Growth of the Energy Spread Due to the Radiative Interaction in a Short Electron Bunch Moving in an Undulator

E.L. Saldin[a], E.A. Schneidmiller[a], M.V. Yurkov[b]

[a] *Automatic Systems Corporation, 443050 Samara, Russia*

[b] *Joint Institute for Nuclear Research, Dubna, 141980 Moscow Region, Russia*

Abstract

This paper presents investigations of the longitudinal radiative force in an electron bunch moving in an undulator (wiggler). Analytical solution is obtained for a Gaussian longitudinal bunch profile. Radiative interaction of the particles in an intensive microbunch induces a correlated energy spread in the electron beam. Numerical estimations presented in the paper show that this effect can be important for free electron lasers.

1. Introduction

The theory of the radiative interaction of the electrons in an intensive microbunch traversing curved trajectory is intensively developed nowadays. This is explained by the practical importance of the radiative effects for the beam dynamics in linear colliders and short-wavelength free electron lasers. When intensive electron bunch passes bending magnets, bunch compressors, wigglers etc, the radiative interaction induces the energy spread in the electron beam and can lead to the transverse emittance dilution in dispersive regions.

In this paper we present the results of analytical calculation of the radiative interaction of particles in a bunch with the line-charge distribution moving in an undulator (a wiggler) [1]. The transient effects (when the bunch enters and leaves the undulator) are withdrawn from the consideration and the shielding effects (influence of a vacuum chamber on the radiative process) are neglected. Practical application of the obtained results is illustrated with the numerical examples for the proposed SASE FELs. Also, the testing of the numerical simulation codes would be difficult without rigorous analytical results.

2. General solution for a bunch moving in an undulator

Let us consider the electron bunch with a linear density of particles $\lambda(s)$ moving along the z direction in the undulator with with magnetic field

$$H_x = H_w \cos(k_w z) \ .$$

The transverse and the longitudinal velocities of an electron can be approximated by

$$\beta_y = \frac{K}{\gamma}\sin(k_w z), \qquad \beta_z = \beta - \frac{K^2}{2\gamma^2}\sin^2(k_w z) \ ,$$

where $K = eH_w/k_w mc^2$ is the undulator parameter, $\gamma = \mathcal{E}/mc^2$ is relativistic factor, $(1+K^2/2)/\gamma^2 \ll 1$ and $\beta \simeq 1 - 1/2\gamma^2$. Transverse coordinate of the electron oscillates as

$$y = -\frac{K}{\gamma k_w}\cos(k_w z) \ .$$

The bunch length is assumed to be much shorter than the undulator period $2\pi/k_w$. We calculate the longitudinal (along the particle's velocity) radiative force assuming the motion of the particles to be given. Only the interaction connected with the curvature is considered and we omit the trivial longitudinal space-charge forces in a bunch moving on a straight line, since they can be calculated separately. The calculations are based on the general algorithm for an arbitrary small-angle trajectory described in ref. [1]. Leaving the details of the calculations, we present the final result for the rate of the energy change as a function of the positions of the electron in the bunch and in the undulator, s and z, respectively [1]:

$$\frac{d\mathcal{E}}{cdt} = e^2 k_w \int_{-\infty}^{s} ds' D(\hat{s} - \hat{s}', K, \hat{z}) \frac{d\lambda(s')}{ds'} \ , \qquad (1)$$

where

$$D(\hat{s}, K, \hat{z}) = \frac{1}{\hat{s}} - 2\ \frac{\Delta - K^2 B(\Delta, \hat{z})}{\Delta^2 + K^2 B^2(\Delta, \hat{z})} \times [\sin\Delta \cos\hat{z} + (1 - \cos\Delta)\sin\hat{z}] \ , \qquad (2)$$

$$B(\Delta, \hat{z}) = (1 - \cos\Delta - \Delta\sin\Delta)\cos\hat{z} + (\Delta\cos\Delta - \sin\Delta)\sin\hat{z} \ , \qquad (3)$$

and Δ is the solution of the transcendental equation:

$$\hat{s} = \frac{\Delta}{2}\left(1 + \frac{K^2}{2}\right) + \frac{K^2}{4\Delta}\{[2(1 - \cos\Delta) - \Delta\sin\Delta] \times (\cos\Delta\cos 2\hat{z} + \sin\Delta\sin 2\hat{z}) - 2(1 - \cos\Delta)\} \ . \quad (4)$$

Here the following reduced variables are introduced: $\hat{s} = \gamma^2 k_w s$ and $\hat{z} = k_w z$.

