
















































































































































































































































































































































































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Measurement of CP asymmetries and branching-fraction ratios for $B^\pm \rightarrow DK^\pm$ and $D\pi^\pm$ with $D \rightarrow K_S^0 K^\pm \pi^\mp$ using Belle and Belle II data

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ABSTRACT:

We measure CP asymmetries and branching-fraction ratios for $B^\pm \rightarrow DK^\pm$ and $D\pi^\pm$ decays with $D \rightarrow K_s^0 K^\pm \pi^\mp$, where D is a superposition of D^0 and \bar{D}^0 . We use the full data set of the Belle experiment, containing 772×10^6 $B\bar{B}$ pairs, and data from the Belle II experiment, containing 387×10^6 $B\bar{B}$ pairs, both collected in electron-positron collisions at the $\Upsilon(4S)$ resonance. Our results provide model-independent information on the unitarity triangle angle ϕ_3 .

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1 Introduction

In the Standard Model, CP violation is described by a single irreducible complex phase of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1, 2]. The CKM matrix is unitary, so $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$, where V_{ij} is the CKM matrix element coupling quark flavor i to quark flavor j . This condition, represented by a triangle in the complex plane, provides a promising way to verify the Standard Model by testing the closure of the triangle. The interior angle ϕ_3 (also known as γ), defined as $\arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$, is independent of top-quark couplings and a benchmark of the Standard Model. Through the interference of $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$ transition amplitudes in tree-level b -hadron decays, where non-Standard-Model effects are negligible [3], ϕ_3 can be studied in a theoretically reliable way [4]. An improved direct measurement of ϕ_3 would provide a key input for Standard Model predictions, which can be compared with predictions of other measured observables that are sensitive to new particles and interactions.

The world average of ϕ_3 measurements, $(65.9_{-3.5}^{+3.3})^\circ$ [5], in which the LHCb experiment contributes most, is dominated by studies of the interference of $B^\pm \rightarrow D^0 K^\pm$ and $B^\pm \rightarrow \bar{D}^0 K^\pm$ decays in which the D^0 and \bar{D}^0 decay to a common final state [6–8]. Grossman, Ligeti, and Soffer (GLS) proposed a method to measure ϕ_3 with singly Cabibbo-suppressed decays of D mesons, $D \rightarrow K_S^0 K^\pm \pi^\mp$ [9], where D is a superposition of D^0 and \bar{D}^0 mesons. Experimentally, one measures seven observables to access ϕ_3 , including four CP asymmetries

and three branching-fraction ratios in $B^\pm \rightarrow Dh^\pm$ decays, where h is a pion or kaon. From these, the ϕ_3 -related information is extracted without model dependent uncertainties from the amplitude model of D decay. However, information about the $D \rightarrow K_S^0 K^\pm \pi^\mp$ dynamics is necessary. The CLEO experiment measured this information using all such D decays and also using only decays in which the $K_S^0 \pi^\mp$ pair has a mass within $100 \text{ MeV}/c^2$ of the known $K^*(892)^\mp$ mass [10].¹ In that region, the interference of $B^\pm \rightarrow D^0 h^\pm$ and $B^\pm \rightarrow \bar{D}^0 h^\pm$ decays is enhanced due to the large strong-phase difference in $D \rightarrow K_S^0 K^\pm \pi^\mp$ decays, and ϕ_3 can be determined more precisely. The principal experimental challenge is extracting the signal given the small branching fractions of these channels. The LHCb collaboration reported the most precise GLS measurement to date [11].

In this paper, we present a similar measurement for $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ using $772 \times 10^6 B\bar{B}$ pairs collected by the Belle experiment [12] and $387 \times 10^6 B\bar{B}$ pairs collected by the Belle II experiment [13], produced in electron-positron collisions at the $\Upsilon(4S)$ resonance.² We fit to the distributions of two signal-discriminating observables, simultaneously in both data sets and all channels to extract the seven GLS observables. An additional measurement restricted to a $K^{*\pm}$ -enriched region of the D meson phase space is also reported, as well as results based on the Belle data only.

2 Formalism

We categorize $B^\pm \rightarrow Dh^\pm$ followed by $D \rightarrow K_S^0 K^\pm \pi^\mp$ as same-sign (SS) or opposite-sign (OS) decays according to the charge of the K^\pm produced by the D meson relative to the charge of the B^\pm meson. The four CP asymmetries are

$$\mathcal{A}_m^{Dh} \equiv \frac{N_m^{Dh^-} - N_m^{Dh^+}}{N_m^{Dh^-} + N_m^{Dh^+}} \quad \text{with } h = \pi, K, \quad (2.1)$$

where $N_m^{Dh^\pm}$ is the number of $B^\pm \rightarrow Dh^\pm$ decays, and m denotes the decay type, which is either SS or OS. The three branching-fraction ratios are

$$\mathcal{R}_m^{DK/D\pi} \equiv \frac{N_m^{DK^-} + N_m^{DK^+}}{N_m^{D\pi^-} + N_m^{D\pi^+}} \quad (2.2)$$

and

$$\mathcal{R}_{\text{SS/OS}}^{D\pi} \equiv \frac{N_{\text{SS}}^{D\pi^-} + N_{\text{SS}}^{D\pi^+}}{N_{\text{OS}}^{D\pi^-} + N_{\text{OS}}^{D\pi^+}}. \quad (2.3)$$

3 Belle and Belle II detectors

The Belle detector [12, 14] was a large-solid-angle magnetic spectrometer at the KEKB accelerator [15, 16], which collided 8 GeV electrons with 3.5 GeV positrons. The subdetectors of Belle most relevant for our study are the silicon vertex detector and the central drift chamber for charged-particle tracking and ionization-energy loss measurement and the

¹The $K^*(892)^\pm$ is henceforth referred to as $K^{*\pm}$.

²Here, B indicates either a B^+ or B^0 .

aerogel threshold Cherenkov counters and time-of-flight scintillation counters for charged particle identification (PID). They were situated in a uniform axial magnetic field of 1.5 T.

The Belle II detector [13] is an upgrade of the Belle detector at the SuperKEKB accelerator [17], which collides 7 GeV electrons with 4 GeV positrons. Innermost is a tracking system, including two layers of silicon pixel sensors, four layers of silicon strip detectors, and the central drift chamber. Only 15% of the azimuthal angle is covered by the second layer of the pixel detector for the Belle II data used in this paper. Outside the drift chamber, the time-of-propagation and aerogel ring-imaging Cherenkov subdetectors cover the barrel and forward endcap regions, respectively. Outside these subdetectors are the electromagnetic calorimeter and a solenoid, which provides a uniform 1.5 T magnetic field. A K_L^0 and muon detector is installed in the iron flux-return yoke of the solenoid.

4 Simulation

We use simulated samples to identify sources of background, optimize selection criteria, calculate selection efficiencies, and determine fit models. We generate $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$ events and simulate particle decays with EVTGEN [18]; we generate continuum $e^+e^- \rightarrow q\bar{q}$ where q is a $u, d, c,$ or s quark with PYTHIA [19] for Belle and KKMC [20] and PYTHIA for Belle II; we simulate final-state radiation with PHOTOS [21]; and we simulate detector response using GEANT3 [22] for Belle and GEANT4 [23] for Belle II. We model our signal processes using both nonresonant $D \rightarrow K_s^0 K^\pm \pi^\mp$ and $D \rightarrow K^{*\mp} K^\pm$ decays. In the Belle simulation, beam backgrounds are taken into account by overlaying random-trigger data. In the Belle II simulation, they are accounted for by simulating the Touschek effect [24], beam-gas scattering, and luminosity-dependent backgrounds from Bhabha scattering and two-photon quantum-electrodynamic processes [25, 26].

5 Event selection

We reconstruct events using the Belle II analysis software for both Belle and Belle II data [27–29]. All events are required to pass the online selection criteria based on either total energy deposition in the electromagnetic calorimeter or the number of charged-particle tracks in the central drift chamber. The efficiency of the online selection is found to be close to 100%.

