DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 86-045
April 1986

GEOMETRIC STRING FIELD THEORY

by

G. Münster

II. Institut für Theoretische Physik, Universität Hamburg

ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

DESY 86-045
April 1986

ISSN 0418-9833

GEOMETRIC STRING FIELD THEORY

Gernot Münster

II. Institut für Theoretische Physik
Universität Hamburg
Luruper Chausse 149
D-2000 Hamburg 50

Abstract:

A reparametrization invariant action for string fields describing non-interacting closed bosonic strings is presented. Its construction is based on geometric principles motivated by Polyakov's functional integral for surfaces. Special care is devoted to the precise definition of singular operators and to reparametrization anomalies.

April 1986

1. Introduction and summary

More than ten years ago Kaku and Kikkawa [1] worked out a field theory of relativistic strings. The basic object is the string field $\Phi[x]$, which is a functional of open or closed curves

$$x = (x^\mu(\sigma)) \quad ; \quad \mu = 1,\dots,d=26 \, , \quad 0 \leq \sigma \leq 2\pi \, .$$

In the quantum theory of string fields $\Phi[x]$ is an operator creating a string, which is located on x, from the vacuum. Therefore string field theory is sometimes referred to as "second quantized string theory". Kaku and Kikkawa consider open bosonic strings and their treatment is based on the light-cone-gauge formulation of relativistic strings [2].

On the other hand the importance of having a Lorentz- and gauge-covariant formulation of string field theory has been pointed out by several authors in the last years. Gauge covariance means covariance under reparametrizations of the string. An early attempt in this direction was made by Marshall and Ramond [3]. In their work, however, the Virasoro anomalies (see [4] for a review) are not taken into account, which spoil the reparametrization invariance of the operators under consideration.

Recently different groups have obtained covariant actions for free and interacting string fields [5-9]. Besides the original string field they contain supplementary fields, whose number is infinite in the most symmetric treatments. Although these approaches satisfy the quest for a covariant string field theory, the following questions remain open:

1.) What is the geometric meaning of the supplementary fields? As string theory is supposed to be a theory of geometrical objects, namely strings, all fields appearing in it should have some geometrical origin.

2.) The local symmetry of the actions, consisting of so-called chordal gauge transformations [6], is not identical to reparametrization invariance. It is only on the subset of string fields satisfying the Virasoro constraints

$$L_n \Phi = 0 \quad , \quad n > 0 \tag{1.1}$$

that both symmetry transformations coincide. Therefore, what happened to reparametrization invariance?

These two questions motivate a different approach, which is presented in this paper. The guiding principles are to introduce only fields with a definite geometric interpretation and to consider reparametrization invariance as the basic symmetry. The first requirement is met by the introduction of a one-dimensional metric

$$g(\sigma) \in \mathbb{R} \quad , \quad 0 \leq \sigma \leq 2\pi$$

on curves $x^{\mu}(\sigma)$. Inspired by the work of Polyakov [10], who considers a two-dimensional metric $g_{ab}(\sigma,\tau)$ on string world-sheets, the metric is treated as an independent variable. Consequently the string field is a functional $\Phi[x,g]$ also depending on $g(\sigma)$.

Below we present a free action for the closed bosonic string field Φ, which is local and reparametrization invariant. The kinetic term involving the string variable x is a generalization of the dual model Hamiltonian $L_o + \tilde{L}_o - 2$. The requirement that Virasoro anomalies are cancelled leads to the presence of a kinetic term for the metric g too. Formally the action reads in d space-time dimensions

$$S = (\Phi, K\Phi)$$

$$K = :\int d\sigma \; g(\sigma)^{-\frac{\beta}{2}} \left\{ L(\sigma) + \varkappa\, g(\sigma)^{-2} \left[\beta^2 g(\sigma)^2 \frac{\delta^2}{\delta(\log g(\sigma))^2} - (\beta^2+4\beta)\, g'(\sigma)^2 + 4\beta\, g(\sigma)\, g''(\sigma) \right] \right\}: \tag{1.2}$$

where

$$L(\sigma) = \frac{1}{\pi} \sum_{n=-\infty}^{\infty} e^{in\sigma} \left(L_n + \bar{L}_{-n} - \frac{d}{12}\delta_{n,0} \right) \quad , \tag{1.3}$$

$\varkappa$ and β are numerical constants and the prime denotes differentiation with respect to σ. The precise definition of the action S, which has to take care of anomalies and singularities in the operator K, is subject of the following sections.

