A New Method for Ultrashort Electron Pulse-shape Measurement Using Synchrotron Radiation from a Bending Magnet

J. Krzywinski[a], E.L. Saldin[b], E.A. Schneidmiller[b], M.V. Yurkov[c]

[a] *Institute of Physics of the Polish Academy of Sciences, 02688 Warszawa, Poland*

[b] *Automatic Systems Corporation, 443050 Samara, Russia*

[c] *Joint Institute for Nuclear Research, Dubna, 141980 Moscow Region, Russia*

Abstract

A new non-destructive method for measurement of the longitudinal profile of subpicosecond electron bunches is proposed. The method is based on measurements in frequency domain of correlations between the fluctuations of synchrotron radiation produced by an electron bunch passing a bending magnet. The proposed device is the combination of a monochromator and a counting interferometer which gives directly the square of the modulus of the Fourier transform of the longitudinal electron bunch profile. Reconstruction of the beam profile from these data is performed by means of the standard technique developed for the image reconstruction from the data obtained by means of the Handbury-Brown and Twiss intensity interferometer. Because the principle of the method is based essentially on the statistical properties of the synchrotron radiation and the detection process itself these topics are also considered in detail. The signal to noise ratio is analyzed in terms of degeneracy parameter. The degeneracy parameter increases approximately as a third power of the wavelength which makes the visible range of synchrotron radiation to be a natural choice for the spectral intensity correlation measurement. In the end of the paper we illustrate with numerical example the potential of the proposed method for on line, non-destructive diagnostics of the electron beam in the accelerator driving the TESLA Test Facility Free Electron Laser at DESY.

1. Introduction

The length of electron bunches for the next generation linear colliders are of an order of $\sigma_z \simeq 0.1 - 1$ mm [1, 2, 3]. The projects of X-ray FELs require even shorter bunches, down to 0.025 mm [1, 4, 5]. These values are less by an order of magnitude than those used in the existent accelerators. Development of non-destructive methods for measurements of longitudinal distribution of the beam current in such a short bunches is a challenging problem.

Recently the method for measuring ultrashort pulses of incoherent radiation has been proposed [6]. The method is based on detecting the fluctuations of the visibility of interference fringes in a two-beam interferometer.

Here we describe a new method for nondestructive measurement of the longitudinal profile of the electron bunch. The method is based on the measurements in frequency domain of correlations between the fluctuations of synchrotron radiation produced by a bunch passing a bending magnet.

The proposed device is the combination of a monochromator and a counting interferometer which gives directly the square of the modulus of the Fourier transform of the longitudinal electron bunch profile. Reconstruction of the beam profile from these data is performed by means of a standard technique developed for the image reconstruction from the data obtained by means of the Hanbury-Brown and Twiss intensity interferometer.

TESLA FEL-Report 1997-06

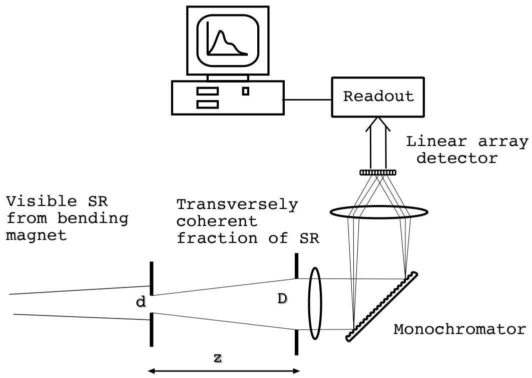

Fig. 1. The layout of the device for measurement the longitudinal profile of the electron pulse.

2. Principle of the method

The layout of the device for measurement the longitudinal profile of the electron pulse is presented in Fig. 1. An ultrarelativistic electron bunch passes a bending magnet and radiates a pulse of synchrotron radiation. The diaphragm of aperture d is used for selection of the transversely coherent fraction of synchrotron radiation which is directed to the monochromator with an aperture D. The monochromator is placed at the distance of z ¿from the diaphragm. The resolution of the monochromator is equal to $\Delta\omega_m$, the central frequency is equal to ω_0 and $\Delta\omega_m/\omega_0 \ll 1$. The radiation reaching the monochromator is transversely coherent when, according to Van Cittert-Zernike theorem, the following condition is fulfilled (see e.g. ref. [7]): $cz/(\omega_0 d) > D$, where c is the velocity of light. A one-dimensional array photodetector is placed at the monochromator exit which performs measurement of the spectral distribution of the radiation energy. The signal from each element of the photodetector is stored in the computer in matrix form. The row index corresponds to the frequency and the column index corresponds to the pulse number. This matrix is used for the calculation of statistical properties of the radiation.

