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Probing Top-Quark Couplings at Polarized NLC

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ABSTRACT

The energy spectrum of the lepton(s) in $e^+e^- \to t\bar{t} \to \ell^\pm \cdots / \ell^+\ell^- \cdots$ at next linear colliders (NLC) is studied for arbitrary longitudinal beam polarizations as a test of possible new physics in top-quark couplings. The most general non-standard form factors are assumed for $\gamma t\bar{t}$, $Zt\bar{t}$ and Wtb vertices to analyze new-physics effects in a model-independent way. Expected precision in determining these form factors is estimated applying the optimal-observable procedure to the spectrum.

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1. Introduction

The discovery of the top quark has completed the fermion spectrum required by the electroweak standard model (SM). It is still an open question, however, if the top-quark interactions obey the SM scheme or there exists any new-physics contribution. The top quark decays immediately after being produced [1] since its huge mass $m_t^{exp} = 175.6 \pm 5.5$ GeV [2] leads to a decay width Γ_t much larger than $\Lambda_{\rm QCD}$. Therefore the decay process is not influenced by any fragmentation effects and the decay products carry lots of information on the top-quark properties.

The energy distribution of the final lepton(s) in $e^+e^- \to t\bar{t} \to \ell^\pm \cdots / \ell^+\ell^- \cdots$ turns out to be a useful tool to analyze the top-quark couplings [3]. Indeed it has been frequently studied in the literature over the past several years [4]–[11] in order to find observables sensitive to CP violation. To illustrate this point, it will be instructive to see how the spectrum is affected by non-conservation of CP in the production process:

Since $t\bar{t}$ are produced mainly through γ/Z exchange, their helicities would be only $(+-)$ or $(-+)$ if m_t were much smaller than $\sqrt{s}$. Fortunately, however, this is not the case and we can expect copious $(++)$ and $(--)$ productions as well even at $\sqrt{s} = 500$ GeV.[♯1] These states transform into each other under CP operation as $\hat{C}\hat{P}|\mp\mp\rangle = |\pm\pm\rangle$, which means that the difference $N(--) - N(++)$ could be a useful measure of CP violation [3]–[6]. This important information cannot be drawn directly since the top decays too rapidly as mentioned, but is transferred to the final-lepton-energy distributions as follows:

(1) The heavy top requires a large fraction ($\sim 70\,\%$) of W bosons are longitudinally polarized in $t \to bW$ since $\bar{b}\gamma_\mu\gamma_5 t \cdot \varepsilon^\mu \sim m_t \bar{b}\gamma_5 t$ when $\varepsilon^\mu = \varepsilon_L^\mu \sim k^\mu$ (ε and k are the polarization and the four-momentum of W, respectively).

(2) The produced b ($\bar{b}$) is left-handed (right-handed) in the SM since $m_b/\sqrt{s} \ll 1$.

[♯1] A rough estimate within the SM gives $N(-+) : N(+-) : N(--) : N(++)$ is $5 : 3.5 : 1 : 1$, where $N(\cdots)$ denotes the number of $t\bar{t}$ pairs with the indicated helicities

(3) Because of (1) and (2), W^+'s three-momentum prefers to be parallel (anti-parallel) to that of $t(+)(t(-))$, where $t(\cdots)$ expresses a top with the indicated helicity. Consequently ℓ^+ in the $t(+)$ decay becomes more energetic than in the $t(-)$ decay, while it is just opposite for the $\bar{t}$ decay, i.e., $\bar{t}(-)$ produces more energetic ℓ^- than $\bar{t}(+)$ does.

(4) Therefore, we expect larger number of energetic ℓ^+ (ℓ^-) for $N(--) < N(++)$ (for $N(--) > N(++)$).

In realistic analyses, one should take into account that other source of non-SM effects may also exist. However, most of the above-mentioned articles focused on *CP*-violating effects in $\gamma/Zt\bar{t}$ vertices (production) only and did not assume the most general form for the interactions of $\gamma t\bar{t}$, $Zt\bar{t}$ and Wtb. Therefore, in our previous paper [9], we have performed a comprehensive analysis taking into account *CP*-violating *and* *CP*-conserving non-standard top-quark couplings contributing both to the production *and* decay process.

In this paper, extending that work for arbitrary longitudinal $e^\pm$ polarizations, we present a systematic way to determine the non-SM parameters describing the general $\gamma/Zt\bar{t}$ and Wtb couplings. In our another recent paper [10] we have discussed how the same process receives non-SM contributions from effective four-Fermi interactions. Therefore, with the present work we will complete a full analysis of anomalous effects in top-quark interactions for polarized e^+e^- beams in a model-independent way, where beyond-the-SM physics is parameterized by the $SU(3)\times SU(2)\times U(1)$ symmetric effective Lagrangian [12].

This paper is organized as follows. First in sec.2 we describe the basic framework of our analysis, and give the normalized single- and double-lepton-energy distributions. Then, in sec.3, we estimate to what precision all the non-standard parameters can be measured using the optimal-observable method [13]. Adopting two sets of non-SM-parameter values we show in detail how effective the use of polarized beams could be for achieving better precision. Finally, we summarize our results in sec.4. In the appendix we collect several functions used in the main text for completeness, though they could also be found in our previous papers [7, 9, 10].

2. The lepton-energy distributions

In this section we briefly present our formalism, and then derive thereby the single- and double-lepton-energy distributions.

We will treat all the fermions except the top quark as massless and adopt the technique developed in [14]. This is a useful method to calculate distributions of final particles appearing in a production process of on-shell particles and their subsequent decays. This technique is applicable when the narrow-width approximation

$$\left| \frac{1}{p^2 - m^2 + im\Gamma} \right|^2 \simeq \frac{\pi}{m\Gamma}\delta(p^2 - m^2)$$

can be adopted for the decaying intermediate particles. In fact, this is very well satisfied for both t and W since $\Gamma_t \simeq 175(m_t/m_W)^3$ MeV $\ll m_t$ and $\Gamma_W \simeq 2$ GeV $\ll M_W$.

Adopting this method, one can derive the following formulas for the inclusive distributions of the single-lepton ℓ^+ and double-lepton $\ell^+\ell^-$ in the reaction $e^+e^- \to t\bar{t}$ [5]:

$$\begin{aligned} &\frac{d^3\sigma}{d^3\boldsymbol{p}_\ell/(2p_\ell^0)}(e^+e^- \to \ell^+ + \cdots) \\ &\qquad = \frac{4}{\Gamma_t}\int d\Omega_t \frac{d\sigma}{d\Omega_t}(n, 0)\frac{d^3\Gamma_\ell}{d^3\boldsymbol{p}_\ell/(2p_\ell^0)}(t \to b\ell^+\nu), \end{aligned} \tag{1}$$

$$\begin{aligned} &\frac{d^6\sigma}{d^3\boldsymbol{p}_\ell/(2p_\ell^0)d^3\boldsymbol{p}'_\ell/(2p_\ell^{0\prime})}(e^+e^- \to \ell^+\ell^- + \cdots) \\ &= \frac{4}{\Gamma_t^2}\int d\Omega_t \frac{d\sigma}{d\Omega_t}(n, m)\frac{d^3\Gamma_\ell}{d^3\boldsymbol{p}_\ell/(2p_\ell^0)}(t \to b\ell^+\nu)\frac{d^3\Gamma_\ell}{d^3\boldsymbol{p}'_\ell/(2p_\ell^{0\prime})}(\bar{t} \to \bar{b}\ell^-\bar{\nu}), \end{aligned} \tag{2}$$

where Γ_ℓ and Γ_t are the leptonic and total widths of *unpolarized* top respectively, and $d\sigma(n, m)/d\Omega_t$ is obtained from the angular distribution of $t\bar{t}$ with spins s_+ and s_- in $e^+e^- \to t\bar{t}$, $d\sigma(s_+, s_-)/d\Omega_t$, by the following replacement:

$$\begin{aligned} s_+^\mu \to n^\mu &= +\left(g^{\mu\nu} - \frac{p_t^\mu p_t^\nu}{m_t^2}\right)\frac{m_t}{p_t p_\ell}p_{\ell\,\nu} \\ s_-^\mu \to m^\mu &= -\left(g^{\mu\nu} - \frac{p_{\bar{t}}^\mu p_{\bar{t}}^\nu}{m_t^2}\right)\frac{m_t}{p_{\bar{t}} p'_\ell}p'_{\ell\,\nu}. \end{aligned} \tag{3}$$

(Exchanging the roles of s_+ and s_- and reversing the sign of n^μ, we get the single distribution of ℓ^-.)

In order to obtain the lepton spectra according to the above formulas we shall first calculate the $t\bar{t}$-production cross section and their decay rates.

$t\bar{t}$ production

Let us start with the $t\bar{t}$ production. We can represent the most general $t\bar{t}$ couplings to the photon and Z boson as

$$\Gamma^\mu_{vt\bar{t}} = \frac{g}{2}\,\bar{u}(p_t)\left[\,\gamma^\mu\{A_v+\delta A_v-(B_v+\delta B_v)\gamma_5\}+\frac{(p_t-p_{\bar{t}})^\mu}{2m_t}(\delta C_v-\delta D_v\gamma_5)\,\right]v(p_{\bar{t}}), \quad (4)$$

where g denotes the $SU(2)$ gauge coupling constant, $v=\gamma, Z$, and

$$A_\gamma=\frac{4}{3}\sin\theta_W,\quad B_\gamma=0,\quad A_Z=\frac{1}{2\cos\theta_W}\Big(1-\frac{8}{3}\sin^2\theta_W\Big),\quad B_Z=\frac{1}{2\cos\theta_W}.$$

Among the above form factors, $\delta A_{\gamma,Z}$, $\delta B_{\gamma,Z}$, $\delta C_{\gamma,Z}$ and $\delta D_{\gamma,Z}$ are parameterizing CP-conserving and CP-violating non-standard interactions, respectively. Note that we dropped two other independent terms proportional to $(p_t+p_{\bar{t}})^\mu$ since their effects vanish in the limit of zero electron mass.

On the other hand, interactions of initial e^+e^- have been assumed untouched by non-standard interactions since their structures are well described within the SM:

- γe^+e^- vertex

$$\Gamma^\mu_{\gamma e^+e^-} = -e\,\bar{v}(p_{e^+})\,\gamma^\mu\,u(p_{e^-})\,, \quad (5)$$

- Ze^+e^- vertex

$$\Gamma^\mu_{Ze^+e^-} = \frac{g}{4\cos\theta_W}\,\bar{v}(p_{e^+})\,\gamma^\mu(v_e+\gamma_5)\,u(p_{e^-})\,, \quad (6)$$

where $v_e\equiv -1+4\sin^2\theta_W$.