Tracks originating from K^\pm and π^\pm are selected by requiring that each have a distance of closest approach to the e^+e^- interaction point smaller than 1.0 cm in the longitudinal direction (parallel to the e^+ beam at Belle and the principal axis of the magnet at Belle II) and smaller than 0.2 cm in the transverse plane. We identify the species of each charged hadron using $\mathcal{L}(K/\pi) = \mathcal{L}(K)/[\mathcal{L}(K) + \mathcal{L}(\pi)]$, where $\mathcal{L}(h)$ is the likelihood for hadron h to have produced the relevant track based on information from the aerogel threshold Cherenkov counters, time-of-flight scintillation counters, and the central drift chamber for Belle and all subdetectors for Belle II.

We identify an h^\pm as a kaon if $\mathcal{L}(K/\pi) > 0.6$. To improve signal efficiency, no PID requirement is applied to the pion candidate from D decay. Only prompt pion candidates,

which are produced directly from B^\pm decay, are required to satisfy $\mathcal{L}(K/\pi) < 0.6$. In Belle II data, we also restrict the polar angle of the prompt h^\pm in the laboratory frame to be within the acceptance of the PID detectors. The kaon-identification efficiency depends on the particle momentum and is in the range of 86%–90%. The probability of misidentifying a pion as a kaon is in the range of 3%–9%.

We reconstruct K_S^0 mesons via their decays to $\pi^+\pi^-$. Each candidate K_S^0 is formed from a pair of oppositely-charged particles with no PID requirements, constrained to come from a common vertex. The resulting dipion mass must be in the range of $[487, 508] \text{ MeV}/c^2$, which corresponds to $\pm 3\sigma$ around the known K^0 mass [30], with σ being the mass resolution. To improve purity, we reject combinatorial background based on the output of a neural network for Belle [31] and a boosted decision tree (BDT) [32] for Belle II. In the latter case, 15 input variables are selected including kinematic quantities and the number of hits in the vertex detector associated to the π^\pm tracks. The most discriminating variables are the angle between the K_S^0 momentum and the direction from the beam interaction point to the decay vertex position in the laboratory frame, and the flight length of the K_S^0 normalized by its uncertainty.

Each neutral D candidate, reconstructed from K_S^0 , K^\pm , and π^\mp candidates, must have a mass in the range of $[1.85, 1.88] \text{ GeV}/c^2$, which corresponds to $\pm 3\sigma$ around the known D^0 mass [30], where σ is the typical D mass resolution.

Each B^\pm candidate is reconstructed from D and prompt h^\pm candidates. To suppress continuum background, we require that the beam-constrained mass,

$$M_{\text{bc}} \equiv c^{-2} \sqrt{E_{\text{beam}}^{*2} - |\vec{p}_{BC}|^2}, \quad (5.1)$$

exceed $5.27 \text{ GeV}/c^2$, where E_{beam}^{*2} is the beam energy and \vec{p}_B is the B momentum, both defined in the e^+e^- center-of-mass (c.m.) frame. We also require $|\Delta E| < 0.15 \text{ GeV}$, where $\Delta E \equiv E_B - E_{\text{beam}}^*$ and E_B is the B energy in the c.m. frame.

The remaining background comes mostly from continuum events, which are topologically distinguishable from $B\bar{B}$ events. Since the momentum of a B is only $333 \text{ MeV}/c$ in the c.m. frame, the final-state particles of a $B\bar{B}$ event are almost isotropically distributed in the c.m. frame. The final-state particles of continuum events form mainly back-to-back jets. We discriminate between $B\bar{B}$ and continuum events using modified Fox-Wolfram moments [33, 34], the thrust of the B decay products [35], the angle between the axis of this thrust and that of the particles in the rest of the event (ROE), the polar angle of the B momentum, the distance between the B decay vertex and that of the ROE in the longitudinal direction, and the output of a B -flavor-tagging algorithm [36, 37]³. All frame-dependent quantities are calculated in the c.m. frame. Simulated samples show that these variables have correlations of 4% or smaller with ΔE , which is used in the signal-extraction fit. We train BDT classifiers with these variables separately for Belle and Belle II. The classifier output, C , is distributed between zero and one. We require $C > 0.3$, which rejects 64.3% of continuum background candidates and retains 95.3% of signal candidates for Belle, and

³For Belle II, a category-based algorithm is adopted.

rejects 63.0% of continuum background candidates and retains 97.2% of signal candidates for Belle II.

We suppress B^\pm candidates in which the D comes from a $D^{*\pm}$ decay by reconstructing possible $D^{*\pm}$ candidates from the D and a charged particle (assumed to be a pion) from the ROE and vetoing any B^\pm candidate whose D forms a $D^{*\pm}$ with a mass difference $M(K_S^0 K^+ \pi^- \pi^\pm) - M(K_S^0 K^+ \pi^-)$ in the range of $[143, 148] \text{ MeV}/c^2$.

Remaining background candidates come from continuum events, cross-feed background from signal events in which the prompt K^\pm is misidentified as a π^\pm or vice-versa, or from other B decays such as $B^\pm \rightarrow D^* h^\pm$ or $B^\pm \rightarrow DK^{*\pm}$. Sidebands in D mass, $[1.73, 1.85] \text{ GeV}/c^2$ and $[1.88, 1.94] \text{ GeV}/c^2$, show no significant backgrounds from B^\pm decays without intermediate D mesons.

On average, 1.02 B^\pm candidates are reconstructed per event. In events with multiple candidates, we keep only the one with the smallest χ^2 calculated from the measured and known masses of the D , M_{bc} , the known B mass, and the resolutions on both measured masses. Simulated samples show that we select the correct candidate in 78% of such events.

6 Signal extraction

To determine the CP asymmetries and branching-fraction ratios, we fit to the two-dimensional distributions of ΔE and C' , a transformation of C that is uniformly distributed between zero and one for signal and peaks at zero for continuum background [7]. We model their distributions independently since they have negligible correlations according to simulation. We perform an unbinned extended maximum-likelihood fit simultaneously in sixteen subsets of the data distinguished by the two charges of the B^\pm , the two relative charges of the K^\pm from the D , the two species of the prompt h^\pm , and the two experiments. The fit function accounts for contributions from the signal decays, continuum background, cross-feed background, and other $B\bar{B}$ backgrounds. We perform the fit separately for the full D phase space and for the $K^{*\pm}$ region.

For the signal component, we model ΔE as a sum of two Gaussian functions and an asymmetric Gaussian function with all parameters fixed to values determined from simulated samples, except for the common mean of all three Gaussian functions and a common multiplier for all their widths, which account for differences in resolution between the experiment and simulation. We model C' as uniformly distributed.

For the continuum component, we model ΔE as a straight line and C' as the sum of two exponential functions. All parameters but the slope of the line and the rate parameter for the steeper exponential are fixed to values determined from simulated samples.

For the cross-feed component, we model ΔE identically to signal, but with its own parameters, and C' as a straight line, with independent parameters for the DK and $D\pi$ data and all parameters fixed to values determined from simulated samples.

For the $B\bar{B}$ -background component, we model ΔE as a sum of two exponential functions and C' as a straight line, with independent parameters for DK and $D\pi$ and all parameters except for the rate parameter of the steeper exponential function fixed to values determined from simulated samples.

Table 1. PID and tracking efficiencies.

	Belle	Belle II
ϵ_{K^+}	0.845 ± 0.009	0.820 ± 0.005
ϵ_{K^-}	0.853 ± 0.008	0.821 ± 0.004
ϵ_{π^+}	0.926 ± 0.008	0.929 ± 0.003
ϵ_{π^-}	0.930 ± 0.009	0.918 ± 0.003
δ	0.973 ± 0.005	0.972 ± 0.004

For each data subset i , the total number of signal events $n_{\text{tot},i}^{Dh}$ with no PID requirement on the K or π from the B are related to the observed numbers of signal events $n_{\text{sig},i}^{Dh}$, cross-feed background yields $n_{\text{bkg},i}^{Dh}$, and the PID efficiencies ϵ_h as follows:

$$n_{\text{sig},i}^{DK} = \epsilon_K n_{\text{tot},i}^{DK}, \quad (6.1)$$

$$n_{\text{sig},i}^{D\pi} = \epsilon_\pi n_{\text{tot},i}^{D\pi}, \quad (6.2)$$

$$n_{\text{bkg},i}^{DK} = (1 - \epsilon_\pi) n_{\text{tot},i}^{D\pi}, \quad (6.3)$$

$$n_{\text{bkg},i}^{D\pi} = (1 - \epsilon_K) n_{\text{tot},i}^{DK}. \quad (6.4)$$

In the fit, the $n_{\text{tot},i}^{DK}$ and $n_{\text{tot},i}^{D\pi}$ yields are expressed in terms of the CP asymmetries and branching-fraction ratios, the sum of all $n_{\text{tot},i}^{D\pi}$, and the ratio δ of the efficiency for detecting $B^\pm \rightarrow DK^\pm$ over that for $B^\pm \rightarrow D\pi^\pm$ decays. The PID efficiencies and δ are fixed from simulated samples and corrected for discrepancies between experiment and simulation that depend on particle momentum, direction, charge, and species. Those corrections range between 0.960 and 0.984, with typical uncertainties of 0.008 to 0.009, as estimated using the control channels $D^{*+} \rightarrow D^0(\rightarrow K^-\pi^+)\pi^+$, and $K_S^0 \rightarrow \pi^+\pi^-$ for K^\pm and π^\pm . Table 1 lists the post-correction efficiencies and δ with uncertainties including those on the corrections.