The principle of the operation of the proposed device is based on the statistical properties of synchrotron radiation . The detailed analysis of this subject and the operation of the device is presented in ref [8]. Here, for clarity we present only the main results.

Let us describe an electron bunch profile by the profile function $F(t)$ which gives the probability of the arrival of an electron at the magnet entrance within the time interval $(t, t+dt)$. The electron bunch function $F(t)$ and its Fourier transform $\bar{F}(\omega)$ are connected by Fourier transformation:

$$\bar{F}(\omega) = \int\limits_{-\infty}^{\infty} e^{i\omega t} F(t) dt \ .$$

The first and the second order field correlation functions are defined as

$$g_1(\omega_1 - \omega_2) \equiv \frac{\langle \bar{E}(\omega_1)\bar{E}(\omega_2)\rangle}{\sqrt{\langle | \bar{E}(\omega_1) |^2\rangle\langle | \bar{E}(\omega_2) |^2\rangle}} \ ,$$

$$g_2(\omega_1 - \omega_2) \equiv \frac{\langle | \bar{E}(\omega_1) |^2 | \bar{E}(\omega_2) |^2\rangle}{\langle | \bar{E}(\omega_1) |^2\rangle\langle | \bar{E}(\omega_2) |^2\rangle} \ ,$$

where $\bar{E}(\omega)$ is Fourier component of electric field of the synchrotron radiation.

It is shown in ref. [8] that synchrotron radiation possesses all the features corresponding to completely chaotic polarized light. In particular, the second order correlation function is expressed via the first order correlation function:

$$g_2(\omega_1 - \omega_2) = 1 + | g_1(\omega_1 - \omega_2) |^2 \ .$$

The explicit expression for the first order correlation function has the form

$$g_1(\omega_1 - \omega_2) = \bar{F}(\omega_1 - \omega_2) \ .$$

We define the interval of spectral coherence $\Delta\omega_c$ as follows:

$$\Delta\omega_c = \int\limits_{-\infty}^{\infty} | g_1(\Delta\omega) |^2 \, d(\Delta\omega) \ .$$

In the case of Gaussian profile of the electron beam:

$$F(t) = \frac{1}{\sqrt{2\pi}\sigma_z} \exp\left(-\frac{t^2}{2\sigma_z^2}\right) \ ,$$

the interval of the spectral coherence is equal to $\Delta\omega_c = \sqrt{\pi}/\sigma_z$.

At sufficient resolution of the monochromator ($\Delta\omega_m \ll \Delta\omega_c$), it seems to be technically feasible to measure the second order spectral correlation function of Fourier components of electric field of the synchrotron radiation.

$$g_2(\omega_1 - \omega_2) \simeq \frac{\langle K_1 K_2\rangle}{\langle K_1\rangle\langle K_2\rangle}$$

where $K_{1,2}$ - numbers of photocounts of two emblements of the photodetector corresponding to frequencies $\omega_{1,2}$ respectively. This function contains the information about the Fourier transform of the electron bunch profile:

$$g_2(\omega_1 - \omega_2) = 1 + \mid \bar{F}(\omega_1 - \omega_2) \mid^2 \ .$$

When resolution of the monochromator is worse than the interval of spectral coherence, $\Delta\omega_c \simeq c/\sigma_z$, the mean value-to-dispersion ratio of the photocounts in a given frequency range $\Delta\omega_m$ can be measured. The square of this ratio is equal to M, the number of coherence intervals inside the monochromator linewidth. Analysis of these measurements allows one to estimate the electron bunch length as $\sigma_z \simeq cM/\Delta\omega_m$.