The angular distribution of polarized $t\bar{t}$ pair in presence of the above non-standard interactions is obtained after a tedious but straightforward calculation. The result is however a bit too lengthy, so we give the explicit form in the appendix

and here instead we describe its structure rather qualitatively:
First, the invariant amplitude can be expressed as

$$\mathcal{M} = \sum_{i,I} C_{iI}\, j^i_\mu J^{I\mu} \tag{7}$$

where

$$j^i_\mu \equiv \bar{v}(p_{e^+})\Gamma^i_\mu u(p_{e^-}) \quad (i = V, A)$$
$$J^I_\mu \equiv \bar{u}(p_t)\Gamma^I_\mu v(p_{\bar{t}}) \quad (I = V, A, S, P)$$

and

$$\Gamma^{V,A,S,P}_\mu \equiv \gamma_\mu,\ \gamma_\mu\gamma_5,\ q_\mu,\ q_\mu\gamma_5 \quad (q \equiv p_t - p_{\bar{t}}).$$

Therefore $|\mathcal{M}|^2$ consists of a number of terms whose coefficients are $C^*_{iI}C_{i'I'}$. In the explicit formula in the appendix, we express $C^*_{iI}C_{iI'}$ $(I, I' = V, A)$, $C^*_{iI}C_{i'I'}$ $(i \neq i'$ and $I, I' = V, A)$, $C^*_{iI}C_{i'P}$ $(i, i', I = V, A)$ and $C^*_{iI}C_{i'S}$ $(i, i', I = V, A)$ as D, E, F and G respectively, and moreover we attach subscripts V, A and VA to D and E according to $[I = I' = V]$, $[I = I' = A]$ and $[I = V, I' = A]$, while F and G are classified by $i = 1 \sim 4$ according to their V/A structure.[♯2]

It is worth to notice that:

- In the SM-limit only $D_{V,A,VA}$ and $E_{V,A,VA}$ remain and all $F_i = G_i = 0$,
- Non-zero F_i's are generated by the CP-violating form factors $\delta D_{\gamma,Z}$,
- Contributions to G_i's are created by the CP-conserving form factors $\delta C_{\gamma,Z}$.

For the initial beam-polarization we follow the convention by Tsai [16]:

$$P_{e^-} = +[N(e^-, +1) - N(e^-, -1)]/[N(e^-, +1) + N(e^-, -1)], \tag{8}$$
$$P_{e^+} = -[N(e^+, +1) - N(e^+, -1)]/[N(e^+, +1) + N(e^+, -1)], \tag{9}$$

where $N(e^{-(+)}, h)$ is the number of $e^-(e^+)$ with helicity h in each beam.[♯3] When the initial e^- and e^+ get polarized, j^V_μ and j^A_μ mix with each other since the spin (helicity) projection operator for $u(p_{e^-})$ and $v(p_{e^+})$ in the massless limit is

[♯2] More explicit formulas will appear in a separate paper [15].
[♯3] Note that P_{e^+} is defined with the opposite overall sign in some other papers (see, e.g., [17]).

$(1 \pm \gamma_5)/2$. Then we obtain the cross section for arbitrarily-polarized e^+e^- beams by replacing D_V, D_A, D_{VA}, E_V, E_A, E_{VA}, F_i and G_i ($i = 1 \sim 4$) with $D_V^{(*)}$, $D_A^{(*)}$, $D_{VA}^{(*)}$, $E_V^{(*)}$, $E_A^{(*)}$, $E_{VA}^{(*)}$, $F_i^{(*)}$ and $G_i^{(*)}$, where

$$
\begin{aligned}
D^{(*)}_{V,\,A,\,VA} &= (1 + P_{e^-}P_{e^+})D_{V,\,A,\,VA} - (P_{e^-} + P_{e^+})E_{V,\,A,\,VA},\\
E^{(*)}_{V,\,A,\,VA} &= (1 + P_{e^-}P_{e^+})E_{V,\,A,\,VA} - (P_{e^-} + P_{e^+})D_{V,\,A,\,VA},\\
F^{(*)}_{1,\,2,\,3,\,4} &= (1 + P_{e^-}P_{e^+})F_{1,\,2,\,3,\,4} - (P_{e^-} + P_{e^+})F_{2,\,1,\,4,\,3},\\
G^{(*)}_{1,\,2,\,3,\,4} &= (1 + P_{e^-}P_{e^+})G_{1,\,2,\,3,\,4} - (P_{e^-} + P_{e^+})G_{2,\,1,\,4,\,3}.
\end{aligned}
$$

t and $\bar{t}$ decays

We will adopt the following parameterization of the Wtb vertex suitable for the $t \to W^+b$ and $\bar{t} \to W^-\bar{b}$ decays:

$$\Gamma^\mu_{Wtb} = -\frac{g}{\sqrt{2}}V_{tb}\,\bar{u}(p_b)\Big[\gamma^\mu(f_1^L P_L + f_1^R P_R) - \frac{i\sigma^{\mu\nu}k_\nu}{M_W}(f_2^L P_L + f_2^R P_R)\Big]u(p_t), \quad (10)$$

$$\bar{\Gamma}^\mu_{Wtb} = -\frac{g}{\sqrt{2}}V_{tb}^*\,\bar{v}(p_{\bar{t}})\Big[\gamma^\mu(\bar{f}_1^L P_L + \bar{f}_1^R P_R) - \frac{i\sigma^{\mu\nu}k_\nu}{M_W}(\bar{f}_2^L P_L + \bar{f}_2^R P_R)\Big]v(p_{\bar{b}}), \quad (11)$$

where $P_{L/R} = (1 \mp \gamma_5)/2$, V_{tb} is the (tb) element of the Kobayashi-Maskawa matrix and k is the momentum of W. Because W is on shell,[♯4] the two additional form factors were not taken into account. It is worth to mention that the above form factors satisfy the following relations [18]:

$$f_1^{L,R} = \pm\bar{f}_1^{L,R}, \qquad f_2^{L,R} = \pm\bar{f}_2^{R,L}, \quad (12)$$

where the upper (lower) signs are those for CP-conserving (-violating) contributions.[♯5]

$Wl\nu$ couplings are treated within the SM as $\gamma/Zt\bar{t}$ couplings:

$$\Gamma^\mu_{Wl\nu} = -\frac{g}{2\sqrt{2}}\,\bar{u}(p_\nu)\gamma^\mu(1-\gamma_5)v(p_{\ell^+}), \quad (13)$$

$$\bar{\Gamma}^\mu_{Wl\nu} = -\frac{g}{2\sqrt{2}}\,\bar{u}(p_{\ell^-})\gamma^\mu(1-\gamma_5)v(p_{\bar{\nu}}). \quad (14)$$

[♯4]Remember that we use the narrow-width approximation also for the W propagator.
[♯5]Assuming CP-conserving Kobayashi-Maskawa matrix.

Assuming that $\overset{(-)}{f_1^L} -1$, $\overset{(-)}{f_1^R}$, $\overset{(-)}{f_2^L}$ and $\overset{(-)}{f_2^R}$ are small and keeping only their linear terms, we obtain for the differential spectrum the following result:

$$\frac{1}{\Gamma_t}\frac{d^2\Gamma_\ell}{dx d\omega}(t \to b\ell^+\nu) = \frac{1+\beta}{\beta}\,\frac{3B_\ell}{W}\omega\left[1 + 2\mathrm{Re}(f_2^R)\sqrt{r}\left(\frac{1}{1-\omega} - \frac{3}{1+2r}\right)\right], \tag{15}$$

where x is the rescaled lepton-energy introduced in [5]

$$x \equiv \frac{2E_\ell}{m_t}\left(\frac{1-\beta}{1+\beta}\right)^{1/2},$$

with E_ℓ being the energy of ℓ in e^+e^- c.m. frame, ω is defined as

$$\omega \equiv (p_t - p_\ell)^2/m_t^2,$$

B_ℓ is the leptonic branching ratio of t ($\simeq 0.22$ for $\ell = e, \mu$), and

$$W \equiv (1-r)^2(1+2r), \qquad r \equiv (M_W/m_t)^2.$$

An analogous formula holds for $\bar{t} \to \bar{b}\ell^-\bar{\nu}$ with f_2^R replaced by $\bar{f}_2^L$.

Lepton-energy distributions

Now let us give the lepton-energy spectra in terms of x. Since we are going to apply the method of optimal observables [13] in order to isolate various non-standard contributions, it is convenient to express the spectrum as a sum of known independent functions multiplied by coefficients parameterizing non-standard physics to be determined. In the following, we use as input data $M_W = 80.43$ GeV, $M_Z = 91.1863$ GeV, $m_t = 175.6$ GeV, $\sin^2\theta_W = 0.2315$ [2] and $\sqrt{s}$ =500 GeV.

1. Single distribution

Adopting the formulas eqs.(1,24,15), keeping only linear terms in non-standard form factors, and integrating over Ω_t and the necessary top-quark-decay phase space, one obtains the following normalized single-lepton-energy spectrum:

$$\frac{1}{B_\ell\sigma_{e\bar{e}\to t\bar{t}}}\frac{d\sigma^\pm}{dx} = \sum_{i=1}^{3} c_i^\pm f_i(x), \tag{16}$$

where $\sigma_{e\bar{e}\to t\bar{t}} \equiv \sigma_{tot}(e^+e^- \to t\bar{t})$ and $\pm$ corresponds to $\ell^\pm$. The first term comes from the SM and the coefficients are

$$c_1^\pm = 1,$$

the second term originates from the anomalous $\gamma/Zt\bar{t}$ couplings (see eq.(4)) contributing to the production process

$$c_2^{\pm} = a_1\,\delta D_V^{(*)} - a_2\,[\,\delta D_A^{(*)} - \mathrm{Re}(G_1^{(*)})\,] + a_3\mathrm{Re}(\delta D_{VA}^{(*)}) \mp \xi^{(*)},$$

and the third term comes from the non-SM Wtb couplings (see eqs.(10,11)) which influence the top-quark decay distribution (see eq.(15))

$$c_3^{+} = \mathrm{Re}(f_2^R), \quad c_3^{-} = \mathrm{Re}(\bar{f}_2^L).$$

Here, $\delta D_{V,A,VA}^{(*)}$ are the non-SM parts of $D_{V,A,VA}^{(*)}$, $\xi^{(*)}$ is a CP-violating parameter in the production process which is defined in a similar way as ξ used in [5, 7]:

$$\xi^{(*)} \equiv 2\,\mathrm{Re}(F_1^{(*)})\,a_{VA}^{(*)},$$

and a_i are defined as

$$a_1 \equiv -\,\eta^{(*)}(3-\beta^2)a_{VA}^{(*)}, \quad a_2 \equiv 2\,\eta^{(*)}\beta^2 a_{VA}^{(*)}, \quad a_3 \equiv 4\,a_{VA}^{(*)},$$

with $a_{VA}^{(*)} \equiv 1/[\,(3-\beta^2)D_V^{(0,*)} + 2\beta^2 D_A^{(0,*)}\,]$ (the superscript "(0)" denotes the SM-part) and

$$\eta^{(*)} \equiv 4\,a_{VA}^{(*)} D_{VA}^{(0,*)}$$

($= 0.2074$ in case of no beam polarization). On the other hand, the functions $f_i(x)$ are

$$f_1(x) = f(x) + \eta^{(*)}\,g(x), \quad f_2(x) = g(x), \quad f_3(x) = \delta f(x) + \eta^{(*)}\,\delta g(x), \qquad (17)$$

where $f(x)$ and $g(x)$ are functions introduced in [5], while $\delta f(x)$ and $\delta g(x)$ are functions derived in our previous work [7], which satisfy the following normalization conditions:

$$\int f(x)dx = 1 \quad \text{and} \quad \int g(x)dx = \int \delta f(x)dx = \int \delta g(x)dx = 0.$$