The fits determine the sums of all $n_{\text{tot},i}^{D\pi}$ yields in the full D phase space to be 2209 ± 59 for Belle and 1210 ± 39 for Belle II, and in the region of the $K^{*\pm}$ to be 1337 ± 42 for Belle and 732 ± 30 for Belle II. The sum of all $n_{\text{tot},i}^{DK}$ yields is calculated using the fit results and is 238 ± 21 for Belle and 131 ± 12 for Belle II for the full D phase space and 126 ± 15 for Belle and 69 ± 9 for Belle II in the region of the $K^{*\pm}$. Figures 1–4 show the data and fit results for the full D phase space, and figures 5–8 show the same distributions for the $K^{*\pm}$ region. We enhance the signal in the plots by displaying ΔE distributions for events with $C' > 0.6$, and the C' distributions for events with $|\Delta E| < 0.05 \text{ GeV}$. In the ΔE distributions of the DK data subset, the cross-feed component from $D\pi$ is visible as a peak at higher ΔE values.

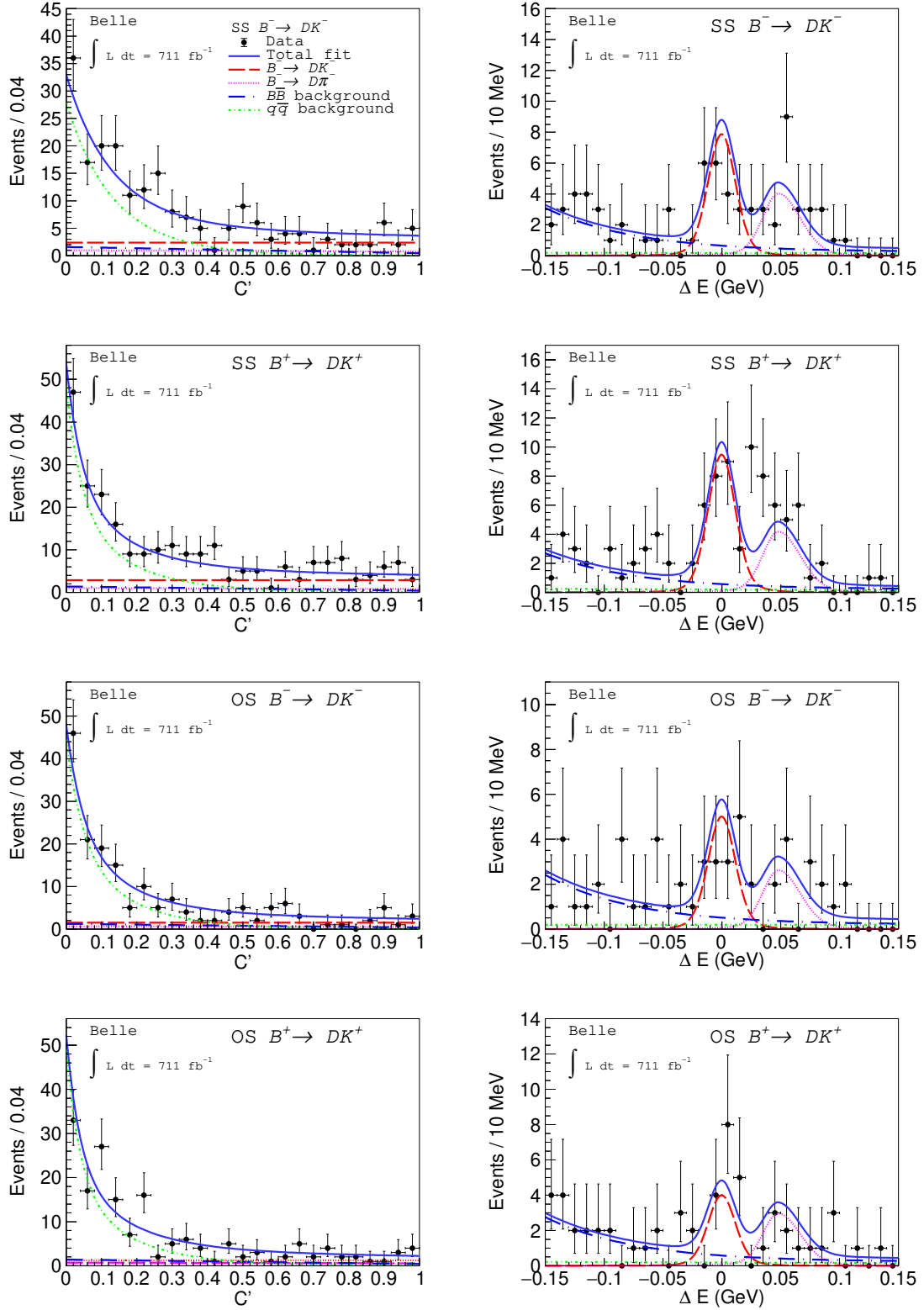


Figure 1. Distributions of C' and ΔE for $B^\pm \rightarrow DK^\pm$ candidates reconstructed in the Belle data for the full D phase space, with the fit results overlaid. The SS or OS indicates the type of the signal decay chain, same-sign or opposite-sign, respectively.