When the string field $\Phi[x,g]$ is expanded in terms of oscillation modes of the metric g, an infinite number of fields $\Upsilon^{\partial}_{I}[x]$ arise, which may be regarded as supplementary fields like in [5-9].

Many problems remain to be investigated:

- construction of the corresponding action for superstrings,
- construction of interactions terms,
- relation to the actions of [5-9].

I also investigated the case of open strings. It turned out that the action corresponding to (1.2) has problems for open strings, which are due to singularities of the operator K at the ends of the string, $\sigma = 0, \pi$. These singularities produce new anomalies which spoil reparametrization invariance. Hopefully some boundary terms might cure this.

2. String fields

In this section the framework and notation is fixed. For more details see [4, 5, 6]. As usual the units are chosen such that the Regge slope does not appear in the equations. We consider Euclidean space-time in d dimensions. Closed parametrized curves are expanded as

$$X^\mu(\sigma) = \sum_{n=-\infty}^{\infty} x_n^\mu e^{in\sigma} \quad , \quad 0 \le \sigma \le 2\pi \ , \quad \mu = 1, \dots, d \tag{2.1}$$

$$x_{-n}^\mu = (x_n^\mu)^* \ .$$

The string field is a real valued functional $\Phi[X] = \Phi[X^\mu(\sigma)]$, which is assumed to be differentiable with respect to the functional derivative operator. This operator is expanded as

$$\frac{\delta}{\delta X^\mu(\sigma)} = \frac{1}{2\pi} \sum_n e^{-in\sigma} \frac{\partial}{\partial x_n^\mu} \ . \tag{2.2}$$

Creation and annihilation operators are defined by

$$\begin{aligned} a_n^\mu &= i\,(4\pi)^{-\frac{1}{2}} \left(\frac{\partial}{\partial x_{-n}^\mu} + 2\pi n\, x_n^\mu \right) \\ \bar{a}_n^\mu &= i\,(4\pi)^{-\frac{1}{2}} \left(\frac{\partial}{\partial x_n^\mu} + 2\pi n\, x_{-n}^\mu \right) \\ a_0^\mu &= \bar{a}_0^\mu \end{aligned} \tag{2.3}$$

and the oscillator algebra is

$$\begin{aligned} [a_n^\mu , a_m^\nu] &= [\bar{a}_n^\mu , \bar{a}_m^\nu] = n\, \delta^{\mu\nu}\, \delta_{n+m} \\ [a_n^\mu , \bar{a}_m^\nu] &= 0 \\ \delta_n &:= \delta_{n,0} \ . \end{aligned} \tag{2.4}$$

A vacuum string field is defined by

$$a_n^\mu \Phi_0 = \bar{a}_n^\mu \Phi_0 = 0 \ , \qquad n > 0 \tag{2.5}$$

and is a Gaussian in the variables x. In the following we consider string fields which are contained in the Fock space generated from Φ_0. The inner product on this space, formally written as a path integral

$$(\Phi_1 , \Phi_2)_X = \int \mathcal{D}[X^\mu(\sigma)] \ \ \Phi_1[X]\, \Phi_2[X] \tag{2.6}$$

is uniquely defined by

$$a_{-n}^\mu = (a_n^\mu)^+ \ , \quad \bar{a}_{-n}^\mu = (\bar{a}_n^\mu)^+ \ , \quad (\Phi_0 , \Phi_0)_X = 1 \ . \tag{2.7}$$

In the following we shall write all equations only for the a-oscillators, if they look the same for the $\bar{a}$-oscillators.