$f(x)$ and $g(x)$ describe the process with the standard top decays while $\delta f(x)$ and $\delta g(x)$ come from the non-standard contribution to the decay process. Here let us remind readers that the $c_2^{\pm}$ term in (16), which is proportional to $g(x)$, originates in the spin dependent part of the lepton spectrum and would vanish if, for instance, hadronization effects would dilute the top-quark polarization. As explained in the introduction the lepton-energy spectrum should depend on the polarization of the parent top quark, that is the reason why all the non-standard effects in the production process manifest themselves as modification of the coefficient in front of $g(x)$ for the normalized spectrum. We recapitulate these functions in the appendix.

It should be emphasized that the coefficients $c_i^{\pm}$ contain both contributions from CP-conserving and CP-violating interactions, therefore their determination does not provide a direct test of CP invariance. However, as was discussed in ref. [7] one can easy combine measurements of the spectra for ℓ^+ and ℓ^- in order to construct purely CP-violating observables like $d\sigma^+/dx - d\sigma^-/dx$. It is also worth to notice here that even though measurement of c_i does not disentangle CP-conserving and CP-violating interactions it allows for discrimination between non-standard effects originating from the production and those from the decay.

The functions $f_i(x)$ are shown in fig.1 for unpolarized beams. Since $f_{1,3}(x)$ have $P_{e^\pm}$ dependence through $\eta^{(*)}$, we also present them in figs.2 and 3 respectively for $P_{e^-} = +1$ vs $P_{e^+} = 0/+1$ and for $P_{e^-} = -1$ vs $P_{e^+} = 0/-1$ as examples.[♯6]

[♯6] $P_{e^+} = 0$ and $+1(-1)$ give the same $\eta^{(*)}$ and consequently the same $f_{1,3}(x)$ when $P_{e^-} = +1(-1)$.

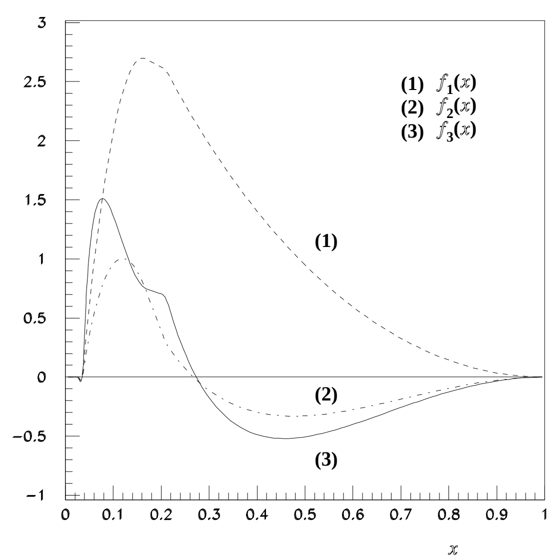

Figure 1: The functions $f_i(x)$ defined in eq. (17) for $P_{e^+} = P_{e^-} = 0$.

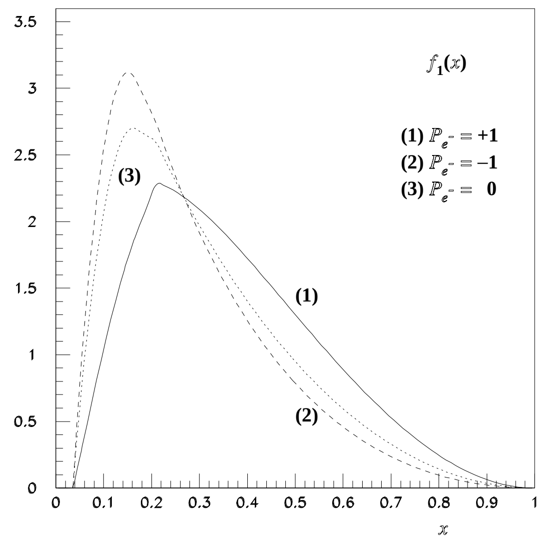

Figure 2: The function $f_1(x)$ for $P_{e^-} = +1$ vs $P_{e^+} = 0/+1$ (solid line), for $P_{e^-} = -1$ vs $P_{e^+} = 0/-1$ (dashed line) and for no polarization (dotted line).

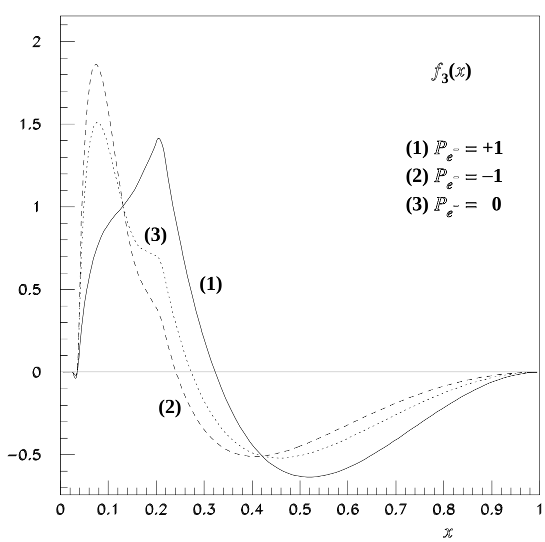

Figure 3: The function $f_3(x)$ for $P_{e^-} = +1$ vs $P_{e^+} = 0/+1$ (solid line), for $P_{e^-} = -1$ vs $P_{e^+} = 0/-1$ (dashed line) and for no polarization (dotted line).

2. Double distribution

Applying the same algorithm as for the single spectrum one finds for the normalized double-lepton-energy spectrum the following formula:

$$\frac{1}{B_\ell^2 \sigma_{e\bar{e}\to t\bar{t}}} \frac{d^2\sigma}{dx d\bar{x}} \;=\; \sum_{i=1}^{6} c_i f_i(x,\bar{x}), \tag{18}$$

where the first term comes from the SM

$$c_1 = 1,$$

the second and third terms are CP-violating non-SM contributions of $\gamma/Zt\bar{t}$ and Wtb couplings respectively,

$$c_2 = \xi^{(*)}, \quad c_3 = \frac{1}{2}\mathrm{Re}(f_2^R - \bar{f}_2^L),$$

the fourth and fifth terms are both CP-conserving non-SM $\gamma/Zt\bar{t}$ contributions

$$c_4 = a_1'\,\delta D_V^{(*)} + a_2'\,\delta D_A^{(*)} + a_3'\mathrm{Re}(G_1^{(*)}),$$

$$c_5 = a_1\,\delta D_V^{(*)} - a_2\,[\,\delta D_A^{(*)} - \mathrm{Re}(G_1^{(*)})\,] + a_3\mathrm{Re}(\delta D_{VA}^{(*)}),$$

while the last term is CP-conserving non-SM Wtb contribution

$$c_6 = \frac{1}{2}\mathrm{Re}(f_2^R + \bar{f}_2^L).$$

The corresponding functions are

$$\begin{aligned}
f_1(x,\bar{x}) &= f(x)f(\bar{x}) + \eta^{(*)}\,[\,f(x)g(\bar{x}) + g(x)f(\bar{x})\,] + \eta'^{(*)}g(x)g(\bar{x}), \\
f_2(x,\bar{x}) &= f(x)g(\bar{x}) - g(x)f(\bar{x}), \\
f_3(x,\bar{x}) &= \delta f(x)f(\bar{x}) - f(x)\delta f(\bar{x}) \\
&+ \eta^{(*)}\,[\,\delta f(x)g(\bar{x}) - f(x)\delta g(\bar{x}) + \delta g(x)f(\bar{x}) - g(x)\delta f(\bar{x})\,] \\
&+ \eta'^{(*)}[\,\delta g(x)g(\bar{x}) - g(x)\delta g(\bar{x})\,], \\
f_4(x,\bar{x}) &= g(x)g(\bar{x}), \\
f_5(x,\bar{x}) &= f(x)g(\bar{x}) + g(x)f(\bar{x}), \\
f_6(x,\bar{x}) &= \delta f(x)f(\bar{x}) + f(x)\delta f(\bar{x}) \\
&+ \eta^{(*)}\,[\,\delta f(x)g(\bar{x}) + f(x)\delta g(\bar{x}) + \delta g(x)f(\bar{x}) + g(x)\delta f(\bar{x})\,] \\
&+ \eta'^{(*)}[\,\delta g(x)g(\bar{x}) + g(x)\delta g(\bar{x})\,],
\end{aligned} \tag{19}$$

with $\eta'^{(*)} \equiv \beta^{-2} a_{VA}^{(*)} [\,(1+\beta^2) D_V^{(0,*)} + 2\beta^2 D_A^{(0,*)}\,] (= 1.2720$ for $P_e = P_{\bar{e}} = 0)$ and a'_i being defined as

$$a'_1 \equiv [\,\beta^{-2}(1+\beta^2) - (3-\beta^2)\eta'^{(*)}\,]\, a_{VA}^{(*)},$$
$$a'_2 \equiv 2(1-\beta^2\eta'^{(*)})\, a_{VA}^{(*)}, \quad a'_3 \equiv 2(1+\beta^2\eta'^{(*)})\, a_{VA}^{(*)}.$$

$f_{1,4,5,6}(x,\bar{x})$ and $f_{2,3}(x,\bar{x})$ are respectively symmetric and antisymmetric in $(x,\bar{x})$, which are signals of *CP* conservation and *CP* violation. Since f_4 and f_5 are both from the *CP*-conserving parts of the production process, we may recombine them, but we chose the above combination so that only f_5 remains in computing the single distributions.