It follows from the geometry of the problem and from eqs. (2) – (4) that function D is periodical in z. with the period equal to the half of the undulator period π/k_w (or, to π in terms of the normalized position $\hat{z}$). In the following we will study the rate of the energy change averaged over the z coordinate:

$$\frac{d\bar{\mathcal{E}}}{cdt} = e^2 k_w \int_{-\infty}^{s} ds' \bar{D}(\hat{s} - \hat{s}', K) \frac{d\lambda(s')}{ds'} \ , \qquad (5)$$

where

$$\bar{D}(\hat{s}, K) = \frac{1}{\pi}\int_0^{\pi} d\hat{z} D(\hat{s}, K, \hat{z}) \ . \qquad (6)$$

In the case of small values of the undulator parameter, $K \ll 1$, the function $\bar{D}(\hat{s}, K)$ takes the simple form:

$$\bar{D}(\hat{s}, K) = -K^2 \left(\frac{\sin^2\hat{s}}{\hat{s}} + \frac{\sin 2\hat{s}}{2\hat{s}^2} - \frac{\sin^2\hat{s}}{\hat{s}^3}\right) \ . \qquad (7)$$

3. Averaged solution for a Gaussian bunch

In this section we consider a bunch with a Gaussian distribution of linear density:

$$\lambda(s) = \frac{N}{\sqrt{2\pi}\sigma}\ \exp\left[-\frac{s^2}{2\sigma^2}\right] \ . \qquad (8)$$

The averaged solution for the Gaussian bunch can be written in the form:

$$\frac{d\bar{\mathcal{E}}}{cdt} = \frac{e^2 N K^2}{\sqrt{2\pi}\sigma^2\gamma^2}\bar{G}(p, K, x) \ , \qquad (9)$$

where $x = s/\sigma$ and p is the bunch length parameter:

$$p = \frac{\gamma^2 k_w \sigma}{1 + K^2/2} \ .$$

In the general case function $\bar{G}$ should be calculated by means of numerical integration of eq. (5). Nevertheless, in some region of parameters it can be expressed analytically. Let us study practically important case of a long bunch, $p \gg 1$. First, we consider the case of small K. Under these conditions function $\bar{G}$ can be calculated analytically using eqs. (5) and (7):

$$\bar{G}(p, x) = \frac{x}{2}\exp\left(-\frac{x^2}{2}\right)\ln p + F(x) \ . \qquad (10)$$

Here parameter p is reduced to $p \simeq \gamma^2 k_w \sigma$, and function $F(x)$ has the form:

$$F(x) = \frac{1}{4}(C + 3\ln 2 - 2)x\exp\left(-\frac{x^2}{2}\right) - \sqrt{\frac{\pi}{8}}\left[1 + \mathrm{erf}\left(\frac{x}{\sqrt{2}}\right) - x\exp\left(-\frac{x^2}{2}\right) \times \int_0^x dx' \exp\left(\frac{(x')^2}{2}\right)\left(1 + \mathrm{erf}\left(\frac{x'}{\sqrt{2}}\right)\right)\right] \ , \qquad (11)$$

where $C = 0.577...$ is the Euler's constant and erf(...) is the error function [2]. The plot of function $F(x)$ is presented in Fig. 1. Figure 2 presents the plots of function $\bar{G}$ calculated at different values of parameter p.

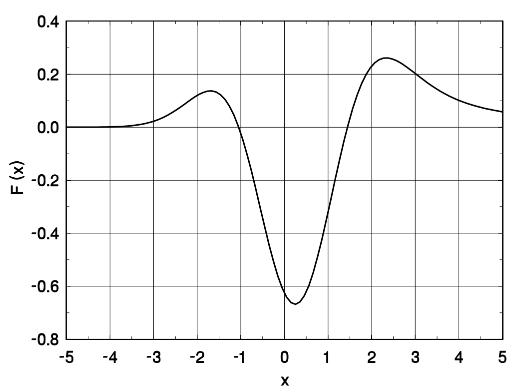

Fig. 1. Function $F(x)$ given by eq. (11).

In the case of an arbitrary value of the undulator parameter K, it is difficult to find explicit analytical solution. Nevertheless, using the results of numerical

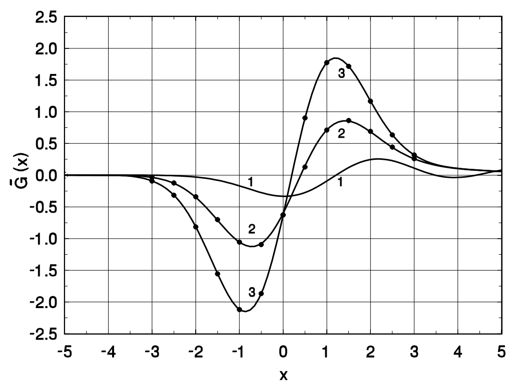

Fig. 2. Function $\bar{G}$ for small value of the undulator parameter K and different values of the bunch length parameter p. Curve (1): $p = 1$, curve (2): $p = 30$, and curve (3): $p = 1000$. The curves are the results of numerical integration of eq. (5) and the circles are calculated with the help of analytical formula (10) for large values of parameter p.

integration of eq. (5), we can write function $\bar{G}$ in the following form ($p \gg 1$):

$$\bar{G}(p, K, x) = \frac{x}{2} \exp\left(-\frac{x^2}{2}\right) [\ln p + g(K)] + F(x) \ , \tag{12}$$

where function $g(K)$ changes from 0 to 1 when K changes from small to large values. The plot of this function is presented in Fig. 3.

