Numerical Study of Performance Limitations of X-ray Free Electron Laser Operation due to Quantum Fluctuation of Undulator Radiation

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Abstract

One of the fundamental limitations towards achieving very short wavelength in a self amplified spontaneous emission free electron laser (SASE FEL) is connected with the energy diffusion in the electron beam due to quantum fluctuations of undulator radiation. Parameters of the LCLS and TESLA X-ray FEL projects are very close to this limit and there exists necessity in upgrading FEL simulation codes for optimization of SASE FEL for operation at a shortest possible wavelength. In this report we describe a one-dimensional FEL simulation code taking into account the effects of incoherent undulator radiation. Using similarity techniques we have calculated universal functions describing degradation of the FEL process due to quantum fluctuations of undulator radiation.

1 Introduction

It has been realized more than ten years ago that single-pass free electron laser (FEL) can provide the possibility to generate powerful, coherent VUV and X-ray radiation [1–3]. Several projects of such FEL amplifiers are under development at present [4–6].

When designing an FEL amplifier operating at the wavelength around 1 Å one should also take into account the effect of energy diffusion in the electron beam due to quantum fluctuations of undulator radiation. Recent study of performance limitations of an X-ray free electron laser has shown that this

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effect leads to the growth of the energy spread in the electron beam, σ_{E}, when the electron beam passes the undulator. This effect imposes fundamental limit towards achieving very short wavelength given by the following estimation [7] (for the case of zero energy spread at the undulator entrance):

$$\lambda_{\min} \simeq 45\pi \left[\lambda\!\!\!^{-}_{\mathrm{c}} r_{\mathrm{e}}\right]^{1/5} L_{\mathrm{w}}^{-7/15} \left[\epsilon_n^2 \frac{I_{\mathrm{A}}}{I}\right]^{8/15} , \qquad (1)$$

or, to a good approximation

$$\lambda_{\min}[\mathring{A}] \simeq 4 \frac{\pi\epsilon_n[\mathrm{mm\ mrad}]}{\sqrt{I[\mathrm{kA}] L_{\mathrm{w}}[\mathrm{m}]}} ,$$

where $\lambda\!\!\!^{-}_{\mathrm{c}} = \hbar/mc$, $\hbar$ is Planck constant, $r_{\mathrm{e}} = e^2/m_{\mathrm{e}}c^2$, $(-e)$ and m_{e} are the charge and the mass of the electron, respectively, c is the spead of light, $I_{\mathrm{A}} \simeq 17$ kA is Alfven's current, L_{w} is the length of the undulator, $\epsilon_n = \gamma\epsilon$ is normalized emittance, and γ is the relativistic Lorentz factor.

All the existent FEL simulation codes do not take into account the effect of the energy diffusion in the electron beam due to quantum fluctuations of undulator radiation. On the other hand, design parameters of existent projects of X-ray FELs (LCLS at SLAC [5] and X-ray FEL at linear collider TESLA [6]) are very close to this limit and there exists an urgent necessity in more rigorous simulations of their parameters.

In this report we describe a one-dimensional FEL simulation code taking into account the effects of incoherent undulator radiation. Using similarity techniques we have calculated universal functions describing degradation of the FEL process due to quantum fluctuations of undulator radiation.

2 Numerical simulation algorithm

In this section we present brief description of the one-dimensional simulation code upgraded with the equations including the effects of incoherent undulator radiation. The self-consistent FEL equations are identical to those described in paper [8] (section 3). To describe the influence of incoherent undulator radiation on the FEL process we have included two physical effects into the FEL code. The first one is additional energy loss which is given by well known classical expression:

$$\mathrm{d} < \mathcal{E} > / \mathrm{d}z = -2r_{\mathrm{e}}^2\gamma^2 H_{\mathrm{w}}^2(z)/3 \ , \tag{2}$$

where H_{w} is the magnetic field of the undulator on axis.