Here, as for the single spectrum, since for a given c_i there is no mixing between the production and decay processes, we will be able to judge if the non-standard contributions originate from the production or from the decay of top quarks. Furthermore, in contrast with the single spectrum, the coefficients c_i receive contributions either from *CP*-conserving $(i = 1,4,5,6)$ or *CP*-violating $(i = 2,3)$ interactions. Therefore determination of the coefficients provides a direct test of *CP* invariance.

The functions $f_i(x,\bar{x})$ are plotted in fig.4 for unpolarized case. Since $f_{1,3,6}(x,\bar{x})$ depend on $P_{e^\pm}$ through $\eta^{(*)}$ and/or $\eta'^{(*)}$, we also show them in fig.5 for $P_{e^-} = +1$ vs $P_{e^+} = 0/+1$ (on the left side) and for $P_{e^-} = -1$ vs $P_{e^+} = 0/-1$ (on the right side) as examples. It can be observed from the figures that the shapes of the functions $f_{1,3}(x)$ and $f_{1,3,6}(x,\bar{x})$ vary substantially with the polarization of the initial beams. Therefore it is justified to consider determination of the coefficients c_i through energy-spectrum measurements for various polarizations since one can hope that carefully-adjusted beam-polarization may increase precision of the analysis.[♯7]

[♯7] Getting higher statistics is also a reason for considering polarized beams.

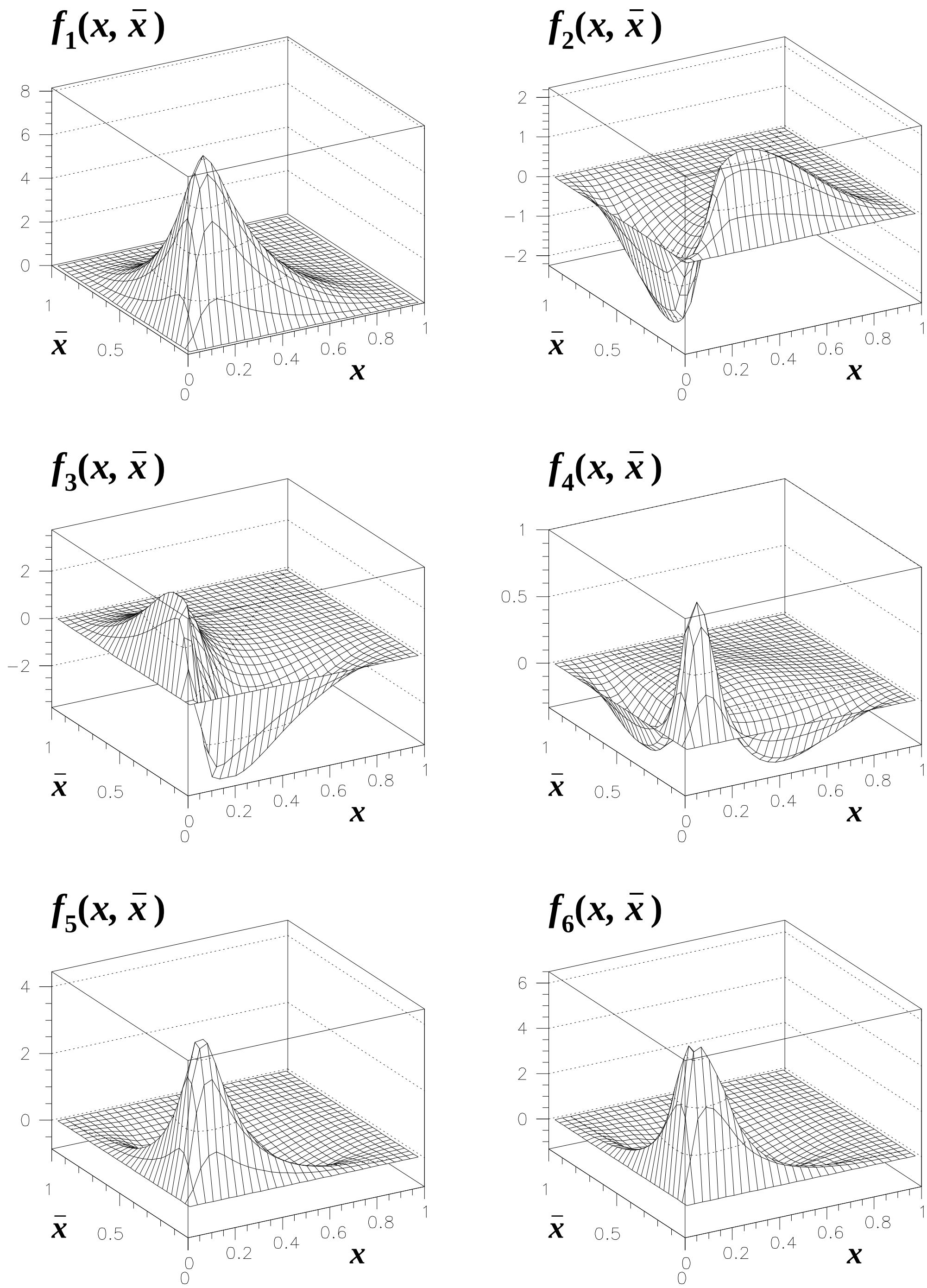

Figure 4: The functions $f_i(x, \bar{x})$ defined in eq.(19) for $P_{e^-} = P_{e^+} = 0$.

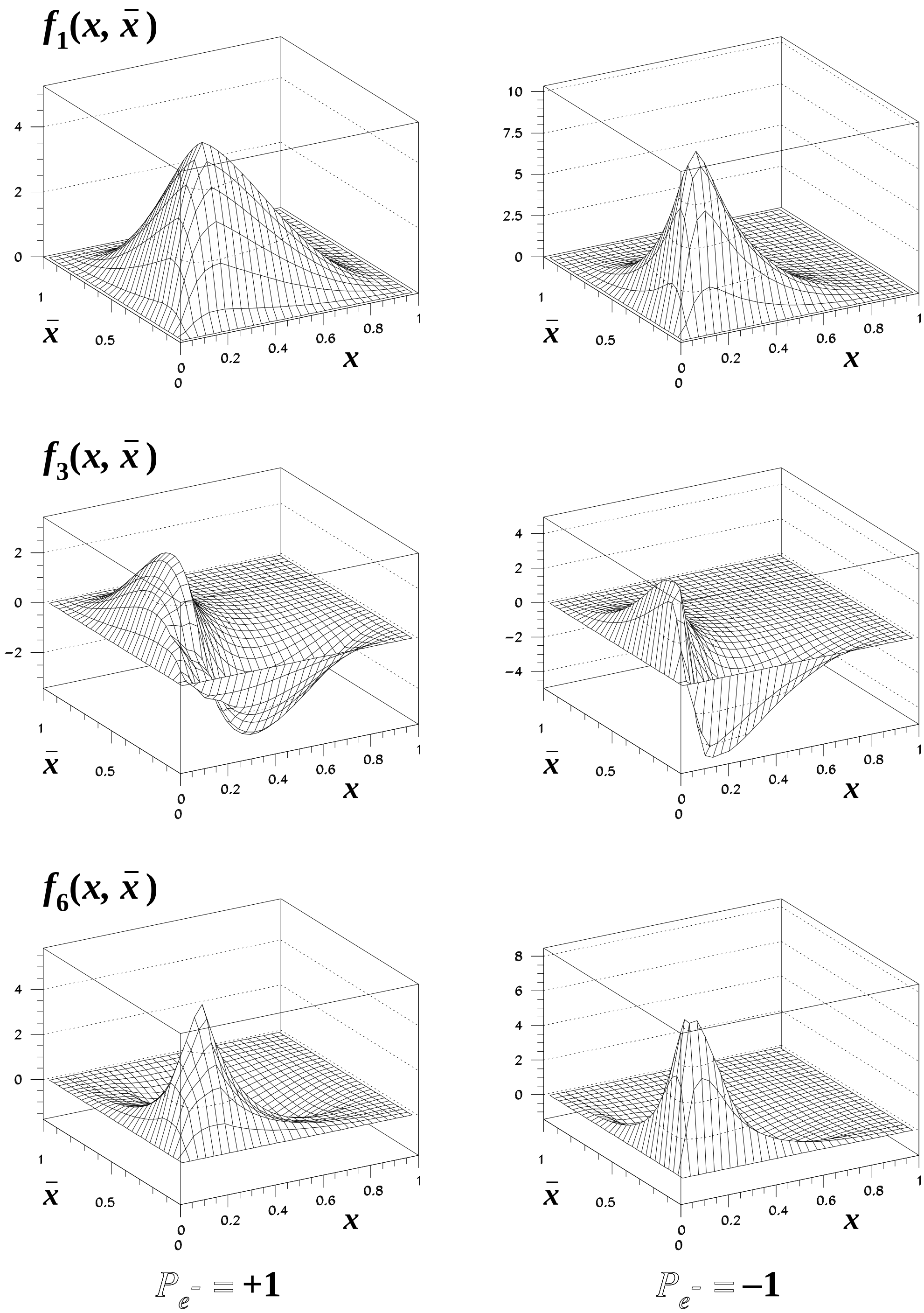

Figure 5: The functions $f_{1,3,6}(x, \bar{x})$ for $P_{e^-} = +1$ vs $P_{e^+} = 0/+1$ (on the left side) and for $P_{e^-} = -1$ vs $P_{e^+} = 0/-1$ (on the right side).

3. The optimal observables

We are now ready to perform a numerical analysis, but let us first summarize the main points of the optimal-observable technique [13]. Suppose we have a cross section

$$\frac{d\sigma}{d\phi}(\equiv \Sigma(\phi)) = \sum_i c_i f_i(\phi)$$

where $f_i(\phi)$ are known functions of the location in final-state phase space ϕ and c_i's are model-dependent coefficients. The goal would be to determine c_i's. It can be done by using appropriate weighting functions $w_i(\phi)$ such that $\int w_i(\phi)\Sigma(\phi)d\phi = c_i$. Generally, different choices for $w_i(\phi)$ are possible, but there is a unique choice so that the resultant statistical error is minimized. Such functions are given by

$$w_i(\phi) = \sum_j X_{ij} f_j(\phi)/\Sigma(\phi)\,, \tag{20}$$

where X_{ij} is the inverse matrix of M_{ij} which is defined as

$$M_{ij} \equiv \int \frac{f_i(\phi)f_j(\phi)}{\Sigma(\phi)} d\phi\,. \tag{21}$$

When we take these weighting functions, the statistical uncertainty of c_i-determination through $d\sigma/d\phi$ measurement becomes

$$\Delta c_i = \sqrt{X_{ii}\,\sigma_T/N}\,, \tag{22}$$

where $\sigma_T \equiv \int(d\sigma/d\phi)d\phi$ and N is the total number of events. It is clear from the definition of the matrix M_{ij}, eq.(21), that M_{ij} has no inverse if the functions $f_i(\phi)$ are linearly dependent, and then we cannot perform any meaningful analysis. One can see it more intuitively as follows: when $f_i(\phi) = f_j(\phi)$ the splitting between c_i and c_j would be totally arbitrary and only $c_i + c_j$ could be determined.