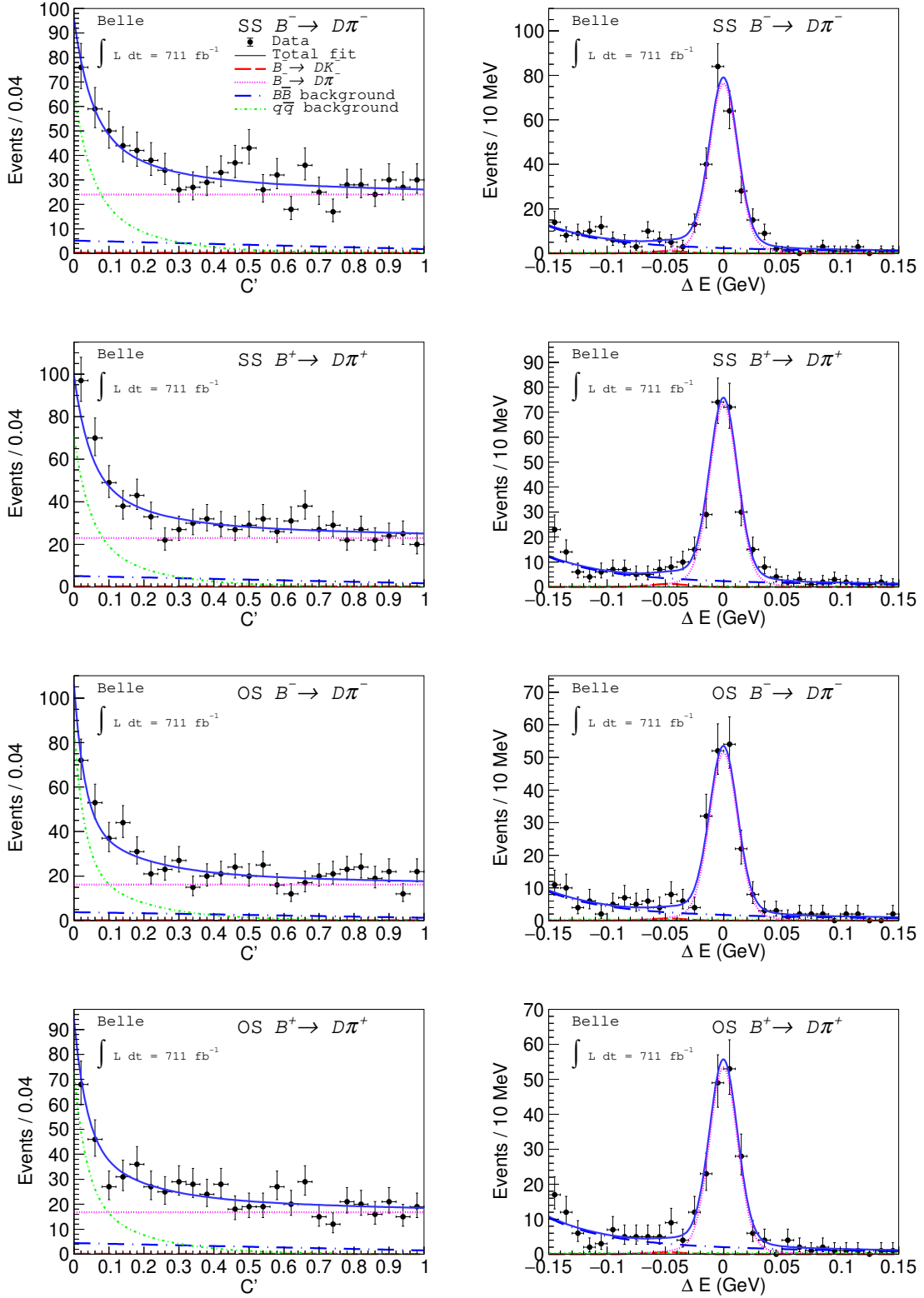


Figure 2. Distributions of C' and ΔE for $B^\pm \rightarrow D\pi^\pm$ candidates reconstructed in the Belle data for the full D phase space, with the fit results overlaid. The SS or OS indicates the type of the signal decay chain, same-sign or opposite-sign, respectively.

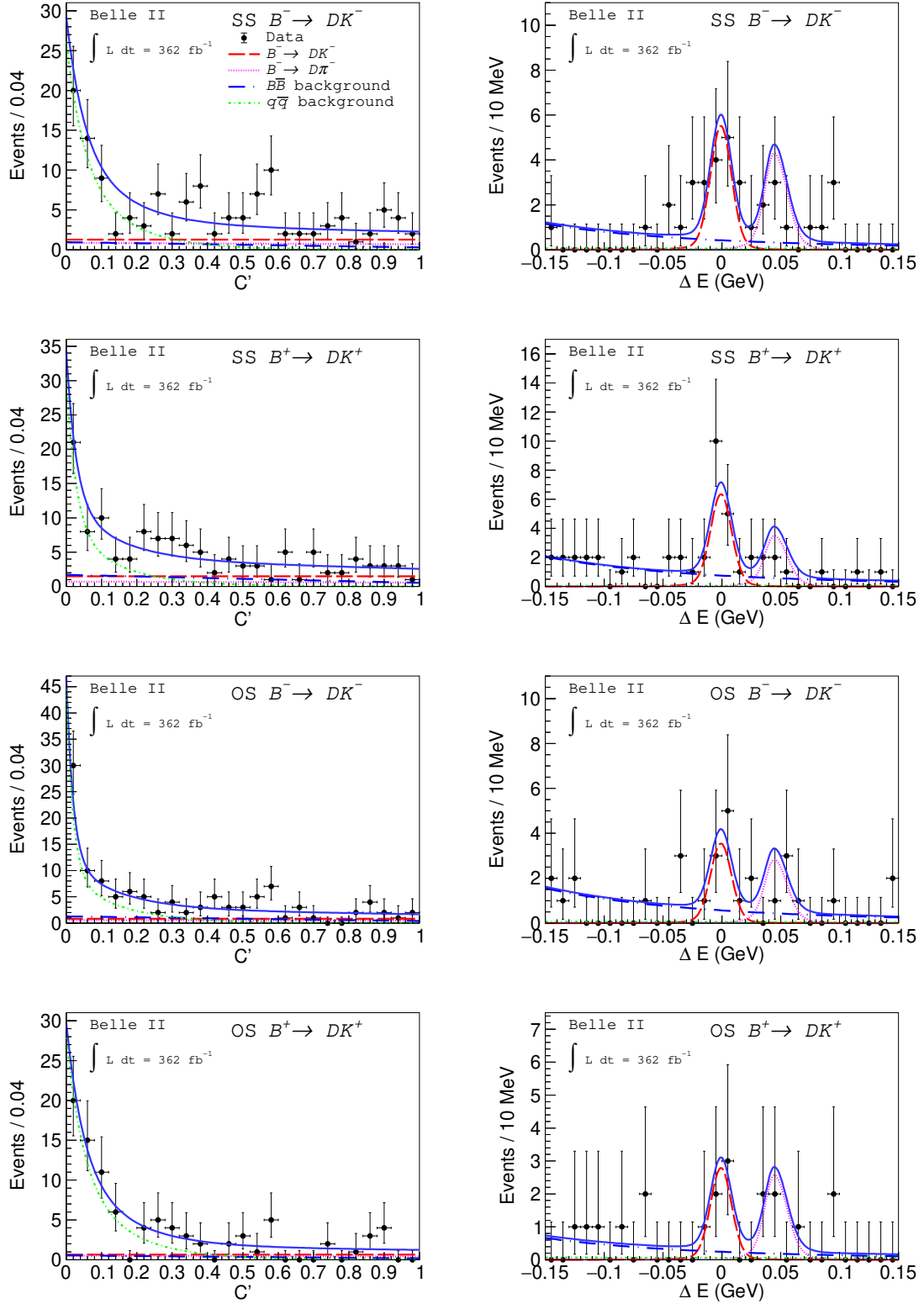


Figure 3. Distributions of C' and ΔE for $B^\pm \rightarrow DK^\pm$ candidates reconstructed in the Belle II data for the full D phase space, with the fit results overlaid. The SS or OS indicates the type of the signal decay chain, same-sign or opposite-sign, respectively.