Another effect is energy diffusion in the electron beam due to quantum fluctuations of the undulator radiation. The rate of energy diffusion is given by the expression:

$$\frac{\mathrm{d} < (\delta\mathcal{E})^2 >}{\mathrm{d}t} = \int \mathrm{d}\omega\hbar\omega\frac{\mathrm{d}I}{\mathrm{d}\omega} \ , \tag{3}$$

where $\mathrm{d}I/\mathrm{d}\omega$ is the spectral intensity of an undulator radiation. Explicit expression for the rate of the energy diffusion has the following form [9]:

$$\frac{\mathrm{d} < (\delta\gamma)^2 >}{\mathrm{d}t} = \frac{14}{15}c\lambda\!\!\!^{-}_{\mathrm{c}} r_{\mathrm{e}}\gamma^4\kappa_{\mathrm{w}}^3 K^2 F(K) \ , \tag{4}$$

where $\kappa_{\mathrm{w}} = 2\pi/\lambda_{\mathrm{w}}$, λ_{w} is the undulator period length, $K = eH_{\mathrm{w}}/m_{\mathrm{e}}c^2\kappa_{\mathrm{w}}$ is the dimensionless undulator parameter, and $F(K)$ is given by the following fitting formulae:

$$F(K) = 1.42K + \frac{1}{1 + 1.50K + 0.95K^2} \qquad \text{for helical undulator} \ , \tag{5a}$$

$$F(K) = 0.60K + \frac{1}{2 + 2.66K + 0.80K^2} \qquad \text{for planar undulator} \ . \tag{5b}$$

The simulation algorithm is organized as follows. Equations of motion of macroparticles and the field equations (see ref. [8], section 3) are integrated by means of Runge-Kutta scheme. An additional loss of the electron energy is calculated with eq. (2) and additional energy spread is introduced by means of random generator after each integration step in accordance with eq. (4). For the latter procedure to be physically correct, one should care about suppression of numerical noise connected with finite number of macroparticles. In other words, all the moments of the distribution function $f(\Psi, P, z)$ (here $P = \mathcal{E} - \mathcal{E}_0$):

$$a_k = \int P^k \exp(-\mathrm{i}\Psi) f(\Psi, P, z) \mathrm{d}P \mathrm{d}\Psi \ , \tag{6}$$

must have the same values before and after performing the procedure of introducing an additional energy spread. Otherwise, an additional (unphysical) bunching due to numerical noise will appear which will produce an error in the results of calculations. The necessity of compensation of the numerical noise can be explained in the following way. Suppose one has a problem to prepare an initial ensemble of the particles corresponding to unmodulated electron beam:

$$a_0(0) = \int \exp(-\mathrm{i}\Psi) f(\Psi, P, 0) \mathrm{d}P \mathrm{d}\Psi = 0 \ , \tag{7}$$

and some arbitrary distribution in the energy. Let us consider the evolution of such a distribution function $f(\Psi, P, z)$ in a drift space (no FEL process):

$$\frac{\mathrm{d}f(\Psi, P, z)}{\mathrm{d}z} - \alpha P f(\Psi, P, z) = 0 \ . \tag{8}$$

It follows from this equation that distribution functions at coordinates z and $z + \Delta z$ are connected by the relation:

$$f(\Psi, P, z + \Delta z) = f(\Psi, P, z) \exp(\alpha P \Delta z) \ . \tag{9}$$

Using eqs. (9) and (7) the evolution of the bunching factor a_0 is as follows:

$$a_0(z) = a_0(0) + \sum_{k=1}^{\infty} a_k \frac{\alpha^k}{k!} z^k \ . \tag{10}$$

It can been seen in eq. (10) that although starting with bunching factor $a_0(0) = 0$, bunching can occur if higher moments exist non equal to zero. In the presence of the FEL process this can introduce an error in the results of the calculation.

For the correction of the moments the phase space is divided up into N stripes with a limited range of the momentum ($P_{min} < P < P_{max}$), where the correction scheme is applied for each stripe. A larger number of stripes improves the results because a correlation between P and $\exp(-\mathrm{i}\Psi)$ is reduced. In general all higher moments, caused by the numerical noise, are reduced in the limit of $N \to \infty$. Numerical simulations show that for practical calculations it is sufficient to compensate only the first two moments of the distribution function, a_0 and a_1. In this case the accuracy of calculations is better than 1 % for $N = 150$ and 10000 macroparticles.