Numerical analysis

We apply the above procedure to the normalized lepton-energy distributions derived in sec.2, eqs(16,18). From the theoretical point of view, perfectly-polarized initial beams ($P_{e^+} = P_{e^-} = \pm 1$) are the most attractive. However, those are difficult to realize in practice, especially for the positron beam. We shall therefore discuss the following two cases:

$$(1)\quad P_{e^+}=0 \quad \text{vs} \quad P_{e^-}=0,\ \pm0.5,\ \pm0.8\ \text{ and }\ \pm1,$$
$$(2)\quad P_{e^+}=P_{e^-}(\equiv P_e)=0,\ \pm0.5,\ \pm0.8\ \text{ and }\ \pm1.$$

Before carrying out detailed computations, we shall briefly discuss how the statistical errors Δc_i depend on $P_{e^\pm}$. For this aim we have to check polarization effects in the lepton spectra. These spectra depend on $P_{e^\pm}$ through the coefficients c_i and the functions f_i in eqs.(17,19) as well, but the strongest dependence comes from the normalization factor since it is proportional to $\sigma_{e\bar{e}\to t\bar{t}}$ which is

$$\begin{aligned}\sigma_{e\bar{e}\to t\bar{t}} \sim (3-\beta^2)\Big[\,(1+P_{e^-}P_{e^+})D_V^{(0)}-(P_{e^-}+P_{e^+})E_V^{(0)}\,\Big]\\ +2\beta^2\Big[\,(1+P_{e^-}P_{e^+})D_A^{(0)}-(P_{e^-}+P_{e^+})E_A^{(0)}\,\Big],\end{aligned} \tag{23}$$

where $D_{V,A}^{(0)}$ and $E_{V,A}^{(0)}$ are the SM parts of $D_{V,A}$ and $E_{V,A}$ in eq.(25). Neglecting the vector-type part of $\gamma e\bar{e}$ coupling v_e ($v_e=-1+4\sin^2\theta_W$ is tiny for $\sin^2\theta_W=0.2315$), we have

$$D_V^{(0)}=C(A_\gamma^2+A_Z^2d'^2),\quad D_A^{(0)}=CB_Z^2d'^2,\quad E_V^{(0)}=2CA_\gamma A_Z d',\quad E_A^{(0)}=0.$$

Since $E_V^{(0)}>0$ for $\sin^2\theta_W=0.2315$, negative polarizations increase $\sigma_{e\bar{e}\to t\bar{t}}$. The matrix M_{ij} is proportional to $\sigma_{e\bar{e}\to t\bar{t}}$ through the normalization factor, which means that negative polarizations would reduce statistical errors, eq.(22), since the matrix $X_{ij}\propto 1/\sigma_{e\bar{e}\to t\bar{t}}$. As it has been mentioned, M_{ij} depends, to a certain extent, on $P_{e^\pm}$ also through c_i and $\eta^{(*)}$ in the functions f_i, therefore even for nearly the same number of detected events (the same $\sigma_{e\bar{e}\to t\bar{t}}$) statistical errors may differ. However, the general tendency is consistent with this naïve expectation as will be observed later in tables presenting our results.

1. Single-distribution analysis

First, we shall consider the single distribution. Using eq.(22) for $d\sigma^\pm/dx$ we can obtain $\Delta c_{2,3}^\pm$, the statistical errors for the determination of $c_{2,3}^\pm$, as a function of the expected number of detected single-lepton events N_ℓ. For a given integrated luminosity L and lepton-tagging efficiency ϵ_ℓ one has $N_\ell=B_\ell\sigma_{e\bar{e}\to t\bar{t}}L_{\rm eff}^\ell$, where $L_{\rm eff}^\ell\equiv\epsilon_\ell L$ (in $\,\mathrm{fb}^{-1}$ units) is the effective luminosity. In the following we use

$\epsilon_\ell = 0.6$ and $L = 100$ fb^{-1} as an example of realistic experimental constraint,[♯8] and estimate $\sigma_{e\bar{e}\to t\bar{t}}$ within the SM by using $\alpha(s)(\simeq 1/126)$.

(1) P_{e^-}	0	+0.5	+0.8	+1.0	−0.5	−0.8	−1.0
$\Delta c_2^{\pm}$	0.13	0.16	0.12	0.09	0.09	0.08	0.07
$\Delta c_3^{\pm}$	0.08	0.10	0.08	0.06	0.06	0.05	0.05
N_ℓ	7676	6259	5409	4843	9093	9943	10509
(2) P_e	0	+0.5	+0.8	+1.0	−0.5	−0.8	−1.0
$\Delta c_2^{\pm}$	0.13	0.11	0.08	0.07	0.07	0.05	0.05
$\Delta c_3^{\pm}$	0.08	0.07	0.05	0.04	0.05	0.04	0.03
N_ℓ	7676	6762	8055	9685	12429	17122	21019

Table 1: Expected statistical errors in $c_{2,3}^{\pm}$ measurements and the number of the single-lepton-inclusive events N_ℓ for beam polarization (1) $P_{e^+} = 0$ vs $P_{e^-} = 0,\ \pm 0.5,\ \pm 0.8$ and ± 1, (2) $P_{e^+} = P_{e^-} (\equiv P_e) = 0,\ \pm 0.5,\ \pm 0.8$ and ± 1 at $\sqrt{s} = 500$ GeV. N_ℓ has been estimated within the SM for $\epsilon_\ell = 0.6$ and $L = 100$ fb^{-1}.

In table 1 we present $\Delta c_{2,3}^{\pm}$ and N_ℓ for the above ϵ_ℓ and L with the described configurations of beam polarization. From table 1, readers might conclude that the use of polarized beam(s) is quite effective for providing higher precision, especially negatively-polarized beams seem to be most suitable since we have smaller $\Delta c_{2,3}^{\pm}$ as anticipated in the above discussion. Indeed, this is the case for $c_3^{\pm}$ measurement. For instance, when $\mathrm{Re}(f_2^R)$, $\mathrm{Re}(\bar{f}_2^L) = \pm 0.1$, then $N_{SD} = |c_3^{\pm}|/\Delta c_3^{\pm}$, statistical significances for an observation of $c_3^{\pm}$, becomes 2.0 for $P_{e^-} = -1$ and 3.3 for $P_e = -1$, which means we can expect 2σ and 3σ confidence level respectively. However, for a given set of non-standard couplings, the coefficients $c_2^{\pm}$ vary with polarization. Therefore we should discuss their N_{SD} inevitably instead of statistical errors only.

[♯8] Assuming $L = 100$ fb^{-1} is in fact quite conservative since the integrated luminosity as high as 500 fb^{-1} is being recently discussed [19] as a realistic possibility in the context of the TESLA collider design for $\sqrt{s} = 500$ GeV .

We will consider the following two sets of the couplings (of the order of 15% of the SM strength) in tables 2 and 3:[♯9]

(a) $\mathrm{Re}(\delta A_{\gamma,Z}) = \mathrm{Re}(\delta B_{\gamma,Z}) = \mathrm{Re}(\delta C_{\gamma,Z}) = \mathrm{Re}(\delta D_{\gamma,Z}) = 0.1$,

(b) $\mathrm{Re}(\delta A_{\gamma}) = \mathrm{Re}(\delta B_{\gamma}) = \mathrm{Re}(\delta C_{\gamma}) = \mathrm{Re}(\delta D_{\gamma}) = 0.1$,
$\mathrm{Re}(\delta A_{Z}) = \mathrm{Re}(\delta B_{Z}) = \mathrm{Re}(\delta C_{Z}) = \mathrm{Re}(\delta D_{Z}) = -0.1$.

(1) P_{e^-}	0	+0.5	+0.8	+1.0	−0.5	−0.8	−1.0
c_2^+	0.39	0.36	0.28	0.17	0.38	0.36	0.34
c_2^-	0.14	0.16	0.12	0.05	0.09	0.06	0.03
$\lvert c_2^+\rvert/\Delta c_2^\pm$	3.03	2.31	2.25	1.83	4.10	4.65	4.96
$\lvert c_2^-\rvert/\Delta c_2^\pm$	1.11	1.04	1.01	0.58	1.00	0.75	0.49
(2) P_e	0	+0.5	+0.8	+1.0	−0.5	−0.8	−1.0
c_2^+	0.39	0.28	0.19	0.17	0.36	0.35	0.34
c_2^-	0.14	0.12	0.06	0.05	0.06	0.04	0.03
$\lvert c_2^+\rvert/\Delta c_2^\pm$	3.03	2.52	2.47	2.59	5.20	6.30	7.01
$\lvert c_2^-\rvert/\Delta c_2^\pm$	1.11	1.13	0.86	0.81	0.83	0.68	0.70

Table 2: Statistical significance of $c_2^\pm$ measurement for beam polarization (1) $P_{e^+} = 0$ vs $P_{e^-} = 0,\ \pm0.5,\ \pm0.8$ and ±1, and (2) $P_{e^+} = P_{e^-}(\equiv P_e) = 0,\ \pm0.5,\ \pm0.8$ and ±1, and the parameter set (a) $\mathrm{Re}(\delta A_\gamma) = \mathrm{Re}(\delta A_Z) = \mathrm{Re}(\delta B_\gamma) = \mathrm{Re}(\delta B_Z) = \mathrm{Re}(\delta C_\gamma) = \mathrm{Re}(\delta C_Z) = \mathrm{Re}(\delta D_\gamma) = \mathrm{Re}(\delta D_Z) = 0.1$ at $\sqrt{s} = 500$ GeV.

[♯9] One may notice that certain entries (some of c_i coefficients) in tables 2 and 3 are identical. Indeed two polarization scenarios considered here provide for these cases exactly same values for c_i. Therefore, comparing statistical significances for them one can see the net effect of different statistics, as the expected number of events is different for the cases. The same will also apply to tables 5 and 6.