The second derivative operator

$$\frac{\delta^2}{\delta X^\mu(\sigma)\, \delta X^\mu(\tau)}$$

is singular in the limit $\sigma \to \tau$. The singularity can be removed by a normal ordering prescription. We define

$$L(\sigma) = -\frac{\delta^2}{\delta x^\mu(\sigma)\,\delta x^\mu(\sigma)} + \left(x'^\mu(\sigma)\right)^2$$

$$= \frac{1}{\pi}\sum_n e^{in\sigma}\left(L_n + \bar{L}_{-n} - \frac{d}{12}\,\delta_n\right) , \tag{2.8}$$

where

$$L_n = \frac{1}{2}\sum_\ell :a_{n+\ell}\cdot a_{-\ell}: \tag{2.9}$$

are the Virasoro operators, obeying the Virasoro algebra

$$[L_m, L_n] = (m-n)\,L_{m+n} + \frac{d}{12}\,m(m^2-1)\,\delta_{m+n} . \tag{2.10}$$

The central term proportional to d, the Virasoro anomaly, is due to the normal ordering. The constant term in $L(\sigma)$ may also be considered to be an effect of normal ordering [11]

$$\text{``}\frac{1}{2}\sum_\ell a_\ell\cdot a_{-\ell}\text{''} = L_0 + \frac{d}{2}\,\zeta(-1) = L_0 - \frac{d}{24} \tag{2.11}$$

and is included for later convenience. In the light cone gauge the free string field action is essentially given by

$$\int d\sigma\ \ L(\sigma) = 2\,(L_0 + \bar{L}_0) + \text{const.} \tag{2.12}$$

3. Reparametrizations

A reparametrization is a mapping in the space of all curves x, given by

$$x^\mu(\sigma) \longrightarrow \tilde{x}^\mu(\sigma) = x^\mu(s(\sigma)) \tag{3.1}$$

with some strictly monotonic function $s(\sigma)$, obeying

$$s(\sigma + 2\pi) = s(\sigma) + 2\pi . \tag{3.2}$$

Infinitesimally we write

$$\begin{aligned} s(\sigma) &= \sigma + \varepsilon\, f(\sigma) \\ \tilde{x}(\sigma) &= x(\sigma) + \varepsilon\, f(\sigma)\, x'(\sigma) + \mathcal{O}(\varepsilon^2) . \end{aligned} \tag{3.3}$$

The corresponding mapping which is induced on string fields

$$\Phi[x] \longrightarrow \tilde{\Phi}[x] = \Phi[\tilde{x}]$$

is infinitesimally given by

$$\tilde{\Phi}[x] = \Phi[x] + \varepsilon\int d\sigma\ f(\sigma)\ R(\sigma)\ \Phi[x] + \mathcal{O}(\varepsilon^2) , \tag{3.4}$$

where

$$R_f = \int d\sigma\ f(\sigma)\, R(\sigma) = \int d\sigma\ f(\sigma)\ x'^\mu(\sigma)\,\frac{\delta}{\delta x^\mu(\sigma)} \tag{3.5}$$

is generator of the reparametrization specified by $f(\sigma)$. R_f is a well defined operator and does not need normal ordering. When f is expanded as

$$f(\sigma) = \sum_n c_n\, e^{-in\sigma} \tag{3.6}$$

we have

$$\begin{aligned} R_f &= -i\sum_n c_n\, R_n \\ R_n &= L_n - \bar{L}_{-n} . \end{aligned} \tag{3.7}$$

The algebra of these operators

$$[R_m, R_n] = (m-n)\, R_{m+n} \tag{3.8}$$

does not contain the Virasoro anomaly. Therefore it is consistent to require that reparametrizations form a symmetry of string field theory.