3 Simulation results

We consider simplified situation of a "cold" electron beam at the undulator entrance and neglect the influence of the space charge field. It is assumed that the FEL amplifier is tuned to the resonance frequency. Under these approximations operation of the conventional FEL amplifier is described in terms of the gain parameter Γ and the efficiency parameter ρ (see, e.g. refs. [2,8,10]):

$$\Gamma = \left[\frac{2\pi^2 j_0 K^2}{I_{\mathrm{A}} \lambda_{\mathrm{w}} \gamma^3} \right]^{1/3} , \qquad \rho = \frac{\lambda_{\mathrm{w}} \Gamma}{4\pi} , \tag{11}$$

where j_0 is the beam current density (for the case of a helical undulator).

We assume that the mean energy loss (see eq. (2)) are compensated by an appropriate undulator tapering and study pure effect of the energy diffusion in the electron beam due to quantum fluctuations of undulator radiation (see eq. (4)). When simulating SASE FEL with steady-state simulation code, one should set the value of the "effective" power of input shot noise which is given approximately by the relation [11,12]:

$$W_{\mathrm{sh}} \simeq \frac{3\sqrt{4\pi}\rho^2 P_{\mathrm{b}}}{N_{\lambda}\sqrt{\ln(N_{\lambda}/\rho)}} \ , \tag{12}$$

where $P_{\mathrm{b}} = \gamma m_{\mathrm{e}} c^2 I/e$ is the power of the electron beam and $N_{\lambda} = I\lambda/(ec)$. For the case of the X-ray SASE FEL at TESLA [6] the value of the reduced input power is of about $\hat{W}_{\mathrm{sh}} = W_{\mathrm{sh}}/\rho P_{\mathrm{b}} \simeq 3 \times 10^{-7}$. This value has been used in the simulations.

Under accepted approximations the value of the reduced power at saturation, $\hat{\eta} = P_{\mathrm{out}}/\rho P_{\mathrm{b}}$, and the saturation length, $\hat{L}_{\mathrm{sat}} = L_{\mathrm{sat}}\Gamma$ are universal functions of the reduced input power $\hat{W}_{\mathrm{sh}}$ and the parameter of quantum fluctuations $\hat{q}$ (see eq. (4)):

$$\hat{q} = \frac{7}{15}\frac{\lambda\!\!\!^{-}_{\mathrm{c}} r_{\mathrm{e}}}{\rho^3}\gamma^2 k_{\mathrm{w}}^2 K^2 F(K) \ . \tag{13}$$

Fig. 1 presents the plots of these universal functions in dependency of $\hat{q}$ for four diffferent settings of the reduced input power $\hat{W}_{\mathrm{sh}}$. It is seen that operation of the FEL amplifier degrades significantly when the value of the parameter of quantum fluctuations is increased.

Operating point of 1 Å FEL at TESLA (50 GeV energy of the electron beam [6]) corresponds to the value of the parameter of quantum fluctuations $\hat{q} = 9 \times 10^{-3}$. Fig. 2 illustrate the degradation of the FEL performance of the 1 ÅFEL at TESLA. It is seen that parameters of the project have been chosen correctly and there is only a slight degradation of the FEL performance due to quantum fluctuations of undulator radiation.