(1) P_{e^-}	0	+0.5	+0.8	+1.0	−0.5	−0.8	−1.0
c_2^+	0.17	0.31	0.46	0.61	0.11	0.08	0.07
c_2^-	$-4{\cdot}10^{-3}$	0.04	0.11	0.19	−0.01	10^{-3}	0.01
$\lvert c_2^+\rvert/\Delta c_2^\pm$	1.33	1.97	3.70	6.63	1.15	1.07	1.02
$\lvert c_2^-\rvert/\Delta c_2^\pm$	0.03	0.24	0.86	2.09	0.06	0.02	0.10
(2) P_e	0	+0.5	+0.8	+1.0	−0.5	−0.8	−1.0
c_2^+	0.17	0.46	0.59	0.61	0.08	0.07	0.07
c_2^-	$-4{\cdot}10^{-3}$	0.11	0.18	0.19	10^{-3}	0.01	0.01
$\lvert c_2^+\rvert/\Delta c_2^\pm$	1.33	4.14	7.86	9.38	1.20	1.31	1.44
$\lvert c_2^-\rvert/\Delta c_2^\pm$	0.03	0.97	2.40	2.95	0.02	0.12	0.14

Table 3: Statistical significance of $c_2^\pm$ measurement for beam polarization (1) $P_{e^+} = 0$ vs $P_{e^-} = 0,\ \pm0.5,\ \pm0.8$ and ±1, and (2) $P_{e^+} = P_{e^-}(\equiv P_e) = 0,\ \pm0.5,\ \pm0.8$ and ±1, and the parameter set (b) $\mathrm{Re}(\delta A_\gamma) = -\mathrm{Re}(\delta A_Z) = \mathrm{Re}(\delta B_\gamma) = -\mathrm{Re}(\delta B_Z) = \mathrm{Re}(\delta C_\gamma) = -\mathrm{Re}(\delta C_Z) = \mathrm{Re}(\delta D_\gamma) = -\mathrm{Re}(\delta D_Z) = 0.1$ at $\sqrt{s} = 500$ GeV.

These tables show that the use of negatively-polarized beam(s) is not always optimal: for the parameter set (a) a good precision in c_2^+ measurement is realized when $P_e < 0$, but even in this case the precision in c_2^- measurement becomes better for $P_e > 0$ or even $P_e = 0$ (table 2). Moreover in case (b) both c_2^+ and c_2^- get the highest precision for $P_e = +1$ (table 3). *Therefore one should carefully adjust optimal polarization to test any given model of physics beyond the SM.* One can conclude (as far as the coefficient sets discussed here are concerned) that the appropriate beam polarization for the set (a) provides measurements of c_2^+ at 5.0σ and 7.0σ level for $P_{e^-} = -1.0$ and $P_e = -1.0$, respectively. For the set (b) maximal statistical significance for c_2^+ determination is 6.6 and 9.4 for $P_{e^-} = +1.0$ and $P_e = +1.0$, respectively. Since $c_2^- \ll c_2^+$ it is seen that the maximal statistical significance for c_2^- is much lower: 1.1 for the set (a) and 3.0 for the set (b).

2. Double-distribution analysis

We can perform similar computations for the double-lepton distribution. Results are presented in tables 4, 5 and 6. We find again in table 4 that negative polarizations give smaller Δc_i. As a result, $|c_{3,6}|/\Delta c_{3,6}$ can be easily estimated from this table once $\mathrm{Re}(f_2^R)$ and $\mathrm{Re}(\bar{f}_2^L)$ are fixed. On the other hand, $c_{2,4,5}$ have polarization dependence themselves, so we need tables 5 and 6 in order to draw a meaningful conclusion, where the statistical significance for $c_{2,4,5}$ has been presented. Again some of c_i in tables 5 and 6 are identical as in the case of the single lepton channel.

(1) P_{e^-}	0	+0.5	+0.8	+1.0	−0.5	−0.8	−1.0
Δc_2	0.20	0.23	0.21	0.17	0.16	0.14	0.13
Δc_3	0.13	0.15	0.14	0.11	0.11	0.09	0.09
Δc_4	0.31	0.35	0.39	0.41	0.30	0.29	0.28
Δc_5	0.22	0.25	0.22	0.17	0.17	0.14	0.13
Δc_6	0.14	0.16	0.14	0.12	0.11	0.09	0.09
$N_{\ell\ell}$	1013	826	714	639	1200	1312	1387
(2) P_e	0	+0.5	+0.8	+1.0	−0.5	−0.8	−1.0
Δc_2	0.20	0.19	0.14	0.12	0.12	0.10	0.09
Δc_3	0.13	0.12	0.09	0.08	0.08	0.07	0.06
Δc_4	0.31	0.34	0.32	0.29	0.26	0.22	0.20
Δc_5	0.22	0.19	0.14	0.12	0.13	0.10	0.09
Δc_6	0.14	0.12	0.09	0.08	0.08	0.07	0.06
$N_{\ell\ell}$	1013	893	1063	1278	1641	2260	2775

Table 4: Expected statistical errors in $c_{2,3,4,5,6}$ measurements and the expected observed numbers of the double-lepton-inclusive events $N_{\ell\ell}$ for beam polarization (1) $P_{e^+} = 0$ vs $P_{e^-} = 0,\ \pm 0.5,\ \pm 0.8$ and ± 1, (2) $P_{e^+} = P_{e^-} (\equiv P_e) = 0,\ \pm 0.5,\ \pm 0.8$ and ± 1 at $\sqrt{s} = 500$ GeV. $N_{\ell\ell}$ has been estimated within the SM for $\epsilon_\ell = 0.6$ and $L = 100$ fb^{-1}.

Among the coefficients for the double-leptonic spectrum, $c_{2,3}$ are CP-violating parameters. Since c_3 does not depend on the beam polarization as already mentioned, one can just say (from table 4) that 3σ effects could be observed for $P_e = -1.0$ if $\mathrm{Re}(f_2^R - \bar{f}_2^L)/2 > 0.18$. On c_2 one has to conclude from tables 5 and 6 that for both sets of non-standard couplings its determination would not be easy for the assumed luminosity, as its statistical significance reaches at most 1.7. This is due to the smaller number of detected events in this channel as it could have been anticipated. Still we can say that the use of polarized beams is very helpful to increase precision. Indeed, if we are able to achieve $L = 500$ fb^{-1} as discussed in [19], then $|c_2|/\Delta c_2$ would reach 3.8 for $P_e = -1$ in case (a) (the same value could be obtained for $P_e = +1$ in case (b)), while we have only $|c_2|/\Delta c_2 = 1.4$ if the beams were unpolarized.

(1) P_{e^-}	0	$+0.5$	$+0.8$	$+1.0$	-0.5	-0.8	-1.0
c_2	-0.12	-0.10	-0.08	-0.06	-0.14	-0.15	-0.16
c_4	0.21	0.15	0.10	0.06	0.25	0.27	0.28
c_5	0.27	0.26	0.20	0.11	0.23	0.21	0.19
$\|c_2\|/\Delta c_2$	0.61	0.42	0.37	0.34	0.89	1.08	1.21
$\|c_4\|/\Delta c_4$	0.67	0.43	0.25	0.14	0.84	0.93	0.99
$\|c_5\|/\Delta c_5$	1.24	1.05	0.92	0.63	1.41	1.45	1.44
(2) P_e	0	$+0.5$	$+0.8$	$+1.0$	-0.5	-0.8	-1.0
c_2	-0.12	-0.08	-0.06	-0.06	-0.15	-0.15	-0.16
c_4	0.21	0.10	0.06	0.06	0.27	0.28	0.28
c_5	0.27	0.20	0.13	0.11	0.21	0.19	0.19
$\|c_2\|/\Delta c_2$	0.61	0.41	0.44	0.48	1.21	1.53	1.71
$\|c_4\|/\Delta c_4$	0.67	0.28	0.19	0.19	1.04	1.26	1.40
$\|c_5\|/\Delta c_5$	1.24	1.03	0.89	0.90	1.62	1.85	2.04

Table 5: Statistical significance of $c_{2,4,5}$ measurement for beam polarization (1) $P_{e^+} = 0$ vs $P_{e^-} = \pm 0.5$, ± 0.8 and ± 1, and (2) $P_{e^+} = P_{e^-} (\equiv P_e) = \pm 0.5$, ± 0.8 and ± 1, and the parameter set (a) $\mathrm{Re}(\delta A_\gamma) = \mathrm{Re}(\delta A_Z) = \mathrm{Re}(\delta B_\gamma) = \mathrm{Re}(\delta B_Z) = \mathrm{Re}(\delta C_\gamma) = \mathrm{Re}(\delta C_Z) = \mathrm{Re}(\delta D_\gamma) = \mathrm{Re}(\delta D_Z) = 0.1$ at $\sqrt{s} = 500$ GeV.

$c_{4,5,6}$ are CP-conserving coefficients. Concerning c_6, 3σ-level measurement is possible for $P_e = -1.0$ when $\mathrm{Re}(f_2^R + \bar{f}_2^L)/2 > 0.18$. On c_4 we are also led to a similar conclusion to c_2, but c_5 determination is different. That is, the statistical significance for c_5 measurement can reach 2.0 for $P_e = -1$ (case (a)) and 3.3 for $P_e = +1$ (case (b)). This is quite in contrast with that for c_4, which is less than 2 as one can see from tables 5 and 6.

(1) P_{e^-}	0	+0.5	+0.8	+1.0	−0.5	−0.8	−1.0
c_2	−0.09	−0.14	−0.18	−0.21	−0.06	−0.04	−0.03
c_4	0.11	0.14	0.18	0.20	0.08	0.07	0.06
c_5	0.08	0.17	0.28	0.40	0.05	0.04	0.04
$\lvert c_2\rvert/\Delta c_2$	0.43	0.58	0.84	1.22	0.35	0.29	0.25
$\lvert c_4\rvert/\Delta c_4$	0.34	0.42	0.45	0.50	0.26	0.23	0.20
$\lvert c_5\rvert/\Delta c_5$	0.39	0.69	1.29	2.30	0.30	0.29	0.30
(2) P_e	0	+0.5	+0.8	+1.0	−0.5	−0.8	−1.0
c_2	−0.09	−0.18	−0.21	−0.21	−0.04	−0.03	−0.03
c_4	0.11	0.18	0.20	0.20	0.07	0.06	0.06
c_5	0.08	0.28	0.38	0.40	0.04	0.04	0.04
$\lvert c_2\rvert/\Delta c_2$	0.43	0.94	1.49	1.73	0.33	0.33	0.35
$\lvert c_4\rvert/\Delta c_4$	0.34	0.51	0.63	0.70	0.25	0.26	0.29
$\lvert c_5\rvert/\Delta c_5$	0.39	1.45	2.72	3.25	0.33	0.38	0.42

Table 6: Statistical significance of $c_{2,4,5}$ measurement for beam polarization (1) $P_{e^+} = 0$ vs $P_{e^-} = \pm 0.5,\ \pm 0.8$ and ± 1, and (2) $P_{e^+} = P_{e^-} (\equiv P_e) = \pm 0.5,\ \pm 0.8$ and ± 1, and the parameter set (b) $\mathrm{Re}(\delta A_\gamma) = -\mathrm{Re}(\delta A_Z) = \mathrm{Re}(\delta B_\gamma) = -\mathrm{Re}(\delta B_Z) = \mathrm{Re}(\delta C_\gamma) = -\mathrm{Re}(\delta C_Z) = \mathrm{Re}(\delta D_\gamma) = -\mathrm{Re}(\delta D_Z) = 0.1$ at $\sqrt{s} = 500$ GeV.