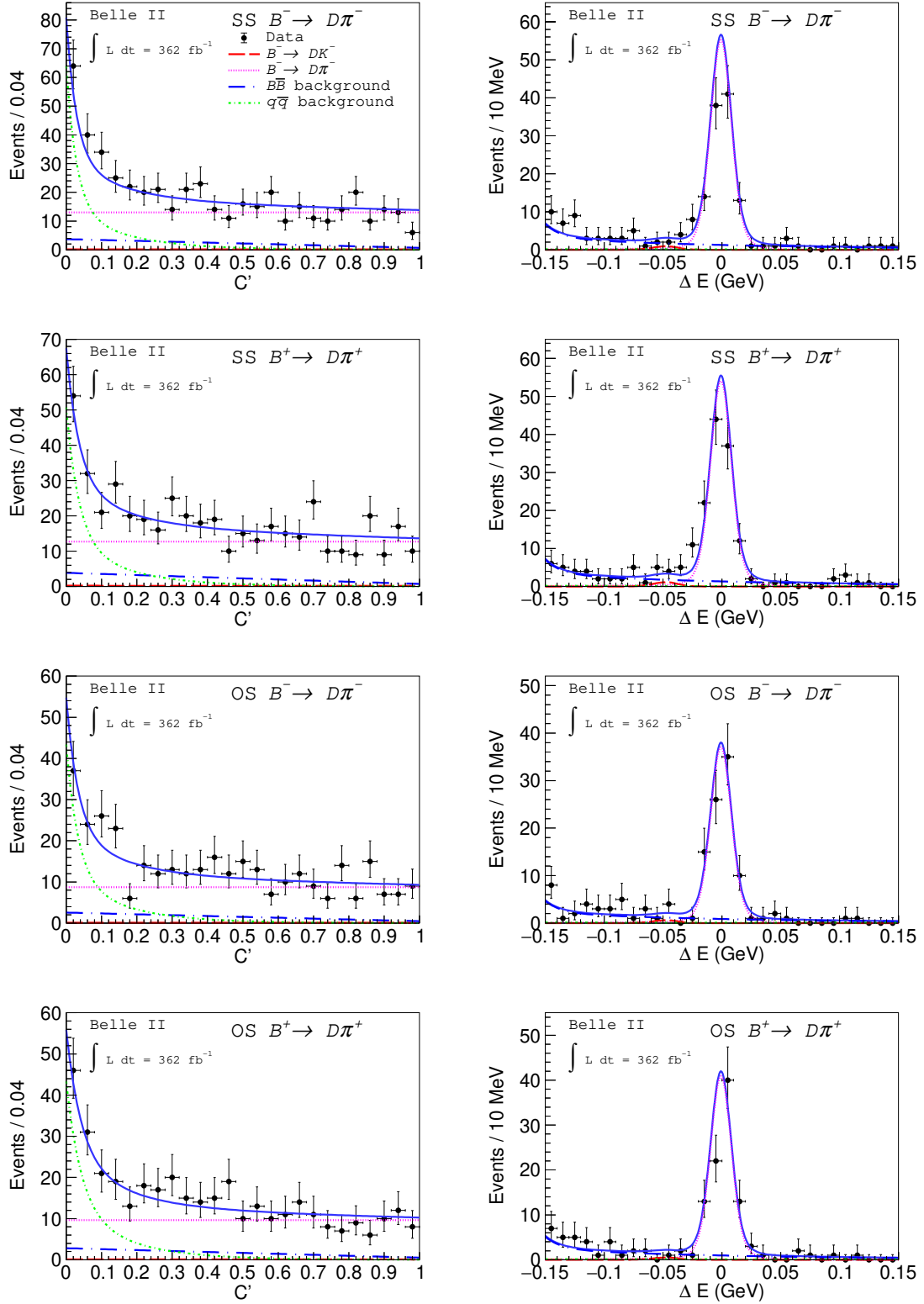


Figure 4. Distributions of C' and ΔE for $B^\pm \rightarrow D\pi^\pm$ candidates reconstructed in the Belle II data for the full D phase space, with the fit results overlaid. The SS or OS indicates the type of the signal decay chain, same-sign or opposite-sign, respectively.

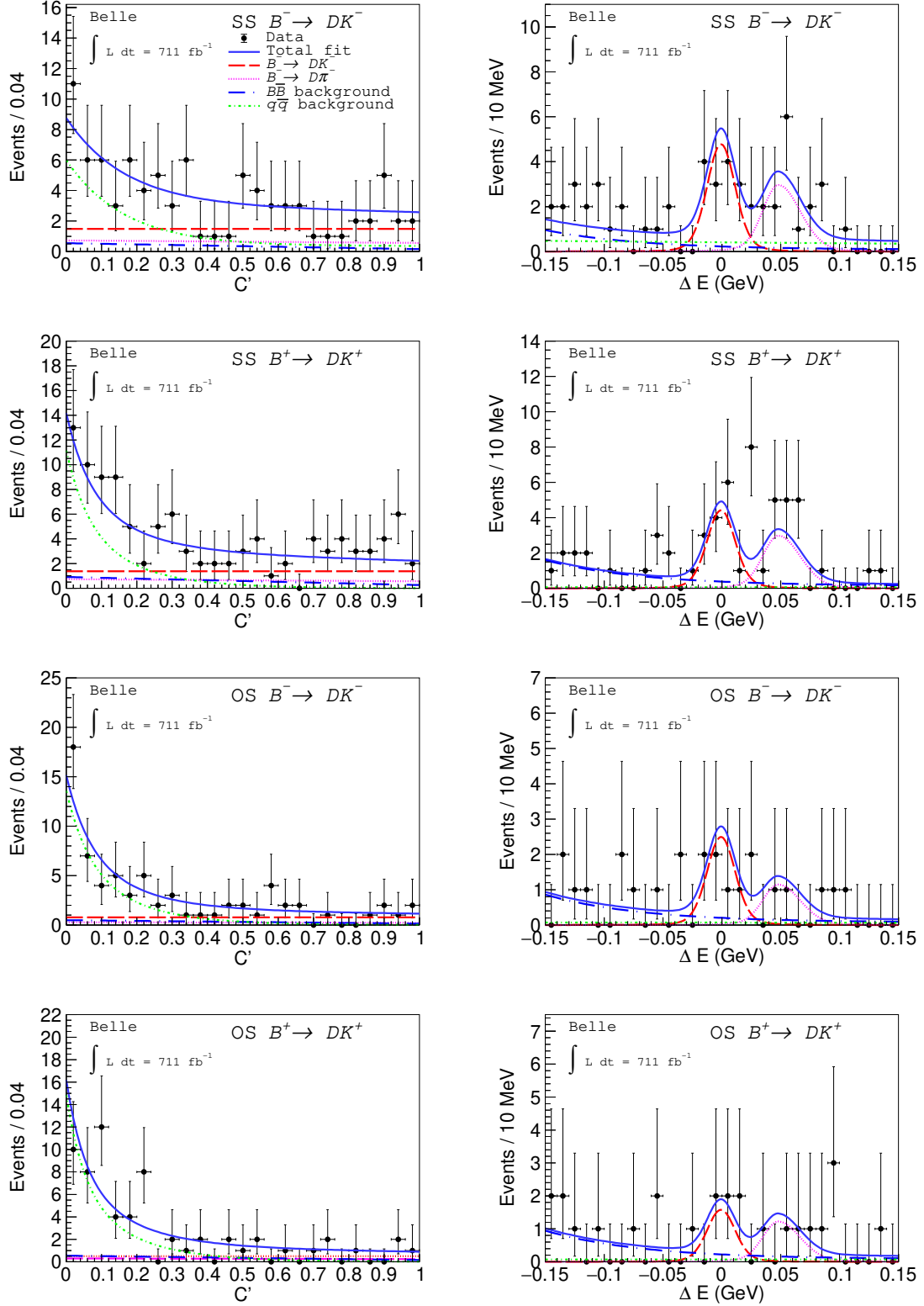


Figure 5. Distributions of C' and ΔE for $B^\pm \rightarrow DK^\pm$ candidates reconstructed in the Belle data for the $K^{*\pm}$ region, with the fit results overlaid. The SS or OS indicates the type of the signal decay chain, same-sign or opposite-sign, respectively.