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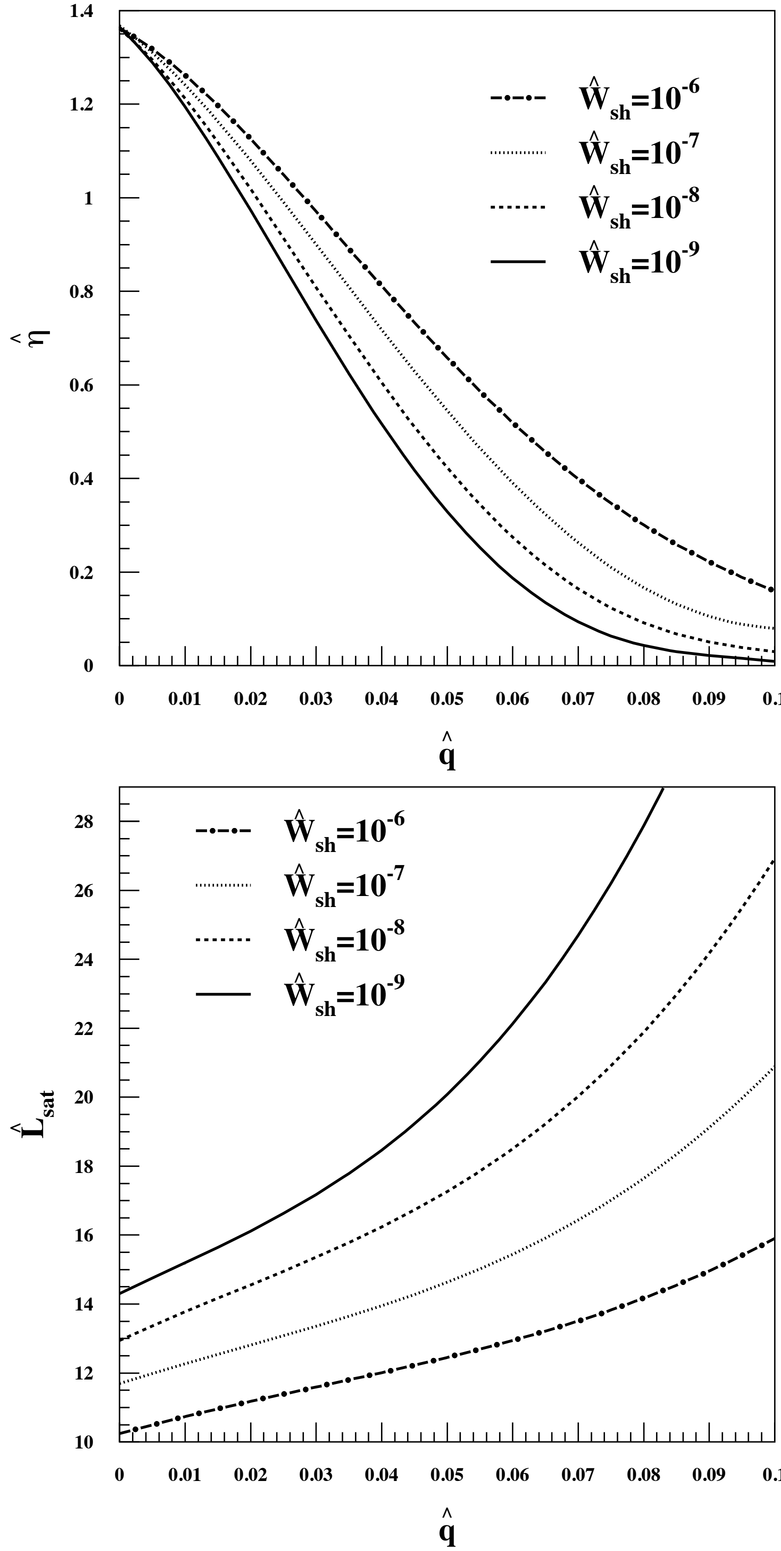

Fig. 1. Dependency of the saturation efficiency $\hat{\eta} = P_{\mathrm{out}}/\rho P_{\mathrm{b}}$ and saturation length $\hat{L}_{\mathrm{sat}} = L_{\mathrm{sat}}\Gamma$ on the value of the parameter of quantum fluctuations $\hat{q}$ and reduced input power $\hat{W}_{\mathrm{sh}}$.