4. Summary and comments

Next-generation linear colliders of e^+e^-, NLC, will provide a cleanest environment for studying top-quark interactions. There, we shall be able to perform detailed tests of the top-quark couplings to the vector bosons and either confirm the SM simple generation-repetition pattern or discover some non-standard interactions. In this paper, assuming the most general (CP-violating *and* CP-conserving) couplings for $\gamma t\bar{t}$, $Zt\bar{t}$ and Wtb, we have studied in a model-independent way the single- and the double-leptonic spectra for arbitrary longitudinal beam polarizations. Then, the optimal-observable technique [13] has been adopted to determine non-standard couplings through measurements of these spectra.

The method applied here, the optimal observables, allows to disentangle various non-standard contributions to the production process ($e^+e^- \to t\bar{t}$) and to the decay ($t \to Wb$). Using the single-leptonic-energy spectrum for $\ell^\pm$ and assuming non-standard couplings of the order of 15% of the SM strength, we have found that an appropriate selection of the initial-beam polarization may provide observable effects for non-standard corrections to *the production process*, $|c_2^+|/\Delta c_2^+$, even at 9.4σ level when both e^- and e^+ beams are polarized and at 6.6σ when only e^- beam is polarized. On the other hand, from the same spectrum measurement one can expect on *non-standard contributions to the top-quark decay* the statistical significance of the signal $N_{SD} = |c_3^\pm|/\Delta c_3^\pm$ of the order of 3.0 and 2.0 for both beams polarized and only electron beam polarized, respectively.

The direct application of the optimal method for the single spectrum does not allow for discrimination between CP-violating and CP-conserving non-standard interactions since their effects mix in coefficients of the spectrum, $c_i^\pm$. However, as it was discussed in ref. [7] one can easily combine measurements of the spectrum for ℓ^+ and ℓ^- in order to construct purely CP-violating observables.

In contrast with the single spectrum, utilizing the method of optimal observables directly for the double-leptonic-energy spectrum one can separately determine and disentangle the CP-violating coupling from the production of $t\bar{t}$ pairs (c_2) and the one from the top-quark decay (c_3). For the typical strength of the non-standard couplings discussed here, the highest statistical significance for CP violation in *the production and/or in the decay* was estimated to be 1.7 for both beams polarized, while we found that the maximal signal from CP-conserving interactions in *the production process* ($|c_5|/\Delta c_5$) could reach 3.3 and 2.3 for both and only electron beam polarized, respectively. For CP-conserving interactions in the decay the expected effect is lower, namely 1.6 for the statistical significance for both considered cases of maximal polarization.

It should be emphasized that we have used in this study very conservative integrated luminosity, namely $L = 100\ \mathrm{fb}^{-1}$. That is, the luminosity considered now as realistic is by factor 5 larger. Therefore one may expect that even though we have not considered any background here and our analysis does not take into account any detector details (to a large extent they are not available yet), the results presented here should serve as a fair estimation of real signals for beyond-the-SM physics.

To summary, we found (i) the use of longitudinal beams could be very effective in order to increase precision of the determination of non-SM couplings, however (ii) optimal polarization depends on the model of new physics under consideration, therefore polarization of the initial beams should be carefully adjusted for each tested model. *For such optimal polarization the maximal non-standard signal should be observable in the single-leptonic spectrum on the effects generated by contributions (both CP-conserving and CP-violating) to the production mechanism of $t\bar{t}$ pairs. On the other hand, the most challenging measurement would be the determination of CP-conserving contributions to the decay process.* Since we have already carried out a similar analysis of possible consequences emerging from effective four-Fermi interactions $e\bar{e} \to t\bar{t}$ and $t(\bar{t}) \to b\ell^+\nu(\bar{b}\ell^-\bar{\nu})$ in [10], this paper completes a full analysis of modifications for lepton-energy distributions by non-standard interactions of the top quark in a model-independent way for polarized e^+e^- experiments.

The results presented here are the most precise ones which could be obtained from the single or double energy distribution alone. It will of course be possible to achieve a higher precision by combing our results with other statistically-independent data. Among them, lepton angular distributions are very promising. Indeed what one could measure via the energy spectra are the real parts of the non-standard form factors, while we would be able to determine their imaginary parts by using, e.g., an up-down asymmetry to the top direction as shown in [4]. However, non-SM effects in the decay process were not taken into account in that study. The lepton angular distributions relative to the initial beam direction will also give us valuable information. Some analysis focusing on the CP violation in the production vertices has been made in [20]. However, comprehensive analysis including non-standard effects both in the production and in the decay process for all measurable distributions of the $t\bar{t}$ decay products seems to be needed [21].

Finally, let us give a brief comment on the effects of radiative corrections. All the non-standard couplings considered here may be generated at the multi-loop level within the SM. In fact, CP-violating couplings $\delta D_{\gamma,Z}$ and $\mathrm{Re}(f_2^R - \bar{f}_2^L)$ requires at least two loops of the SM, so they are negligible. However, CP-conserving couplings $\delta A_{\gamma,Z}$, $\delta B_{\gamma,Z}$, $\delta C_{\gamma,Z}$ and $\mathrm{Re}(f_2^R + \bar{f}_2^L)$ could be generated already at the one-loop level approximation of QCD. Therefore, in order to disentangle non-SM interactions and the one-loop QCD effects it is important to calculate and subtract the QCD contributions from the lepton-energy spectrum, this is however beyond the scope of this paper.

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Appendix

The angular distribution of polarized $t\bar{t}$ pair is given by the following formula:

$$
\begin{aligned}
&\frac{d\sigma}{d\Omega_t}(e^+e^- \to t(s_+)\bar{t}(s_-)) \\
&= \frac{3\beta\alpha^2}{16s^3}\Big[\ D_V\,[\,\{4m_t^2 s + (lq)^2\}(1 - s_+s_-) + s^2(1 + s_+s_-) \\
&\qquad\qquad +2s(ls_+\,ls_- - Ps_+\,Ps_-) +\ 2\,lq(ls_+\,Ps_- - ls_-\,Ps_+)\,] \\
&\quad + D_A\,[\,(lq)^2(1 + s_+s_-) - (4m_t^2 s - s^2)(1 - s_+s_-) \\
&\qquad\qquad -2(s - 4m_t^2)(ls_+\,ls_- - Ps_+\,Ps_-) -\ 2\,lq(ls_+\,Ps_- - ls_-\,Ps_+)\,] \\
&\quad -4\,\mathrm{Re}(D_{VA})\,m_t\,[\,s(Ps_+ - Ps_-) + lq(ls_+ + ls_-)\,] \\
&\quad +2\,\mathrm{Im}(D_{VA})\,[\,lq\,\epsilon(s_+, s_-, q, l) + ls_-\epsilon(s_+, P, q, l) + ls_+\epsilon(s_-, P, q, l)\,]
\end{aligned}
$$

$$
\begin{aligned}
&+4\,E_V\,m_t s(ls_+ + ls_-) + 4\,E_A\,m_t\,lq(Ps_+ - Ps_-)\\
&+4\,\mathrm{Re}(E_{VA})\,[\,2m_t^2(ls_+\,Ps_- - ls_-\,Ps_+) - lq\,s\,]\\
&+4\,\mathrm{Im}(E_{VA})\,m_t[\,\epsilon(s_+,P,q,l) + \epsilon(s_-,P,q,l)\,]\\
&-\mathrm{Re}(F_1)\,\frac{1}{m_t}[\,lq\,s(ls_+ - ls_-) - \{(lq)^2 + 4m_t^2 s\}(Ps_+ + Ps_-)\,]\\
&+2\,\mathrm{Im}(F_1)\,[\,s\,\epsilon(s_+,s_-,P,q) + lq\,\epsilon(s_+,s_-,P,l)\,]\\
&+2\,\mathrm{Re}(F_2)\,s(Ps_+\,ls_- + Ps_-\,ls_+)\\
&-\mathrm{Im}(F_2)\,\frac{s}{m_t}[\,\epsilon(s_+,P,q,l) - \epsilon(s_-,P,q,l)\,]\\
&-2\,\mathrm{Re}(F_3)\,lq(Ps_+\,ls_- + Ps_-\,ls_+)\\
&+\mathrm{Im}(F_3)\,\frac{lq}{m_t}[\,\epsilon(s_+,P,q,l) - \epsilon(s_-,P,q,l)\,]\\
&-\mathrm{Re}(F_4)\,\frac{s}{m_t}[\,lq\,(Ps_+ + Ps_-) - (s-4m_t^2)(ls_+ - ls_-)\,]\\
&-2\,\mathrm{Im}(F_4)\,[\,Ps_+\epsilon(s_-,P,q,l) + Ps_-\epsilon(s_+,P,q,l)\,]\\
&+2\,\mathrm{Re}(G_1)\,[\,\{4m_t^2 s + (lq)^2 - s^2\}(1 - s_+s_-) - 2s\,Ps_+Ps_-\\
&\qquad +lq(ls_+\,Ps_- - ls_-\,Ps_+)\,]\\
&-\mathrm{Im}(G_1)\,\frac{lq}{m_t}[\,\epsilon(s_+,P,q,l) + \epsilon(s_-,P,q,l)\,]\\
&-\mathrm{Re}(G_2)\,\frac{s}{m_t}[\,(s-4m_t^2)(ls_+ + ls_-) - lq\,(Ps_+ - Ps_-)\,]\\
&-2\,\mathrm{Im}(G_2)\,[\,Ps_+\epsilon(s_-,P,q,l) - Ps_-\epsilon(s_+,P,q,l)\,]\\
&-\mathrm{Re}(G_3)\,\frac{lq}{m_t}[\,lq\,(Ps_+ - Ps_-) - (s-4m_t^2)(ls_+ + ls_-)\,]\\
&-2\,\mathrm{Im}(G_3)\,lq\,\epsilon(s_+,s_-,q,l)\\
&+2\,\mathrm{Re}(G_4)\,[\,(s-4m_t^2)(Ps_+\,ls_- - Ps_-\,ls_+) + 2\,lq\,Ps_+Ps_-\,]\\
&+\mathrm{Im}(G_4)\,\frac{1}{m_t}(s-4m_t^2)[\,\epsilon(s_+,P,q,l) + \epsilon(s_-,P,q,l)\,]\,\Big], \qquad (24)
\end{aligned}
$$

where $\beta(\equiv \sqrt{1-4m_t^2/s})$ is the velocity of t in e^+e^- c.m. frame,