The quantum nature of the photodetection process should be considered when the number K of photocounts in the spectral coherence interval is low . The expression for variance of photocounts fluctuations contains two terms:

$$\sigma_K^2 = \frac{\langle K^2\rangle - \langle K\rangle^2}{\langle K\rangle^2} = \frac{1}{\langle K\rangle} + \frac{1}{M} \ . \qquad (1)$$

The first term corresponds to the "photon shot noise" and its origin is in the Poisson distribution. The second term corresponds to the classical fluctuations of the energy in the radiation bunch and takes its origin from the shot noise in the electron bunch. The ratio of the classical variance to the "photon shot noise" variance is named as the photocount degeneracy parameter δ_c [7]:

$$\delta_c = \frac{\langle K\rangle}{M} \ . \qquad (2)$$

The formula for calculation of the photocount degeneracy parameter δ_c is given by:

$$\delta_c = \eta R_m \frac{\Delta\omega_m}{M\Delta\omega_c}\delta_W \ , \qquad (3)$$

where η is the quantum efficiency, R_m is the integral reflection coefficient of the monochromator mirrors and the dispersive element and δ_W is the wave degeneracy parameter which is equal to the average number of transversely coherent photons radiated by the electron bunch inside the spectral interval of coherence $\Delta\omega_c$. Physically this parameter describes the average number of photons which can interfere, or, according to the quantum theory, the number of photons in one quantum state (one "mode"). It has been shown in ref. [8] that the value of the wave degeneracy parameter for synchrotron radiation can be estimated as:

$$\delta_W \simeq \frac{B_{peak}\lambda^3}{4c} \ , \qquad (4)$$

where $B_{peak}$is the peak value of spectral brightness and λ is the radiation wavelength.

The signal-to-noise ratio associated with the output of the counting interferometer depends on the photocount degeneracy parameter δ_c as [8]
at $\delta_c \gg 1$:

$$\frac{S}{N} \simeq \frac{\sqrt{N_b} \mid \bar{F}(\omega_1 - \omega_2) \mid^2}{\sqrt{11 \mid \bar{F}(\omega_1 - \omega_2) \mid^4 - 12 \mid \bar{F}(\omega_1 - \omega_2) \mid^2 + 9}} \ , \qquad (5a)$$

at $\delta_c \ll 1$:

$$\frac{S}{N} \simeq \frac{\sqrt{N_b}\delta_c \mid \bar{F}(\omega_1 - \omega_2) \mid^2}{\sqrt{1 + \mid \bar{F}(\omega_1 - \omega_2) \mid^2}} \ , \qquad (5b)$$

where N_b is the number of independent measurements.

For this method to be applicable the radiation wavelength must be much smaller than the bunch length. At a bunch length of the order of 0.1 mm one can use a wide interval of the radiation spectrum, from the infrared down to X-rays. The choice of the optimal value of the operating frequency is influenced by such issues as tolerable value of the signal-to-noise ratio, required resolution of the monochromator ($\Delta\omega_m/\omega \ll c/(\omega\sigma_z)$) and the existence of commercially available detectors and optical elements with the required parameters. The choice of the visible range possesses the following advantages. The degeneracy parameter can reach a value larger than unity and therefore can ensure short measurement time. The required resolution of the monochromator ($\Delta\omega_m/\omega \ll c/(\omega\sigma_z) \simeq 0.1\%$) can be achieved without significant efforts. There is also a highly developed technology of electrooptical devices (optical fibers, fast detectors, electrooptical switches, etc.) operating in the visible wavelength range.

3. Numerical example

The operation of the new method for the electron pulse-shape measurement is illustrated for the TESLA Test Facility which is under construction at DESY [5]. It is supposed to use synchrotron radiation from the last bending magnet of the third bunch compressor (see Table 1). The value of the peak spectral brightness is equal to 10^{17} Phot./(sec$\times$mrad$^2\times$mm$^2\times$0.1% bandw.) and the degeneracy parameter is about of $\delta_W \simeq 10^4$ at the chosen operating wavelength of $\lambda = 5000$ Å .