The transformation of operators under reparametrizations is determined by their commutators with R_f. If an operator $A(\sigma)$ obeys

$$[R_f, A(\sigma)] = J\, f'(\sigma)\, A(\sigma) + f(\sigma)\, A'(\sigma) \tag{3.9}$$

then it is said to have reparametrization-spin (R-spin) J. For example for the following multiplication operators we find

$$x^\mu(\sigma) \quad \text{has} \quad J=0 \quad , \quad x'^\mu(\sigma) \quad \text{has} \quad J=1$$

and for the derivative

$$\frac{\delta}{\delta x^\mu(\sigma)} \quad : \quad J = 1 \qquad .$$

If $A(\sigma)$ has R-spin 1 the integral

$$\int d\sigma \;\; A(\sigma)$$

is reparametrization invariant.

The operator $L(\sigma)$, which is naively expected to have R-spin 2, transforms anomalous:

$$\begin{aligned}[R_f, L(\sigma)] &= 2\, f'(\sigma)\, L(\sigma) + f(\sigma)\, L'(\sigma) \\ &\quad - \frac{d}{6\pi}\, f'''(\sigma) \qquad .\end{aligned} \tag{3.10}$$

It is precisely this anomaly which causes difficulties in finding reparametrization invariant actions. Without the anomaly the action

$$\begin{aligned} S_{MR} &= (\Phi, K_{MR}\, \Phi)_x \\ K_{MR} &= \int d\sigma \;\; (x'(\sigma)^2)^{-\frac{1}{2}}\; L(\sigma) \end{aligned} \tag{3.11}$$

which has been considered by Marshall and Ramond [3] would be formally reparametrization invariant.

4. Reparametrization invariant action

Apart from the Virasoro anomaly another problem with the action S_{MR} is the appearance of the inverse of x' in it, which would make a treatment with Fock space methods difficult. Motivated by Polyakov's approach to the first quantized string theory [10] we looked for a solution to both problems through the introduction of an independent metric $g(\sigma)$ as a new variable. Under reparametrizations it transforms according to

$$\tilde{g}(\sigma)\, d\sigma^2 = g(s(\sigma))\;\; ds(\sigma)^2 \tag{4.1}$$

or infinitesimally

$$\tilde{g}(\sigma) = g(\sigma) + \varepsilon \left\{ 2 f'(\sigma)\, g(\sigma) + f(\sigma)\, g'(\sigma) \right\} \quad . \tag{4.2}$$

The string field $\Phi[x,g]$ now also depends on $g(\sigma)$. The generators of reparametrizations on Φ are extended correspondingly and read

$$R_f = \int d\sigma \left\{ f(\sigma)\, x'^\mu(\sigma) \frac{\delta}{\delta x^\mu(\sigma)} + \left(2 f'(\sigma)\, g(\sigma) + f(\sigma)\, g'(\sigma)\right) \frac{\delta}{\delta g(\sigma)} \right\} \quad . \tag{4.3}$$

In order to be able to give precise definitions of operators involving $g(\sigma)$, we have to specify the space of string fields and its inner product again. Formally the inner product would be given by a path integral of the form

$$(\Phi_1, \Phi_2) = \int d\mu[g(\sigma)] \; (\Phi_1[x,g], \Phi_2[x,g])_x \quad , \tag{4.4}$$

where the measure is required to be reparametrization invariant. Arguments similar to those in [10] suggest

$$d\mu[g(\sigma)] = \prod_\sigma \frac{dg(\sigma)}{g(\sigma)} = \prod_\sigma d\varphi(\sigma) \tag{4.5}$$

where

$$g(\sigma) = e^{\varphi(\sigma)} \quad . \tag{4.6}$$

Therefore we use $\varphi(\sigma)$ instead of $g(\sigma)$ in the following and define a Fock space of string fields with respect to $\varphi(\sigma)$. Let

$$\varphi(\sigma) = \sum_n \varphi_n \, e^{in\sigma} \, , \qquad \varphi_{-n} = \varphi_n^* \tag{4.7}$$

and define oscillators analogous to (2.3)

$$\begin{aligned} b_n &= i(4\pi)^{-\frac{1}{2}} \left(\frac{\partial}{\partial \varphi_{-n}} + 2\pi n \varphi_n \right) \\ \bar{b}_n &= i(4\pi)^{-\frac{1}{2}} \left(\frac{\partial}{\partial \varphi_n} + 2\pi n \varphi_{-n} \right) \end{aligned} \tag{4.8}$$

satisfying

$$[b_n, b_m] = [\bar{b}_n, \bar{b}_m] = n\,\delta_{m+n} \, , \quad [b_n, \bar{b}_m] = 0 \quad . \tag{4.9}$$