$$
P \equiv p_e + p_{\bar{e}}\,(= p_t + p_{\bar{t}})\,, \quad l \equiv p_e - p_{\bar{e}}\,, \quad q \equiv p_t - p_{\bar{t}}\,,
$$

the symbol $\epsilon(a,b,c,d)$ means $\epsilon_{\mu\nu\rho\sigma}a^\mu b^\nu c^\rho d^\sigma$ for $\epsilon_{0123} = +1$,

$$
\begin{aligned}
D_V \;\equiv\; & C\,[\,A_\gamma^2 - 2A_\gamma A_Z v_e d' + A_Z^2(1+v_e^2)d'^2 + 2(A_\gamma - A_Z v_e d')\mathrm{Re}(\delta A_\gamma)\\
&-2\{A_\gamma v_e d' - A_Z(1+v_e^2)d'^2\}\mathrm{Re}(\delta A_Z)\,],
\end{aligned}
$$

$$
\begin{aligned}
D_A \;&\equiv\; C\,[\,B_Z^2(1+v_e^2)d'^2 - 2B_Z v_e d'\mathrm{Re}(\delta B_\gamma) + 2B_Z(1+v_e^2)d'^2\mathrm{Re}(\delta B_Z)\,],\\
D_{VA} \;&\equiv\; C\,[\,-A_\gamma B_Z v_e d' + A_Z B_Z(1+v_e^2)d'^2 - B_Z v_e d'(\delta A_\gamma)^*\\
&\quad +(A_\gamma - v_e d' A_Z)\delta B_\gamma + B_Z(1+v_e^2)d'^2(\delta A_Z)^*\\
&\quad -\{A_\gamma v_e d' - A_Z(1+v_e^2)d'^2\}\delta B_Z\,],\\
E_V \;&\equiv\; 2C\,[\,A_\gamma A_Z d' - A_Z^2 v_e d'^2 + A_Z d'\mathrm{Re}(\delta A_\gamma) + (A_\gamma d' - 2A_Z v_e d'^2)\mathrm{Re}(\delta A_Z)\,],\\
E_A \;&\equiv\; 2C\,[\,-B_Z^2 v_e d'^2 + B_Z d'\mathrm{Re}(\delta B_\gamma) - 2B_Z v_e d'^2\mathrm{Re}(\delta B_Z)\,],\\
E_{VA} \;&\equiv\; C\,[\,A_\gamma B_Z d' - 2A_Z B_Z v_e d'^2 + B_Z d'(\delta A_\gamma)^* + A_Z d'\delta B_\gamma\\
&\quad -2B_Z v_e d'^2(\delta A_Z)^* + (A_\gamma d' - 2A_Z v_e d'^2)\delta B_Z\,],\\
F_1 \;&\equiv\; C\,[\,-(A_\gamma - A_Z v_e d')\delta D_\gamma + \{A_\gamma v_e d' - A_Z(1+v_e^2)d'^2\}\delta D_Z\,],\\
F_2 \;&\equiv\; C\,[\,-A_Z d'\delta D_\gamma - (A_\gamma d' - 2A_Z v_e d'^2)\delta D_Z\,],\\
F_3 \;&\equiv\; C\,[\,B_Z v_e d'\delta D_\gamma - B_Z(1+v_e^2)d'^2\delta D_Z\,],\\
F_4 \;&\equiv\; C\,[\,-B_Z d'\delta D_\gamma + 2B_Z v_e d'^2\delta D_Z\,],\\
G_1 \;&\equiv\; C\,[\,(A_\gamma - A_Z v_e d')\delta C_\gamma - \{A_\gamma v_e d' - A_Z(1+v_e^2)d'^2\}\delta C_Z\,],\\
G_2 \;&\equiv\; C\,[\,A_Z d'\delta C_\gamma + (A_\gamma d' - 2A_Z v_e d'^2)\delta C_Z\,],\\
G_3 \;&\equiv\; C\,[\,-B_Z v_e d'\delta C_\gamma + B_Z(1+v_e^2)d'^2\delta C_Z\,],\\
G_4 \;&\equiv\; C\,[\,B_Z d'\delta C_\gamma - 2B_Z v_e d'^2\delta C_Z\,] \qquad (25)
\end{aligned}
$$

and

$$
C \equiv 1/(4\sin^2\theta_W), \quad d' \equiv s/[4\sin\theta_W\cos\theta_W(s-M_Z^2)].
$$

In the above formulas, only linear terms in non-standard couplings have been kept.

The functions $f(x)$, $g(x)$, $\delta f(x)$ and $\delta g(x)$ in eqs.(17) and (19) are given as

$$
\begin{aligned}
f(x) &= C_1\Big\{\,r(r-2) + 2x\frac{1+\beta}{1-\beta} - x^2\Big(\frac{1+\beta}{1-\beta}\Big)^2\,\Big\},\\
&\qquad\qquad \text{(for the interval } I_1,\ I_4)\\
&= C_1\,(1-r)^2, \qquad \text{(for the interval } I_2)\\
&= C_1\,(1-x)^2, \qquad \text{(for the interval } I_3,\ I_6)\\
&= C_1\,x\Big\{\,x + \frac{4\beta}{1-\beta} - x\Big(\frac{1+\beta}{1-\beta}\Big)^2\,\Big\},\\
&\qquad\qquad \text{(for the interval } I_5)
\end{aligned}
$$

$$g(x) = C_2\Big[-rx + x^2\frac{1+\beta}{1-\beta} - x\ln\frac{x(1+\beta)}{r(1-\beta)}$$
$$+ \frac{1}{2(1+\beta)}\Big\{ r(r-2) + 2x\frac{1+\beta}{1-\beta} - x^2\Big(\frac{1+\beta}{1-\beta}\Big)^2 \Big\} \Big],$$
(for the interval $I_1,\ I_4$)
$$= C_2\Big\{ (1-r+\ln r)x + \frac{1}{2(1+\beta)}(1-r)^2 \Big\},$$
(for the interval I_2)
$$= C_2\Big\{ (1-x+\ln x)x + \frac{1}{2(1+\beta)}(1-x)^2 \Big\},$$
(for the interval $I_3,\ I_6$)
$$= C_2\, x\Big[\frac{2\beta x}{1-\beta} - \ln\frac{1+\beta}{1-\beta}$$
$$+ \frac{1}{2(1+\beta)}\Big\{ x + \frac{4\beta}{1-\beta} - x\Big(\frac{1+\beta}{1-\beta}\Big)^2 \Big\} \Big],$$
(for the interval I_5)

where
$$C_1 \equiv \frac{3}{2W}\frac{1+\beta}{\beta}, \qquad C_2 \equiv \frac{3}{W}\frac{(1+\beta)^2}{\beta},$$
and $W \equiv (1-r)^2(1+2r)$ with $r \equiv (M_W/m_t)^2$ as defined in the main text,
$$\delta f(x) = C_3\Big\{ \frac{1}{2}r(r+8) - 2x(r+2)\frac{1+\beta}{1-\beta} + \frac{3}{2}x^2\Big(\frac{1+\beta}{1-\beta}\Big)^2$$
$$+ (1+2r)\ln\frac{x(1+\beta)}{r(1-\beta)} \Big\},$$
(for the interval $I_1,\ I_4$)
$$= C_3\Big\{ \frac{1}{2}(r-1)(r+5) - (1+2r)\ln r \Big\},$$
(for the interval I_2)
$$= C_3\Big\{ \frac{1}{2}(x-1)(5+4r-3x) - (1+2r)\ln x \Big\},$$
(for the interval $I_3,\ I_6$)
$$= C_3\Big\{ (1+2r)\ln\frac{1+\beta}{1-\beta} - \frac{4\beta x}{1-\beta}(r+2) + \frac{6\beta}{(1-\beta)^2}x^2 \Big\},$$
(for the interval I_5)

$$\delta g(x) = C_3\Big[1-\beta+2(3-\beta)r + \frac{1}{2}r^2 - \frac{3}{2}(1-2\beta)\Big(\frac{1+\beta}{1-\beta}\Big)^2 x^2$$

$$
\begin{aligned}
&+ (1+\beta)x\Big\{\frac{1}{r}(r-1)(3r+1) - \frac{2(r+2)}{1-\beta}\Big\} \\
&+ \{1+2r+2(1+\beta)(r+2)x\}\ln\frac{x(1+\beta)}{r(1-\beta)}\ \Big], \\
&\qquad\qquad\qquad\qquad\qquad \text{(for the interval } I_1,\ I_4) \\
= C_3\Big[\ &\frac{1}{2}(r-1)(r+5) - (1+2r)\ln r \\
&+ (1+\beta)x\Big\{\frac{1}{r}(r-1)(5r+1) - 2(r+2)\ln r\Big\}\ \Big], \\
&\qquad\qquad\qquad\qquad\qquad \text{(for the interval } I_2) \\
= C_3\Big[\ &-\frac{7}{2} - 4r - \beta(2r+1) + 2x\{1-\beta+r(2+\beta)\} \\
&+ \frac{3}{2}(1+2\beta)x^2 - \{2r+1+2(1+\beta)(r+2)x\}\ln x\ \Big], \\
&\qquad\qquad\qquad\qquad\qquad \text{(for the interval } I_3,\ I_6) \\
= C_3\Big[\ &-(1+2r)\Big(2\beta - \ln\frac{1+\beta}{1-\beta}\Big) + \frac{6\beta^3}{(1-\beta)^2}x^2 \\
&- 2(r+2)x\Big\{\frac{2\beta}{1-\beta} - (1+\beta)\ln\frac{1+\beta}{1-\beta}\Big\}\ \Big], \\
&\qquad\qquad\qquad\qquad\qquad \text{(for the interval } I_5)
\end{aligned}
$$

where

$$C_3 \equiv \frac{6}{W}\frac{1+\beta}{\beta}\frac{\sqrt{r}}{1+2r}.$$

The intervals I_i $(i = 1 \sim 6)$ of x are given by

$$
\begin{aligned}
I_1 :&\quad r(1-\beta)/(1+\beta) \le x \le (1-\beta)/(1+\beta), \\
I_2 :&\quad (1-\beta)/(1+\beta) \le x \le r, \\
I_3 :&\quad r \le x \le 1, \\
&\qquad\qquad (I_{1,2,3} \text{ are for } r \ge (1-\beta)/(1+\beta)) \\
I_4 :&\quad r(1-\beta)/(1+\beta) \le x \le r, \\
I_5 :&\quad r \le x \le (1-\beta)/(1+\beta), \\
I_6 :&\quad (1-\beta)/(1+\beta) \le x \le 1. \\
&\qquad\qquad (I_{4,5,6} \text{ are for } r \le (1-\beta)/(1+\beta))
\end{aligned}
$$

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