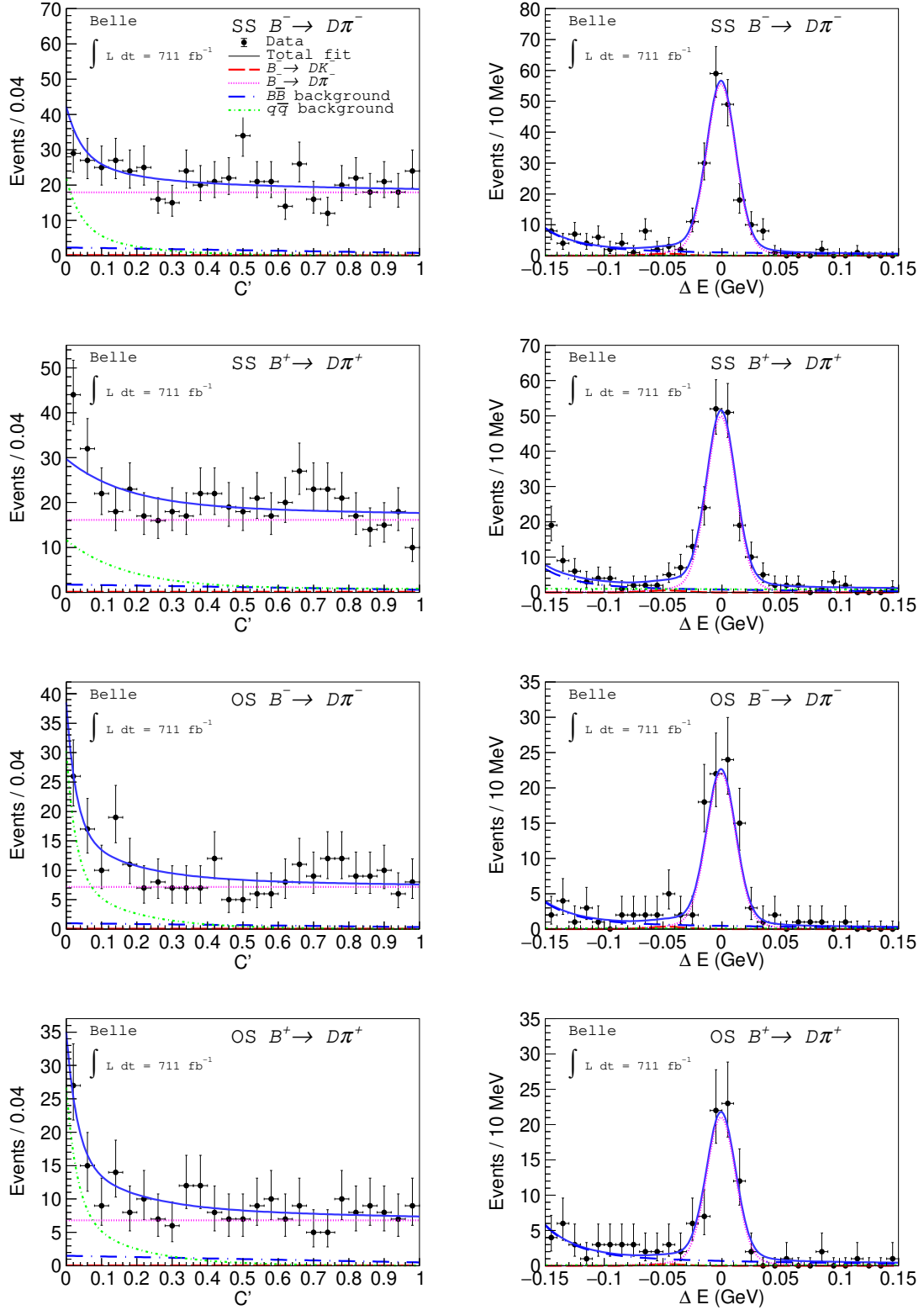


Figure 6. Distributions of C' and ΔE for $B^\pm \rightarrow D\pi^\pm$ candidates reconstructed in the Belle data for the $K^{*\pm}$ region, with the fit results overlaid. The SS or OS indicates the type of the signal decay chain, same-sign or opposite-sign, respectively.

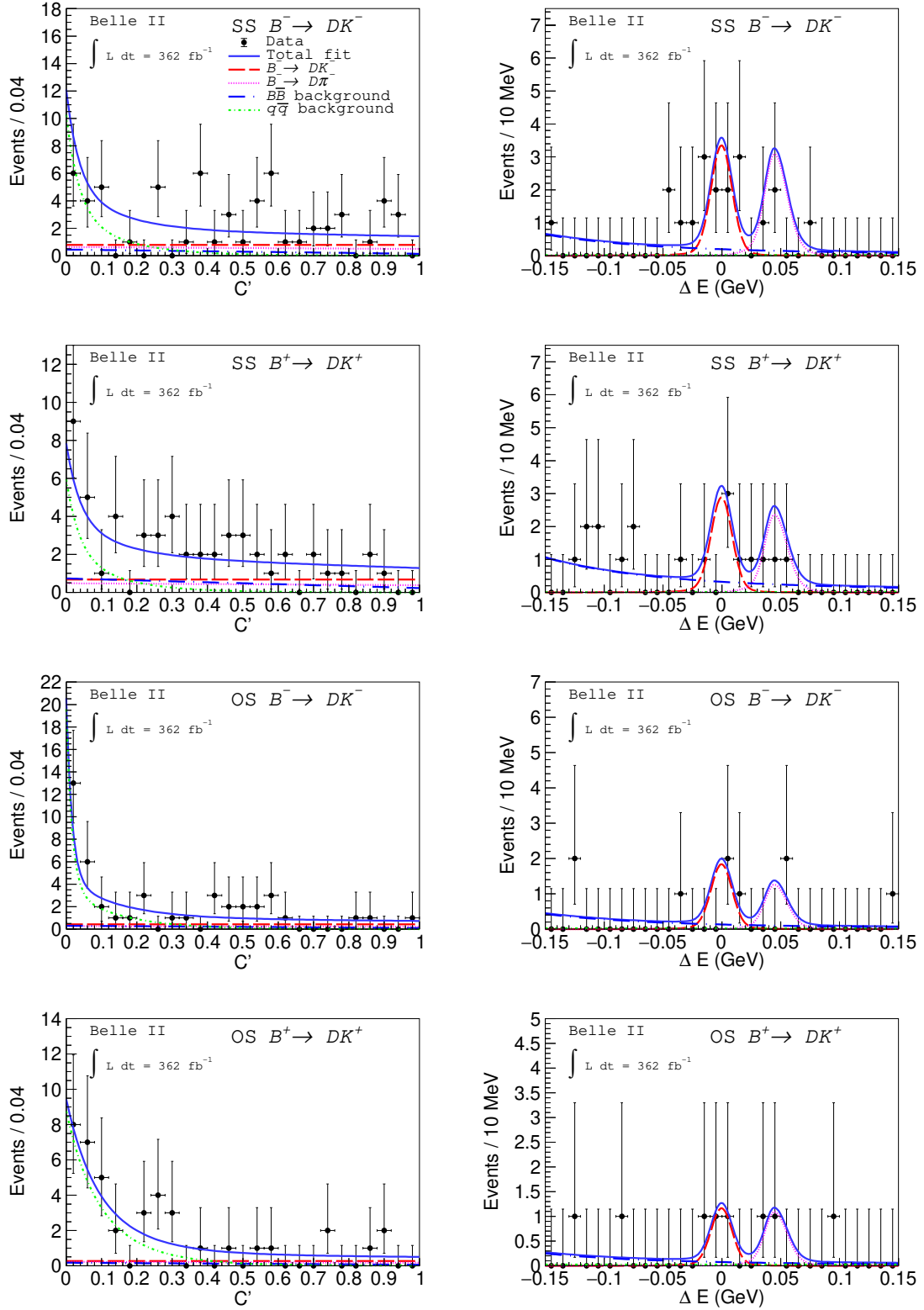


Figure 7. Distributions of C' and ΔE for $B^\pm \rightarrow DK^\pm$ candidates reconstructed in the Belle II data for the $K^{*\pm}$ region, with the fit results overlaid. The SS or OS indicates the type of the signal decay chain, same-sign or opposite-sign, respectively.