Table 1. Parameters of SR source

Electron beam	
Energy, $\mathcal{E}_0$	500 MeV
Peak current, I_0	2.5 kA
rms bunch length, σ_z	50 μm
Normalized rms emittance , ϵ_n	2π mm mrad
Number of bunches per train	7200
Repetition rate	10 Hz
Bending magnet	
Length of magnet, L_w	0.5 m
Magnetic field, H	1.3 T
β-function,	11 m
Radiation	
Wavelength, λ_0	5000 Å
Spectral coherence $\Delta\omega_c/\omega_0$	0.3 %
Spectral brightness*	10^{17}
Wave degeneracy parameter, δ_W	10^4

* In units of Phot./(sec$\times$ mrad^2$\times$mm^2$\times$0.1% bandw.)

We assume the use of a commercially available monochromator with the resolution of $\Delta\omega_m/\omega \simeq 3 \times 10^{-4}$. The electron bunch length is equal to 0.05 mm which corresponds to the relative value of the interval of spectral coherence of $\Delta\omega_c/\omega \simeq 3 \times 10^{-3}$. Comparing $\Delta\omega_m$ with $\Delta\omega_c$ one can conclude that ten pixels of the photodetector is sufficient to cover the interval of the spectral coherence. The photocount degeneracy parameter δ_c (see eq. (3)) will be of the order of 10^2 at the quantum efficiency of the detector of $\eta \simeq 0.3$ and the value of the integral reflection coefficient of the monochromator mirrors and dispersive elements of $R_m \simeq 0.3$.

The design value of the emittance of the electron bunch in the TTF accelerator is small, so the visible fraction of the SR from the magnet is always transversely coherent and there is no need in the installation of additional diaphragm (see Fig.1). If the monochromator is placed at the distance of $L = 2$ m from the radiation source, the aperture of the monochromator in the vertical direction should be not smaller than $\Delta\theta_c L \simeq 1$ cm. If vertical aperture is less than this value, the flux of coherent photons is decreased. The aperture of the monochromator in the horizontal plane can not be larger than $\Delta\theta_c L$ due to uniform distribution of the radiation in the horizontal plane.

Let us now estimate the number of shots required for achieving a given accuracy of the formfactor measurement. Suppose we wish to achieve a signal-to-noise ratio of 100. The number of independent measurements required to achieve this accuracy depends on the value of the formfactor at given frequency. Using eq. (5a) we find that the number of shots should be about 10^5 at the value of the formfactor about of unity. If the linear array detector covers, for instance, the frequency range of $10\Delta\omega_c$, one can perform ten independent measurements per one shot (per one radiation pulse). As a result, one can decrease the number of shots which is required for a given accuracy by a factor of ten. In this case, one macropulse of the TTF accelerator should be sufficient to obtain the required accuracy. This requires 10 MHz data acquisition system. One can use photomultipliers or pin-photodiodes as photodetectors. In this case the light from the monochromator exit can be distributed to photomultipliers (or photodiodes) by means of optical fibers.

References

[1] "Conceptual Design of a 500 GeV e+e− Linear Collider with Integrated X-ray Laser Facility" (Editors R.Brinkmann, G. Materlik, J. Rossbach, A. Wagner), DESY 97-048, Hamburg, 1997.

[2] "Zeroth-Order Design Report for the Next Linear Collider", LBNL-PUB-5424,SLAC Report 474, UCRL-ID-124161 (May 1996).

[3] Status reports on CLIC, JLC and VLEPP are given in: Proceedings of the LC'92 ECFA Workshop on e^+e^- Linear Colliders (July,25-August,2,1993,Germany), MPI-PhE/93-14, ECFA 93-154.

[4] R. Tatchyn et. al., Nucl. Instr. and Methods **A375**(1996)274.

[5] J. Rossbach, Nucl. Instr. and Methods **A375**(1996)269.

[6] M.S. Zolotorev and G.V. Stupakov, "Fluctuational Interferometry for Measurement of Short Pulses of Incoherent Radiation", SLAC-PUB-7132, March 1996.

[7] J. Goodman, Statistical Optics, (John Willey and Sons, New York, 1985).

[8] J. Krzywinski, E.L. Saldin, E.A. Schneidmiller and M.V. Yurkov, "A new method for Ultrashort Electron Pulse -shape Measurements Using Synchrotron Radiation from a Bending Magnet," Nucl. Instr. and Methods **A**, in press.