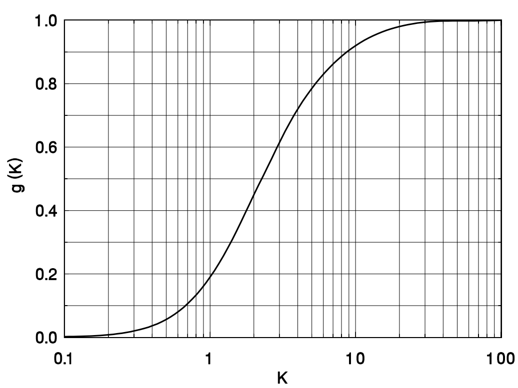

Fig. 3. Function $g(K)$ entering eq. (12).

In conclusion to this section we present the formula for the induced correlated energy spread in the Gaussian bunch due to the radiative interaction. Using expression (12) we write down this formula in the form convenient for practical calculations ($p \gg 1$):

$$\sigma_\gamma = 0.219 \frac{I K^2 L_w}{I_{\mathrm{A}} \sigma \gamma^2} \times$$

$$\sqrt{[\ln p + g(K)]^2 + 0.933[\ln p + g(K)] - 0.786} \ , \tag{13}$$

where $I = Nec/\sqrt{2\pi}\sigma$ is the peak current, $I_A = 17$ kA is Alfven current, L_w is the undulator length and

$$mc^2 \sigma_\gamma = \sqrt{\langle \bar{\mathcal{E}}^2 \rangle - \langle \bar{\mathcal{E}} \rangle^2} \ .$$

4. Discussion

Let us perform estimations of the applicability region of the obtained results for the practically important case of a long bunch, $\sigma \gg (1 + K^2/2)/\gamma^2 k_w$. First, we consider the transient effects. When the behaviour of the radiative forces after the bunch leaves the undulator is not important from practical point of view (which is true for FELs), only the entrance transient effect is of interest. In this case we can write down the following limitation on the undulator (wiggler) length L_w, allowing us to neglect transient effects:

$$L_w \gg \sigma \gamma_z^2 \ , \tag{14}$$

where $\sigma \gamma_z^2 = \sigma \gamma^2 / (1 + K^2/2)$ is typical formation length of the radiation.

Second, we estimate the region of parameters where we can neglect the influence of the bunch transverse size and of the vacuum pipe. It follows from simple geometrical consideration that a characteristic measure distinguishing these effects is the mean geometric value of the bunch length and the formation length of the radiation. Thus, we can roughly estimate the region where the considered effects can be neglected:

$$\sigma_\perp \ll \sigma \gamma_z \ll b \ . \tag{15}$$

Here $\sigma_\perp$ and b are transverse dimensions of the bunch and of the vacuum chamber, respectively.

When the above mentioned limitations are not satisfied, the considered effects become to be important leading to suppression of the radiative interaction. In other words, the model considered in this paper describes the worst-case approximation, which makes it useful for quick estimations of the radiative interaction effects. Also, analytical results presented in the paper can serve as primary standards for testing numerical simulation codes.

In conclusion to this paper let us illustrate practical application of the obtained results with two numerical examples. The first one is the 6 nm SASE FEL being under construction at the TESLA Test Facility at DESY [3]. Parameters of the project are: the energy is 1 GeV, the rms bunch length is 50 μm, the peak current is 2.5 kA, the undulator period is 2.73 cm, K is 1.27 and the undulator length is 27 m. Substituting these values into formula (13), we obtain that the induced correlated energy spread is equal to

$\sigma_\gamma/\gamma = 4 \times 10^{-5}$ which is negligible. Besides, condition (14) and the condition (15) for the shielding are not satisfied, since the diameter of vacuum chamber is equal to 1 cm. This will lead to further reduction of the effect. It should be noticed that such a situation is typical for the projects of VUV and X-ray FELs.

The second example is the proposal by the Duke university [4] to construct 1.4 μm SASE FEL using the PALADIN wiggler. The energy is 200 MeV, the rms bunch length is 50 μm, the bunch radius is 0.125 cm, the peak current is 2.5 kA, the wiggler period is 8 cm, K is 3 and the wiggler length is 15 m. Assuming the size of the vacuum chamber to be about 2 cm, we obtain that conditions (14) and (15) are met and our simple model provides correct estimation of the effect. According to formula (13), we obtain $\sigma_\gamma/\gamma = 8.4\times 10^{-3}$. Effective operation of the free electron laser requires $\sigma_\gamma/\gamma \ll \rho$, where ρ is the FEL parameter [5]. For the Duke SASE FEL $\rho = 4.8 \times 10^{-3}$, and the latter condition is strongly violated. Besides, the SASE coherence length [6] is of the order of the bunch length. Thus, the large energy spread will be induced within the coherence length and the FEL process will be destroyed.

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