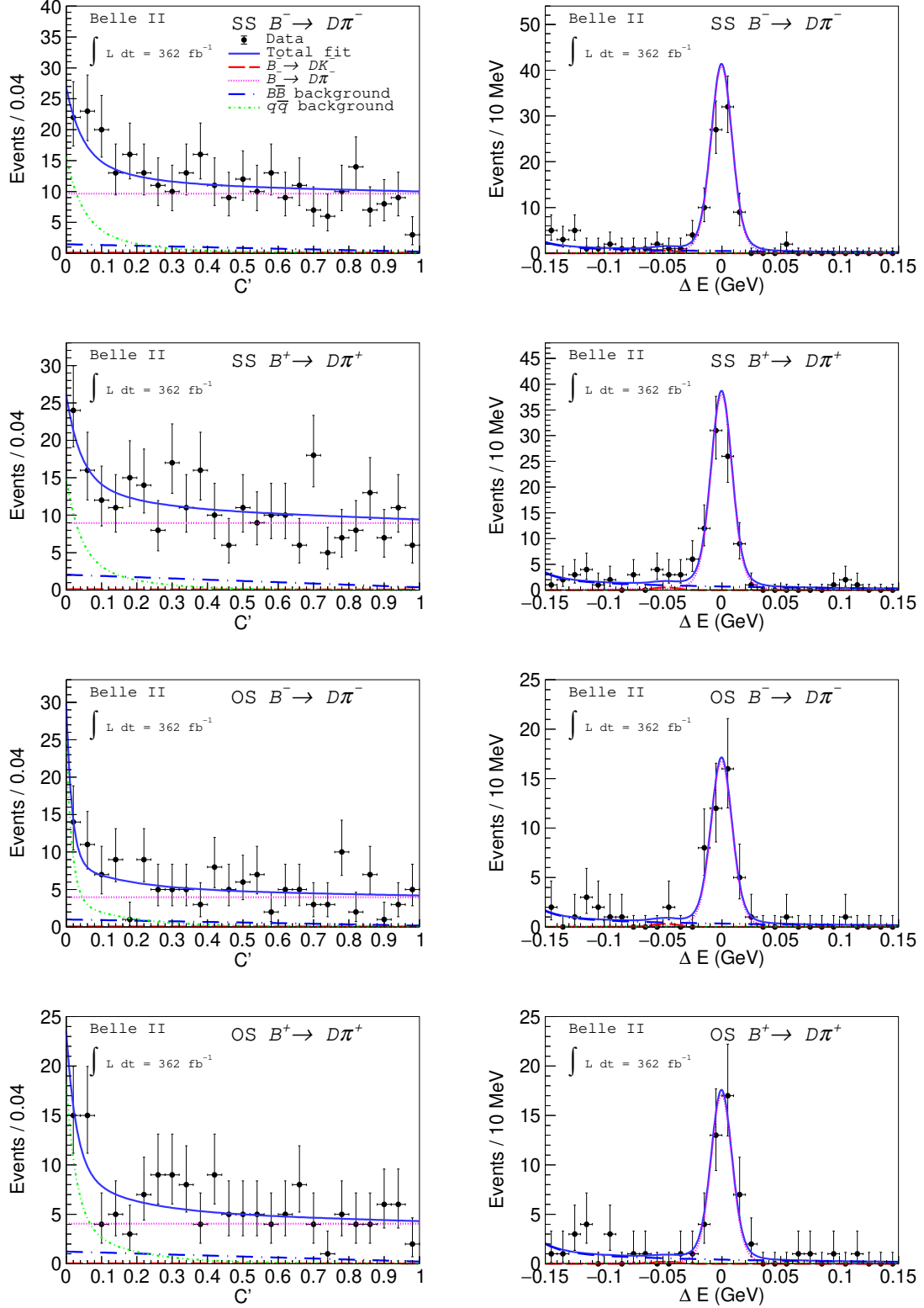


Figure 8. Distributions of C' and ΔE for $B^\pm \rightarrow D\pi^\pm$ candidates reconstructed in the Belle II data for the $K^{*\pm}$ region, with the fit results overlaid. The SS or OS indicates the type of the signal decay chain, same-sign or opposite-sign, respectively.

Table 2. Systematic and statistical uncertainties in units of 10^{-2} . The entries “—” indicate uncertainties smaller than 10^{-4} .

	A_{SS}^{DK}	A_{OS}^{DK}	$A_{SS}^{D\pi}$	$A_{OS}^{D\pi}$	$R_{SS}^{DK/D\pi}$	$R_{OS}^{DK/D\pi}$	$R_{SS/OS}^{D\pi}$
Full D phase space							
$\epsilon_{K^\pm}, \epsilon_{\pi^\pm}$	0.38	0.56	0.19	0.14	0.05	0.06	0.09
δ	—	0.03	—	—	0.04	0.03	0.02
Model	0.62	0.78	0.02	0.02	0.30	0.22	0.07
$\epsilon_{K_S^0 K^- \pi^+} / \epsilon_{K_S^0 K^+ \pi^-}$	0.82	0.83	0.82	0.83	0.01	0.01	0.02
Total syst. unc.	1.1	1.3	0.9	0.9	0.4	0.3	0.2
Stat. unc.	9.1	13.3	2.6	3.1	1.2	1.3	5.7
$K^{*\pm}$ region							
$\epsilon_{K^\pm}, \epsilon_{\pi^\pm}$	0.37	0.61	0.17	0.15	0.03	0.08	0.13
δ	0.02	0.02	0.01	0.01	0.03	0.04	0.04
Model	1.04	0.97	0.20	0.03	0.46	0.49	0.61
$\epsilon_{K_S^0 K^- \pi^+} / \epsilon_{K_S^0 K^+ \pi^-}$	1.6	0.8	1.6	0.8	0.1	0.1	1.7
Total syst. unc.	2.0	1.4	1.6	0.9	0.5	0.6	1.9
Stat. unc.	11.9	18.4	2.9	4.6	1.2	2.0	13.2

7 Systematic uncertainties

We consider the sources of systematic uncertainties listed in table 2. For the first three sources in the table, we estimate the systematic effects associated with the fixed efficiencies in the fit and the uncertainties in the choice of model parameters. We vary the values for the fixed parameters one thousand times, sampling them from a multivariate Gaussian distribution with the known uncertainties and correlations. We repeat the fit for each variation and inspect the distributions of results. If they are approximately Gaussian, we take the standard deviations as systematic uncertainties. If they are non-Gaussian, we conservatively take the full ranges of the distributions as the systematic uncertainties. For the ΔE shape of the continuum component, which does not have any fixed parameters, we repeat the fit using a second-order polynomial function as an alternative model and take the changes of the fit results as systematic uncertainties.

In our fits, we assume equal efficiencies for detecting and reconstructing $D \rightarrow K_S^0 K^- \pi^+$ and $D \rightarrow K_S^0 K^+ \pi^-$ decays. In simulation, the ratio of the former to the latter is 0.98. We repeat the analysis using this value and assign the differences in the results as systematic uncertainties. Our simulated samples are generated assuming the D decay products are evenly distributed in the available phase space, which is not the distribution expected in the data. However, the efficiency ratios calculated with an alternative decay model of $D \rightarrow K^{*\pm} K^\mp$, which is the dominant process in $D \rightarrow K_S^0 K^\mp \pi^\pm$ [30], are equivalent to the nominal ones.

8 Results

The results from the Belle and Belle II data for the full D phase space are

$$A_{\text{SS}}^{DK} = -0.089 \pm 0.091 \pm 0.011, \quad (8.1)$$

$$A_{\text{OS}}^{DK} = 0.109 \pm 0.133 \pm 0.013, \quad (8.2)$$

$$A_{\text{SS}}^{D\pi} = 0.018 \pm 0.026 \pm 0.009, \quad (8.3)$$

$$A_{\text{OS}}^{D\pi} = -0.028 \pm 0.031 \pm 0.009, \quad (8.4)$$

$$R_{\text{SS}}^{DK/D\pi} = 0.122 \pm 0.012 \pm 0.004, \quad (8.5)$$

$$R_{\text{OS}}^{DK/D\pi} = 0.093 \pm 0.013 \pm 0.003, \quad (8.6)$$

$$R_{\text{SS/OS}}^{D\pi} = 1.428 \pm 0.057 \pm 0.002, \quad (8.7)$$

and for the $K^{*\pm}$ region are

$$A_{\text{SS}}^{DK} = 0.055 \pm 0.119 \pm 0.020, \quad (8.8)$$

$$A_{\text{OS}}^{DK} = 0.231 \pm 0.184 \pm 0.014, \quad (8.9)$$

$$A_{\text{SS}}^{D\pi} = 0.046 \pm 0.029 \pm 0.016, \quad (8.10)$$

$$A_{\text{OS}}^{D\pi} = 0.009 \pm 0.046 \pm 0.009, \quad (8.11)$$

$$R_{\text{SS}}^{DK/D\pi} = 0.093 \pm 0.012 \pm 0.005, \quad (8.12)$$

$$R_{\text{OS}}^{DK/D\pi} = 0.103 \pm 0.020 \pm 0.006, \quad (8.13)$$

$$R_{\text{SS/OS}}^{D\pi} = 2.412 \pm 0.132 \pm 0.019, \quad (8.14)$$

where the first uncertainty is statistical and the second is systematic. Tables 3–6 list the statistical and systematic correlations of all results. Our results are consistent with LHCb’s results [11], with less precision due to smaller sample size. This study is also performed with the Belle data set alone and the results are reported in Appendix A.

Our results alone do not allow for an unambiguous determination of ϕ_3 , but combined with other results in global fits they constrain it. The enhancement of the $D \rightarrow K_s^0 K^\pm \pi^\mp$ strong-phase difference (δ_D) in the $K^{*\pm}$ region indicated by the current CLEO measurement [10] suggests that GLS results restricted to that region may better constrain ϕ_3 than the results for the full D phase space. More precise δ_D determinations might challenge this finding in the future.

9 Summary

We measure the CP asymmetries and branching-fraction ratios for $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ decays with $D \rightarrow K_s^0 K^\pm \pi^\mp$ using the full Belle data set containing 772×10^6 $B\bar{B}$ pairs and a Belle II data set containing 387×10^6 $B\bar{B}$ pairs. We extract these observables simultaneously through a simultaneous fit across data sets and channels for the full D phase space and in the region of the $K^{*\pm}$. These results, combined with other ϕ_3 -related results, constrain the unitarity triangle angle ϕ_3 .