The space of string fields and its inner product are defined in the same way as in sect. 2, including the b-oscillators. Furthermore we introduce the counterpart of $L(\sigma)$, namely

$$\begin{aligned} K(\sigma) &= - \frac{\delta^2}{\delta\varphi(\sigma)\,\delta\varphi(\sigma)} + (\varphi'(\sigma))^2 \\ &:= \frac{1}{\pi} \sum_n e^{in\sigma} \left(K_n + \bar{K}_{-n} - \frac{1}{12}\delta_n \right) \end{aligned} \tag{4.10}$$

with

$$K_n = \frac{1}{2} \sum_\ell : b_{n+\ell}\, b_{-\ell} : \tag{4.11}$$

$$[K_m, K_n] = (m-n) K_{m+n} + \frac{1}{12} m(m^2-1)\,\delta_{m+n} \quad . \tag{4.12}$$

The generators of reparametrization are in terms of φ

$$\begin{aligned} R_f = \int d\sigma \Big\{ & f(\sigma)\, x'^\mu(\sigma) \frac{\delta}{\delta x^\mu(\sigma)} \\ & + \left(2f'(\sigma) + f(\sigma)\varphi'(\sigma)\right) \frac{\delta}{\delta\varphi(\sigma)} \Big\} \end{aligned} \tag{4.13}$$

$$R_n = L_n - \bar{L}_{-n} + K_n - \bar{K}_{-n} - i\sqrt{4\pi}\, n\,(b_n + \bar{b}_{-n}) \tag{4.14}$$

and still satisfy the algebra (3.8). The operator $K(\sigma)$ transforms as follows

$$\begin{aligned} [R_f, K(\sigma)] = {} & 2f'(\sigma) K(\sigma) + f(\sigma) K'(\sigma) \\ & + 4 f''(\sigma)\varphi'(\sigma) - \frac{1}{6\pi} f'''(\sigma) \quad . \end{aligned} \tag{4.15}$$

The last term is the Virasoro anomaly again, whereas the third term is due to the fact that already classically $(\varphi')^2$ does not transform like an operator with R-spin. Trying various possibilities we found that the simplest modification of $K(\sigma)$ which naively (disregarding the Virasoro anomaly) has R-spin 2, is

$$K(\sigma) - 4\varphi''(\sigma) \quad . \tag{4.16}$$

This operator offers the possibility to cancel the Virasoro anomaly of $L(z)$. The operator

$$H(z) = L(z) - \varkappa\,(K(z) - 4\varphi''(z)) \tag{4.17}$$

obeys

$$\begin{aligned}[R_f, H(z)] &= 2f'(z)\,H(z) + f(z)\,H'(z) \\ &\quad - \frac{1}{6\pi}\left[d - \varkappa(48\pi+1)\right] f'''(z)\end{aligned} \tag{4.18}$$

where the extra 48π-term comes from $\varphi''(z)$. For the choice of $\varkappa$

$$d - \varkappa(48\pi+1) = 0 \tag{4.19}$$

the anomaly is absent and $H(z)$ is an operator with R-spin 2.

