range exceeding a few keV) calculated by formula (4).

Let us consider an example for typical experimental conditions described in [3,4]. In those works, the PXR spectral peak from the (111) crystallographic planes of a Si single-crystal was observed in the vicinity of the center of the PXR reflection at a fixed observation angle $\theta = 305.9\, mrad$, and the angle of the crystal rotation was around $\theta / 2$. PXR was generated by a 25.0 MeV electron beam in a thin Si plate in Laue geometry. The value l in (4) is $l = \frac{2\pi}{g_{(111)} \cdot \sin\frac{\theta}{2}} = 20.7\ Angsroms$ in the center of the PXR reflection, where $g_{(111)}$ is the module of the reciprocal lattice vector for the (111) Si crystallographic plane. The energy of the observed radiation [3,4] and the calculated one by the approximate formula (4) in the center of the PXR reflection spectral peak is $\hbar\omega_0 \approx 12.9\, keV$. The energy $\hbar\omega_1$ calculated by formula (7) is $\hbar\omega_1 \approx 0.8\, eV$ for the plasma energy in Si crystal $\hbar\omega_p = 31.1\, eV$ and n=1. The energy of the low-frequency radiation (7) does not satisfy the condition (3) and cannot be considered as a realistic solution. The energy of the high-frequency radiation (8) $\hbar\omega_2 = \hbar\omega_0 - \hbar\omega_1$ is less than $\hbar\omega_0 \approx 12.9\, keV$ for 0.8 eV. The relative correction is $< 10^{-4}$. The difference between the exact high-frequency solution $\hbar\omega_2$ (8) and the approximate solution (4) amounts to only 0.8 eV, which is much less than the observed width of the PXR spectral peak of 166 eV in [3,4]. Thus, the difference between the exact (8) and the approximate (4) solution is negligibly small for PXR, and the approximate solution (4) is practically correct.

3. RESULTS AND DISCUSSION

We demonstrated a good agreement between the exact (8) and the approximate vacuum solution (4) for PXR frequencies produced by relativistic particles in a crystal. Let us discuss the reason, why the exact and approximate vacuum solutions for radiation frequencies are practically the same for PXR in a crystal, and they are sufficiently different at emission of radiation in forward direction from macroscopic periodical structures, as transition radiation from a stack of thick foils [5,6], undulator radiation from an undulator filled by amorphous medium [7], and undulator radiation from a volume reflection undulator [8].

The important thing is the period length l of the structure. In the case of PXR the period is determined by the distance between crystallographic planes, which is usually in in the Angstrom region. Therefore, the values of the low-frequency solution (7) and the correction in the high-frequency solution (8) are insignificant. In the case of macroscopic structures [5-8], the period lengths l can be in sub-mm or even larger range. Therefore, the low-frequency solution and the related correction of the high-frequency solution for macroscopic structures [5-8] can be sufficient to lead to significant changes of the spectral distribution of the emitted radiation.

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ЧАСТОТА ПАРАМЕТРИЧНОГО РЕНТГЕНІВСЬКОГО ВИПРОМІНЮВАННЯ

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Отримано точний розв'язок рівняння частоти параметричного рентгенівського випромінювання релятивістських заряджених частинок, що рухаються в кристалі, та порівняно з наближеним розв'язком. Встановлено, що точний розв'язок добре узгоджується з приблизним рішенням і що приблизний частотний розв'язок практично коректний для порівняння з експериментальними даними.