Table 3. Statistical correlations for results in the full D phase space using Belle and Belle II data.

	A_{OS}^{DK}	$A_{SS}^{D\pi}$	$A_{OS}^{D\pi}$	$R_{SS}^{DK/D\pi}$	$R_{OS}^{DK/D\pi}$	$R_{SS/OS}^{D\pi}$
A_{SS}^{DK}	0.001	-0.010	0.000	0.001	-0.000	0.003
A_{OS}^{DK}		0.000	-0.006	0.003	0.008	-0.001
$A_{SS}^{D\pi}$			0.000	0.006	-0.001	-0.004
$A_{OS}^{D\pi}$				0.001	0.001	0.001
$R_{SS}^{DK/D\pi}$					0.038	-0.176
$R_{OS}^{DK/D\pi}$						0.178

Table 4. Systematic correlations for results in the full D phase space using Belle and Belle II data.

	A_{OS}^{DK}	$A_{SS}^{D\pi}$	$A_{OS}^{D\pi}$	$R_{SS}^{DK/D\pi}$	$R_{OS}^{DK/D\pi}$	$R_{SS/OS}^{D\pi}$
$A_{SS,DK}$	0.390	-0.002	0.065	0.409	0.447	0.129
$A_{OS,DK}$		0.025	0.001	0.205	0.234	0.032
$A_{SS,D\pi}$			0.036	0.052	0.043	-0.048
$A_{OS,D\pi}$				0.094	0.086	0.063
$R_{SS,DK/D\pi}$					0.953	-0.072
$R_{OS,DK/D\pi}$						0.089

Table 5. Statistical correlations for results in the region of the $K^{*\pm}$ using Belle and Belle II data.

	A_{OS}^{DK}	$A_{SS}^{D\pi}$	$A_{OS}^{D\pi}$	$R_{SS}^{DK/D\pi}$	$R_{OS}^{DK/D\pi}$	$R_{SS/OS}^{D\pi}$
A_{SS}^{DK}	0.003	-0.012	0.001	-0.052	-0.013	0.002
A_{OS}^{DK}		0.001	-0.011	-0.004	-0.034	0.002
$A_{SS}^{D\pi}$			0.000	0.002	-0.004	-0.011
$A_{OS}^{D\pi}$				-0.002	-0.002	0.014
$R_{SS}^{DK/D\pi}$					0.034	-0.132
$R_{OS}^{DK/D\pi}$						0.208

Table 6. Systematic correlations for results in the region of the $K^{*\pm}$ using Belle and Belle II data.

	A_{OS}^{DK}	$A_{SS}^{D\pi}$	$A_{OS}^{D\pi}$	$R_{SS}^{DK/D\pi}$	$R_{OS}^{DK/D\pi}$	$R_{SS/OS}^{D\pi}$
$A_{SS,DK}$	0.195	0.046	0.013	0.120	-0.053	0.191
$A_{OS,DK}$		0.038	0.004	0.344	0.210	0.006
$A_{SS,D\pi}$			0.023	-0.004	-0.037	0.017
$A_{OS,D\pi}$				-0.016	-0.023	0.006
$R_{SS,DK/D\pi}$					0.914	0.015
$R_{OS,DK/D\pi}$						-0.097

A Belle data results

The results using only Belle data for the full D phase space, are

$$A_{SS}^{DK} = -0.121 \pm 0.120 \pm 0.013, \quad (\text{A.1})$$

$$A_{OS}^{DK} = -0.016 \pm 0.182 \pm 0.014, \quad (\text{A.2})$$

$$A_{SS}^{D\pi} = 0.014 \pm 0.032 \pm 0.011, \quad (\text{A.3})$$

$$A_{OS}^{D\pi} = 0.001 \pm 0.039 \pm 0.011, \quad (\text{A.4})$$

$$R_{SS}^{DK/D\pi} = 0.112 \pm 0.014 \pm 0.002, \quad (\text{A.5})$$

$$R_{OS}^{DK/D\pi} = 0.085 \pm 0.016 \pm 0.002, \quad (\text{A.6})$$

$$R_{SS/OS}^{D\pi} = 1.472 \pm 0.074 \pm 0.002, \quad (\text{A.7})$$

and for the $K^{*\pm}$ region are

$$A_{SS}^{DK} = 0.028 \pm 0.163 \pm 0.022, \quad (\text{A.8})$$

$$A_{OS}^{DK} = 0.220 \pm 0.245 \pm 0.014, \quad (\text{A.9})$$

$$A_{SS}^{D\pi} = 0.041 \pm 0.036 \pm 0.011, \quad (\text{A.10})$$

$$A_{OS}^{D\pi} = 0.041 \pm 0.059 \pm 0.011, \quad (\text{A.11})$$

$$R_{SS}^{DK/D\pi} = 0.082 \pm 0.014 \pm 0.002, \quad (\text{A.12})$$

$$R_{OS}^{DK/D\pi} = 0.097 \pm 0.027 \pm 0.002, \quad (\text{A.13})$$

$$R_{SS/OS}^{D\pi} = 2.592 \pm 0.180 \pm 0.005. \quad (\text{A.14})$$

Tables 7–10 list the statistical and systematic correlations.

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Table 7. Statistical correlations for results in the full D phase space using Belle data alone.

	A_{OS}^{DK}	$A_{SS}^{D\pi}$	$A_{OS}^{D\pi}$	$R_{SS}^{DK/D\pi}$	$R_{OS}^{DK/D\pi}$	$R_{SS/OS}^{D\pi}$
A_{SS}^{DK}	0.000	-0.015	0.000	0.027	0.002	0.004
A_{OS}^{DK}		-0.000	-0.013	0.004	0.051	0.000
$A_{SS}^{D\pi}$			0.000	0.009	-0.003	-0.008
$A_{OS}^{D\pi}$				-0.002	-0.000	0.007
$R_{SS}^{DK/D\pi}$					0.048	-0.166
$R_{OS}^{DK/D\pi}$						0.168

Table 8. Systematic correlations for results in the full D phase space using Belle data alone.

	A_{OS}^{DK}	$A_{SS}^{D\pi}$	$A_{OS}^{D\pi}$	$R_{SS}^{DK/D\pi}$	$R_{OS}^{DK/D\pi}$	$R_{SS/OS}^{D\pi}$
$A_{SS,DK}$	0.326	-0.015	-0.073	-0.179	0.026	0.298
$A_{OS,DK}$		0.025	-0.053	-0.125	0.065	0.163
$A_{SS,D\pi}$			0.058	0.083	0.059	-0.022
$A_{OS,D\pi}$				0.036	-0.026	-0.060
$R_{SS,DK/D\pi}$					0.804	-0.313
$R_{OS,DK/D\pi}$						0.183

Table 9. Statistical correlations for results in the region of the $K^{*\pm}$ using Belle data alone.

	A_{OS}^{DK}	$A_{SS}^{D\pi}$	$A_{OS}^{D\pi}$	$R_{SS}^{DK/D\pi}$	$R_{OS}^{DK/D\pi}$	$R_{SS/OS}^{D\pi}$
$A_{SS,DK}$	-0.003	-0.013	-0.001	-0.072	0.014	-0.001
$A_{OS,DK}$		-0.000	-0.021	-0.006	-0.019	0.003
$A_{SS,D\pi}$			0.000	0.005	0.001	-0.009
$A_{OS,D\pi}$				-0.002	-0.006	0.020
$R_{SS,DK/D\pi}$					0.036	-0.121
$R_{OS,DK/D\pi}$						0.196

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Table 10. Systematic correlations for results in the region of the $K^{*\pm}$ using Belle data alone.

	A_{OS}^{DK}	$A_{SS}^{D\pi}$	$A_{OS}^{D\pi}$	$R_{SS}^{DK/D\pi}$	$R_{OS}^{DK/D\pi}$	$R_{SS/OS}^{D\pi}$
A_{SS}^{DK}	0.197	0.176	-0.049	0.299	0.007	0.283
A_{OS}^{DK}		0.056	-0.025	0.051	0.152	0.070
$A_{SS}^{D\pi}$			0.056	0.035	-0.002	0.126
$A_{OS}^{D\pi}$				0.009	-0.049	0.059
$R_{SS}^{DK/D\pi}$					0.583	0.313
$R_{OS}^{DK/D\pi}$						-0.059

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