In order to obtain a reparametrization invariant kinetic term K as in (3.11) we must define a quantity of the type

$$K = \int dz\,\left(g(z)\right)^{-\frac{1}{2}} H(z)$$

taking care of singularities of the operator product. The ansatz which we shall try in the following is to use normal ordering again. Let

$$\begin{aligned}\varphi(z) &= \varphi_+(z) + \varphi_-(z) \\ \varphi_\pm(z) &= \tfrac{1}{2}\varphi_0 \pm i(4\pi)^{-\frac{1}{2}} \sum_{n=1}^{\infty} \tfrac{1}{n}\left(b_{\mp n}\, e^{\mp inz} + \bar{b}_{\mp n}\, e^{\pm inz}\right).\end{aligned} \tag{4.20}$$

Then

$$:g(z)^{\gamma}: \;=\; \exp(\gamma\varphi_+(z))\,\exp(\gamma\varphi_-(z)) \tag{4.21}$$

is well defined. However, it aquires an anomalous dimension such that its R-spin comes out to be

$$J = 2\gamma - \frac{\gamma^2}{4\pi}\,. \tag{4.22}$$

We account for this by introducing the rescaled field

$$\phi = \beta\varphi \tag{4.23}$$

where

$$\beta + \frac{\beta^2}{16\pi} = 1 \tag{4.24}$$

such that the operator

$$:e^{-\frac{1}{2}\phi(z)}:$$

has R-spin J = - 1, equal to the naive R-spin of $g(z)^{-\frac{1}{2}}$. Furthermore we define the rescaled

$$\tilde{K}(z) = \beta^2 K(z) \cong -\beta^4 \frac{\delta^2}{\delta\phi(z)\,\delta\phi(z)} + \phi'(z)^2\,. \tag{4.25}$$

In products like

$$:e^{-\frac{1}{2}\phi(z)}\,\tilde{K}(z):$$

the normal ordering produces new anomalies, which change the numerical coefficients of the Virasoro anomaly. For

$$\tilde{H}(z) = \;:e^{-\frac{1}{2}\phi(z)}\left\{L(z) - \varkappa\left[\tilde{K}(z) - 4\phi''(z) + \lambda\right]\right\}: \tag{4.26}$$

we obtain

$$[R_f, \tilde{H}(z)] = f'(z)\tilde{H}(z) + f(z)\tilde{H}'(z)$$
$$- \frac{1}{6\pi}(d - 48\pi\varkappa) f'''(z) :e^{-\frac{1}{2}\phi(z)}:$$
$$+ \frac{\varkappa}{6\pi}(12\pi\lambda - \beta^2) f'(z) :e^{-\frac{1}{2}\phi(z)}: . \tag{4.27}$$

If we choose the constants $\varkappa$ and λ suitably

$$\varkappa = \frac{d}{48\pi} , \quad \lambda = \frac{\beta^2}{12\pi} \tag{4.28}$$

the anomaly is cancelled and $\tilde{H}(z)$ transforms with R-spin J = 1. Note the difference between (4.19) and (4.28).

Now the way is paved completely for writing down a reparametrization invariant action for closed string fields.

$$S = (\Phi, K\Phi)$$
$$K = \int dz \; \tilde{H}(z) \tag{4.29}$$

with $\tilde{H}(z)$ defined in (4.26).

If one tries to write down the analogous action for open string fields, it turns out that in this case normal ordering introduces some additional anomalies, which would have to be cancelled by some boundary terms in the action.

Returning to the case of closed strings, let $\hat{\Phi}^0_0[\varphi]$ be that string field which is the Fock space vacuum with respect to $\varphi(z)$.

$$b_n \hat{\Phi}^0_0 = \bar{b}_n \hat{\Phi}^0_0 = 0 , \quad n \geq 0 \tag{4.30}$$

When $\Phi[x,\varphi]$ is expanded like

$$\Phi[x,\varphi] = \sum_{I,J} \Psi^J_I[x,\varphi_0] \; \hat{\Phi}^J_I[\varphi]$$
$$= \Psi^0_0 \hat{\Phi}^0_0 + \Psi^0_1 \hat{\Phi}^0_1 + \Psi^1_0 \hat{\Phi}^1_0 + \Psi^0_{1,1} \hat{\Phi}^0_{1,1} + \dots \tag{4.31}$$