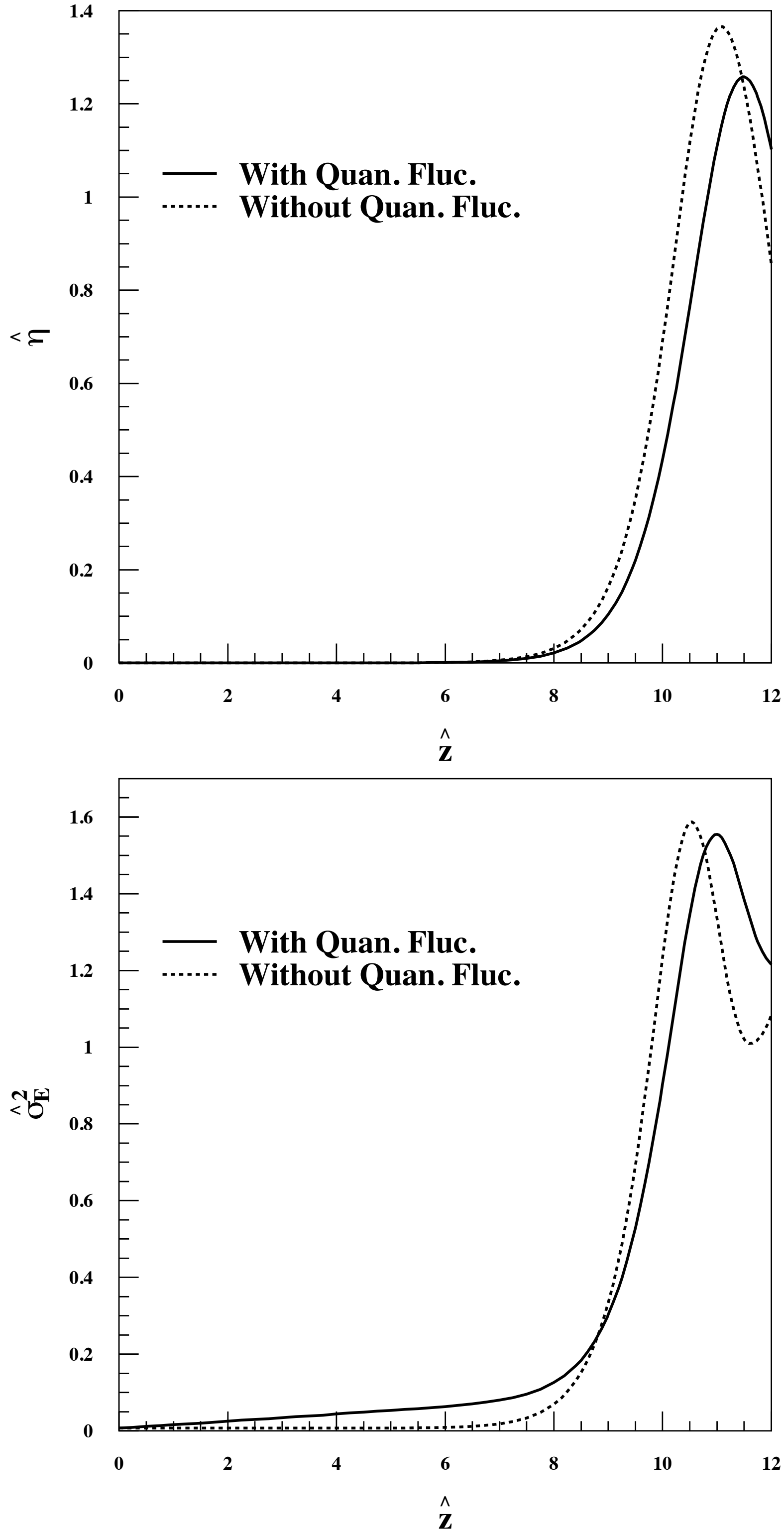

Fig. 2. Reduced efficiency $\hat{\eta} = P_{\mathrm{out}}/\rho P_{\mathrm{b}}$ and induced energy spread $\hat{\sigma}_E^2 = \langle \mathcal{E}^2 - \langle \mathcal{E} \rangle^2 \rangle / \rho^2 \langle \mathcal{E} \rangle^2$ for different position $\hat{z}$ in the TESLA 50 GeV FEL. Results including quantum fluctuations are drawn by a solid line ($\hat{q} = 9 \times 10^{-3}$), without – by a dashed line.