with

$$I = (i_1, \dots, i_k) , \quad 1 \leq i_1 \leq \dots \leq i_k$$

and J being multiindices, and

$$\hat{\Phi}^0_1 = b_{-1}\hat{\Phi}^0_0 , \quad \hat{\Phi}^1_0 = \bar{b}_{-1}\hat{\Phi}^0_0 , \quad \hat{\Phi}^0_{1,1} = b^2_{-1}\hat{\Phi}^0_0$$
$$\hat{\Phi}^0_2 = \frac{1}{\sqrt{2}} b_{-2}\hat{\Phi}^0_0 , \quad \text{etc.} \tag{4.32}$$

we obtain an infinite number of string fields $\Psi^J_I[x,\varphi_0]$. Since S transforms trivially under shifts of φ_0 , we may restrict ourselves to string fields independent of φ_0 and obtain the action in terms of the component fields

$$S = \sum_{I,J,I',J'} \Big\{ \sum_n A^{nJJ'}_{II'} \left(\Psi^J_I , \left[L_n + \bar{L}_{-n} - \left(\tfrac{d}{12} + \pi\lambda\varkappa\right)\delta_n \right] \Psi^{J'}_{I'} \right)_x$$
$$- \varkappa B^{JJ'}_{II'} \left(\Psi^J_I , \Psi^{J'}_{I'} \right)_x \Big\} \tag{4.33}$$

with coefficients

$$A^{nJJ'}_{II'} = \left(\hat{\Phi}^J_I , :\int \frac{dz}{\pi} e^{inz} e^{-\frac{1}{2}\phi(z)} : \hat{\Phi}^{J'}_{I'} \right) \tag{4.34}$$

$$B^{JJ'}_{II'} = \left(\hat{\Phi}^J_I , :\int dz \; e^{-\frac{1}{2}\phi(z)} \left(\tilde{K}(z) - 4\phi''(z) \right) : \hat{\Phi}^{J'}_{I'} \right) \tag{4.35}$$

which can be evaluated straightforwardly.

5. Conclusions

We have obtained a reparametrization invariant action for non-interacting closed string fields, which allows to interpret supplementary fields as stemming from an extra metric variable on the string.

Our approach is motivated by Polyakov's treatment of the path integral for surfaces [10]. Although I believe that there is some relation to it, it is not yet clear what the connection is. In particular the action itself does not single out the number of dimensions d = 26 particularly. A possibility would be that d = 26 comes out as a special case, when the gauge is fixed à la Faddeev-Popov and the theory is quantized. If this produces additional contributions to the Virasoro anomaly, the value

$$\varkappa = \frac{d-26}{48\pi}$$

as suggested by Polyakov's calculation could result. This question deserves further study.

The next step would be to define a proper interaction term. Questions of spontaneous symmetry breaking etc. could then be investigated.

Acknowledgement: I thank Peter Weisz for discussions and reading the manuscript.

References:

1. M. Kaku, K. Kikkawa, Phys. Rev. D10 (1974) 1110, 1823
2. J. Goldstone et al., Nucl. Phys. B56 (1973) 109
3. C. Marshall, P. Ramond, Nucl. Phys. B85 (1975) 375
4. J. Scherk, Rev. Mod. Phys. 47 (1975) 123
5. A. Neveu, P. West, CERN-TH.4200/85;
   A. Neveu, H. Nicolai, P. West, Nucl. Phys. B264 (1986) 573;
   A. Neveu, J. Schwarz, P. West, Phys. Letters B164 (1985) 51;
   A. Neveu, P. West, Phys. Letters B165 (1985) 63;
   A. Neveu, H. Nicolai, P. West, Phys. Letters B167 (1986) 307;
   A. Neveu, P. West, Phys. Letters B168 (1986) 192
6. T. Banks, M. Peskin, Nucl. Phys. B264 (1986) 513
7. M. Kaku, J. Lykken, CUNY preprint, New York, 1985
8. W. Siegel, B. Zwiebach, Nucl. Phys. B263 (1986) 105
9. P. Ramond, preprint UFTP-85-18, Florida, 1985
10. A. Polyakov, Phys. Letters B103 (1981) 207
11. L. Brink, H.B. Nielsen, Phys. Letters B45 (1973) 332